Decentralized Event-Driven Constrained Control Using Adaptive Critic Designs

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***Abstract*— We study the decentralized event-driven control problem of nonlinear dynamical systems with mismatched interconnections and asymmetric input constraints. To begin with, by introducing a discounted cost function for each auxiliary subsystem, we transform the decentralized event-driven constrained control problem into a group of nonlinear H_2-constrained optimal control problems. Then, we develop the event-driven Hamilton–Jacobi–Bellman equations (ED-HJBEs), which arise in the nonlinear H_2-constrained optimal control problems. Meanwhile, we demonstrate that all the solutions of the ED-HJBEs together keep the overall system stable in the sense of uniform ultimate boundedness (UUB). To solve the ED-HJBEs, we build a critic-only architecture under the framework of adaptive critic designs. The architecture only employs critic neural networks and updates their weight vectors via the gradient descent method. After that, based on the Lyapunov approach, we prove that the UUB stability of all signals in the closed-loop auxiliary subsystems is assured. Finally, simulations of an illustrated nonlinear interconnected plant are provided to validate the present designs.**

***Index Terms*— Adaptive critic designs (ACDs), adaptive dynamic programming (ADP), decentralized event-driven control, input constraint, reinforcement learning (RL).**

I. Introduction

INTERCONNECTIONS are typical features of many complex systems, such as cooperating robotic systems, intelligent transportation systems, and water distribution systems. These characteristics often make it intractable to design stabilizing controllers for such systems via one-shot methods. To tackle this difficulty, the decentralized control methodology was introduced [1]. For the late few years, applications of optimal control methods to decentralized control have been an attractive field [2]–[4]. This is because the decentralized control of the whole system can be obtained through solving a series of optimal control problems of isolated subsystems. More importantly, adaptive critic designs (ACDs) have been widely exploited and become powerful tools to solve optimal control problems (note: the detailed description of ACDs was provided in [5]). A noticeable advantage of using ACDs lies in that "the curse of dimensionality" is obviated. There are some synonyms for ACDs, such as adaptive dynamic programming (ADP) [6]–[8] and reinforcement learning (RL) [9]–[11]. In this article, we will develop an ACD-based decentralized control strategy for a class of nonlinear systems having mismatched interconnections and asymmetric input constraints.

It is well known that there are two types of interconnections involved in complex nonlinear systems, namely, matched interconnections [12] and mismatched interconnections [13]. For matched interconnected systems, one often introduces a set of nominal subsystems (or rather, isolated nominal subsystems) to assist obtaining the decentralized controller when using ADP-based optimal control methods (see [12]), whereas for mismatched interconnected plants, one generally proposes a group of auxiliary subsystems to help deriving the decentralized controller (see [13]). The key characteristic distinguishing the nominal subsystem and the auxiliary subsystem is that an additional control (namely, the auxiliary control) is introduced to the auxiliary subsystem. Because of this feature, there are two issues arising in designing ACD-based decentralized controllers for mismatched interconnected systems. First, the auxiliary control must satisfy certain inequalities, such as the inequalities $\|v_i^*(x_i(t))\|^2 \leq Q_i(x_i(t)), i = 1, 2, \ldots, N$, given in [13, Th. 1]. Unfortunately, these inequalities often cannot be proved analytically. Then, a question to be asked: can we remove such inequalities? This article will tackle this issue. Second, the control policy and the auxiliary control policy for each auxiliary subsystem are often tuned in different triggering mechanisms. Specifically, when designing the decentralized event-driven controllers for mismatched interconnected systems, one often updates the control policy and the auxiliary control policy in the event-driven mechanism and the time-driven mechanism, respectively. According to [14], the event-driven control policies have better performance than the time-driven control policies in reducing computational burden because they are executed aperiodically. In view of this fact, what are the effects on control performance if we update the control policy and the auxiliary control policy simultaneously in the event-driven mechanism? This article will also address this problem.

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Input constraints widely exist in engineering applications because of intrinsic physical properties, such as air pressure, temperature, and voltage. To assure the safety or stability of the controlled systems, one has to consider plants with input constraints. Until now, several studies have been conducted to present decentralized adaptive controllers for nonlinear interconnected systems in the presence of *symmetric* input constraints (see the related work in the following). However, rather few works have been reported on studying the decentralized control problem of nonlinear interconnected systems with *asymmetric* input constraints. When considering the interconnected systems with asymmetric input constraints, we can observe that the obtained decentralized controller does not stay at zero when the state vector arrives at the equilibrium point (see $u_i^*(x_i)$ defined in later formula (14) and Remark 2). This characteristic leads to a significant difference with the case that the interconnected systems have symmetric input constraints, where the decentralized controller must stay at zero with the steady state being obtained. Thus, how to propose novel methods to handle this feature is another issue to be tackled in this article.

A. Related Work

For nonlinear systems having no input constraints, Wang *et al.* [15] introduced an ACD-based optimal control approach to obtain the decentralized stabilizing controller for matched interconnected systems. The decentralized control algorithm in [15] makes the initial admissible control no longer necessary for its implementation. By taking a similar technique as [15], Qu *et al.* [16] suggested a decentralized tracking control strategy for large-scale nonlinear systems with matched interconnections. In [15] and [16], the nominal subsystems were required to help deriving the decentralized control (or the decentralized tracking control). Note that the control methods for *matched* interconnected systems are not always suitable for *mismatched* interconnected systems. Zhao *et al.* [17] studied the decentralized control problem of large-scale mismatched interconnected systems by using the local policy iteration. To tackle the mismatched interconnections, they introduced a cost function containing its partial derivative for each isolated subsystem. Recently, Narayanan *et al.* [18] used ADP to solve nonzero-sum games in order to acquire a distributed nearly optimal event-triggered control for mismatched interconnected systems. Actually, the distributed optimal event-triggered control is a Nash equilibrium point of the nonzero-sum games. More recently, Yang and He [19] developed a decentralized event-driven control strategy for mismatched interconnected systems via adaptive critic learning together with experience replay. Just as previously mentioned, a group of auxiliary subsystems have to be presented in [19] to assist obtaining the decentralized event-driven control. Meanwhile, the auxiliary control policies have to satisfy certain inequalities.

For nonlinear systems having input constraints, Liu *et al.* [20] proposed an integral RL to derive the decentralized optimal tracking control for matched interconnected systems with totally unavailable *priori* knowledge. After that, Yang and He [21] developed a policy iteration-based ADP to obtain the decentralized optimal control law for partially unknown mismatched interconnected systems. Similar to [15]–[17], nominal subsystems and auxiliary subsystems were presented in [20] and [21] to help obtaining the decentralized optimal tracking control and the decentralized optimal control, respectively. Actually, with the introduction of nominal (or auxiliary) subsystems, the decentralized optimal (tracking) control was derived via solving a group of H_2-constrained optimal (tracking) control problems. Recently, Tan [22] solved a set of H_∞-constrained control problems to derive the distributed event-triggered control for interconnected strict-feedback systems with partially unavailable dynamics. However, Liu *et al.* [20], Yang and He [21], and Tan [22] all considered the nonlinear interconnected systems suffering from symmetric input constraints.

B. Contribution

The main contributions of this article are fourfold.

1) The restrictive inequalities imposed on auxiliary control policies (such as $\|v_i^*(x_i(t))\|^2 \le Q_i(x_i(t))$ given in [13]) are removed when we design the decentralized event-driven controller by using auxiliary subsystems (see Theorem 1). Thus, the present decentralized event-driven control scheme could be more flexible for applications.
2) Rather than solving the event-driven H_∞-constrained optimal control problems or nonzero-sum games, we obtain the decentralized event-driven control via solving a group of event-driven H_2-constrained optimal control problems. Accordingly, our method does not need to assure the existence of the saddle point or the Nash equilibrium point in advance, which is a necessary and challenging task in solving the H_∞-constrained optimal control problems or nonzero-sum games.
3) When implementing the present decentralized control strategy, we update the control policy and the auxiliary control policy simultaneously in the event-driven mechanism. This makes the computational burden further lower down in comparison with the case only updating the control policy in the event-driven mechanism.
4) With a discount term and a nonquadratic function being together introduced to the cost function, we can tackle the decentralized event-driven control problem of mismatched interconnected systems with asymmetric input constraints via the present ACDs. Hence, the ACDs proposed in this article are applicable for general interconnected plants, especially those mismatched interconnected systems having asymmetric input constraints.

C. Notation

$\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{Z}^+$ denote the sets of real numbers, positive real numbers, and positive integral numbers, respectively. $\mathbb{R}^{m_i}$ and $\mathbb{R}^{n_i \times m_i}$ denote the spaces of real vectors of size $m_i \times 1$ and real matrices of size $n_i \times m_i$, respectively. $\Omega_i \subset \mathbb{R}^{n_i}$ is a compact set and $\mathscr{A}(\Omega_i)$ denotes the set of admissible control defined on Ω_i. "T" and "$\triangleq$" are symbols for "transposition" and "equal by definition," respectively. The matrices $Q_i > 0$

and $A > 0$ mean that Q_i and A are positive definite. For the vector $x_i = [x_{i1}, x_{i2}, \ldots, x_{im_i}]^\mathsf{T} \in \mathbb{R}^{m_i}$, its norm is denoted by $\|x_i\| = (\sum_{p=1}^{m_i} x_{\mathrm{ip}}^2)^{1/2}$. For the matrix $D = (b_{\mathrm{kp}})_{n_i \times m_i}$, its Frobenius norm is denoted by $\|D\| = (\sum_{\kappa=1}^{n_i} \sum_{p=1}^{m_i} b_{\mathrm{kp}})^{1/2}$.

II. Problem Description and Preliminaries

A. Problem Description

We consider the interconnected continuous-time system, which is composed of N nonlinear subsystems given by

$$\begin{aligned}\dot{x}_i(t) &= f_i(x_i(t)) + g_i(x_i(t))u_i(t) + \Delta f_i(x(t))\\ x_i(0) &= x_{i0}, \quad i = 1, 2, \ldots, N\end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state vector of ith subsystem with $x_{i0} \in \mathbb{R}^{n_i}$ being its initial state, $u_i \in \mathfrak{U}_i \subset \mathbb{R}^{m_i}$ is the input vector of ith subsystem, and $\mathfrak{U}_i$ denotes the set of input vectors having asymmetric bounds, that is,

$$\mathfrak{U}_i = \left\{ (u_{i1}, u_{i2}, \ldots, u_{im_i})^\mathsf{T} \in \mathbb{R}^{m_i} : u_{\min}^i \le u_{\mathrm{ip}} \le u_{\max}^i, \; |u_{\min}^i| \ne |u_{\max}^i|, \; p = 1, 2, \ldots, m_i \right\}$$

where $u_{\min}^i \in \mathbb{R}$ and $u_{\max}^i \in \mathbb{R}$ denote the minimum and maximum bound of input variables $u_{\mathrm{ip}} \in \mathbb{R}$, $p = 1, 2, \ldots, m_i$, used in the ith subsystem, respectively, $f_i(x_i) \in \mathbb{R}^{n_i}$ and $g_i(x_i) \in \mathbb{R}^{n_i \times m_i}$ are known vector-valued function and matrix function, respectively, and $\Delta f_i(x)$ is an uncertain interconnection with $x = [x_1^\mathsf{T}, x_2^\mathsf{T}, \ldots, x_N^\mathsf{T}]^\mathsf{T} \in \mathbb{R}^n$ (note: $n = \sum_{i=1}^N n_i$) being the whole state.

Assumption 1: For every $i \in \{1, 2, \ldots, N\}$, the ith subsystem defined in (1) is controllable. Meanwhile, $x_i = 0$ is the unique equilibrium point of the ith subsystem over Ω_i with $f_i(0) = 0$. In addition, $\Delta f_i(x)$ meets the mismatched condition, which is presented in the form

$$\Delta f_i(x) = h_i(x_i) d_i(x), \quad i = 1, 2, \ldots, N$$

where $h_i(x_i) \in \mathbb{R}^{n_i \times l_i}$ is a known matrix function (note: $h_i(x_i) \ne g_i(x_i)$ for every $x_i \in \mathbb{R}^{n_i}$ if $l_i = m_i$), $d_i(x) \in \mathbb{R}^{l_i}$ is an uncertain function satisfying

$$\|d_i(x)\| \le \sum_{j=1}^N a_{ij} P_{ij}(x_j), \quad i = 1, 2, \ldots, N \tag{2}$$

where $a_{ij} \ge 0$, $j = 1, 2, \ldots, N$, are adjustable parameters and $P_{ij} : \mathbb{R}^{n_j}/\{0\} \to \mathbb{R}^+$ with $P_{ij}(x_j)$ being positive definite functions. Moreover, $d_i(0) = 0$ and $P_{ij}(0) = 0$.

Letting

$$P_i(x_i) = \max_{1 \le j \le N} \{P_{ji}(x_i)\} \tag{3}$$

we have (2) further relaxed as

$$\|d_i(x)\| \le \sum_{j=1}^N b_{ij} P_j(x_j), \quad i = 1, 2, \ldots, N \tag{4}$$

where $b_{ij} \ge a_{ij} P_{ij}(x_j)/P_j(x_j) \ge 0$, $j = 1, 2, \ldots, N$.

Assumption 2: For each ith subsystem ($i \in \{1, 2, \ldots, N\}$), $\mathrm{rank} g_i(x_i) = m_i (m_i < n_i)$ and $g_i(0) = 0$. Meanwhile, for every $x_i \in \mathbb{R}^{n_i}$, $g_i^\mathsf{T}(x_i) h_i(x_i) = 0$. In addition, for arbitrary, $x_i \in \mathbb{R}^{n_i}$ $\|g_i(x_i)\| \le b_{g_i}$ and $\|h_i(x_i)\| \le b_{h_i}$ with b_{g_i} and b_{h_i} being the positive constants.

Control Objective: The goal of this article is to develop a decentralized control strategy in an appropriate mechanism for interconnected system (1), which meets Assumptions 1 and 2, such that uniform ultimate boundedness (UUB) stability of the entire closed-loop system is ensured.

Due to system (1) having mismatched interconnected terms and asymmetric input constraints, one often finds it hard to acquire the decentralized control using a direct method. Thus, we will present an indirect method. To be specific, we transform the decentralized control problem of input-constrained interconnected system (1) into a series of H_2-constrained optimal control problems of auxiliary subsystems.

B. Hamilton–Jacobi–Bellman Equation for the ith Constrained Auxiliary Subsystem

In order to convert the decentralized constrained control problem into a group of H_2-constrained optimal control problems of auxiliary subsystems, we first need to present the ith ($i = 1, 2, \ldots, N$) auxiliary subsystem, that is, the auxiliary system for the ith subsystem of interconnected system (1). According to [23], the auxiliary system associated with the ith subsystem defined in (1) has the form

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i + \big(I_{n_i} - g_i(x_i) g_i^+(x_i)\big) h_i(x_i) \nu_i \tag{5}$$

with $g_i^+(x_i) \in \mathbb{R}^{m_i \times n_i}$ being the Moore–Penrose pseudoinverse of $g_i(x_i)$ and $\nu_i \in \mathbb{R}^{l_i}$ being the auxiliary control. Noting that u_i has asymmetric bounds (that is, $u_i \in \mathfrak{U}_i$), we call (5) the ith constrained auxiliary subsystem.

Using Assumption 2 (or rather, $\mathrm{rank} g_i(x_i) = m_i$) and the theory of matrix computations [24, Ch. 5.5], we can calculate $g_i^+(x_i)$ via

$$g_i^+(x_i) = \big(g_i^\mathsf{T}(x_i) g_i(x_i)\big)^{-1} g_i^\mathsf{T}(x_i). \tag{6}$$

Inserting (6) into (5) and noticing that $g_i^\mathsf{T}(x_i) h_i(x_i) = 0$ (see Assumption 2), we have the ith constrained auxiliary subsystem (5) simplified as

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i + h_i(x_i)\nu_i. \tag{7}$$

The discounted cost function for the ith constrained auxiliary subsystem (7) has the form

$$V_i^{u_i, \nu_i}(x_i) = \int_t^\infty e^{-\alpha_i(\tau - t)} \mathcal{R}_i(x_i(\tau), u_i(\tau), \nu_i(\tau)) \mathrm{d}\tau \tag{8}$$

where $\alpha_i > 0$ is the discount factor

$$\mathcal{R}_i(x_i, u_i, \nu_i) = \rho_i P_i^2(x_i) + Q_i(x_i) + \mathcal{W}_i(u_i) + \eta_i \|\nu_i\|^2 \tag{9}$$

with $\rho_i > 0$ and $0 < \eta_i \le 1$ being the adjustable parameters, $P_i(x_i) \in \mathbb{R}$ being defined in (3), $Q_i(x_i) = x_i^\mathsf{T} Q_i x_i$ and $Q_i > 0$, $\mathcal{W}_i(u_i) \in \mathbb{R}$ being the semipositive definite function, and $\|\nu_i\|^2 = \nu_i^\mathsf{T} \nu_i$. Due to $u_i \in \mathfrak{U}_i$, motivated by the works of [25] and [26], we let $\mathcal{W}_i(u_i)$ be the nonquadratic function with respect to u_i in order to tackle asymmetric constraints. Specifically, $\mathcal{W}_i(u_i)$ is defined as

$$\mathcal{W}_i(u_i) = 2\beta_i \sum_{p=1}^{m_i} \int_{c_i}^{u_{ip}} \psi^{-1}\big(\beta_i^{-1}(\xi_p - c_i)\big) \mathrm{d}\xi_p$$

where

$$\beta_i = \left(u^i_{\max} - u^i_{\min}\right)/2, \quad c_i = \left(u^i_{\max} + u^i_{\min}\right)/2 \tag{10}$$

and $\psi^{-1}(\cdot) \in C^1(\Omega_i)$ is an odd monotonic function with $\psi^{-1}(0) = 0$. To facilitate discussion, we let $\psi(\cdot) = \tanh(\cdot)$ [note: $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$].

Remark 1: Due to the decay term $e^{-\alpha_i(\tau - t)}$ involved in the integral term on the right-hand side of (8), $V_i^{u_i,\nu_i}(x_i)$ is called the discounted cost function. If we ignore such a decay term (that is, let $\alpha_i = 0$), then $V_i^{u_i,\nu_i}(x_i)$ might be divergent. This is because u_i will not converge to zero when the state arrives at the equilibrium point $x_i = 0$ (note: the detailed explanation is given in Remark 2). This is why we present a discounted cost function like (8).

Let the optimal value of $V_i^{u_i,\nu_i}(x_i)$ be stated as

$$V_i^*(x_i) = \min_{u_i,\nu_i \in \mathscr{A}(\Omega_i)} V_i^{u_i,\nu_i}(x_i). \tag{11}$$

Then, according to [27], $V_i^*(x_i)$ satisfies the Hamilton–Jacobi–Bellman equation (HJBE)

$$\min_{u_i,\nu_i \in \mathscr{A}(\Omega_i)} H\left(x_i, \nabla V_i^*(x_i), u_i, \nu_i\right) = 0 \tag{12}$$

where $H(x_i, \nabla V_i^*(x_i), u_i, \nu_i)$ is called the Hamiltonian and expressed as

$$\begin{aligned} &H\left(x_i, \nabla V_i^*(x_i), u_i, \nu_i\right) \\ &\quad = \left(\nabla V_i^*(x_i)\right)^{\mathsf{T}} \left(f_i(x_i) + g_i(x_i) u_i + h_i(x_i)\nu_i\right) - \alpha_i V_i^*(x_i) \\ &\qquad + \rho_i P_i^2(x_i) + Q_i(x_i) + \mathcal{W}_i(u_i) + \eta_i \|\nu_i\|^2 \end{aligned} \tag{13}$$

with $\nabla V_i^*(x_i) = \partial V_i^*(x_i)/\partial x_i$ and $\nabla V_i^*(0) = 0$.

Let the optimal control be denoted by $u_i^*(x_i)$. Then, according to the stationary condition [28, Ch. 5.9], we have

$$\partial H\left(x_i, \nabla V_i^*(x_i), u_i^*(x_i), \nu_i\right) \big/ \partial u_i^*(x_i) = 0.$$

Together with (13), this yields that the optimal control $u_i^*(x_i)$ has the form

$$u_i^*(x_i) = -\beta_i \tanh\left(\frac{1}{2\beta_i} g_i^{\mathsf{T}}(x_i) \nabla V_i^*(x_i)\right) + \mathcal{C}_i \tag{14}$$

where $\mathcal{C}_i = [c_i, c_i, \ldots, c_i]^{\mathsf{T}} \in \mathbb{R}^{m_i}$.

Remark 2: Due to the existence of asymmetric constraints (i.e., $|u^i_{\min}| \neq |u^i_{\max}|$), there holds $c_i \neq 0$ [see c_i defined in (10)]. Then, it follows from (14) that $u_i^*(0) = \mathcal{C}_i \neq 0$. Thus, for guaranteeing Assumption 1 to make sense (or rather, to ensure that $x_i = 0$ is the equilibrium point), we need the condition $g_i(0) = 0$ to be presented in Assumption 2.

In a way similar to deriving $u_i^*(x_i)$ in (14), we deduce that the optimal auxiliary control $\nu_i^*(x_i)$ is

$$\nu_i^*(x_i) = -\frac{1}{2\eta_i} h_i^{\mathsf{T}}(x_i) \nabla V_i^*(x_i). \tag{15}$$

Inserting (14) and (15) into (12), we have the HJBE for the ith constrained auxiliary subsystem (7) restated as

$$\begin{aligned} &\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}} f_i(x_i) + \rho_i P_i^2(x_i) + Q_i(x_i) \\ &\quad + \mathcal{W}_i\left(-\beta_i \tanh\left(\frac{1}{2\beta_i} g_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i)\right) + \mathcal{C}_i\right) \\ &\quad - \beta_i \left(\nabla V_i^*(x_i)\right)^{\mathsf{T}} g_i(x_i) \tanh\left(\frac{1}{2\beta_i} g_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i)\right) \\ &\quad + \left(\nabla V_i^*(x_i)\right)^{\mathsf{T}} g_i(x_i)\mathcal{C}_i - \alpha_i V_i^*(x_i) \\ &\quad - \left\| \frac{1}{2\sqrt{\eta_i}} h_i^{\mathsf{T}}(x_i)\nabla V_i^*(x_i)\right\|^2 = 0. \end{aligned} \tag{16}$$

Equation (16) is called the time-driven HJBE, for it is solved in a time-driven mechanism. As proved in [21], the solutions of the N time-driven HJBEs like (16) all together constitute the decentralized control. Apparently, the decentralized control law developed in [21] was implemented in the time-driven mechanism. As pointed out in [29], the time-driven control laws updated the control policies periodically and often resulted in heavy computational burden. To avoid such a deficiency, we shall present a decentralized event-driven control law for interconnected system (1).

III. Decentralized Event-Driven Constrained Control Strategy

We first introduce an event-driven mechanism used in [30] and develop the event-driven HJBE (ED-HJBE) for the ith constrained auxiliary subsystem. Then, we build a bridge between the decentralized event-driven control of input-constrained interconnected system (1) (note: we call it the decentralized event-driven constrained control for brevity) and the solutions of N ED-HJBEs. After that, for obtaining the decentralized event-driven constrained control, we solve the N ED-HJBEs via a critic-only architecture under the framework of ACDs. Finally, we present the stability analysis of the ith closed-loop auxiliary subsystem.

A. Event-Driven Mechanism and ED-HJBE for ith Constrained Auxiliary Subsystem

Let the triggering instants for ith constrained auxiliary subsystem be written as t_k^i ($k = 0, 1, 2, \ldots$) and satisfy $t_k^i < t_{k+1}^i$. Then, the sequence of triggering instants is $\{t_k^i\}_{k=0}^{\infty}$. At the triggering instant t_k^i, we say that the ith constrained auxiliary subsystem state is sampled and denote it by

$$\bar{x}_{i,k} = x_i\left(t_k^i\right), \ k = 0, 1, 2, \ldots$$

Before the next triggering instant t_{k+1}^i releases, there usually generates a gap between the two states $\bar{x}_{i,k}$ and $x_i(t)$ over the interval $[t_k^i, t_{k+1}^i)$. The gap is described by an error function $e_{i,k}(t)$, which is in the form

$$e_{i,k}(t) = \bar{x}_{i,k} - x_i(t), \ t \in \left[t_k^i, t_{k+1}^i\right). \tag{17}$$

The main characteristics of the present event-driven mechanism could be described via (17) as follows: 1) if the event in the ith constrained auxiliary subsystem is triggered (such as $t = t_k^i$), then we have $e_{i,k}(t_k^i) = 0$ and thus update the control policy, and 2) if the event given in the ith constrained auxiliary subsystem is not triggered (such as $t \neq t_k^i$), we get $e_{i,k}(t_k^i) \neq 0$ and thus keep the control policy unchanged over the interval $[t_k^i, t_{k+1}^i)$, $k = 0, 1, 2, \ldots$ The latter needs the zero-order hold technique [31], which is expressed as

$$\mu_i(\bar{x}_{i,k}, t) = u_i(\bar{x}_{i,k}) = u_i(x_i(t_k^i)),\ \ t \in [t_k^i, t_{k+1}^i).$$

Under the present event-driven mechanism, we can obtain from (14) that the optimal event-driven control for the ith constrained auxiliary subsystem (7) and the cost function (8) is

$$\begin{aligned}\mu_i^*(\bar{x}_{i,k}, t) &= u_i^*(\bar{x}_{i,k}) \\ &= -\beta_i \tanh\left(\frac{g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla V_i^*(\bar{x}_{i,k})}{2\beta_i}\right) + \mathcal{C}_i \end{aligned} \quad (18)$$

with $t \in [t_k^i, t_{k+1}^i)$ and $\nabla V_i^*(\bar{x}_{i,k}) = (\partial V_i^*(x_i)/\partial x_i)|_{x_i = \bar{x}_{i,k}}$.

Likewise, based on (15), we have the optimal event-driven auxiliary control expressed as [note: $t \in [t_k^i, t_{k+1}^i)$]

$$\upsilon_i^*(\bar{x}_{i,k}, t) = \nu_i^*(\bar{x}_{i,k}) = -\frac{1}{2\eta_i} h_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla V_i^*(\bar{x}_{i,k}). \quad (19)$$

Replacing u_i and ν_i in (12) with $u_i^*(\bar{x}_{i,k})$ in (18) and $\nu_i^*(\bar{x}_{i,k})$ in (19), respectively, we obtain that the ED-HJBE for the ith constrained auxiliary subsystem at triggering instants $t = t_k^i$, $k = 0, 1, 2, \ldots$, is

$$\begin{aligned}&(\nabla V_i^*(x_i))^{\mathsf{T}} f_i(x_i) + \rho_i P_i^2(x_i) + Q_i(x_i) \\ &\quad + \mathcal{W}_i\left(-\beta_i \tanh\left(\frac{1}{2\beta_i} g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla V_i^*(\bar{x}_{i,k})\right) + \mathcal{C}_i\right) \\ &\quad -\beta_i (\nabla V_i^*(x_i))^{\mathsf{T}} g_i(x_i) \tanh\left(\frac{1}{2\beta_i} g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla V_i^*(\bar{x}_{i,k})\right) \\ &\quad + (\nabla V_i^*(x_i))^{\mathsf{T}} g_i(x_i)\mathcal{C}_i - \alpha_i V_i^*(x_i) \\ &\quad -\frac{1}{2\eta_i}(\nabla V_i^*(x_i))^{\mathsf{T}} h_i(x_i) h_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla V_i^*(\bar{x}_{i,k}) \\ &\quad + \left\|\frac{1}{2\sqrt{\eta_i}} h_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla V_i^*(\bar{x}_{i,k})\right\|^2 = 0. \end{aligned} \quad (20)$$

Remark 3: If there is no confusion of symbols, we omit the time variable "t" in both $\mu_i^*(\bar{x}_{i,k}, t)$ and $\upsilon_i^*(\bar{x}_{i,k}, t)$. To be specific, we denote $\mu_i^*(\bar{x}_{i,k}, t)$ and $\upsilon_i^*(\bar{x}_{i,k}, t)$ as $\mu_i^*(\bar{x}_{i,k})$ and $\upsilon_i^*(\bar{x}_{i,k})$, respectively.

B. Relationship Between Decentralized Event-Driven Constrained Control and the Solutions of N ED-HJBEs

Prior to proceeding, we present an assumption for $u_i^*(x_i)$ in (14) and $\nu_i^*(x_i)$ in (15). The assumption was widely employed in the literature, such as [30], [32], and [33].

Assumption 3: There are two positive constants $K_{u_i^*}$ and $K_{\nu_i^*}$ (i.e., Lipschitz constants) such that, for any $x_i, \bar{x}_{i,k} \in \boldsymbol{\Omega}_i$,

$$\begin{aligned}\|u_i^*(x_i) - u_i^*(\bar{x}_{i,k})\| &\le K_{u_i^*}\|x_i - \bar{x}_{i,k}\| = K_{u_i^*}\|e_{i,k}\| \\ \|\nu_i^*(x_i) - \nu_i^*(\bar{x}_{i,k})\| &\le K_{\nu_i^*}\|x_i - \bar{x}_{i,k}\| = K_{\nu_i^*}\|e_{i,k}\|.\end{aligned}$$

Remark 4: Together with (18), (19), and Remark 3, we find that Assumption 3 yields

$$\begin{aligned}\|\mu_i^*(\bar{x}_{i,k}) - u_i^*(x_i)\| &\le K_{u_i^*}\|e_{i,k}\| \\ \|\upsilon_i^*(\bar{x}_{i,k}) - \nu_i^*(x_i)\| &\le K_{\nu_i^*}\|e_{i,k}\|.\end{aligned}$$

Theorem 1: Consider N constrained auxiliary subsystems defined as (7) with their relevant cost functions described as (8). Let Assumptions 1–3 hold. Then, there must have N positive constants ρ_i^*, $i = 1, 2, \ldots, N$, such that, for each $\rho_i \ge \rho_i^*$, the N optimal event-driven control policies $\mu_1^*(\bar{x}_{1,k}), \mu_2^*(\bar{x}_{2,k}), \ldots, \mu_N^*(\bar{x}_{N,k})$ together [note: $\mu_i^*(\bar{x}_{i,k})$ is defined in (18)] can keep interconnected system (1) stable in the sense of UUB as long as the triggering condition is

$$\|e_{i,k}(t)\|^2 \le \left(\frac{1 - 2\gamma_i}{4K_{\max}^2}\right) Q_i(x_i(t)) \triangleq \check{e}_{i,T}(t) \quad (21)$$

where $0 < \gamma_i < 1/2$, $K_{\max} = \max\{K_{u_i^*}, K_{\nu_i^*}\}$ with $K_{u_i^*}$ and $K_{\nu_i^*}$ being defined in Assumption 3, and $\check{e}_{i,T}(t)$ is the triggering threshold.

Proof: See Appendix I. ∎

Remark 5: Some notes for Theorem 1 are given as follows.

1) As shown in Theorem 1, there was no restrictive inequality imposed on the auxiliary control policies. Specifically, Theorem 1 removed strictly restrictive inequalities $\|\nu_i^*(x_i(t))\|^2 \le Q_i(x_i(t))$, $i = 1, 2, \ldots, N$, used in [19]. This is an advantage of Theorem 1. Note that Theorem 1 developed in this article only assured the UUB stability rather than asymptotical stability of the entire closed-loop system like [19]. Thus, there might exist a tradeoff between the control performance and the restrictive inequalities.
2) The triggering condition (21) makes sense only when it excludes the Zeno behavior. According to [34, Th. III.1] and [35, Th. 2], The Zeno behavior can be prevented from happening when the minimum intersample time is positive. Fortunately, under Assumptions 2 and 3, there holds the minimum intersample time $(\Delta t_k^i)_{\min_k} > 0$, where $\Delta t_k^i = t_{k+1}^i - t_k^i$, $i = 1, 2, \ldots, N$. Because the proof is analogous to [30] and [36], we omit it here for avoiding redundancy.

According to Theorem 1, the decentralized event-driven constrained control could be obtained by finding N optimal event-driven control policies $\mu_1^*(\bar{x}_{1,k})$, $\mu_2^*(\bar{x}_{2,k})$, …, $\mu_N^*(\bar{x}_{N,k})$. To this end, we solve the N ED-HJBEs like (20) by using a critic-only architecture in the framework of ACDs.

C. Critic-Only Architecture for Solving N ED-HJBEs

Note that solving N ED-HJBEs is in a way similar to solving the ED-HJBE (20) for ith constrained auxiliary subsystem. Thus, we only provide the procedure of solving (20).

According to [37, Th. 3.1], neural networks (NNs) could approximate any continuous function over the compact set. Thus, we could use an NN to reconstruct $V_i^*(x_i)$ over $\boldsymbol{\Omega}_i$ as follows:

$$V_i^*(x_i) = \omega_{c_i}^{\mathsf{T}}\sigma_{c_i}(x_i) + \varepsilon_{c_i}(x_i) \quad (22)$$

where $\omega_{c_i} \in \mathbb{R}^{\ell_i}$ is the ideal weight vector generally unavailable in advance, $\ell_i \in \mathbb{Z}^+$ is the number of neurons, $\sigma_{c_i}(x_i) = [\sigma_{c_{i1}}(x_i), \sigma_{c_{i2}}(x_i), \ldots, \sigma_{c_{i\ell_i}}(x_i)]^{\mathsf{T}} \in \mathbb{R}^{\ell_i}$ is the vector activation function, which is continuously differentiable over $\boldsymbol{\Omega}_i$ with its elements $\sigma_{c_{i1}}(x_i), \sigma_{c_{i2}}(x_i), \ldots, \sigma_{c_{i\ell_i}}(x_i)$ being linearly independent for any $x_i \neq 0$ (*note:* $\sigma_{c_{is}}(0) = 0$, $s = 1, 2, \ldots, \ell_i$), and $\varepsilon_{c_i}(x_i) \in \mathbb{R}$ is the approximation error. According to [25], $\varepsilon_{c_i}(x_i)$ can be small enough if ℓ_i is sufficiently large. Specifically, $\varepsilon_{c_i}(x_i) \to 0$ when $\ell_i \to \infty$.

The derivative of $V_i^*(x_i)$ in (22) at the sampled state $\bar{x}_{i,k}$ is

$$\nabla V_i^*(\bar{x}_{i,k}) = \nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\omega_{c_i} + \nabla\varepsilon_{c_i}(\bar{x}_{i,k}) \tag{23}$$

where

$$\nabla\breve{\mathcal{B}}(\bar{x}_{i,k}) = \frac{\partial\breve{\mathcal{B}}(x_i)}{\partial x_i}\bigg|_{x_i=\bar{x}_{i,k}}$$

with $\breve{\mathcal{B}}(\cdot)$ denoting $\sigma_{c_i}(\cdot)$ or $\varepsilon_{c_i}(\cdot)$.

Inserting $\nabla V_i^*(\bar{x}_{i,k})$ in (23) into (18) and using Remark 3, we have $\mu_i^*(\bar{x}_{i,k})$ restated as [note: $t \in [t_k^i, t_{k+1}^i)$]

$$\mu_i^*(\bar{x}_{i,k}) = -\beta_i \tanh\big(\Phi_1(\bar{x}_{i,k})\big) + \varepsilon_{\mu_i^*}(\bar{x}_{i,k}) + \mathcal{C}_i \tag{24}$$

where

$$\Phi_1(\bar{x}_{i,k}) = \frac{1}{2\beta_i} g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\omega_{c_i}$$
$$\varepsilon_{\mu_i^*}(\bar{x}_{i,k}) = -(1/2)\big(I_{m_i} - \mathcal{E}(\varsigma(\bar{x}_{i,k}))\big) g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla\varepsilon_{c_i}(\bar{x}_{i,k})$$

and $\mathcal{E}(\varsigma(\bar{x}_{i,k})) = \mathrm{diag}\{\tanh^2(\varsigma_p(\bar{x}_{i,k}))\}$ $(p = 1, 2, \ldots, m_i)$ with $\varsigma(\bar{x}_{i,k}) = [\varsigma_1(\bar{x}_{i,k}), \varsigma_2(\bar{x}_{i,k}), \ldots, \varsigma_{m_i}(\bar{x}_{i,k})]^{\mathsf{T}} \in \mathbb{R}^{m_i}$ being selected between $(1/(2\beta_i))g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla V_i^*(\bar{x}_{i,k})$ and $\Phi_1(\bar{x}_{i,k})$.

Similarly, using $\nabla V_i^*(\bar{x}_{i,k}))$ in (23), we have $\upsilon_i^*(\bar{x}_{i,k})$ in (19) represented as [note: $t \in [t_k^i, t_{k+1}^i)$]

$$\upsilon_i^*(\bar{x}_{i,k}) = -\frac{1}{2\eta_i} h_i^{\mathsf{T}}(x_i)\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\omega_{c_i} + \varepsilon_{\upsilon_i^*}(\bar{x}_{i,k}) \tag{25}$$

where $\varepsilon_{\upsilon_i^*}(\bar{x}_{i,k}) = -h_i^{\mathsf{T}}(x_i)\nabla\varepsilon_{c_i}(\bar{x}_{i,k})\big/(2\eta_i)$.

Note that ω_{c_i} in (22) is unknown beforehand. Thus, it makes $\mu_i^*(\bar{x}_{i,k})$ in (24) and $\nu_i^*(x_i)$ in (25) infeasible for practical implementation. A promising way to tackle this issue is to replace ω_{c_i} with its estimated value $\hat{\omega}_{c_i}$. Specifically, rather than using $V_i^*(x_i)$ in (22), we utilize its estimated value $\hat{V}_i^*(x_i)$, which is the output of the critic NN, that is,

$$\hat{V}_i^*(x_i) = \hat{\omega}_{c_i}^{\mathsf{T}}\sigma_{c_i}(x_i). \tag{26}$$

Then, making use of (26) and taking a way similar to obtaining $\mu_i^*(\bar{x}_{i,k})$ in (24), we have the estimated value of $\mu_i^*(\bar{x}_{i,k})$ expressed as

$$\hat{\mu}_i(\bar{x}_{i,k}) = -\beta_i \tanh\big(\Phi_2(\bar{x}_{i,k})\big) + \mathcal{C}_i,\ t \in \big[t_k^i, t_{k+1}^i\big) \tag{27}$$

where

$$\Phi_2(\bar{x}_{i,k}) = \frac{1}{2\beta_i} g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\hat{\omega}_{c_i}.$$

Likewise, we could obtain the estimated value of $\upsilon_i^*(\bar{x}_{i,k})$ in (25) as

$$\hat{\upsilon}_i(\bar{x}_{i,k}) = -\frac{1}{2\eta_i} h_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\hat{\omega}_{c_i},\ t \in \big[t_k^i, t_{k+1}^i\big). \tag{28}$$

Letting $V_i^*(x_i)$, u_i, and ν_i in (13) be replaced with $\hat{V}_i^*(x_i)$ in (26), $\hat{\mu}_i(\bar{x}_{i,k})$ in (27), and $\hat{\upsilon}_i(\bar{x}_{i,k})$ in (28), respectively, we deduce that the approximation Hamiltonian is [note: $t \in [t_k^i, t_{k+1}^i)$]

$$\hat{H}\big(x_i, \nabla\hat{V}_i^*(x_i), \hat{\mu}_i(\bar{x}_{i,k}), \hat{\upsilon}_i(\bar{x}_{i,k})\big) = \hat{\omega}_{c_i}^{\mathsf{T}}\phi_i + \rho_i P_i^2(x_i) + Q_i(x_i) + \mathcal{W}_i(\hat{u}_i(\bar{x}_{i,k})) + \eta_i \big\|\hat{\upsilon}_i(\bar{x}_{i,k})\big\|^2$$

where

$$\phi_i = \nabla\sigma_{c_i}(x_i)\big(f_i(x_i) + g_i(x_i)\hat{\mu}_i(\bar{x}_{i,k}) + h_i(x_i)\hat{\upsilon}_i(\bar{x}_{i,k})\big) - \alpha_i\sigma_{c_i}(x_i). \tag{29}$$

Recall that the time-driven HJBE defined in (12) implies

$$H\big(x_i, \nabla V_i^*(x_i), u_i^*(x_i), \nu_i^*(x_i)\big) = 0.$$

We, therefore, get an error function of Hamiltonian, denoted by e_{c_i}, formulated as

$$\begin{aligned} e_{c_i} =\ & \hat{H}\big(x_i, \nabla\hat{V}_i^*(x_i), \hat{\mu}_i(\bar{x}_{i,k}), \hat{\upsilon}_i(\bar{x}_{i,k})\big) \\ & - H\big(x_i, \nabla V_i^*(x_i), u_i^*(x_i), \nu_i^*(x_i)\big) \\ =\ & \hat{\omega}_{c_i}^{\mathsf{T}}\phi_i + \rho_i P_i^2(x_i) + Q_i(x_i) + \mathcal{W}_i\big(\hat{u}_i(\bar{x}_{i,k})\big) \\ & + \eta_i\big\|\hat{\upsilon}_i(\bar{x}_{i,k})\big\|^2. \end{aligned} \tag{30}$$

To make $\hat{\omega}_{c_i} \to \omega_{c_i}$, we need to force $e_{c_i} \to 0$, that is, e_{c_i} in (30) must be made small enough. To attain this goal, we tune $\hat{\omega}_{c_i}$ to minimize $E(e_{c_i}) = (1/2)e_{c_i}^{\mathsf{T}}e_{c_i}$. The tuning rule for $\hat{\omega}_{c_i}$ is derived via applying both the gradient descent approach and the normalization technique to $E(e_{c_i})$, that is, [note: $t \in [t_k^i, t_{k+1}^i)$]

$$\dot{\hat{\omega}}_{c_i} = -\frac{\lambda_i}{2\big(1 + \phi_i^{\mathsf{T}}\phi_i\big)^2}\frac{\partial E(e_{c_i})}{\partial\hat{\omega}_{c_i}} = -\frac{\lambda_i\phi_i}{\big(1 + \phi_i^{\mathsf{T}}\phi_i\big)^2}e_{c_i} \tag{31}$$

where $\lambda_i > 0$ is a designable parameter, $(1 + \phi_i^{\mathsf{T}}\phi_i)^{-2}$ is the normalization term, and ϕ_i is defined in (29).

Define the weight error as $\tilde{\omega}_{c_i} = \omega_{c_i} - \hat{\omega}_{c_i}$. Then, together with (31), we have $\tilde{\omega}_{c_i}$ satisfied

$$\dot{\tilde{\omega}}_{c_i} = -\lambda_i\varphi_i\varphi_i^{\mathsf{T}}\tilde{\omega}_{c_i} + \lambda_i\frac{\varphi_i}{1 + \phi_i^{\mathsf{T}}\phi_i}\varepsilon_{H_i} \tag{32}$$

where $\varphi_i = \phi_i\big/\big(1 + \phi_i^{\mathsf{T}}\phi_i\big)$ and $\varepsilon_{H_i} = -\nabla\varepsilon_{c_i}^{\mathsf{T}}(x_i)\big(f_i(x_i) + g_i(x_i)\hat{\mu}_i(\bar{x}_{i,k}) + h_i(x_i)\hat{\upsilon}_i(\bar{x}_{i,k})\big) + \alpha_i\varepsilon_{c_i}(x_i)$ is the residual error (note: because of obtaining ε_{H_i} in a way similar to [32], we omit the detailed process here).

D. Stability Analysis

Before conducting the stability analysis, we present some basic assumptions used in [38]–[40].

Assumption 4: For all $x_i \in \Omega_i$, $\|\nabla\sigma_{c_i}(x_i)\| \le b_{\sigma_{c_i}}$ with $b_{\sigma_{c_i}}$ being the positive constant. Meanwhile, for all $x_i \in \Omega_i$, $\|\varepsilon_{\mu_i^*}(x_i)\| \le b_{\varepsilon_{\mu_i^*}}$, $\|\varepsilon_{\upsilon_i^*}(x_i)\| \le b_{\varepsilon_{\upsilon_i^*}}$, and $\|\varepsilon_{H_i}\| \le b_{\varepsilon_{H_i}}$ with $b_{\varepsilon_{\mu_i^*}}$, $b_{\varepsilon_{\upsilon_i^*}}$, and $b_{\varepsilon_{H_i}}$ being the positive constants.

Assumption 5: Over the interval $[t, t + T_0]$, φ_i defined in (32) is persistently exciting. Specifically, we have constants $0 < \varrho_1 \le \varrho_2$ and $T_0 > 0$ such that

$$\varrho_1\ I_{\ell_i} \le \int_t^{t+T_0}\varphi_i(\tau)\varphi_i^{\mathsf{T}}(\tau)\mathrm{d}s \le \varrho_2 I_{\ell_i} \tag{33}$$

where I_{ℓ_i} is the identity matrix of size $\ell_i \times \ell_i$.

Theorem 2: Consider the ith constrained auxiliary subsystem (7) with its related ED-HJBE (20). Let Assumptions 1–5 hold and let the event-driven control policies be proposed as (27) and (28). If the initial control policies are admissible and the weight vector is tuned via (31), then the UUB stability of the ith auxiliary subsystem state x_i and the weight error $\tilde{\omega}_{c_i}$

is ensured, provided that the triggering condition is (21) and the following inequality holds:

$$3\lambda_i\theta_{\min}\left(\varphi_i\varphi_i^{\mathsf{T}}\right) - 4\left(\frac{b_{h_i}b_{\sigma_{c_i}}}{\eta_i}\right)^2 > 0 \tag{34}$$

with $\theta_{\min}\left(\varphi_i\varphi_i^{\mathsf{T}}\right)$ being the minimum eigenvalue of symmetric matrix $\varphi_i\varphi_i^{\mathsf{T}}$.

Proof: See Appendix II. ■

Remark 6: There are two reasons for presenting Assumption 5. First, according to [32, Th. 2], we have to keep φ_i persistently exciting in order to make $\hat{\omega}_{c_i}$ converge to the ideal weight vector ω_{c_i}. Second, by using Assumption 5 [or rather, the inequality (33)], we can deduce that $\theta_{\min}(\varphi_i\varphi_i^{\mathsf{T}})$ defined in (34) is positive. In this circumstance, by properly selecting λ_i, we can keep the inequality (34) valid.

To make the present decentralized event-driven constrained control scheme better for understanding, we summarize it as Algorithm 1.

Algorithm 1 Decentralized Event-Driven Constrained Control Strategy for Mismatched Interconnected Systems

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initialization: Choose the positive parameters α_i, ρ_i, ℓ_i, and λ_i; Determine the parameters $b_{ij} \geq 0$, c_i, $0 < \eta_i \leq 1$, $\beta_i > 0$, $0 < \gamma_i < 1/2$, $K_{\max} > 0$, and the matrix Q_i; Set the initial states x_{i0} and $\bar{x}_{i0}$, the computational accuracy $\epsilon_0 > 0$, the maximum iteration step $q_{\max} \in \mathbb{Z}^+$, the initial iteration index $q = 0$, and the initial weight vector $\hat{\omega}_{c_i}^{(0)}$.

repeat

$\hat{\mu}_i^{(q)}(\bar{x}_{i,k}) \leftarrow$ Eq. (27); $\hat{\upsilon}_i^{(q)}(\bar{x}_{i,k}) \leftarrow$ Eq. (28);

$\hat{\omega}_{c_i}^{(q+1)} \leftarrow$ Eq. (31);

$q \leftarrow q + 1$;

$e_{i,k} \leftarrow \bar{x}_{i,k} - x_i$; $\check{e}_{i,T} \leftarrow (1 - 2\gamma_i)Q_i(x_i)/(4K_{\max}^2)$;

if $\|e_{i,k}\| > \sqrt{\check{e}_{i,T}}$ **then**

$$\hat{\mu}_i^{(q)}(\bar{x}_{i,k}) \leftarrow -\beta_i \tanh\left(\Phi_2^{(q)}(\bar{x}_{i,k})\right) + \mathcal{C}_i$$

$$\triangleright\ \Phi_2^{(q)}(\bar{x}_{i,k}) = \frac{1}{2\beta_i}g_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\hat{\omega}_{c_i}^{(q)};$$

$$\hat{\upsilon}_i^{(q)}(\bar{x}_{i,k}) \leftarrow -\frac{1}{2\eta_i}h_i^{\mathsf{T}}(\bar{x}_{i,k})\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\hat{\omega}_{c_i}^{(q)};$$

end if

until $q \geq q_{\max}$ or $\left\|\hat{\omega}_{c_i}^{(q+1)} - \hat{\omega}_{c_i}^{(q)}\right\| < \epsilon_0$;

Insert $\hat{\omega}_{c_i}^{(q+1)}$ into (27), and thus obtain the approximate optimal event-driven control $\hat{\mu}_i^*(\bar{x}_{i,k})$. Then, $\hat{\mu}_1^*(\bar{x}_{1,k})$, $\hat{\mu}_2^*(\bar{x}_{2,k}), \ldots, \hat{\mu}_N^*(\bar{x}_{N,k})$ all together constitute the decentralized event-driven constrained control.

Remark 7: Some notes for the decentralized event-driven constrained control and the present ACDs are given as follows.

1) The present decentralized event-driven constrained control scheme is obtained by using ACDs incorporated with adaptive control methods, such as Laypunov stability analyses and gradient descent approaches. Thus, the ACDs developed in this article are totally different from the ADP method proposed in [6], which does not look into stability analyses of the closed-loop system.
2) The present ACDs are actually in a similar spirit as [5]. In this article, we extend the work of [5] to obtain the decentralized event-driven control of nonlinear systems with mismatched interconnections and asymmetric input constraints. There are two main characteristics distinguishing this article and [5].
    a) First, in this article, we have to introduce an event-driven auxiliary control $\hat{\upsilon}_i(\bar{x}_{i,k})$ defined as (28), which was not required in [5].
    b) Second, the event-driven control $\hat{\mu}_i(\bar{x}_{i,k})$ defined in (27) is more complex than the event-driven control proposed in [5].

Due to the above mentioned two core differences [in particular, the added event-driven auxiliary control $\hat{\upsilon}_i(\bar{x}_{i,k})$ defined in (28)], it is much more difficult and complex than [5] to employ ACDs to derive the decentralized event-driven control law for nonlinear systems with *mismatched* interconnections and asymmetric input constraints (note: [5] only considered nonlinear plants with *matched* perturbations and ignored the interconnections). Therefore, the work of this article is nontrivial.

IV. Simulation Experiment

We study the interconnected nonlinear systems modified from [41], which have the form

$$\begin{aligned}
\dot{x}_1 &= \begin{bmatrix} -x_{11} + x_{12} \\ -0.5(x_{11} + x_{12}) + 0.5x_{11}^2 x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ \sin(x_{12}) \end{bmatrix} u_1 \\
&\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix}(x_{11} + x_{22})\sin^2(q_1 x_{12})\cos(0.5x_{21}) \\
\dot{x}_2 &= \begin{bmatrix} 0.5x_{22} \\ -x_{21} - 0.5x_{22} + 0.5x_{21}\cos^2(x_{22}) \end{bmatrix} \\
&\quad + \begin{bmatrix} 0 \\ \sin(x_{21})\cos(x_{22}) \end{bmatrix} u_2 \\
&\quad + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\left(0.6(x_{12} + x_{22})\cos\left(q_2 e^{x_{11}^2}\right)\right)
\end{aligned} \tag{35}$$

where $x_1 = [x_{11}, x_{12}]^{\mathsf{T}}$ and $x_2 = [x_{21}, x_{22}]^{\mathsf{T}}$ are the state vectors of subsystems 1 and 2, respectively, and their initial state vectors are $x_{10} = [0.5, -0.5]^{\mathsf{T}}$ and $x_{20} = [1, -1]^{\mathsf{T}}$, $u_1 \in \mathfrak{U}_1 = \{u_1 \in \mathbb{R}\colon -2 \leq u_1 \leq 4\}$ and $u_2 \in \mathfrak{U}_2 = \{u_2 \in \mathbb{R}\colon -2 \leq u_2 \leq 3\}$ are input variables for subsystems 1 and 2, respectively, and q_1 and q_2 are unknown scalar parameters (note: we select q_1 and q_2 randomly within the interval $[-3, 3]$).

To make Assumption 1 [or rather, the inequality (4)] hold, we choose $P_1(x_1) = \|x_1\|$, $P_2(x_2) = \|x_2\|$, $b_{11} = 1$, $b_{12} = 1$, $b_{21} = 0.6$, and $b_{22} = 0.6$. Meanwhile, we get from (35) that $g_1(x_1) = [0, \sin(x_{12})]^{\mathsf{T}}$, $g_2(x_2) = [0, \sin(x_{21})\cos(x_{22}]^{\mathsf{T}}$, and $h_i(x_i) = [1, 0]^{\mathsf{T}}$, $i = 1, 2$. Thus, there hold $g_i(0) = 0$ and $\mathrm{rank}(g_i(x_i)) < 2$ as well as $g_i^{\mathsf{T}}(x_i)h_i(x_i) = 0$, $i = 1, 2$. Apparently, $g_i(x_i)$ and $h_i(x_i)$ $(i = 1, 2)$ are bounded. Hence, Assumption 2 holds.

Two constrained auxiliary subsystems associated with interconnected system (35) could be obtained via (7) (note: here, functions for constrained auxiliary we call them the constrained auxiliary subsystems 1 and 2). As shown in Theorem 1, we need to find the optimal control policies for constrained auxiliary subsystems 1 and 2 in order to derive the decentralized event-driven control of system (35). To this end, we first present two cost functions like (8) for constrained auxiliary subsystems 1 and 2, respectively. We set $\alpha_1 = 0.6$, $\alpha_2 = 0.6$, $\eta_1 = 0.25$, $\eta_2 = 0.25$, and $Q_1 = Q_2 = I_2$ with I_2 being the identity matrix of size 2×2. Meanwhile, we let $\rho_1 = 4$ and $\rho_2 = 4$ in order to ensure the matrix A defined in later (45) to be positive definite. Then, according to (8), the cost functions for constrained auxiliary subsystems 1 and 2 are given (note: $V_i(x_i)$ denotes $V_i^{u_i,\nu_i}(x_i)$, $i = 1, 2$. Besides, x_i, $\mathcal{W}_i(u_i)$, and ν_i separately denote $x_i(\tau)$, $\mathcal{W}_i(u_i(\tau))$, and $\nu_i(\tau)$ without mentioning the variable τ), respectively

$$V_1(x_1) = \int_t^{\infty} e^{-0.6(\tau - t)}\big(5\|x_1\|^2 + \mathcal{W}_1(u_1) + 0.25\nu_1^2\big)\mathrm{d}\tau$$
$$V_2(x_2) = \int_t^{\infty} e^{-0.6(\tau - t)}\big(5\|x_2\|^2 + \mathcal{W}_2(u_2) + 0.25\nu_2^2\big)\mathrm{d}\tau$$

where (note: $i = 1, 2$)

$$\begin{aligned}\mathcal{W}_i(u_i) =\ & 2\beta_i \int_{c_i}^{u_i} \tanh^{-1}\big(\beta_i^{-1}(s - c_i)\big)\mathrm{d}s \\ =\ & 2\beta_i(u_i - c_i)\tanh^{-1}((u_i - c_i)/\beta_i) \\ & + \beta_i^2 \ln\big(1 - (u_i - c_i)^2/\beta_i^2\big).\end{aligned} \tag{36}$$

Performing calculations through (10), we have $\beta_1 = 3$, $c_1 = 1$, $\beta_2 = 2.5$, and $c_2 = 0.5$.

Now, we use critic NNs defined as (26) to solve the two ED-HJBEs like (20) for constrained auxiliary subsystems 1 and 2. To this end, motivated by the work of [25], we choose two different vector activation functions $\sigma_{c_1}(x_1)$ and $\sigma_{c_2}(x_2)$ used in (26) as (note: $\ell_1 = 8$ and $\ell_2 = 8$)

$$\begin{aligned}\sigma_{c_1}(x_1) = \Big[&x_{11}^2, x_{12}^2, x_{11}x_{12}, x_{11}^4, \\ &x_{12}^4, x_{11}^3 x_{12}, x_{11}^2 x_{12}^2, x_{11}x_{12}^3\Big]^{\mathsf{T}} \\ \sigma_{c_2}(x_2) = \Big[&x_{21}^2, x_{22}^2, x_{21}x_{22}, x_{21}^4, \\ &x_{22}^4, x_{21}^3 x_{22}, x_{21}^2 x_{22}^2, x_{21}x_{22}^3\Big]^{\mathsf{T}}.\end{aligned}$$

Meanwhile, we write the two associated weight vectors (namely, $\hat{\omega}_{c_1}$ and $\hat{\omega}_{c_2}$) used in (26) as $\hat{\omega}_{c_1} = [\hat{\omega}_{c_{11}}, \hat{\omega}_{c_{12}}, \ldots, \hat{\omega}_{c_{18}}]^{\mathsf{T}}$ and $\hat{\omega}_{c_2} = [\hat{\omega}_{c_{21}}, \hat{\omega}_{c_{22}}, \ldots, \hat{\omega}_{c_{28}}]^{\mathsf{T}}$, respectively. To keep initial control policies (including the initial auxiliary control policy) admissible for constrained auxiliary subsystems 1 and 2, we let the initial weight vectors of critic NNs be $\hat{\omega}_{c_1}^{\text{initial}} = [0.437, 0.218, 0.2283, 0.4225, 0.3815, 0.3565, 0.174, 0.4197]^{\mathsf{T}}$ and $\hat{\omega}_{c_2}^{\text{initial}} = [0.131, 0.3593, 0.4742, 0.0708, 0.4969, 0.0535, 0.0155, -0.1693]^{\mathsf{T}}$, respectively. Here, $\hat{\omega}_{c_1}^{\text{initial}}$ and $\hat{\omega}_{c_2}^{\text{initial}}$ are obtained via trail and error. The parameters given in the triggering condition (21) and the weight tuning rule (31) are $K_{\max} = 3.5$, $\gamma_i = 0.25$, and $\lambda_i = 0.6$, $i = 1, 2$.

With the MATLAB 2017(a) soft package being applied to the simulation study, we obtain the performance of critic NN weight vectors $\hat{\omega}_{c_1}$ and $\hat{\omega}_{c_2}$ shown in Figs. 1 and 2. Observing Fig. 1, we find that $\hat{\omega}_{c_1}$ is convergent after

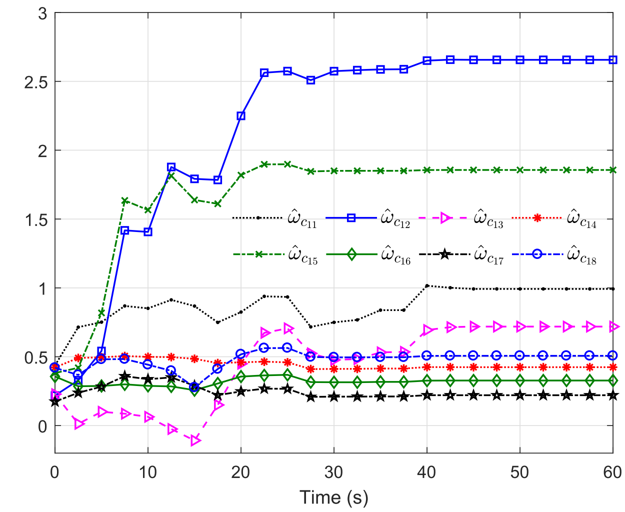

Fig. 1. Performance of weight vector $\hat{\omega}_{c_1} = [\hat{\omega}_{c_{11}}, \hat{\omega}_{c_{12}}, \ldots, \hat{\omega}_{c_{18}}]^{\mathsf{T}}$.

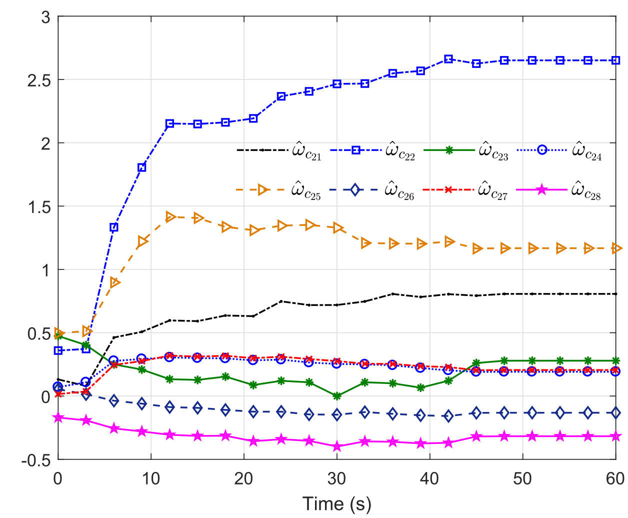

Fig. 2. Performance of weight vector $\hat{\omega}_{c_2} = [\hat{\omega}_{c_{21}}, \hat{\omega}_{c_{22}}, \ldots, \hat{\omega}_{c_{28}}]^{\mathsf{T}}$.

the first 45 s and its converged value is $\hat{\omega}_{c_1}^{\text{converged}} = [0.9924, 2.656, 0.7189, 0.424, 1.8564, 0.3273, 0.221, 0.5066]^{\mathsf{T}}$. Meanwhile, we can see from Fig. 2 that $\hat{\omega}_{c_2}$ is convergent after the first 50 s and its converged value is $\hat{\omega}_{c_2}^{\text{converged}} = [0.807, 2.651, 0.279, 0.1915, 1.1676, -0.132, 0.205, -0.317]^{\mathsf{T}}$. Fig. 3(a) and (b) shows the event-driven control $\hat{\mu}_1(\bar{x}_{1,k})$ and the auxiliary event-driven control $\hat{\upsilon}_1(\bar{x}_{1,k})$ for constrained auxiliary subsystem 1. As shown in Fig. 3(a), $\hat{\mu}_1(\bar{x}_{1,k})$ does not exceed the lower bound $(u_1)_{\min} = -2$ and the upper bound $(u_1)_{\max} = 4$. Fig. 4(a) and (b) shows the event-driven control $\hat{\mu}_2(\bar{x}_{2,k})$ and the auxiliary event-driven control $\hat{\upsilon}_2(\bar{x}_{2,k})$ for constrained auxiliary subsystem 2. According to Fig. 4(a), $\hat{\mu}_2(\bar{x}_{2,k})$ does not surpass the asymmetric bounds, namely, $(u_2)_{\min} = -2$ and $(u_2)_{\max} = 3$. It is verified in Figs. 3(a) and 4(a) that the asymmetric input constraints are overcome with the nonquadratic function defined in (36). It can also be seen from Figs. 3(a) and 4(a) that $\hat{\mu}_1(\bar{x}_{1,k})$ and $\hat{\mu}_2(\bar{x}_{2,k})$ converge to 1 and 0.5, respectively. In other words, both $\hat{\mu}_1(\bar{x}_{1,k})$ and $\hat{\mu}_2(\bar{x}_{2,k})$ converge to nonzero points. This characteristic is consistent with Remark 2. Fig. 5(a) describes the norm of $e_{1,k}(t)$ defined in (17) (i.e., $\|e_{1,k}\|$) and the square root of $\breve{e}_{1,T}$ defined in (21) (i.e., $\sqrt{\breve{e}_{1,T}}$) for

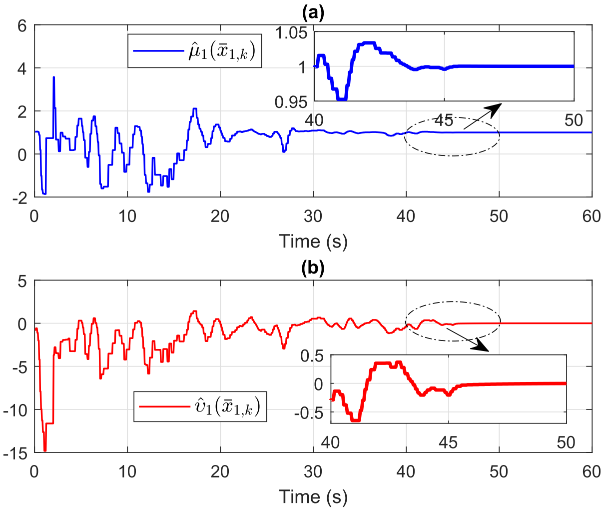

Fig. 3. (a) Event-driven control $\hat{\mu}_1(\bar{x}_{1,k})$. (b) Event-driven auxiliary control $\hat{\upsilon}_1(\bar{x}_{1,k})$.

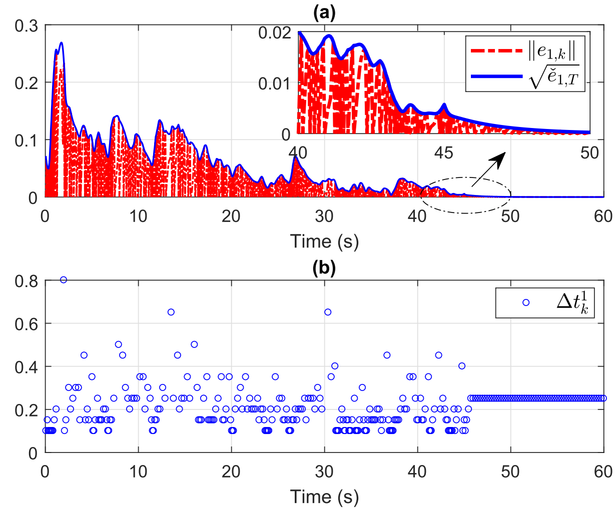

Fig. 5. (a) Norm of $e_{1,k}$ (i.e., $\|e_{1,k}\|$) and the square root of $\check{e}_{1,T}$ (i.e., $\sqrt{\check{e}_{1,T}}$). (b) Intersample time $\Delta t_k^1 = t_{k+1}^1 - t_k^1$.

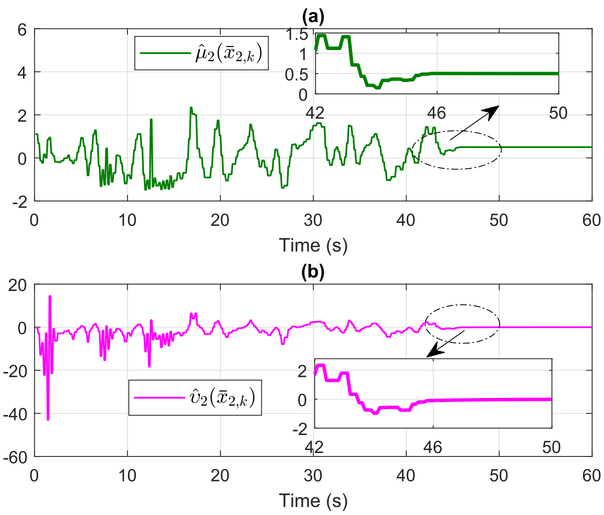

Fig. 4. (a) Event-driven control $\hat{\mu}_2(\bar{x}_{2,k})$. (b) Event-driven auxiliary control $\hat{\upsilon}_2(\bar{x}_{2,k})$.

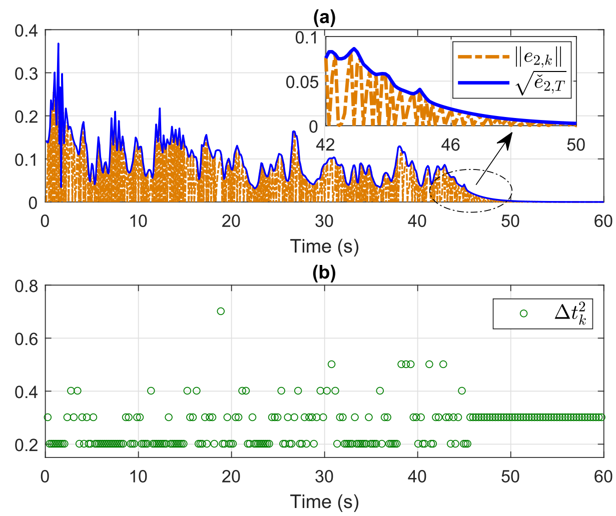

Fig. 6. (a) Norm of $e_{2,k}$ (i.e., $\|e_{2,k}\|$) and the square root of $\check{e}_{2,T}$ (i.e., $\sqrt{\check{e}_{2,T}}$). (b) Intersample time $\Delta t_k^2 = t_{k+1}^2 - t_k^2$ (*note:* the index "2" denotes the constrained auxiliary subsystem 2).

constrained auxiliary subsystem 1. Meanwhile, the associated intersample time, denoted by $\Delta t_k^1 = t_{k+1}^1 - t_k^1$, is shown in Fig. 5(b). Likewise, Fig. 6(a) shows $\|e_{2,k}\|$ and $\sqrt{\check{e}_{2,T}}$ for constrained auxiliary subsystem 2. The related intersample time $\Delta t_k^2 = t_{k+1}^2 - t_k^2$ is shown in Fig. 6(b). It is observed from Figs. 5(b) and 6(b) that $\min\{\Delta t_k^1, \Delta t_k^2\} = 0.1$ s. According to Remark 5, this guarantees the Zeno behavior not to happen. Inserting previously obtained weight vectors $\hat{\omega}_{c_1}^{\text{converged}}$ and $\hat{\omega}_{c_2}^{\text{converged}}$ into (27), respectively, we derive the approximate optimal event-driven control policies, denoted by $\hat{\mu}_1^*(\bar{x}_{1,k})$ and $\hat{\mu}_2^*(\bar{x}_{2,k})$, for constrained auxiliary subsystems 1 and 2. Then, according to Theorem 1, $\hat{\mu}_1^*(\bar{x}_{1,k})$ and $\hat{\mu}_2^*(\bar{x}_{2,k})$ together constitute the decentralized event-driven control of system (35). With the obtained decentralized event-driven control, the entire state of system (35) is stable (see Fig. 7).

To show the advantage of the present event-driven ACDs in reducing the computational burden, we make a comparison with the time-driven ACDs proposed in [41] (see Tables I and II). According to Table I (or Table II), the total numbers of computation are 100 800 and 26 680 (or 51 000 and 21 111) when we implement the time-driven ACDs developed in [41] and the present event-driven ACDs for constrained auxiliary subsystem 1 (or auxiliary subsystem 2), respectively.

TABLE I

COMPARISON OF THE COMPUTATIONAL LOAD FOR SUBSYSTEM 1 BETWEEN THE PRESENT EVENT-DRIVEN ACDS AND THE TIME-DRIVEN ACDS PROPOSED IN [41]

Method		Time-Driven ACDs in [41]	Event-Driven ACDs
Sampled States for Auxiliary Subsystem 1		1200	290
Number of Additions and Multiplications at Each Sampled State for Control Policy and Auxiliary Control Policy as well as Triggering Condition	Controller Update	43	43
	Auxiliary Controller Update	41	41
	Triggering Condition	0	8
Total Number of Computation		100800	26680

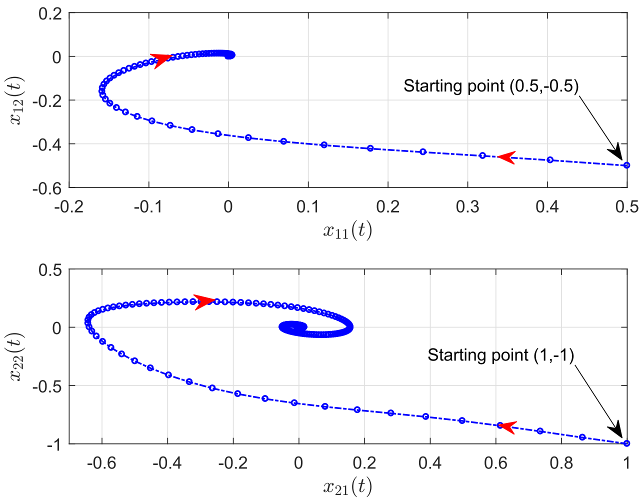

Fig. 7. Overall state vector $x(t) = [x_{11}(t), x_{12}(t), x_{21}(t), x_{22}(t)]^{\mathsf{T}}$.

TABLE II

COMPARISON OF THE COMPUTATIONAL LOAD FOR SUBSYSTEM 2 BETWEEN THE PRESENT EVENT-DRIVEN ACDs AND THE TIME-DRIVEN ACDs PROPOSED IN [41]

Method		Time-Driven ACDs in [41]	Event-Driven ACDs
Sampled States for Auxiliary Subsystem 2		600	227
Number of Additions and Multiplications at Each Sampled State for Control Policy and Auxiliary Control Policy as well as Triggering Condition	Controller Update	44	44
	Auxiliary Controller Update	41	41
	Triggering Condition	0	8
Total Number of Computation		51000	21111

TABLE III

COMPUTATIONAL BURDEN OF ACDs WITH TIME-DRIVEN AUXILIARY CONTROL AND ACDs WITH EVENT-DRIVEN AUXILIARY CONTROL

Approach	Total Number of Computation for Auxiliary Subsystems 1 and 2 (Including Number of Additions and Multiplications for Control, Auxiliary Control, and Triggering Condition at Each Sampled State)
ACDs With Event-Driven Control and *Time-Driven* Auxiliary Control	100394
ACDs With Event-Driven Control and *Event-Driven* Auxiliary Control	47791

This indicates that we only need to make 47791 (i.e., 26 680 + 21 111) computations for implementing the present event-driven ACDs for auxiliary subsystems 1 and 2 together. However, there are 151 800 (i.e., 100 800 + 51 000) computations made to implement the time-driven ACDs proposed in [41] for auxiliary subsystems 1 and 2 together. Thus, the event-driven ACDs make the computational load cut up to 68.52% (i.e., (151 800 − 47 791)/151 800). In addition, to show that updating the control policies and the auxiliary policies simultaneously in the event-driven mechanism for auxiliary subsystems 1 and 2 leads to less computational burden than only tuning the control policies in the event-driven mechanism, we provide comparison results in Table III. According to Table III, the computational burden cuts up to 52.40% (i.e., (100 394 − 47 791)/100 394) when we utilize the ACDs together with the event-driven control and the event-driven auxiliary control.

V. CONCLUSION AND FUTURE WORK

This article has presented a novel ACD-based decentralized event-driven control strategy for mismatched interconnected systems having asymmetric input constraints. The newly proposed decentralized event-driven control scheme not only removes the restrictive inequalities imposed on its implementation but also remarkably lowers down the computational load. It is worth noting here that there probably exists a tradeoff between the control performance and the restrictive conditions (see Remark 5). In addition, due to the restrictive conditions imposed on interconnected system (1) (such as $g_i(0) = 0$ in Assumption 2) and the requirement of accurate mathematical models, it is challengeable to make the present decentralized event-driven control strategy applicable for general practical engineering systems. Thus, how to relax the restrictive conditions (such as Assumptions 1 and 2) at no cost of weakening the control performance and make the present decentralized event-driven control scheme suitable for general practical engineering systems is a direction of our future studies.

Though the decentralized event-driven constrained control strategy is developed to deal with regulation problems, it can be extended to handle the decentralized tracking control problem of interconnected systems if the tracking error is defined similar to [16]. It is worth emphasizing that system (1) suffers from the *mismatched* interconnections rather than the *matched* interconnections in [16]. This feature will give rise to some differences between this article and [16] when we consider the decentralized tracking control problem of interconnected system (1). Due to the space limit, we cannot look deep into this issue in this article. Recently, an adaptive actor-critic tracking control scheme has been proposed for nonlinear systems with the quantized input [42]. In comparison with asymmetric input constraints (see $\mathfrak{U}_i$ defined in Section II), the quantized input imposes more restrictions on the systems' input. Therefore, how to make an extension of the present event-driven control method to cope with the decentralized tracking control problem of interconnected systems with quantized input constraints is also one direction of our future works.

More recently, adaptive NN-based control methods together with backstepping techniques have been suggested to derive decentralized control laws for a class of uncertain strict-feedback (or nonstrict-feedback switched) interconnected systems (see [43], [44]). Then, how to combine these NN-based control approaches and the present event-driven control strategy to tackle decentralized event-driven constrained control problems of strict-feedback (or nonstrict-feedback switched) interconnected systems is another direction of our future research.

APPENDIX I
PROOF OF THEOREM 1

Let the Lyapunov function candidate be

$$\mathcal{L}(x) = \sum_{i=1}^{N} V_i^*(x_i) \tag{37}$$

where $V_i^*(x_i)$, $i = 1, 2, \ldots, N$, are defined in (11). The definition of $V_i^*(x_i)$ in (11) implies that $V_i^*(x_i) > 0$ for $x_i \neq 0$, and $V_i^*(0) = 0$, $i = 1, 2, \ldots, N$. Thus, $V_i^*(x_i)$, $i = 1, 2, \ldots, N$, are positive definite functions [45]. We thus have the conclusion that the function $\mathcal{L}(x)$ is positive definite.

Using N trajectories generated from $\dot{x}_i = f_i(x_i) + g_i(x_i)\mu_i^*(\bar{x}_{i,k}) + \Delta f_i(x)$, $i = 1, 2, \ldots, N$, and Assumption 1, we deduce that the time derivative of $\mathcal{L}(x)$ in (37) yields [note: $\Delta f_i(x) = h_i(x_i)d_i(x)$]

$$\begin{aligned}\dot{\mathcal{L}}(x) =&\ \mathrm{d}\mathcal{L}(x)/\mathrm{d}t = \sum_{i=1}^{N}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\dot{x}_i\\ =& \sum_{i=1}^{N}\Big\{\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i)+g_i(x_i)\mu_i^*(\bar{x}_{i,k})\big)\\ &+\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}h_i(x_i)d_i(x)\Big\}\\ =& \sum_{i=1}^{N}\Big\{\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i)+g_i(x_i)u_i^*(x_i)\big)\\ &+\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}g_i(x_i)\big(\mu_i^*(\bar{x}_{i,k})-u_i^*(x_i)\big)\\ &+\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}h_i(x_i)d_i(x)\Big\}.\end{aligned} \tag{38}$$

Meanwhile, it follows from (12), (13), and (15) that:

$$\begin{cases}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}\big(f_i(x_i)+g_i(x_i)u_i^*(x_i)\big)\\ \quad = -\rho_i P_i^2(x_i) - Q_i(x_i) - \mathcal{W}_i(u_i^*(x_i))\\ \quad +\alpha_i V_i^*(x_i) + \eta_i\big\|\nu_i^*(x_i)\big\|^2\\ \big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}h_i(x_i) = -2\eta_i\big(\nu_i^*(x_i)\big)^{\mathsf{T}}.\end{cases} \tag{39}$$

Inserting (39) into (38), we have

$$\begin{aligned}\dot{\mathcal{L}}(x) = \sum_{i=1}^{N}\Big\{&\alpha_i V_i^*(x_i) - \rho_i P_i^2(x_i) - Q_i(x_i) - \mathcal{W}_i(u_i^*)\\ &+\underbrace{\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}g_i(x_i)\big(\mu_i^*(\bar{x}_{i,k})-u_i^*(x_i)\big)}_{\Pi_1}\\ &+\eta_i\big\|\nu_i^*(x_i)\big\|^2\underbrace{-2\eta_i\big(\nu_i^*(x_i)\big)^{\mathsf{T}}d_i(x)}_{\Pi_2}\Big\}.\end{aligned} \tag{40}$$

Using Young's inequality $2\bar{a}^{\mathsf{T}}\bar{b} \le \|\bar{a}\|^2 + \|\bar{b}\|^2$ and the first inequality in Remark 4, we deduce from Π_1 in (40) that [note: $\bar{a} = -(1/2)(\nabla V_i^*(x_i))^{\mathsf{T}}g_i(x_i)$ and $\bar{b} = u_i^*(x_i) - \mu_i^*(\bar{x}_{i,k})$]

$$\begin{aligned}\Pi_1 &\le \left\|\frac{1}{2}\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}g_i(x_i)\right\|^2 + \big\|u_i^*(x_i)-\mu_i^*(\bar{x}_{i,k})\big\|^2\\ &\le \frac{1}{4}\left\|\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}g_i(x_i)\right\|^2 + K_{u_i^*}^2\|e_{i,k}\|^2.\end{aligned}$$

Meanwhile, using the Cauchy–Schwarz inequality [28, Th. 10.55] and Assumption 1 [or rather, the inequality (4)], we find that Π_2 in (40) yields

$$\Pi_2 \le 2\eta_i\big\|\nu_i^*(x_i)\big\|\|d_i(x)\| \le 2\eta_i\big\|\nu_i^*(x_i)\big\|\sum_{j=1}^{N}b_{ij}P_j(x_j).$$

Thus, we deduce from (40) that

$$\begin{aligned}\dot{\mathcal{L}}(x) \le& -\sum_{i=1}^{N}\left\{\frac{(3+2\gamma_i)}{4}Q_i(x_i) - \Gamma_i\big(x_i, V_i^*, \nu_i^*(x_i)\big)\right\}\\ &-\sum_{i=1}^{N}\left\{\frac{(1-2\gamma_i)}{4}Q_i(x_i) - K_{u_i^*}^2\big\|e_{i,k}(t)\big\|^2\right\}\\ &-\sum_{i=1}^{N}\mathcal{W}_i(u_i^*) - \sum_{i=1}^{N}\eta_i(1-\eta_i)\big\|\nu_i^*(x_i)\big\|^2\\ &-\sum_{i=1}^{N}\Big\{\rho_i P_i^2(x_i) + \eta_i^2\big\|\nu_i^*(x_i)\big\|^2\\ &-2\eta_i\big\|\nu_i^*(x_i)\big\|\sum_{j=1}^{N}b_{ij}P_j(x_j)\Big\}\end{aligned} \tag{41}$$

where

$$\begin{aligned}\Gamma_i\big(x_i, V_i^*, \nu_i^*(x_i)\big) =&\ \alpha_i V_i^*(x_i) + 2\eta_i\big\|\nu_i^*(x_i)\big\|^2\\ &+(1/4)\left\|\big(\nabla V_i^*(x_i)\big)^{\mathsf{T}}g_i(x_i)\right\|^2.\end{aligned} \tag{42}$$

According to [27], for every $x_i \in \Omega_i$, $V_i^*(x_i)$ is continuously differentiable. We, therefore, obtain that $V_i^*(x_i)$ and $\nabla V_i^*(x_i)$ are bounded over Ω_i. Here, we denote

$$\max\{\|V_i^*(x_i)\|, \|\nabla V_i^*(x_i)\|\} \le b_{V_i^*}$$

where $b_{V_i^*} > 0$ is a constant. Then, recalling the expression $\nu_i^*(x_i)$ given in (15) and using Assumption 2, we have $\Gamma_i(x_i, V_i^*, \nu_i^*(x_i))$ in (42) bounded as

$$\Gamma_i\big(x_i, V_i^*, \nu_i^*(x_i)\big) \le \underbrace{\alpha_i b_{V_i^*} + \frac{1}{2\eta_i}b_{h_i}^2 b_{V_i^*}^2 + \frac{1}{4}b_{g_i}^2 b_{V_i^*}^2}_{\delta_{M_i}}. \tag{43}$$

To simplify notations, we let

$$\begin{cases}\bar{\boldsymbol{\rho}} = \ \mathrm{diag}\{\rho_1, \rho_2, \ldots, \rho_N\}\\ \bar{\mathbf{1}} = \ \mathrm{diag}\{1_1, 1_2, \ldots, 1_N\} \quad (1_i = 1,\ i = 1, 2, \ldots, N)\\ z(x) = \big[-P_1(x_1), -P_2(x_2), \ldots, -P_N(x_N),\\ \eta_1\big\|\nu_1^*(x_1)\big\|, \eta_2\big\|\nu_2^*(x_2)\big\|, \ldots, \eta_N\big\|\nu_N^*(x_N)\big\|\big]^{\mathsf{T}}.\end{cases}$$

Then, combining (21) with (43) and observing the facts that $Q_i(x_i) = x_i^{\mathsf{T}}Q_i x_i \ge \theta_{\min}(Q_i)\|x_i\|^2$ (note: $\theta_{\min}(Q_i)$ denotes the minimum eigenvalue of Q_i) and $0 < \eta_i \le 1$ as well as $-\mathcal{W}_i(u_i^*) \le 0$ (note: the function $\mathcal{W}_i(u_i)$ defined in (9) is semipositive definite), we further develop (41) as

$$\begin{aligned}\dot{\mathcal{L}}(x) \le -\sum_{i=1}^{N}\bigg(&\frac{(3+2\gamma_i)}{4}\theta_{\min}(Q_i)\|x_i\|^2 - \delta_{M_i}\bigg)\\ &-z^{\mathsf{T}}(x)Az(x)\end{aligned} \tag{44}$$

where δ_{M_i} is defined in (43) and

$$A=\begin{bmatrix}\bar{\boldsymbol{\rho}} & B^{\mathsf{T}}\\ B & \bar{\mathbf{1}}\end{bmatrix} \text{ with } B=\begin{bmatrix}b_{11} & b_{12} & \cdots & b_{1N}\\ b_{21} & b_{22} & \cdots & b_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ b_{N1} & b_{N2} & \cdots & b_{NN}\end{bmatrix}. \tag{45}$$

Apparently, A defined in (45) is a symmetric real-valued matrix. According to [46, Th. 2.5.6], the matric A is orthogonally diagonalizable. Thus, we can choose large positive constants ρ_i, $i=1,2,\ldots,N$, to make $A>0$ due to the fact that these parameters lie in the principle diagonal of A. Or rather, there must have constants $\rho_i^*>0$, $i=1,2,\ldots,N$, such that $\rho_i\geq\rho_i^*$, $i=1,2,\ldots,N$, ensure that $-z^{\mathsf{T}}(x)Az(x)<0$ for any $z(x)\neq 0$. Then, (44) leads to

$$\dot{\mathcal{L}}(x)\leq-\sum_{i=1}^{N}\left(\frac{(3+2\gamma_i)}{4}\theta_{\min}(Q_i)\|x_i\|^2-\delta_{M_i}\right).$$

Therefore, we have $\dot{\mathcal{L}}(x)<0$ if letting each ith subsystem state $x_i\notin\mathcal{D}_{x_i}$ with $\mathcal{D}_{x_i}$ being given as

$$\mathcal{D}_{x_i}=\left\{x_i:\|x_i\|\leq 2\sqrt{\frac{\delta_{M_i}}{(3+2\gamma_i)\theta_{\min}(Q_i)}}\right\}.$$

This verifies the UUB stability of interconnected system (1) based on the Lyapunov theorem extension [47]. The proof is completed.

Appendix II
Proof of Theorem 2

Note that system (7) with event-driven control policies (27) and (28) constitutes the ith closed-loop auxiliary system. Hence, the ith closed-loop auxiliary system contains x_i and $\bar{x}_{i,k}$ as well as $\tilde{\omega}_{c_i}$ (note: $\hat{\omega}_{c_i}=\omega_{c_i}-\tilde{\omega}_{c_i}$). Due to this fact, we consider the Lyapunov function candidate having the form

$$\mathcal{L}_1(t)=\underbrace{V_i^*(\bar{x}_{i,k})}_{\mathcal{L}_{11}(t)}+\underbrace{V_i^*(x_i(t))}_{\mathcal{L}_{12}(t)}+\underbrace{(1/2)\tilde{\omega}_{c_i}^{\mathsf{T}}\tilde{\omega}_{c_i}}_{\mathcal{L}_{13}(t)}. \tag{46}$$

Because there are continuous state $x_i(t)$ and discrete state (or sampled state) $\bar{x}_{i,k}$ in $\mathcal{L}_1(t)$, we present the discussion from the two following cases.

Case I: Let $t\in[t_k^i,t_{k+1}^i)$, $k=0,1,2,\ldots$ Then, according to the definition of derivative [28, Ch. 5], we have

$$\dot{\mathcal{L}}_{11}(t)=\mathrm{d}V_i^*(\bar{x}_{i,k})/\mathrm{d}t=0. \tag{47}$$

Taking the derivation of $\mathcal{L}_{12}(t)$ and using the trajectory of system $\dot{x}_i=f_i(x_i)+g_i(x_i)\hat{\mu}_i(\bar{x}_{i,k})+h_i(x_i)\hat{v}_i(\bar{x}_{i,k})$, we get

$$\begin{aligned}\dot{\mathcal{L}}_{12}(t)&=\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}\left(f_i(x_i)+g_i(x_i)\hat{\mu}_i(\bar{x}_{i,k})\right)\\&\quad+\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}h_i(x_i)\hat{v}_i(\bar{x}_{i,k})\\&=\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}\left(f_i(x_i)+g_i(x_i)u_i^*(x_i)\right)\\&\quad+\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}g_i(x_i)\left(\hat{\mu}_i(\bar{x}_{i,k})-u_i^*(x_i)\right)\\&\quad+\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}h_i(x_i)\hat{v}_i(\bar{x}_{i,k}).\end{aligned}$$

Together with (39), this yields

$$\begin{aligned}\dot{\mathcal{L}}_{12}(t)&=-\rho_iP_i^2(x_i)-Q_i(x_i)-\mathcal{W}_i(u_i^*(x_i))+\alpha_iV_i^*(x_i)\\&\quad+\eta_i\left\|\nu_i^*(x_i)\right\|^2-2\eta_i\left(\nu_i^*(x_i)\right)^{\mathsf{T}}\hat{v}_i(\bar{x}_{i,k})\\&\quad+\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}g_i(x_i)\left(\hat{\mu}_i(\bar{x}_{i,k})-u_i^*(x_i)\right).\end{aligned} \tag{48}$$

Using Young's inequality $2\bar{a}^{\mathsf{T}}\bar{b}\leq\|\bar{a}\|^2+\|\bar{b}\|^2$, we find that the last term on the right-hand side of (48) satisfies [note: here, $\bar{a}=(1/2)\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}g_i(x_i)$ and $\bar{b}=\hat{\mu}_i(\bar{x}_{i,k})-u_i^*(x_i)$]

$$\begin{aligned}&\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}g_i(x_i)\left(\hat{\mu}_i(\bar{x}_{i,k})-u_i^*(x_i)\right)\\&\quad\leq(1/4)\left\|\left(\nabla V_i^*(x_i)\right)^{\mathsf{T}}g_i(x_i)\right\|^2+\left\|\hat{\mu}_i(\bar{x}_{i,k})-u_i^*(x_i)\right\|^2.\end{aligned}$$

Then, (48) yields

$$\begin{aligned}\dot{\mathcal{L}}_{12}(t)&\leq-\rho_iP_i^2(x_i)-Q_i(x_i)-\mathcal{W}_i(u_i^*(x_i))\\&\quad-\eta_i\left\|\hat{v}_i(\bar{x}_{i,k})\right\|^2+\eta_i\left\|\hat{v}_i(\bar{x}_{i,k})-\nu_i^*(x_i)\right\|^2\\&\quad-2\eta_i\left\|\nu_i^*(x_i)\right\|^2+\left\|\hat{\mu}_i(\bar{x}_{i,k})-u_i^*(x_i)\right\|^2\\&\quad+\Gamma_i\left(x_i,V_i^*,\nu_i^*(x_i)\right)\end{aligned} \tag{49}$$

with $\Gamma_i(x_i,V_i^*,\nu_i^*(x_i))$ being defined in (42). Apparently, there hold $-\rho_iP_i^2(x_i)\leq 0$ and $-\mathcal{W}_i(u_i^*(x_i))\leq 0$ as well as $-\eta_i\left(\|\hat{v}_i(\bar{x}_{i,k})\|^2+2\|\nu_i^*(x_i)\|^2\right)\leq 0$. Thus, (49) implies

$$\begin{aligned}\dot{\mathcal{L}}_{12}(t)&\leq-Q_i(x_i)+\delta_{M_i}+\underbrace{\left\|\hat{\mu}_i(\bar{x}_{i,k})-u_i^*(x_i)\right\|^2}_{\Lambda_1}\\&\quad+\underbrace{\eta_i\left\|\hat{v}_i(\bar{x}_{i,k})-\nu_i^*(x_i)\right\|^2}_{\Lambda_2}\end{aligned} \tag{50}$$

where δ_{M_i} defined in (43) is the bound of $\Gamma_i(x_i,V_i^*,\nu_i^*(x_i))$.

Applying the inequality $\|\bar{a}+\bar{b}\|^2\leq 2\|\bar{a}\|^2+2\|\bar{b}\|^2$ to Λ_1 in (50) and using (24) and (27) as well as the first inequality given in Remark 4, we have [note: here $\bar{a}=\hat{\mu}_i(\bar{x}_{i,k})-\mu_i^*(\bar{x}_{i,k})$ and $\bar{b}=\mu_i^*(\bar{x}_{i,k})-u_i^*(x_i)$]

$$\begin{aligned}\Lambda_1&=\left\|\left(\hat{\mu}_i(\bar{x}_{i,k})-\mu_i^*(\bar{x}_{i,k})\right)+\left(\mu_i^*(\bar{x}_{i,k})-u_i^*(x_i)\right)\right\|^2\\&\leq 2\left\|\hat{\mu}_i(\bar{x}_{i,k})-\mu_i^*(\bar{x}_{i,k})\right\|^2+2\left\|\mu_i^*(\bar{x}_{i,k})-u_i^*(x_i)\right\|^2\\&\leq 4\left\|F(\Phi_1(\bar{x}_{i,k}))-F(\Phi_2(\bar{x}_{i,k}))\right\|^2+4\left\|\varepsilon_{\mu_i^*}(\bar{x}_{i,k})\right\|^2\\&\quad+2K_{u_i^*}^2\|e_{i,k}\|^2\\&\leq 8\left(\left\|F(\Phi_1(\bar{x}_{i,k}))\right\|^2+\left\|F(\Phi_2(\bar{x}_{i,k}))\right\|^2\right)\\&\quad+2K_{u_i^*}^2\|e_{i,k}\|^2+4b_{\varepsilon_{\mu_i^*}}^2\end{aligned} \tag{51}$$

where $F(\Phi_s(\bar{x}_{i,k}))=\beta_i\tanh(\Phi_s(\bar{x}_{i,k}))$ $(s=1,2)$ with $\Phi_1(\bar{x}_{i,k})$ and $\Phi_2(\bar{x}_{i,k})$ defined in (24) and (27), respectively. Noticing that $\Phi_s(\bar{x}_{i,k})\in\mathbb{R}^{m_i}$, we denote $\Phi_s(\bar{x}_{i,k})=[\Phi_{s1}(\bar{x}_{i,k}),\Phi_{s2}(\bar{x}_{i,k}),\ldots,\Phi_{sm_i}(\bar{x}_{i,k})]^{\mathsf{T}}$ with $\Phi_{\mathrm{sp}}(\bar{x}_{i,k})\in\mathbb{R}$, $p=1,2,\ldots,m_i$. Then, using the fact that $|\tanh(y)|\leq 1$ holds for every $y\in\mathbb{R}$, we obtain (note: $s=1,2$)

$$\left\|F(\Phi_s(\bar{x}_{i,k}))\right\|=\beta_i\left(\sum_{p=1}^{m_i}\tanh^2(\Phi_{\mathrm{sp}}(\bar{x}_{i,k}))\right)^{1/2}\leq\beta_i\sqrt{m_i}.$$

Thus, it follows from (51) that:

$$\Lambda_1\leq 2K_{u_i^*}^2\|e_{i,k}\|^2+16m_i\beta_i^2+4b_{\varepsilon_{\mu_i^*}}^2. \tag{52}$$

Taking a way similar to deriving (51) to calculate Λ_2 and using (25) and (28) as well as the second inequality in Remark 4, we have

$$\begin{aligned}\Lambda_2 \leq\ & 2\left\| \hat{\upsilon}_i(\bar{x}_{i,k}) - \upsilon_i^*(\bar{x}_{i,k})\right\|^2 + 2\left\| \upsilon_i^*(\bar{x}_{i,k}) - \nu_i^*(x_i)\right\|^2 \\ \leq\ & 4\left\| -\frac{1}{2\eta_i} h_i^{\mathsf{T}}(x_i)\nabla\sigma_{c_i}^{\mathsf{T}}(\bar{x}_{i,k})\tilde{\omega}_{c_i}\right\|^2 \\ & +2K_{\upsilon_i^*}^2\|e_{i,k}\|^2 + 4\left\|\varepsilon_{\upsilon_i^*}(\bar{x}_{i,k})\right\|^2 \\ \leq\ & \left(b_{h_i}^2 b_{\sigma_{c_i}}^2/\eta_i^2\right)\left\|\tilde{\omega}_{c_i}\right\|^2 + 2K_{\upsilon_i^*}^2\|e_{i,k}\|^2 + 4b_{\varepsilon_{\upsilon_i^*}}^2. \end{aligned} \tag{53}$$

Combining (52) and (53), we deduce from (50) that

$$\begin{aligned}\dot{\mathcal{L}}_{12}(t) \leq\ & -Q_i(x_i) + 4K_{\max}^2\|e_{i,k}\|^2 \\ & +\left(b_{h_i}^2 b_{\sigma_{c_i}}^2/\eta_i^2\right)\left\|\tilde{\omega}_{c_i}\right\|^2 + 16m_i\beta_i^2 \\ & +4b_{\varepsilon_{\mu_i^*}}^2 + 4b_{\varepsilon_{\upsilon_i^*}}^2 + \delta_{M_i}\end{aligned} \tag{54}$$

where $K_{\max} = \max\{K_{u_i^*}, K_{\nu_i^*}\}$.

The derivative of $\mathcal{L}_{13}(t)$ along the solution of (32) is

$$\dot{\mathcal{L}}_{13}(t) = -\lambda_i\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varphi_i^{\mathsf{T}}\tilde{\omega}_{c_i} + \lambda_i\frac{\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i}{1+\phi_i^{\mathsf{T}}\phi_i}\varepsilon_{H_i}. \tag{55}$$

Applying the inequality $2\bar{a}^{\mathsf{T}}\bar{b} \leq \bar{a}^{\mathsf{T}}\bar{a} + \bar{b}^{\mathsf{T}}\bar{b}$ to the second term on the right-hand side of (55) and noting $1/(1+\phi_i^{\mathsf{T}}\phi_i) \leq 1$, we get [note: here, $\bar{a}^{\mathsf{T}} = (1/2)\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i$ and $\bar{b} = \varepsilon_{H_i}$)]

$$\begin{aligned}\frac{\lambda_i\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varepsilon_{H_i}}{1+\phi_i^{\mathsf{T}}\phi_i} \leq\ & \frac{\lambda_i}{1+\phi_i^{\mathsf{T}}\phi_i}\left(\frac{1}{4}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varphi_i^{\mathsf{T}}\tilde{\omega}_{c_i} + \varepsilon_{H_i}^{\mathsf{T}}\varepsilon_{H_i}\right) \\ \leq\ & \frac{\lambda_i}{4}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varphi_i^{\mathsf{T}}\tilde{\omega}_{c_i} + \lambda_i\varepsilon_{H_i}^{\mathsf{T}}\varepsilon_{H_i}.\end{aligned}$$

Then, (55) yields

$$\begin{aligned}\dot{\mathcal{L}}_{13}(t) \leq\ & -\frac{3\lambda_i}{4}\tilde{\omega}_{c_i}^{\mathsf{T}}\varphi_i\varphi_i^{\mathsf{T}}\tilde{\omega}_{c_i} + \lambda_i\varepsilon_{H_i}^{\mathsf{T}}\varepsilon_{H_i} \\ \leq\ & -\frac{3\lambda_i}{4}\theta_{\min}\left(\varphi_i\varphi_i^{\mathsf{T}}\right)\|\tilde{\omega}_{c_i}\|^2 + \lambda_i b_{\varepsilon_{H_i}}^2.\end{aligned} \tag{56}$$

Combining (47), (54), and (56), we have the derivative of $\mathcal{L}(t)$ in (46) satisfied

$$\begin{aligned}\dot{\mathcal{L}}_1(t) \leq\ & -2\gamma_i Q_i(x_i) - (1-2\gamma_i)Q_i(x_i) \\ & -\left(\frac{3\lambda_i}{4}\theta_{\min}\left(\varphi_i\varphi_i^{\mathsf{T}}\right) - \frac{b_{h_i}^2 b_{\sigma_{c_i}}^2}{\eta_i^2}\right)\left\|\tilde{\omega}_{c_i}\right\|^2 \\ & +4K_{\max}^2\|e_{i,k}\|^2 + \pi_0\end{aligned} \tag{57}$$

with $\theta_{\min}(\varphi_i\varphi_i^{\mathsf{T}})$ being given in (34) and π_0 being defined as

$$\pi_0 = 16m_i\beta_i^2 + 4b_{\varepsilon_{\mu_i^*}}^2 + 4b_{\varepsilon_{\upsilon_i^*}}^2 + \lambda_i b_{\varepsilon_{H_i}}^2 + \delta_{M_i}. \tag{58}$$

Using (21) and noticing

$$Q_i(x_i) = x_i^{\mathsf{T}}Q_ix_i \geq \theta_{\min}(Q_i)\|x_i\|^2$$

we obtain from (57) that

$$\begin{aligned}\dot{\mathcal{L}}_1(t) \leq\ & -2\gamma_i\theta_{\min}(Q_i)\|x_i\|^2 + \pi_0 \\ & -\frac{1}{4}\left(3\lambda_i\theta_{\min}\left(\varphi_i\varphi_i^{\mathsf{T}}\right) - 4\ b_{h_i}^2 b_{\sigma_{c_i}}^2/\eta_i^2\right)\left\|\tilde{\omega}_{c_i}\right\|^2.\end{aligned}$$

Therefore, under the condition (34), $\dot{\mathcal{L}}_1(t) < 0$ holds if either $x_i \notin \mathcal{B}_{x_i}$ or $\tilde{\omega}_{c_i} \notin \mathcal{B}_{\tilde{\omega}_{c_i}}$ with $\mathcal{B}_{x_i}$ and $\mathcal{B}_{\tilde{\omega}_{c_i}}$ being, respectively, defined as

$$\begin{aligned}\mathcal{B}_{x_i} &= \left\{x_i : \|x_i\| \leq \sqrt{\frac{\pi_0}{2\gamma_i\theta_{\min}(Q_i)}}\right\} \\ \mathcal{B}_{\tilde{\omega}_{c_i}} &= \left\{\tilde{\omega}_{c_i} : \|\tilde{\omega}_{c_i}\| \leq \sqrt{\frac{4\pi_0}{3\lambda_i\theta_{\min}\left(\varphi_i\varphi_i^{\mathsf{T}}\right) - 4b_{h_i}^2 b_{\sigma_{c_i}}^2/\eta_i^2}}\right\}\end{aligned}$$

where π_0 is defined in (58). Then, using the Lyapunov theorem extension [47], we obtain the UUB stability of both x_i and $\tilde{\omega}_{c_i}$.

Case II: Let $t = t_{k+1}^i$, $k = 0, 1, 2, \ldots$ Then, we need to consider the Lyapunov function candidate (46) in the form of difference, that is,

$$\Delta\mathcal{L}_1\left(t_{k+1}^i\right) = V_i^*(\bar{x}_{i,k+1}) - V_i^*(\bar{x}_{i,k}) + \Theta_i \tag{59}$$

where

$$\begin{aligned}\Theta_i =\ & V_i^*\left(x_i\left(t_{k+1}^i\right)\right) - V_i^*\left(x_i(t_{k+1}^{i-})\right) \\ & +\frac{1}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}\left(t_{k+1}^i\right)\tilde{\omega}_{c_i}\left(t_{k+1}^i\right) - \frac{1}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}\left(t_{k+1}^{i-}\right)\tilde{\omega}_{c_i}\left(t_{k+1}^{i-}\right)\end{aligned}$$

and $x_i(t_{k+1}^{i-}) = \lim_{\imath\to 0^+} x_i(t_{k+1}^i - \imath)$, $\tilde{\omega}_{c_i}(t_{k+1}^{i-}) = \lim_{\imath\to 0^+}\tilde{\omega}_{c_i}(t_{k+1}^i - \imath)$, with $\imath \in (0, t_{k+1}^i - t_k^i)$.

As proved in Case I, when either $x_i \notin \mathcal{B}_{x_i}$ or $\tilde{\omega}_{c_i} \notin \mathcal{B}_{\tilde{\omega}_{c_i}}$ we have $\dot{\mathcal{L}}_1(t) < 0$ for $t \in [t_k^i, t_{k+1}^i)$. This implies $\mathrm{d}(\mathcal{L}_{12}(t) + \mathcal{L}_{13}(t))/\mathrm{d}t < 0$ for all $t \in [t_k^i, t_{k+1}^i)$ [note: $\mathcal{L}_{12}(t)$ and $\mathcal{L}_{13}(t)$ are defined in (46)]. Hence, $\mathcal{L}_{12}(t) + \mathcal{L}_{13}(t)$ is strictly monotonically decreasing over $[t_k^i, t_{k+1}^i)$. Apparently, $\mathcal{L}_{12}(t) + \mathcal{L}_{13}(t)$ is continuous over $[t_k^i, t_{k+1}^i]$. We thus deduce that $\mathcal{L}_{12}(t) + \mathcal{L}_{13}(t)$ is monotonically decreasing over $[t_k^i, t_{k+1}^i]$. Then, letting $\imath \in (0, t_{k+1}^i - t_k^i)$, we get

$$\mathcal{L}_{12}(t_{k+1}^i - \imath) + \mathcal{L}_{13}(t_{k+1}^i - \imath) \geq \mathcal{L}_{12}(t_{k+1}^i) + \mathcal{L}_{13}(t_{k+1}^i). \tag{60}$$

It follows by taking the right limit (that is, $\imath \to 0^+$) over both sides of (60) that:

$$\begin{aligned}& \mathcal{L}_{12}\left(t_{k+1}^{i-}\right) + \mathcal{L}_{13}\left(t_{k+1}^{i-}\right) \\ &\quad = \lim_{\imath\to 0^+}\mathcal{L}_{12}\left(t_{k+1}^i - \varsigma\right) + \lim_{\imath\to 0^+}\mathcal{L}_{13}\left(t_{k+1}^i - \varsigma\right) \\ &\quad \geq\ \mathcal{L}_{12}\left(t_{k+1}^i\right) + \mathcal{L}_{13}\left(t_{k+1}^i\right).\end{aligned}$$

Together with $\mathcal{L}_{12}(t)$ and $\mathcal{L}_{13}(t)$ defined in (46), this yields

$$\begin{aligned}& V_i^*\left(x_i(t_{k+1}^{i-})\right) + \frac{1}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}\left(t_{k+1}^{i-}\right)\tilde{\omega}_{c_i}\left(t_{k+1}^{i-}\right) \\ &\qquad\qquad \geq V_i^*\left(x_i\left(t_{k+1}^i\right)\right) + \frac{1}{2}\tilde{\omega}_{c_i}^{\mathsf{T}}\left(t_{k+1}^i\right)\tilde{\omega}_{c_i}\left(t_{k+1}^i\right)\end{aligned}$$

that is, $\Theta_i \leq 0$ [note: Θ_i is defined in (59)]. On the other hand, UUB stability of $x_i(t)$ in Case I implies that $V_i^*(\bar{x}_{i,k+1}) \leq V_i^*(\bar{x}_{i,k})$. Thus, we have $\Delta\mathcal{L}_1(t_{k+1}^i) < 0$ if $x_i \notin \mathcal{B}_{x_i}$ or $\tilde{\omega}_{c_i} \notin \mathcal{B}_{\tilde{\omega}_{c_i}}$. Then, the UUB stability of x_i and $\tilde{\omega}_{c_i}$ is guaranteed based on the Lyapunov theorem extension [47]. The proof is completed.

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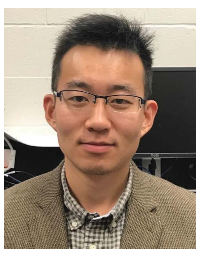

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