Adaptive Fuzzy Backstepping Tracking Control for Flexible Robotic Manipulator

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***Abstract*—In this paper, an adaptive fuzzy state feedback control method is proposed for the single-link robotic manipulator system. The considered system contains unknown nonlinear function and actuator saturation. Fuzzy logic systems (FLSs) and a smooth function are used to approximate the unknown nonlinearities and the actuator saturation, respectively. By combining the command-filter technique with the backstepping design algorithm, a novel adaptive fuzzy tracking backstepping control method is developed. It is proved that the adaptive fuzzy control scheme can guarantee that all the variables in the closed-loop system are bounded, and the system output can track the given reference signal as close as possible. Simulation results are provided to illustrate the effectiveness of the proposed approach.**

***Index Terms*—Actuator saturation, backstepping design, command-filter technique, flexible robotic manipulator, fuzzy adaptive control.**

I. Introduction

WITH the development of industrial processes automation in recent years, some of the work that based on human labor was replaced by robots in fields like medical, industrial production, military, aerospace etc. Therefore, the modeling and control design problems for the flexible robotic manipulators are of essential importance, and receiving considerable attentions. Some effective control methods concerning this issue are adaptive sliding mode technique [1], the feedback linearization method [2], the passivity approach [3], the proportional-derivative control approach [4] and so on. However, the exact dynamic model of the complex flexible joint manipulator is difficult to obtain due to the existence of the uncertainties and nonlinear terms. Thus, the fuzzy logic systems (FLSs) [5]−[10] are introduced in this paper to solve the aforementioned problem of nonlinear terms.

In recent years, some adaptive fuzzy backstepping control schemes have been developed for the robotic manipulator systems [11]−[12]. However, the adaptive fuzzy control strategies in [11]−[12] are based on the traditional backstepping design technique that is subject to the so called "explosion of complexity" problem, which is caused by repeated differentiations of virtual signals. To cope with this problem, a command-filtered-based fuzzy adaptive backstepping control scheme is proposed in [13]−[17] for a class of nonlinear systems by introducing error compensation signals.

It is noted that many engineering systems are often driven by the actuator. Because of the physical limitations of the actuator, the actuator's output cannot be arbitrarily large, which results in the saturation nonlinearity in the actuator. The physical plants may even experience catastrophic accidents when the actuator's saturation is not well addressed [18]−[24]. Although many adaptive intelligent control methods for the single-link robotic manipulator system have been proposed, there are no results on fuzzy adaptive backstepping control of the flexible robotic manipulator with actuator saturation, which motivates the current study.

In this paper, a command-filter-based adaptive fuzzy backstepping control scheme is designed to achieve accurate trajectory tracking for a single-link flexible manipulator in presence of actuator saturation. The proposed adaptive fuzzy backstepping control approach can guarantee that all the signals in the closed-loop system are bounded, but also the system output can track a given reference signal as close as possible. Simulation results are given to further validate the effectiveness of the proposed control method.

II. Problem Statement and Preliminaries

A. System Descriptions

The dynamic equation of single-link robotic manipulator coupled with a brushed direct current motor based on a nonrigid joint (Fig. 1) is expressed as follows

$$\begin{aligned}&J_1\ddot{q}_1+F_1\dot{q}_1+K(q_1-\tfrac{q_2}{N})+mgd\cos q_1=0\\&J_2\ddot{q}_2+F_2\dot{q}_2-\tfrac{K}{N}(q_1-\tfrac{q_2}{N})=K_t i\\&L\dot{i}+Ri+K_b\dot{q}_2=u\end{aligned}\tag{1}$$

where J_1 and J_2 are the inertias, q_1 is the angular positions of the link, q_2 is the motor shaft, R and L are the armature resistance and inductance respectively. i is the armature current, K is the spring constant, K_t is the torque constant, $u(v)$ is the armature voltage, g is the acceleration of gravity, d is the position of the link's center of gravity, F_1 and F_2 are the viscous friction constants, K_b is the back-emf constant, M is the link mass, and N is the gear ratio.

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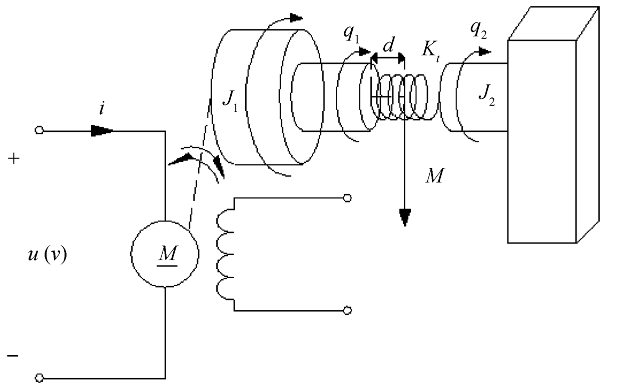

Fig. 1. Single link flexible joint robot.

By introducing the state variables, $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$, $x_5 = i$, and defining $K_t K = J_1 J_2 N L$, the dynamic equation of system (1) becomes

$$\begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{mgd}{J_1}\cos x_1 - \frac{F_1}{J_1}x_2 - \frac{K}{J_1}(x_1 - \frac{x_3}{N}) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{K}{J_2 N}(x_1 - \frac{x_3}{N}) - \frac{F_2}{J_2}x_4 + \frac{K_t}{J_2}x_5 \\ \dot{x}_5 = -\frac{R}{L}x_5 - \frac{K_b}{L}x_4 - \frac{1}{L}u \\ y = x_1. \end{array} \tag{2}$$

System (2) is equivalent to the following pure-feedback form

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \delta_2(x_1, x_2, x_3) + x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \delta_4(x_1, x_2, x_3, x_4, x_5) + x_5 \\ \dot{x}_5 = \delta_5(x_1, x_2, x_3, x_4, x_5, u) + u \\ y = x_1 \end{array} \right. \tag{3}$$

where $\delta_2(x_1, x_2, x_3) = -\frac{mgd}{J_1}\cos x_1 - \frac{F_1}{J_1}x_2 - \frac{K}{J_1}(x_1 - \frac{x_3}{N}) - x_3$, $\delta_4(x_1, x_2, x_3, x_4, x_5) = \frac{K}{J_2 N}(x_1 - \frac{x_3}{N}) - \frac{F_2}{J_2}x_4 + \frac{K_t}{J_2}x_5 - x_5$, $\delta_5(x_1, x_2, x_3, x_4, x_5, u) = -\frac{R}{L}x_5 - \frac{K_b}{L}x_4 - \frac{1}{L}u - u$.

Note that system (3) is of pure-feedback nonlinear form, we introduce the butterworth low-pass filter (LPF) [24] to transform system (3) to

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \delta_2(x_1, x_2, x_{3,f}) + x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \delta_4(x_1, x_2, x_3, x_4, x_{5,f}) + x_5 \\ \dot{x}_5 = \delta_5(x_1, x_2, x_3, x_4, x_5, u_f) + u(v) \\ y = x_1 \end{array} \right. \tag{4}$$

where $x_{3,f} = H_L(s)x_3 \approx x_3$, $x_{5,f} = H_L(s)x_5 \approx x_5$, $u_f = H_L(s)u \approx u$, $H_L(s)$ is a butterworth low-pass filter (LPF). The corresponding filter parameters of Butterworth filters with the cut off frequency $w_c = 1$ rad/s for different values of n can be obtained as in [24].

It should be mentioned that most actuators have low-pass properties, and the Butterworth low-pass filter (LPF) is used to eliminate the interference of high frequency signals. Furthermore, owing to the physical limitations of a DC motor, the armature voltage will no longer change when the voltage increases to a certain extent, namely the DC motor rotor voltage $u(v(t))$ reaches saturation.

According to [25]−[27], $u(v(t))$ denotes the plant input subject to saturation type nonlinearly, which is described as follows:

$$u(v(t)) = sat(v(t)) = \left\{ \begin{array}{ll} sign(v(t))u_N, & |v(t)| \geq u_N \\ v(t), & |v(t)| < u_N \end{array} \right. \tag{5}$$

where u_N is the bound of $u(v(t))$. Clearly, the relationship between the applied control $u(v(t))$ and the control input $v(t)$ has a sharp corner when $|v(t)| = u_N$. Thus backstepping technique cannot be directly applied. Therefore, the saturation $sat(v(t))$ can be approximated by the following smooth function.

$$\tau(v) = u_N \times \tanh(\frac{v}{u_N}) = u_N \frac{e^{v/u_N} - e^{-v/u_N}}{e^{v/u_N} + e^{-v/u_N}} \tag{6}$$

Then, saturation $u(v(t))$ in (5) becomes

$$sat(v) = \tau(v) + \beta(v) = u_N \times \tanh(\frac{v}{u_N}) + \beta(v) \tag{7}$$

where $\beta(v) = sat(v) - \tau(v)$ is a bounded function in time and its bound can be obtained as

$$|\beta(v)| \leq u_N(1 - \tanh(v/u_N)) = D_1. \tag{8}$$

In this section, $0 \leq |v(t)| \leq u_N$ the bound $\beta(v)$ increases from 0 to D_1 as $|v(t)|$ changes from 0 to u_N, and outside this range, the bound $\beta(v)$ decreases from D_1 to 0.

The control objective of this study is to design an adaptive fuzzy controller such that the system output angular position y can track the reference signal y_r as close as possible. Moreover, all the signals that are involved in the resulting closed-loop system are bounded.

Before further proceeding, the following Lemma is first introduced.

Lemma 1 [13], [14]: The command filter is defined as

$$\dot{\kappa}_1 = \omega_n \kappa_2 \tag{9}$$

$$\dot{\kappa}_2 = -2\varsigma\omega_n\kappa_2 - \omega_n(\kappa_1 - \alpha_1). \tag{10}$$

If the input signal α_1 satisfies $|\dot{\alpha}_1| \leq p_1$ and $|\ddot{\alpha}_1| \leq p_2$ for all $t \geq 0$, where p_1 and p_2 are positive constants and $\kappa_1(0) = \alpha_1(0)$, $\kappa_2(0) = 0$. Then, for any $\delta > 0$, there exist $\omega_n > 0$ and $\varsigma \in (0, 1]$, such that $|\kappa_1 - \alpha_1| \leq \delta$, $|\dot{\kappa}_1|$, $|\ddot{\kappa}_1|$ and $|\dddot{\kappa}_1|$ are bounded.

B. Fuzzy Logic Systems

A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy If-then rules of the following form:

R^l : If x_1 is F_1^l and x_2 is F_2^l and … andx_n is F_n^l,
Then y is G^l, $l = 1, 2, \ldots, N$

where $x = (x_1, \ldots, x_n)^T$ and y are the fuzzy logic system input and output, respectively. F_i^l and G^l are fuzzy sets, associating with the membership functions $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$, respectively. N is the number of rules. Through singleton function, center average defuzzification and product inference[28], [29], the fuzzy logic system can be expressed as

$$y(x) = \frac{\sum_{l=1}^{N} \bar{y}_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} [\prod_{i=1}^{n} \mu_{F_i^l}(x_i)]} \tag{11}$$

where $\bar{y}_l = \max_{y \in R} \mu_{G^l}(y)$. Define the fuzzy basis functions as

$$\phi_l = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^l}(x_i))}. \tag{12}$$

Denote $\theta^T = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_N] = [\theta_1, \theta_2, \ldots, \theta_N]$ and $\varphi(x) = [\varphi_1(x), \ldots, \varphi_N(x)]^T$.

The common form of fuzzy logic systems is described as $y(x) = \theta^T \varphi(x)$.

Lemma 2 [28]–[30]: Let $\delta(x)$ be a real smooth function defined on a compact set $\Omega \subseteq \mathbb{R}^N$, and for a positive constant ε, there exists a FLS $y(x) = \theta^T \varphi(x)$ such that

$$\sup_{x \in \Omega} \left| \delta(x) - \theta^T \varphi(x) \right| \leq \varepsilon. \tag{13}$$

According to [28], [29], we define the optimal parameter as

$$\theta^* = \arg \min_{\theta \in R^N} \{ \sup_{x \in \Omega} \left| \delta(x) - \theta^T \varphi(x) \right| \}. \tag{14}$$

Then, one has

$$\delta(x) = \theta^{*T} \varphi(x) + \varepsilon \tag{15}$$

where ε is the fuzzy minimum approximation error satisfying $|\varepsilon| \leq \varepsilon^*$.

III. Adaptive Fuzzy Control Design and Stability Analysis

In this section, an adaptive fuzzy state-feedback controller, compensating signals and parameter adaptive laws are obtained by utilizing command filter backstepping technique. The stability of the closed-loop system is proved by Lyapunov function stability theory [31]–[33].

The 5-step adaptive fuzzy backstepping state feedback control design is based on the following changes of coordinates:

$$\begin{aligned} \lambda_1 &= x_1 - y_r \\ \lambda_i &= x_i - x_{i,c} \\ \lambda_5 &= x_5 - x_{5,c} - \bar{\lambda} \end{aligned} \tag{16}$$

$$\begin{aligned} v_1 &= \lambda_1 - r_1 \\ v_i &= \lambda_i - r_i, \quad i = 2, \ldots, 4 \\ v_5 &= \lambda_5 - r_5 \end{aligned} \tag{17}$$

where λ_i ($i = 1, \ldots, 5$) are the tracking errors for command filter, $x_{i,c}$ are the outputs of command filter, α_{i-1} are the inputs of command filter. The purpose of the compensating signals r_i is to reduce the effect of the errors $(x_{i+1,c} - \alpha_i)$, which is caused by the command filter. y_r is the desired trajectory, v_i are the compensating tracking error signals and $\bar{\lambda}$ is an auxiliary function, which will be given in Step 5. The command filter is defined as:

$$\dot{\kappa}_1 = \omega \kappa_2 \tag{18}$$

$$\dot{\kappa}_2 = -2\varsigma\omega\kappa_2 - \omega(\kappa_1 - \alpha_i) \tag{19}$$

where $\omega > 0$ and $\varsigma \in (0, 1]$ are parameters to be designed, $x_{i,c}(t) = \kappa_1(t)$ is the output of each filter, and the initial conditions are $\kappa_1(0) = \alpha_i(0)$ and $\kappa_2(0) = 0$.

Step 1: The time derivative of v_1 is

$$\dot{v}_1 = \dot{\lambda}_1 - \dot{r}_1 = \lambda_2 + x_{2,c} - \dot{y}_r - \dot{r}_1. \tag{20}$$

Consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2} v_1^2 \tag{21}$$

The time derivative of V_1 is

$$\dot{V}_1 = v_1(v_2 + r_2 + \alpha_1 - \alpha_1 + x_{2,c} - \dot{y}_r - \dot{r}_1). \tag{22}$$

Choose the first intermediate control function α_1 and the compensating signal $\dot{r}_1$ as

$$\alpha_1 = -c_1 \lambda_1 + \dot{y}_r \tag{23}$$

$$\dot{r}_1 = -c_1 r_1 + r_2 + (x_{2,c} - \alpha_1) \tag{24}$$

where $c_1 > 0$ is a parameter to be designed.

By substituting (23)–(24) into (22), we have

$$\dot{V}_1 \leq -c_1 v_1^2 + v_1 v_2. \tag{25}$$

Step 2: From (16)–(17), the time derivative of v_2 is

$$\begin{aligned} \dot{v}_2 &= \dot{\lambda}_2 - \dot{r}_2 = \dot{x}_2 - \dot{x}_{2,c} - \dot{r}_2 \\ &= \theta_2^{*^T} \varphi_2(\bar{x}_2) + x_3 - \dot{x}_{2,c} - \dot{r}_2 + \varepsilon_2 \\ &= (\theta_2^T + \tilde{\theta}_2^T) \varphi_2(\bar{x}_2) + \alpha_2 - \alpha_2 \\ &\quad + x_3 - \dot{x}_{2,c} - \dot{r}_2 + \varepsilon_2. \end{aligned} \tag{26}$$

Consider the following Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2} v_2^2 + \frac{1}{2\eta_2} \tilde{\theta}_2^T \tilde{\theta}_2 \tag{27}$$

where $\eta_2 > 0$ is a parameter to be designed.

The time derivative of V_2 is

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + v_2((\theta_2^T + \tilde{\theta}_2^T)\varphi_2(\bar{x}_2) + \alpha_2 \\ &\quad - \alpha_2 + x_3 - \dot{x}_{2,c} - \dot{r}_2 + \varepsilon_2) - \tfrac{1}{\eta_2} \tilde{\theta}_2^T \dot{\theta}_2 \\ &\leq -c_1 v_1^2 + v_1 v_2 + v_2((\theta_2^T + \tilde{\theta}_2^T)\varphi_2(\bar{x}_2) + \alpha_2 \\ &\quad - \alpha_2 + x_3 - \dot{x}_{2,c} - \dot{r}_2 + \varepsilon_2) - \tfrac{1}{\eta_2} \tilde{\theta}_2^T \dot{\theta}_2. \end{aligned} \tag{28}$$

By applying Young's inequality, we have

$$v_2 \varepsilon_2 \leq \frac{1}{2} v_2^2 + \frac{1}{2} \varepsilon_2^{*^2}. \tag{29}$$

Substituting (29) into (28) results in

$$\begin{aligned} \dot{V}_2 &\leq -c_1 v_1^2 + v_2(\theta_2^T \varphi_2(\bar{x}_2) + \alpha_2 - \alpha_2 + v_3 \\ &\quad + r_3 + x_{3,c} - \dot{x}_{2,c} - \dot{r}_2 + \tfrac{1}{2} v_2 + v_1) \\ &\quad + (v_2 \tilde{\theta}_2^T \varphi_2(\bar{x}_2) - \tfrac{1}{\eta_2} \tilde{\theta}_2^T \dot{\theta}_2) + \tfrac{1}{2} \varepsilon_2^{*^2}. \end{aligned} \tag{30}$$

Choose the intermediate control function α_2, the compensating signal $\dot{r}_2$ and the parameter adaptation law $\dot{\theta}_2$ as

$$\alpha_2 = -c_2 \lambda_2 - \theta_2^T \varphi_2(\bar{x}_2) - \frac{1}{2} v_2 - v_1 + \dot{x}_{2,c} \tag{31}$$

$$\dot{r}_2 = -c_2 r_2 + r_3 + (x_{3,c} - \alpha_2) \tag{32}$$

$$\dot{\theta}_2 = v_2\eta_2\varphi_2(\bar{x}_2) - \sigma_2\theta_2 \tag{33}$$

where $c_2 > 0$ and $\sigma_2 > 0$ are design parameters.

By substituting (31)−(33) into (30), we have

$$\dot{V}_2 \le -c_1v_1^2 - c_2v_2^2 + +v_2v_3 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \frac{1}{2}{\varepsilon_2^*}^2. \tag{34}$$

Step 3: Similar to step 2, from (16)−(17), the time derivative of v_3 is

$$\dot{v}_3 = \dot{\lambda}_3 - \dot{r}_3 = \dot{x}_3 - \dot{x}_{3,c} - \dot{r}_3 = x_4 - \dot{x}_{3,c} - \dot{r}_3. \tag{35}$$

Consider the following Lyapunov function candidate:

$$V_3 = V_2 + \frac{1}{2}v_3^2. \tag{36}$$

The time derivative of V_3 is

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + v_3(x_4 - \dot{x}_{3,c} - \dot{r}_3)\\ &\le -c_1v_1^2 - c_2v_2^2 + +v_2v_3 + \tfrac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \tfrac{1}{2}{\varepsilon_2^*}^2\\ &\quad +v_3(v_4 + r_4 + x_{4,c} + \alpha_3 - \alpha_3 - \dot{x}_{3,c} - \dot{r}_3).\end{aligned} \tag{37}$$

Choose the intermediate control function α_3 and the compensating signal $\dot{r}_3$

$$\alpha_3 = -c_3\lambda_3 - v_2 + \dot{x}_{3,c} \tag{38}$$

$$\dot{r}_3 = -c_3r_3 + r_4 + (x_{4,c} - \alpha_3). \tag{39}$$

Substituting (38)−(39) into (37) results in

$$\dot{V}_3 \le -c_1v_1^2 - c_2v_2^2 + \frac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \frac{1}{2}{\varepsilon_2^*}^2 + v_3v_4 - c_3v_3^2 \tag{40}$$

Step 4: From (16)−(17), the time derivative of v_4 is

$$\begin{aligned}\dot{v}_4 &= \dot{\lambda}_4 - \dot{r}_4 = \dot{x}_4 - \dot{x}_{4,c} - \dot{r}_4\\ &= {\theta_4^*}^T\varphi_4(\bar{x}_4) + x_5 + \varepsilon_4 - \dot{x}_{4,c} - \dot{r}_4\\ &= (\theta_4^T + \tilde{\theta}_4^T)\varphi_4(\bar{x}_4) + \varepsilon_4 + x_5 - \dot{x}_{4,c} - \dot{r}_4.\end{aligned} \tag{41}$$

Consider the following Lyapunov function candidate:

$$V_4 = V_3 + \frac{1}{2}v_4^2 + \frac{1}{2\eta_4}\tilde{\theta}_4^T\tilde{\theta}_4. \tag{42}$$

From (41)−(42), the time derivative of V_4 is

$$\begin{aligned}\dot{V}_4 &= \dot{V}_3 + v_4((\theta_4^T + \tilde{\theta}_4^T)\phi_4(\bar{x}_4) + \varepsilon_4\\ &\quad +x_5 - \dot{x}_{4,c} - \dot{r}_4) - \tfrac{1}{\eta_4}\tilde{\theta}_4^T\dot{\theta}_4\\ &\le -c_1v_1^2 - c_2v_2^2 + \tfrac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \tfrac{1}{2}{\varepsilon_2^*}^2 - c_3v_3^2 + v_4(v_3\\ &\quad +\theta_4^T\phi_4(\bar{x}_4) + \varepsilon_4 + v_5 + r_5 + x_{5,c} + \alpha_4 - \alpha_4\\ &\quad -\dot{x}_{4,c} - \dot{r}_4) + (v_4\tilde{\theta}_4^T\phi_4(\bar{x}_4) - \tfrac{1}{\eta_4}\tilde{\theta}_4^T\dot{\theta}_4).\end{aligned} \tag{43}$$

By using Young's inequality, we have

$$v_4\varepsilon_4 < \frac{1}{2}v_4^2 + \frac{1}{2}{\varepsilon_4^*}^2. \tag{44}$$

Substituting (44) into (43) results in

$$\begin{aligned}\dot{V}_4 &\le -c_1v_1^2 - c_2v_2^2 + \tfrac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \tfrac{1}{2}{\varepsilon_2^*}^2 - c_3v_3^2 + v_4(v_3\\ &\quad +\theta_4^T\phi_4(\bar{x}_4) + \tfrac{1}{2}v_4 + v_5 + r_5 + x_{5,c} + \alpha_4 - \alpha_4\\ &\quad -\dot{x}_{4,c} - \dot{r}_4) + \tfrac{1}{2}{\varepsilon_4^*}^2 + (v_4\tilde{\theta}_4^T\phi_4(\bar{x}_4) - \tfrac{1}{\eta_4}\tilde{\theta}_4^T\dot{\theta}_4).\end{aligned} \tag{45}$$

Choose the intermediate control function α_4 , the compensating signal $\dot{r}_4$ and parameter adaptation law $\dot{\theta}_4$ as

$$\alpha_4 = -c_4\lambda_4 - v_3 - \frac{1}{2}v_4 - \theta_4^T\varphi_4(\bar{x}_4) + \dot{x}_{4,c} \tag{46}$$

$$\dot{r}_4 = -c_4r_4 + r_5 + (x_{5,c} - \alpha_4) \tag{47}$$

$$\dot{\theta}_4 = v_4\eta_4\varphi_4(\bar{x}_4) - \sigma_4\theta_4. \tag{48}$$

Substituting (46)−(48) into (45) results in

$$\begin{aligned}\dot{V}_4 &\le -c_1v_1^2 - c_2v_2^2 + \tfrac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \tfrac{1}{2}{\varepsilon_2^*}^2\\ &\quad -c_3v_3^2 - c_4v_4^2 + v_4v_5^2 + \tfrac{1}{2}{\varepsilon_4^*}^2 + \tfrac{\sigma_4}{\eta_4}\tilde{\theta}_4^T\theta_4\end{aligned} \tag{49}$$

Step 5: The time derivative of v_5 is

$$\begin{aligned}\dot{v}_5 &= \dot{x}_5 - \dot{x}_{5,c} - \dot{r}_5 - \dot{\bar{\lambda}}\\ &= {\theta_5^*}^T\varphi_5(\bar{x}_5) + u + \varepsilon_5 - \dot{x}_{5,c} - \dot{r}_5 - \dot{\bar{\lambda}}\\ &= (\theta_5^T + \tilde{\theta}_5^T)\varphi_5(\bar{x}_5) + \varepsilon_5 + u - \dot{x}_{5,c} - \dot{r}_5 - \dot{\bar{\lambda}}\end{aligned} \tag{50}$$

where auxiliary function $\bar{\lambda}$ is defined as $\dot{\bar{\lambda}} = -\bar{\lambda} + (\tau(v) - v)$, which is used to solve actuator saturation problem.

Consider the following Lyapunov function candidate:

$$V_5 = V_4 + \frac{1}{2}v_5^2 + \frac{1}{2\eta_5}\tilde{\theta}_5^T\tilde{\theta}_5. \tag{51}$$

From (50)−(51), the time derivative of V_4 is

$$\begin{aligned}\dot{V}_5 &= \dot{V}_4 + v_5((\theta_5^T + \tilde{\theta}_5^T)\varphi_5(\bar{x}_5) + \varepsilon_5\\ &\quad +\tau(v) + \beta(v) - \dot{x}_{5,c} - \dot{r}_5 - \dot{\bar{\lambda}}) - \tfrac{1}{\eta_5}\tilde{\theta}_5^T\dot{\theta}_5\\ &\le -c_1v_1^2 - c_2v_2^2 + \tfrac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \tfrac{1}{2}{\varepsilon_2^*}^2 - c_3v_3^2 - c_4v_4^2\\ &\quad +v_4v_5 + \tfrac{1}{2}{\varepsilon_4^*}^2 + \tfrac{\sigma_4}{\eta_4}\tilde{\theta}_4^T\theta_4 + v_5(\theta_5^T\varphi_5(\bar{x}_5) + \varepsilon_5\\ &\quad +\beta(v) - \dot{x}_{5,c} - \dot{r}_5 + \bar{\lambda} + v) + (v_5\tilde{\theta}_5^T\varphi_5(\bar{x}_5)\\ &\quad -\tfrac{1}{\eta_5}\tilde{\theta}_5^T\dot{\theta}_5).\end{aligned} \tag{52}$$

By using Young's inequality, we have

$$v_5\beta(v) \le \frac{1}{2}v_5^2 + \frac{1}{2}D_1^2 \tag{53}$$

$$v_5\varepsilon_5 \le \frac{1}{2}v_5^2 + \frac{1}{2}{\varepsilon_5^*}^2. \tag{54}$$

Choose the controller v, the compensating signal $\dot{r}_5$ and parameter adaptive law $\dot{\theta}_5$ as

$$v = -c_5\lambda_5 - v_4 - \theta_5^T\varphi_5(\bar{x}_5) - v_5 - \bar{\lambda} + \dot{x}_{5,c} \tag{55}$$

$$\dot{r}_5 = -c_5r_5 \tag{56}$$

$$\dot{\theta}_5 = v_5\eta_5\varphi_5(\bar{x}_5) - \sigma_5\theta_5. \tag{57}$$

By substituting (53)−(57) into (52), we have

$$\begin{aligned}\dot{V}_5 &\le -c_1v_1^2 - c_2v_2^2 + \tfrac{\sigma_2}{\eta_2}\tilde{\theta}_2^T\theta_2 + \tfrac{1}{2}{\varepsilon_2^*}^2\\ &\quad -c_3v_3^2 - c_4v_4^2 + \tfrac{1}{2}{\varepsilon_4^*}^2 + \tfrac{\sigma_4}{\eta_4}\tilde{\theta}_4^T\theta_4\\ &\quad -c_5v_5^2 + \tfrac{\sigma_5}{\eta_5}\tilde{\theta}_5^T\theta_5 + \tfrac{1}{2}{\varepsilon_4^*}^2 + \tfrac{1}{2}D_1^2\\ &\le -\sum_{i=1}^{5}c_iv_i^2 + \sum_{j=2,4,5}\tfrac{\sigma_j}{\eta_j}\tilde{\theta}_j^T\theta_j\\ &\quad +\tfrac{1}{2}\sum_{j=2,4,5}{\varepsilon_j^*}^2 + \tfrac{1}{2}D_1^2.\end{aligned} \tag{58}$$

Then, (58) can be rewritten as

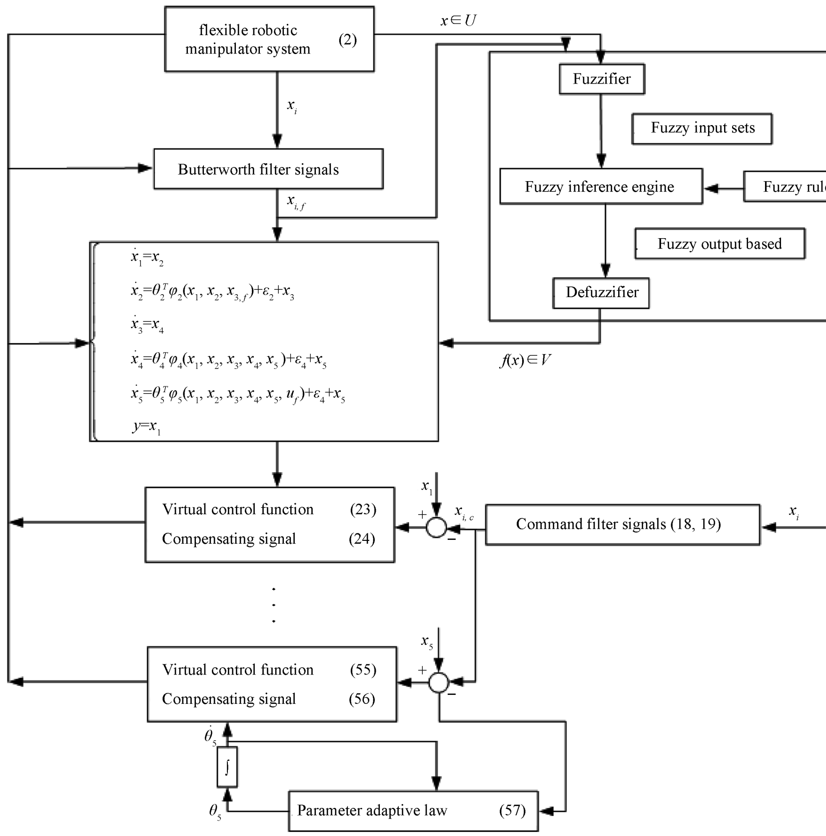

Fig. 2. Adaptive fuzzy backstepping control scheme.

$$\dot{V}_5 \le -CV_5 + D \tag{59}$$

where $C = \min\{2c_i, 2\sigma_j\},\ i = 1, \ldots, 5,\ j = 2, 4, 5$.

By integrating (59) over $[0, t]$, we can get the solution of the above inequality

$$0 \le V_5(t) \le e^{-Ct}V_5(0) + \mu(1 - e^{-Ct}) \tag{60}$$

where $\mu = D/C$.

According to (60), it can be shown that all the signals in the closed-loop system are bounded. Meanwhile, we have:

$$|z_1(t)| \le \sqrt{2(V_5(0)\exp(-Ct) + D/C)}. \tag{61}$$

Since as $t \to \infty$, $\lim\limits_{t\to\infty} \exp(-Ct) = 0$, it follows that $\lim\limits_{t\to\infty} |z_1(t)| \le \sqrt{2D/C}$.

Hence, according to (61), we conclude that the tracking error can be made small by increasing the values of design parameters c_i , σ_j or decreasing η_j and $(i = 1, \ldots, 5, j = 2, 4, 5)$.

From above analysis and design, we can summarize the following Theorem.

Theorem 1. For the single-link flexible robotic manipulator system (1), the proposed adaptive fuzzy backstepping control design scheme can guarantee that the tracking errors converge to a small neighborhood of the origin and all variables in the closed-loop system are bounded.

The configuration of the aforementioned adaptive fuzzy control scheme is shown in Fig. 2.

IV. Simulation

The parameters for the flexible robotic manipulator with the parameters[2] are given as $J_1 = J_2 = 40\,\mathrm{kgm}^2$, $K_t = 10\mathrm{Nm/A}$, $\mathrm{K_b} = 0.976\mathrm{Nm/A}$, $g = 9.8\mathrm{N/Kg}$, $m = 0.102\mathrm{kg}$, $F_1 = F_2 = 0.05\mathrm{Nms/rad}$, $R = 4.5\,\Omega$, $K = 30$, $L = 300\mathrm{H}$, $N = 1$, $d = 0.4\mathrm{m}$.

The reference signal is chosen as $y_r = \sin(t - 1)$.

The input $u(v(t))$ is described by

$$u(v(t)) = sat(v(t)) = \begin{cases} sign(v(t))u_N, & |v(t)| \ge u_N \\ v(t), & |v(t)| < u_N \end{cases}$$

with $u_N = 10$.

In the simulation, fuzzy If-then rules are chosen as:

R_1: If x_1 is F_1^1 $\cdots$ and u_f is F_6^1, then y is G_1;

R_2: If x_1 is F_1^2 $\cdots$ and u_f is F_6^2, then y is G_2;

R_3: If x_1 is F_1^3 $\cdots$ and u_f is F_6^3, then y is G_3;

R_4: If x_1 is F_1^4 $\cdots$ and u_f is F_6^4, then y is G_4;

R_5: If x_1 is F_1^5 $\cdots$ and u_f is F_6^5, then y is G_5;

where fuzzy sets are chosen as $F_i^1 = (NL)$, $F_i^2 = (NS)$, $F_i^3 = (ZE$, $F_i^4 = (PS)$, $F_i^5 = (PL)$, which are defined over the intervals $[-2, 2]$ for each variable. By choosing the partitioning points as -2, -1, 0, 1, 2, and the corresponding fuzzy membership functions (shown by Fig. 3) are given by

$$\mu_{F_2^l}(x_1, x_2, x_{3,f}) = \exp[\tfrac{(x_1+3-l)^2}{2}] \times \exp[-\tfrac{(x_2+3-l)^2}{2}]$$
$$\times \exp[-\tfrac{(x_{3,f}+3-l)^2}{2}],$$
$$\mu_{F_4^l}(x_1, x_2, x_3, x_4, x_{5,f}) = \exp[-\tfrac{(x_1+3-l)^2}{2}]$$
$$\times \exp[-\tfrac{(x_2+3-l)^2}{2}] \times \exp[-\tfrac{(x_3+3-l)^2}{2}]$$
$$\times \exp[-\tfrac{(x_4+3-l)^2}{2}] \times \exp[-\tfrac{(x_{5,f}+3-l)^2}{2}],$$

$$\mu_{F_5^l}(x_1,x_2,x_3,x_4,x_5,u_f)=\exp[-\tfrac{(x_1+3-l)^2}{2}]$$
$$\times\exp[-\tfrac{(x_2+3-l)^2}{2}]\times\exp[-\tfrac{(x_3+3-l)^2}{2}]$$
$$\times\exp[-\tfrac{(x_4+3-l)^2}{2}]\times\exp[-\tfrac{(x_5+3-l)^2}{2}]$$
$$\times\exp[-\tfrac{(u_f+3-l)^2}{2}],$$
$$l=1,\ldots,5$$

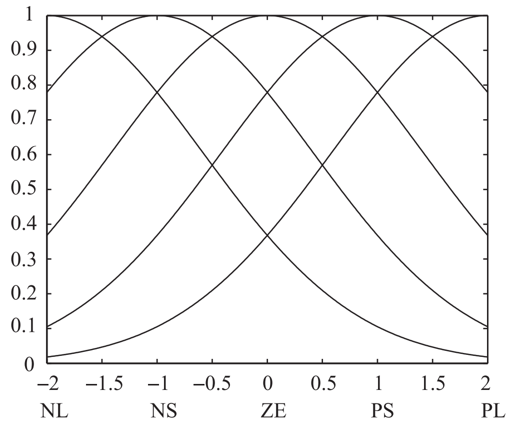

Fig. 3. The fuzzy rules.

Design the command filter as:

$$\dot{\kappa}_1=200\kappa_2 \tag{62}$$

$$\dot{\kappa}_2=-2\times0.9\omega_n\kappa_2-200(\kappa_1-\alpha_1). \tag{63}$$

The compensating signals $\dot{r}_j$, $j=2,4,5$ are designed in the following:

$$\dot{r}_2=-13r_2+r_3+(x_{3,c}-\alpha_2) \tag{64}$$

$$\dot{r}_4=-10r_4+r_5+(x_{5,c}-\alpha_4) \tag{65}$$

$$\dot{r}_5=-10r_5. \tag{66}$$

The virtual control function $\alpha_i, i=1,\ldots,4$, the controller v are chosen as follows:

$$\alpha_1=-100\lambda_1+\dot{y}_r \tag{67}$$

$$\alpha_2=-13\lambda_2-\theta_2^T\varphi_2(\bar{x}_2)-\frac{1}{2}v_2-v_1+\dot{x}_{2,c} \tag{68}$$

$$\alpha_3=-100\lambda_3-v_2+\dot{x}_{3,c} \tag{69}$$

$$\alpha_4=-10\lambda_4-v_3-\frac{1}{2}v_4-\theta_4^T\varphi_4(\bar{x}_4)+\dot{x}_{4,c} \tag{70}$$

$$v=-10\lambda_5-v_4-\theta_5^T\varphi_5(\bar{x}_5)-v_5-\bar{\lambda}+\dot{x}_{5,c}. \tag{71}$$

The parameter adaptation laws $\dot{\theta}_j$, $j=2,4,5$ are chosen as follows:

$$\dot{\theta}_2=2v_2\varphi_2(\bar{x}_2)-50\theta_2 \tag{72}$$

$$\dot{\theta}_4=v_4\varphi_4(\bar{x}_4)-50\theta_4 \tag{73}$$

$$\dot{\theta}_5=v_5\varphi_5(\bar{x}_5)-50\theta_5. \tag{74}$$

The initial conditions of the states are chosen as $x_1(0)=0.03$, $x_2(0)=0.01$ and the other initial values are chosen as zero. Choose the Butterworth low-pass filter as $H_L(s)=1/(s^2+1.414s+1)$.

Simulation results in Figs. 4−9 are obtained by the proposed scheme, where Fig. 4 expresses the tracking trajectories of the output and the given reference signal. It is shown that under the actions of controller (55), the system output follows the desired reference signal well; Figs. 5−6 show the states $x_i,\ i=1,\ldots,5$; Fig. 7 shows the trajectory of $u(v)$; From Figs. 4−7, it can be seen that boundedness of $x_i,\ i=1,\ldots,5$, $u(v)$ is verified. Furthermore, to demonstrate the adaptive learning performance, the norms of the system adaptive laws are demonstrated in Figs. 8−10.

Remark 1: It should be mentioned that [11], [12] proposed different adaptive fuzzy control methods for a single-link robotic manipulator system. However, [11], [12] did not consider the problem of actuator saturation. In addition, the references [11], [12] did not solve the so-called "explosion of complexity" problem which is caused by repeating differentiations of virtual control. In this paper, the problems of "explosion of complexity" and actuator saturation have been solved for the single-link robotic manipulator system.

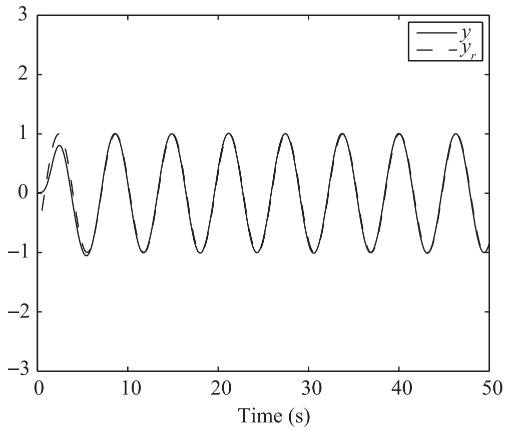

Fig. 4. The trajectories of y (solid) and y_r (dashed).

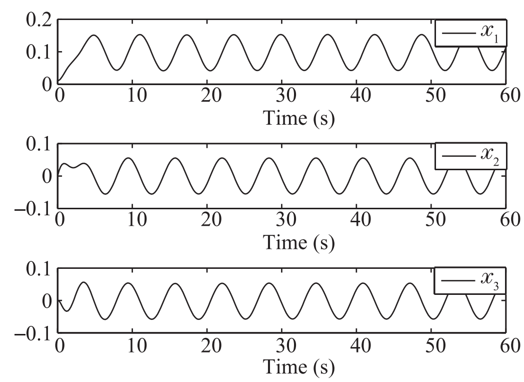

Fig. 5. The trajectories of $x_i, i=1,2,3$.

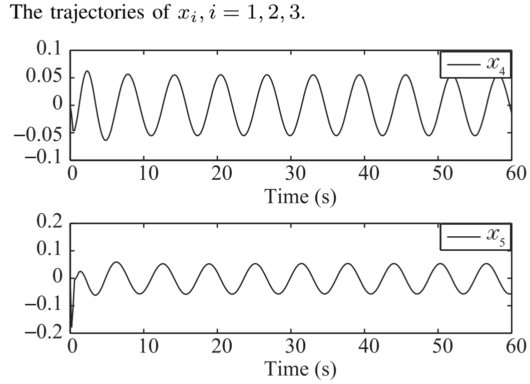

Fig. 6. The trajectories of $x_i, i=4,5$.

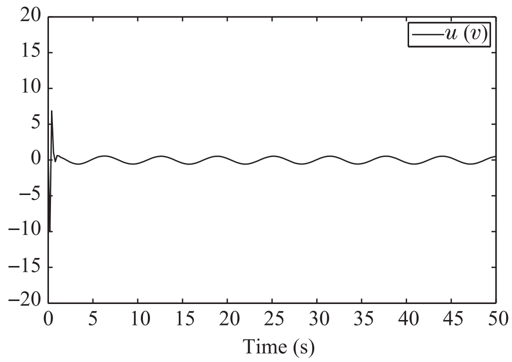

Fig. 7. The trajectory of $u(v)$.

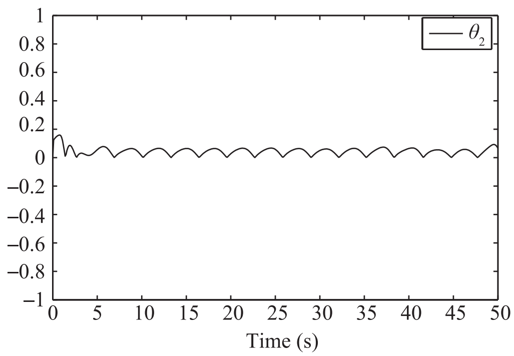

Fig. 8. Norm of θ_2.

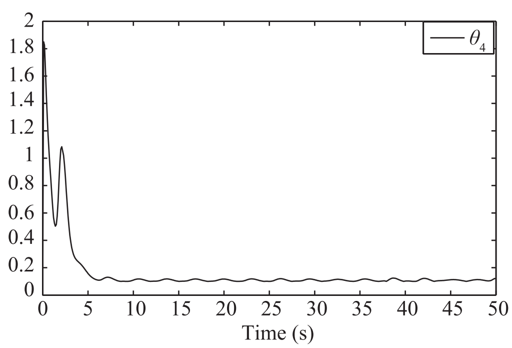

Fig. 9. Norm of θ_4.

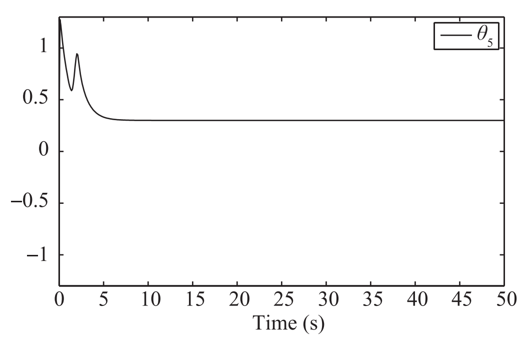

Fig. 10. Norm of θ_5.

To further demonstrate the effectiveness of the proposed control method, we apply the adaptive fuzzy tracking control scheme in [11] to the system (2). The simulation results are also depicted in Figs. 11−12, where Fig. 11 expresses the tracking trajectories of the output and the given reference signal, Fig. 12 shows the trajectory of $u(v)$. From Figs. 11−12, it can be seen that the control method in [11] cannot obtain a better control performances, since there exists the actuator saturation.

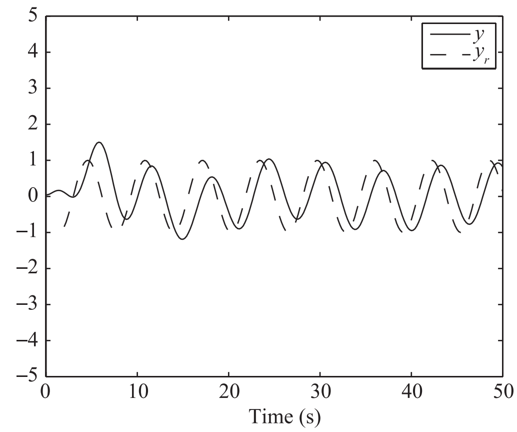

Fig. 11. The trajectories of y (solid) and y_r (dashed).

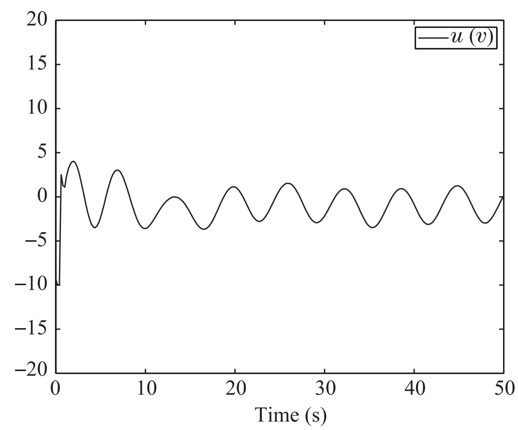

Fig. 12. The trajectory of $u(v)$.

V. Conclusion

In this paper, an adaptive fuzzy backstepping control design method has been presented for a single-link robotic manipulator in the presence of actuator saturation. By combining the command filtered technique and FLSs, an effective adaptive fuzzy backstepping control approach is developed and the stability of the closed-loop system is proved. The main features of the proposed method are as follows. 1) It solved the problem of actuator saturation by introducing the auxiliary design signal. 2) By incorporating the command filter technique into the adaptive fuzzy backstepping design technique, the proposed control scheme solved the problem of "explosion of complexity" inherent in the traditional backsteping control algorithms. Future research works will concentrate on the adaptive fuzzy output feedback control for the two-link flexible manipulator system on the basis of this study.

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