Coordination of Networked Nonlinear Multi-Agents Using a High-Order Fully Actuated Predictive Control Strategy

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***Abstract*—This paper is concerned with the coordinative control problem of networked nonlinear multi-agents (NNM) with communication delays. A high-order fully actuated (HOFA) model is introduced to describe the nonlinear multi-agents. Based on this model, a HOFA predictive coordination method is proposed to compensate for the communication delays actively and achieve simultaneous stability and consensus. This method largely simplifies the design of networked nonlinear multi-agents and makes the control performance be same for networked nonlinear multi-agents with and without communication delays. The analysis on the closed-loop systems derives the simultaneous stability and consensus criteria of networked nonlinear multi-agents using the HOFA predictive coordination method. With the presented way of designing HOFA predictive coordination controllers, a simulated example demonstrates the advantages of the proposed method.**

***Index Terms*—High-order fully actuated systems, networked control systems, networked predictive control, nonlinear multi-agents.**

I. Introduction

THE rapid development of network technology accelerates the development of networked multi-agent systems, such as the Internet of things and industrial Internet systems. A networked multi-agent system is a multi-dimensional complex system integrating communication networks and physical environments. Through the integration of computing, communication and control technologies, it can realize the real-time perception, dynamic control and information service of large-scale engineering systems, make the systems more efficient and coordinative, and have important and wide application prospects [1]–[3]. Networked multi-agent systems have widely been used in the fields of energy, manufacturing, aerospace, telemedicine, etc. Due to the introduction of networks, there are inevitably communication constraints, such as delays, loss, disorder and attacks on data, which bring great challenges to the design and analysis of networked multi-agent control systems [1].

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Much research work has been carried out on the influence of communication constraints (particularly, network delays) on the control performance (e.g., consensus) of networked multi-agent systems [4], [5]. The main methods of dealing with the consensus problem of networked multi-agent systems are the time-delay system method, switching system method, Markov jump system method, stochastic system method, event-triggered control method, etc. The time-delay system method transforms the networked multi-agent control system into a system with variable time-delays so that the system can tolerate a maximum time-delay upper bound and maintain a certain expected system performance [6]. The switching system method describes the networked multi-agent system under bounded uncertain data delay and packet loss effects as a discrete-time system with arbitrary switching so that the existing switching control system theory can be applied directly [7]. The Markov jump system method focuses on the Markov chain characteristics of network delays, and constructs the networked multi-agent system as a Markov jump system for consensus analysis [8]. The stochastic system method provides a powerful tool for analysing the system consensus of networked multi-agent systems with random communication constraints [9]. The event-triggered control method can greatly improve the resource utilization of the networked multi-agent system, but the control performance changes little [10]–[12]. Generally speaking, most control methods of networked multi-agent systems use a passive way to suppress communication constraints, which makes the consensus conditions of the system relatively conservative.

For the communication constraints in networked multi-agent systems, the predictive control strategy has incomparable advantages over other control methods. Considering how to compensate for communication constraints actively and taking advantage of the characteristics that a network can transmit data in packets, the networked predictive control method has been proposed [13]. This method breaks through the traditional control mode of point-to-point single data transmission, and adopts the idea of predictive control to compensate actively for the communication constraints in a networked multi-agent system so that its control performance is almost the same as that of the system without networks. Following the networked predictive control strategy, a networked multi-agent predictive control method has been proposed to compensate for network delays of multi-agents with a directed topology and non-uniform agents via a distributed dynamic output feedback protocol [14]. To solve the simultaneous stability and consensus problem of networked multi-agent systems with communication delays and data loss, further design and analysis of the networked multi-agent predictive control systems has provided the necessary and sufficient conditions of achieving both output consensus and input–output stability [15]. Then, a cloud predictive control scheme for networked multi-agent systems has been presented to reduce the expenses for establishment, operation, and maintenance of the systems tremendously based on its computational efficiency and speed via cloud computing [16]. Although a great number of research achievements have been made in networked multi-agent predictive control [17], [18], most of them focus on linear multi-agents. How to deal with nonlinearities of multi-agents needs further research.

Nowadays, most of nonlinear control systems are generally described in the form of a first-order state space model. Based on this model, several nonlinear control methods have been applied to networked nonlinear multi-agent systems, such as the feedback linearization method [19], the back-stepping method [20], the sliding mode method [21], [22], the Lyapunov method [23]. But, there are still various restrictions on those methods. For example, the feedback linearization method needs strong Lie differentiable conditions, the back-stepping method is employed only for a special class of systems with a triangular model form, and the Lyapunov method needs to find appropriate Lyapunov functions which are not unique. Generally, it is hardly to realize the global stabilisation and consensus of networked nonlinear multi-agent control systems even in the case of no communication constraint.

According to the high-order fully actuated (HOFA) system approach [24], most of physical nonlinear systems can be expressed as a HOFA model, which is another system description form and has more universality, simplicity and flexibility for nonlinear system design and analysis. Based on the HOFA model, this paper studies the coordinative control problem of networked nonlinear multi-agents. Two cases are studied: One is the multi-agents without communication delays and the other is the multi-agents with communication delays. For the first case, a HOFA coordination scheme is presented. For the second case, a HOFA predictive coordination method is proposed to compensate for communication delays actively. Both the HOFA coordination scheme and HOFA predictive coordination method achieve simultaneous stability and consensus of networked nonlinear multi-agents.

II. Coordinated Control of Multi-Agents Without Communication Constraints

There are various mathematical models to describe physical control systems, such as the first order state space model and the transfer function model. Following the HOFA system approach [24], the n-th order fully actuated discrete-time model is utilised to represent nonlinear multi-agents as follows:

$$\begin{aligned} y_i(t+1) &= f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \\ &\quad + g_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), u_i(t)) \\ &\quad t \geq 0, \quad \forall i \in \mathbb{N} \end{aligned} \tag{1}$$

where

$$y_i^{[n_i-1]}(t) = \left[y_i^T(t),\ y_i^T(t-1),\ \ldots,\ y_i^T(t-n_i+1)\right]^T \tag{2}$$

$$u_i^{[m_i-1]}(t-1) = \left[u_i^T(t-1),\ u_i^T(t-2),\ \ldots,\ u_i^T(t-m_i)\right]^T \tag{3}$$

the initial values of the nonlinear agents are given by $y_i(t) = \varphi_i(t)$, $u_i(t) = \psi_i(t)$, $t \leq 0$, $y_i(t) \in \mathbb{R}^{p\times 1}$ is the output, $u_i(t) \in \mathbb{R}^{p\times 1}$ the control input, $f_i(\cdot) \in \mathbb{R}^{p\times 1}$ and $g_i(\cdot) \in \mathbb{R}^{p\times 1}$ the nonlinear functions, $\varphi_i(t) \in \mathbb{R}^{p\times 1}$ and $\psi_i(t) \in \mathbb{R}^{p\times 1}$ the initial functions, $\mathbb{N} = \{1, 2, \ldots, N\}$ the agent set, N the total number of agents, and n_i, m_i, p are positive integers. In addition, it is assumed that $f_i(\cdot)$, $g_i(\cdot)$, $\varphi_i(t)$, $\psi_i(t)$, n_i, m_i and p are known, which are needed for the design of predictive coordination controllers.

For the i-th agent, it is assumed that the following map:

$$h_i(t) = g_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), u_i(t)), \quad \forall t \geq 0 \tag{4}$$

forms a differential homeomorphism from $u_i(t)$ to $h_i(t)$ for all $y_i^{[n_i-1]}(t)$, and $u_i^{[m_i-1]}(t-1)$, $t \geq 0$ and there also exists its inverse map:

$$u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), h_i(t)), \quad \forall t \geq 0. \tag{5}$$

In the case where there is no communication constraint in networked nonlinear multi-agents, following the idea of both the HOFA controller [24] and coordinated controller [15], a HOFA coordination scheme is presented below:

$$u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), v_i(t))\ ,\ t > 0,\ \forall i \in \mathbb{N} \tag{6}$$

where

$$\begin{aligned} v_i(t) &= K_{y_i}^{[q_i]}(t) + K_i^I z_i(t) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \\ &\quad + \sum_{j=1, j\neq i}^{N} K_{ij}^C \left(y_i(t) - y_j(t)\right) \end{aligned} \tag{7}$$

$$K_{y_i}^{[q_i]}(t) = \sum_{j=0}^{q_i} K_{ij}^P y_i(t-j) \tag{8}$$

$K_{ij}^P \in \mathbb{R}^{p\times p}$, for $j = 0, 1, \ldots, q_i$, are the proportional parameter matrices, $K_i^I \in \mathbb{R}^{p\times p}$ the integral parameter matrix and $K_{ij}^C \in \mathbb{R}^{p\times p}$, for $j \in \mathbb{N} - \{i\}$ the coordinative parameter matrices of controller (7) to be designed, and $z_i(t) \in \mathbb{R}^{p\times 1}$ is the integration of the error between the output and desired reference input of the i-th agent and is calculated by

$$z_i(t+1) = z_i(t) + y_i(t) - r(t) \tag{9}$$

$r(t) \in \mathbb{R}^{p\times 1}$ is the known reference input vector. Then, applying the HOFA coordination scheme (6) to multi-agents (1) leads to the following outputs of the agents:

$$y_i(t+1) = K_{y_i}^{[q_i]}(t) + K_i^I z_i(t) + \sum_{j=1, j\neq i}^{N} K_{ij}^C \left(y_i(t) - y_j(t)\right),\ \forall i \in \mathbb{N}. \tag{10}$$

Actually, the combination of (9) and (10) forms the closed-loop networked nonlinear multi-agent system without communication constraints using the HOFA coordination scheme.

III. Coordinated Control of Multi-Agents With Communication Delays

In networked multi-agents, there usually exist various communication constraints, for example, delays, data loss, attacks, quantisation, synchronisation, etc. For the sake of simplicity, only the communication delays are considered here. Let the communication delay from the j-th agent to the i-th agent be fixed and denoted by s_{ij}, and

$$\bar{s}_i = \max\left\{s_{ij},\ \forall j \in \mathbb{N} - \{i\}\right\} \tag{11}$$

be the largest communication delay from all other agents to the i-th agent.

To simplify the presentation, it is also assumed that all the agents of networked nonlinear multi-agents are fully connected via communication networks. This will also make the calculations of the output predictions of all the agents much easier. The most effective way of compensating for the communication delays is the predictive control strategy. In the case of the communication delays, following scheme (6), a HOFA predictive coordination scheme of the agents is proposed as follows:

$$u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), v_i(t)), \quad \forall i \in \mathbb{N} \tag{12}$$

where

$$v_i(t) = K_{y_i}^{[q_i]}(t) + K_i^I z_i(t) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) + \sum_{j=1, j\neq i}^{N} K_{ij}^C\left(y_i(t) - \hat{y}_j(t|t-s_{ij})\right) \tag{13}$$

$$z_i(t) = z_i(t-1) + y_i(t-1) - r(t-1) \tag{14}$$

where $\hat{y}_j(t|t-s_{ij})$ represents the prediction of the output $y_j(t)$ for time t using the information up to time $t-s_{ij}$, which can similarly be applied to denote the predictions of other variables, e.g., $\hat{z}_j(t \mid t-s_{ij})$.

For $l = 1, 2, \ldots, N$ $(l \neq j)$ and $k = 0, 1, \ldots, \bar{s}_i$, let

$$\hat{y}_l(t+k-s_{ij}|t-s_{il}) = y_l(t+k-s_{ij}), \text{ if } s_{il} \leq s_{ij} - k \tag{15}$$

because no prediction is needed. In the i-th agent, without loss of generality, it assumes

$$s_{i1} \geq s_{i2} \geq \cdots \geq s_{iN}. \tag{16}$$

As all the agents of networked nonlinear multi-agents are fully connected via communication networks, following (9) and (10) and employing the input-output information available up to time $t - s_{ij}$, the one-step ahead predictions of the output $y_j(t)$ and variable $z_j(t)$ can be calculated by

$$\hat{y}_j(t-s_{ij}+1|t-s_{ij}) = K_{y_j}^{[q_j]}(t-s_{ij}) + K_j^I z_j(t-s_{ij}) + \sum_{l=1, l\neq j}^{N} K_{jl}^C\left(y_j(t-s_{ij}) - \hat{y}_l(t-s_{ij}|t-s_{il})\right) \tag{17}$$

and

$$\hat{z}_j(t-s_{ij}+1 \mid t-s_{ij}) = z_j(t-s_{ij}) + y_j(t-s_{ij}) - r(t-s_{ij}) \tag{18}$$

where

$$\hat{y}_l(t-s_{ij}|t-s_{il}) = y_l(t-s_{ij}), \quad \text{if } s_{il} \leq s_{ij}. \tag{19}$$

Based on the above predictions of the output $y_j(t)$ and variable $z_j(t)$, the similar strategy is applied to the calculation of their k-step ahead predictions, which results in

$$\begin{aligned}\hat{y}_j(t-s_{ij}+k|t-s_{ij}) = {} & K_{\hat{y}_j}^{[q_j]}(t-s_{ij}+k-1|t-s_{ij}) \\ & + K_j^I \hat{z}_j(t-s_{ij}+k-1 \mid t-s_{ij}) \\ & + \sum_{l=1, l\neq j}^{N} K_{jl}^C\big(\hat{y}_j(t-s_{ij}+k-1|t-s_{ij}) \\ & - \hat{y}_l(t-s_{ij}+k-1|t-s_{il})\big)\end{aligned} \tag{20}$$

for $k = 2, 3, \ldots, \bar{s}_i$, and

$$\begin{aligned}\hat{z}_j(t-s_{ij}+k \mid t-s_{ij}) = {} & \hat{z}_j(t-s_{ij}+k-1 \mid t-s_{ij}) \\ & + \hat{y}_j(t-s_{ij}+k-1 \mid t-s_{ij}) - r(t-s_{ij}+k-1)\end{aligned} \tag{21}$$

for $k = 2, 3, \ldots, \bar{s}_i$, where

$$K_{\hat{y}_j}^{[q_j]}(t+k-s_{ij}|t-s_{ij}) = \sum_{l=0}^{q_j} K_{jl}^P \hat{y}_j(t+k-l-s_{ij}|t-s_{ij}) \tag{22}$$

$$\hat{y}_j(t+k-l-s_{ij}|t-s_{ij}) = y_j(t+k-l-s_{ij}), \quad \text{if } k \leq l \tag{23}$$

for $k = 2, 3, \ldots, \bar{s}_i - 1$.

Employing the input-output data available up to time $t-s_{ij}$ and formulae (20) and (21), the predictions of the output $y_j(t)$ and variable $z_j(t)$ of all the agents are calculated recursively. In the i-th agent, the output predictions of an agent with a greater communication delay must be calculated before the ones of an agent with a less communication delay for the same time so that the following predictions:

$$\hat{y}_l(t-s_{ij}+k-1|t-s_{il}), \quad \text{for } l = 1, 2, \ldots, N, \ \ l \neq j$$

are available before (20) and (21) are carried out. In this way, the output prediction $\hat{y}_j(t|t-s_{ij})$ is finally obtained in the following form:

$$\hat{y}_j(t|t-s_{ij}) = K_{\hat{y}_j}^{[q_j]}(t-1|t-s_{ij}) + K_j^I \hat{z}_j(t-1 \mid t-s_{ij}) + \sum_{l=1, l\neq j}^{N} K_{jl}^C\left(\hat{y}_j(t-1|t-s_{ij}) - \hat{y}_l(t-1|t-s_{il})\right) \tag{24}$$

which is needed in (13). The stability and consensus of the closed-loop networked nonlinear multi-agent system using the HOFA predictive coordination method proposed in this section will be analysed in the next section.

IV. Simultaneous Stability and Consensus Analysis of Closed-Loop Multi-Agent Control Systems

A networked multi-agent control system can achieve consensus, but it does not imply that the stability of the system is guaranteed, which is usually ignored by most researchers. For practical applications, both the consensus and stability of a networked multi-agent control system should simultaneously be analysed. Following Definition 1 in [15], a definition is introduced below.

Definition 1: Networked multi-agent control system (1) with controller (12) achieves input-output stability and output consensus simultaneously if

1) $\lim_{t\to\infty}\|y_i(t)\| < \infty$, if $\|r(t)\| < \infty$, for $t \geq 0$

2) $\lim_{t\to\infty}\left\|y_i(t)-y_j(t)\right\| = 0$

for $\forall i, j \in \mathbb{N}$.

Usually, there are several communication constraints in networked multi-agents. In the case of communication delays, the closed-loop system via the HOFA predictive coordination method is described by (1) and (12)–(14), i.e.,

$$\begin{cases} y_i(t+1) = f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \\ \qquad + g_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), u_i(t)) & (25) \\ u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), v_i(t)) & (26) \\ v_i(t) = K_{y_i}^{[q_i]}(t) + K_i^I z_i(t) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \\ \qquad + \sum_{j=1, j\neq i}^{N} K_{ij}^C\left(y_i(t) - \hat{y}_j(t|t-s_{ij})\right) & (27) \end{cases}$$

for $\forall i \in \mathbb{N}$.

Application of controller (26) and (27) to agent (25) makes the closed-loop system be

$$y_i(t+1) = K_{y_i}^{[q_i]}(t) + K_i^I z_i(t) + \sum_{j=1, j\neq i}^{N} K_{ij}^C\left(y_i(t) - \hat{y}_j(t|t-s_{ij})\right)$$
$$i \in \mathbb{N}. \tag{28}$$

To simplify the presentation, in the i-th agent, it assumes that

$$s_{ij} \geq s_{il}, \text{ for } l = 1, 2, \ldots, N \ (l \neq j) \tag{29}$$

which can be done by resorting the order of other agents connected to the i-th agents. In terms of (19) used in the HOFA predictive coordination method presented in Section III, for $k = 0$, there exist

$$\hat{y}_l(t-s_{ij}|t-s_{il}) = y_l(t-s_{ij}), \text{ for } l = 1, 2, \ldots, N \ (l \neq j). \tag{30}$$

So, (17) and (18) become

$$\begin{aligned} \hat{y}_j(t-s_{ij}+1|t-s_{ij}) &= K_{y_j}^{[q_j]}(t-s_{ij}) + K_j^I z_j(t-s_{ij}) \\ &\quad + \sum_{l=1, l\neq j}^{N} K_{jl}^C\left(y_j(t-s_{ij}) - y_l(t-s_{ij})\right) \\ &= y_j(t-s_{ij}+1) \end{aligned} \tag{31}$$

and

$$\begin{aligned} \hat{z}_j(t-s_{ij}+1 \mid t-s_{ij}) &= z_j(t-s_{ij}) + y_j(t-s_{ij}) - r(t-s_{ij}) \\ &= z_j(t-s_{ij}+1) \end{aligned} \tag{32}$$

which are induced from (9) and (10), respectively. Similarly, the following results for (20) and (21) can recursively be derived:

$$\begin{aligned} \hat{y}_j(t-s_{ij}+k|t-s_{ij}) &= K_{y_j}^{[q_j]}(t-s_{ij}+k-1) \\ &\quad + K_j^I z_j(t-s_{ij}+k-1) \\ &\quad + \sum_{l=1, l\neq j}^{N} K_{jl}^C\Big(y_j(t-s_{ij}+k-1) \\ &\quad - y_l(t-s_{ij}+k-1)\Big) \\ &= y_j(t-s_{ij}+k), \ k = 2, 3, \ldots, \bar{s}_i \end{aligned} \tag{33}$$

and

$$\begin{aligned} \hat{z}_j(t-s_{ij}+k \mid t-s_{ij}) &= z_j(t-s_{ij}+k-1) \\ &\quad + y_j(t-s_{ij}+k-1) - r(t-s_{ij}+k-1) \\ &= z_j(t-s_{ij}+k), \quad k = 2, 3, \ldots, \bar{s}_i. \end{aligned} \tag{34}$$

Finally, for $k = s_{ij}$, the above yields

$$\hat{y}_j(t \mid t-s_{ij}) = y_j(t) \tag{35}$$

and

$$\hat{z}_j(t \mid t-s_{ij}) = z_j(t) = z_j(t-1) + y_j(t-1) - r(t-1) \tag{36}$$

which is equivalent to (9) and also implies

$$\Delta z_j(t) = y_j(t-1) - r(t-1). \tag{37}$$

Then, the closed-loop system (28) becomes

$$y_i(t+1) = K_{y_i}^{[q_i]}(t) + K_i^I z_i(t) + \sum_{j=1, j\neq i}^{N} K_{ij}^C\left(y_i(t) - y_j(t)\right)$$
$$i \in \mathbb{N} \tag{38}$$

which is the same as (10) of the networked nonlinear multi-agents without communication delays. So, the closed-loop systems for the two cases (one is with communication delays and the other is without communication delays) are exactly the same when the HOFA coordination scheme or HOFA predictive coordination method is employed.

Equation (38) can compactly be expressed as

$$Y_i(t+1) = K_i Y_i(t) + \sum_{j=1, j\neq i}^{N} K_{ij} Y_j(t) + \bar{K}_i^I z_i(t), \quad i \in \mathbb{N} \tag{39}$$

where

$$Y_i(t) = \begin{bmatrix} y_i^T(t) & y_i^T(t-1) & \cdots & y_i^T(t-q_i) \end{bmatrix}^T$$

$$\bar{K}_i^I = \begin{bmatrix} (K_i^I)^T & 0 & \cdots & 0 \end{bmatrix}^T$$

$$K_i = \begin{bmatrix} K_{i0}^P + \sum_{j=1, j\neq i}^{N} K_{ij}^c & K_{i1}^P & \cdots & K_{iq_i}^P \\ I & 0 & \cdots & 0 \\ & \ddots & & \vdots \\ & & I & 0 \end{bmatrix}$$

$$K_{ij} = \operatorname{diag}\left\{ -K_{ij}^C \quad 0 \quad \cdots \quad 0 \right\}.$$

Hence, the further compacting on (39) gives the description below:

$$Y(t+1) = K_P Y(t) + K_I z(t) \tag{40}$$

where

$$Y(t) = \begin{bmatrix} Y_1^T(t) & Y_2^T(t) & \ldots & Y_N^T(t) \end{bmatrix}^T$$

$$z(t) = \begin{bmatrix} z_1^T(t) & z_2^T(t) & \ldots & z_N^T(t) \end{bmatrix}^T$$

$$K_P = \begin{bmatrix} K_1 & K_{12} & \ldots & K_{1N} \\ K_{21} & K_2 & \ldots & K_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ K_{N1} & K_{N2} & \ldots & K_N \end{bmatrix}$$

$$K_I = \mathrm{diag}\left\{ \bar{K}_1^I \quad \bar{K}_2^I \quad \ldots \quad \bar{K}_N^I \right\}.$$

Let the reference input be constant, i.e., $r(t) = r_0$. Taking the Δ operation on both (40) and (36) leads to

$$\Delta Y(t+1) = K_P \Delta Y(t) + K_I \Delta z(t) \tag{41}$$

and

$$\Delta z(t+1) = \Delta z(t) + D\Delta Y(t) \tag{42}$$

where

$$D = \mathrm{diag}\{ \ [I, 0, \ldots, 0] \quad [I, 0, \ldots, 0] \quad \cdots \quad [I, 0, \ldots, 0] \ \}.$$

As a result, the closed-loop equation of the networked nonlinear multi-agents utilising the HOFA predictive coordination method is finally expressed as

$$X(t+1) = HX(t) \tag{43}$$

where

$$X(t) = \begin{bmatrix} \Delta Y(t) \\ \Delta z(t) \end{bmatrix}, \quad H = \begin{bmatrix} K_p & K_I \\ D & I \end{bmatrix}.$$

If matrix H is Schur stable, the closed-loop system is stable and also implies

$$\Delta z(t) = 0 \quad \text{as} \quad t \to \infty.$$

For $r(t) = r_0$, it can be seen from (37) that the outputs of the agents are

$$y_i(t) - y_j(t) = r_0 - r_0 = 0, \quad \forall i \in \mathbb{N}, \ \text{as } t \to \infty.$$

Clearly, all the networked nonlinear agents achieve consensus. According to Definition 1, it can be concluded that the networked nonlinear multi-agents are of simultaneous stability and consensus using the HOFA predictive coordination method. Therefore, summarising the above gives the following theorem.

Theorem 1: Networked nonlinear multi-agent (1) with the HOFA coordination controller (6)–(8) or HOFA predictive coordination controller (12)–(14) achieves simultaneous stability and consensus if and only if matrix H in (43) is Schur stable.

Remark 1: The key advantage of the HOFA system approach is to remove the nonlinearities of a nonlinear system and transform it to a desired linear system through the controller design. When this approach is applied to design the controller of a networked nonlinear multi-agent system, Theorem 1 shows that the stability and consensus conditions of the closed-loop system are related only to both its transformed linear system and the linear part parameters of the controller.

V. Design of the Agent Controller Parameters

There are many ways to design the parameters of the HOFA predictive coordination controllers of networked nonlinear multi-agents. This section presents two steps to determine those parameters.

As a special case where there is no coordination between agents, let $K_{ij}^C = 0$, for $i, j \in \mathbb{N}$ and $r(t) = 0$. It can be obtained from (38) and (14) that the closed-loop individual agents are expressed by

$$\begin{cases} y_i(t+1) = K_{y_i}^{[q_i]}(t) + K_i^I z_i(t) \\ z_i(t) = z_i(t-1) + y_i(t-1), \qquad i \in \mathbb{N}. \end{cases} \tag{44}$$

Taking the Δ operation on the first sub-equation and rearranging the second sub-equation in (44) result in

$$\begin{cases} \Delta y_i(t+1) = \Delta K_{y_i}^{[q_i]}(t) + K_i^I \Delta z_i(t) \\ \Delta z_i(t) = y_i(t-1), \qquad i \in \mathbb{N} \end{cases} \tag{45}$$

which leads to

$$\Delta y_i(t+1) = \Delta K_{y_i}^{[q_i]}(t) + K_i^I y_i(t-1), \quad i \in \mathbb{N}. \tag{46}$$

Substituting (8) into (46) and replacing t by $t-1$ yield

$$\Delta y_i(t) - \sum_{j=0}^{q_i} K_{ij}^P \Delta y_i(t-1-j) + K_i^I y_i(t-2) = 0, \quad i \in \mathbb{N} \tag{47}$$

which implies that the characteristic equation of each agent in the z-domain is

$$1 - z^{-1} - \sum_{j=0}^{q_i} K_{ij}^P (1 - z^{-1}) z^{-1-j} + K_i^I z^{-2} = 0, \quad i \in \mathbb{N}. \tag{48}$$

Actually, the above is equivalent to

$$1 - z^{-1} + K_i^I z^{-2} - \sum_{j=1}^{q_i+1} K_{ij-1}^P z^{-j} + \sum_{j=2}^{q_i+2} K_{ij-2}^P z^{-j} = 0, \quad i \in \mathbb{N}. \tag{49}$$

Thus, the design of the HOFA predictive coordination controllers can be carried out in two steps as follows:

Step 1: Design of the proportional integral (PI) parameters

Let the coordinative parameters $K_{ij}^C = 0$, for $i, j \in \mathbb{N}$. This implies there is no coordination between the agents. Each agent controller (13) is actually a PI controller. Then, the controllers of all individual agents can be designed independently. Utilising a control method, e.g., an eigenstructure assignment method, the PI parameters K_{ij}^P and K_i^I, for $j = 0, 1, \ldots, q_i$, $\forall i \in \mathbb{N}$, are determined so that the roots of the characteristic equation (49) are within the unit circle and the desired control performance is satisfied.

Step 2: Design of the coordinative parameters

Based on the PI parameters given in Step 1, the coordinative parameters K_{ij}^C, for $i, j \in \mathbb{N}$, $i \neq j$ are designed to ensure the eigenvalues of matrix H in (43) are within the unit circle. Thus, according to Theorem 1, the closed-loop networked nonlinear multi-agent system achieves simultaneous stability and consensus.

Clearly, the first step guarantees that the individual agents are stable and have the desired control performance when there is no coordination between the agents. The second step ensures that all the networked multi-agents coordinate with simultaneous stability and consensus.

VI. An Example

To illustrate the performance of the HOFA predictive coordination method for networked nonlinear multi-agents with communication delays proposed in this paper, an example is provided in this section. Three different order fully-actuated discrete nonlinear agents are considered as follows:

$$\begin{cases} y_1(t+1) = \dfrac{0.35y_1(t)-0.17y_1(t)y_1(t-1)-0.2y_1(t-1)}{1+0.2u_1^2(t-1)} \\ \qquad\qquad - \dfrac{3u_1(t)+0.1u_1(t-1)}{1+0.15y_1^2(t-2)} \\ y_2(t+1) = \dfrac{0.5y_2(t)y_2(t-1)}{1+y_2^2(t-1)} + 1.5u_2(t)+0.3u_2(t-1) \\ y_3(t+1) = -0.9y_3(t)y_3(t-2) + \dfrac{1.8u_3(t)+0.6u_3(t-1)}{1+0.1y_3^2(t-1)} \end{cases}$$

where all the variables of the three agents have zero initial conditions. From the HOFA networked multi-agent (1), the nonlinear functions $f_i(\cdot)$ and $g_i(\cdot)$ for i = 1, 2, 3 can simply be constructed as

$$f_1(y_1^{[1]}(t), u_1^{[0]}(t-1)) = \frac{0.35y_1(t)-0.17y_1(t)y_1(t-1)-0.2y_1(t-1)}{1+0.2u_1^2(t-1)}$$

$$f_2(y_2^{[1]}(t)) = \frac{0.5y_2(t)y_2(t-1)}{1+y_2^2(t-1)}$$

$$f_3(y_3^{[2]}(t)) = -0.9y_3(t)y_3(t-2)$$

and

$$g_1(y_1^{[2]}(t), u_1^{[0]}(t-1), u_1(t)) = -\frac{3u_1(t)+0.1u_1(t-1)}{1+0.15y_1^2(t-2)}$$

$$g_2(u_2^{[0]}(t-1), u_2(t)) = 1.5u_2(t)+0.3u_2(t-1)$$

$$g_3(y_3^{[1]}(t), u_3^{[0]}(t-1), u_3(t)) = \frac{1.8u_3(t)+0.6u_3(t-1)}{1+0.1y_3^2(t-1)}.$$

The inverse functions $g_i^{-1}(\cdot)$, for $i = 1,2,3$ are

$$g_1^{-1}(y_1^{[2]}(t), u_1^{[0]}(t-1), v_1(t)) = -\frac{(1+0.15y_1^2(t-2))v_1(t)+0.1u_1(t-1)}{3}$$

$$g_2^{-1}(u_2^{[0]}(t-1), v_2(t)) = \frac{v_2(t)-0.3u_2(t-1)}{1.5}$$

$$g_3^{-1}(y_3^{[1]}(t), u_3^{[0]}(t-1), v_3(t)) = \frac{(1+0.1y_3^2(t-1))v_3(t)-0.6u_3(t-1)}{1.8}.$$

The communication graph of the networked three-agent system is assumed to be fully connected, as shown in Fig.1.

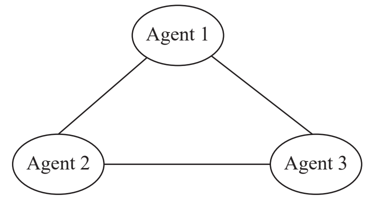

Fig. 1. The communication graph.

Following the two steps of designing the PI and coordinative parameters introduced in Section V, firstly, let the coordinative parameters be zeros and choose $q_1 = q_2 = q_3 = 0$ in (8) to make the characteristic equation (49) of the individual agents become

$$1-(1+K_{i0}^P)z^{-1}-(K_i^I-K_{i0}^P)z^{-2} = 0,\ \ i = 1,2,3.$$

To assign the closed-loop poles of the three agents at 0.91±0.21j (Agent 1), 0.92±0.22j (Agent 2) and 0.93±0.23j (Agent 3), which provide good transient dynamical performance, using the pole assignment method leads to the following PI parameters:

$$K_{10}^P = 0.82,\quad K_{20}^P = 0.84,\quad K_{30}^P = 0.86$$
$$K_1^I = -0.0522,\quad K_2^I = -0.0548,\quad K_3^I = -0.0578.$$

Secondly, when the following coordinative parameters $K_{ij}^C = -0.56$, for $i, j = 1,2,3,\ i \neq j$ (by the trial and error method) are applied, the eigenvalues of matrix H in (43) are

$$-0.8224,\ -0.7971,\ 0.9708,\ 0.9686,\ 0.9201 \pm 0.2204j$$

which are within the unit circle. So, according to Theorem 1, the closed-loop networked three-agent system is stable and all the three agents also achieve the output consensus.

Let the reference input $r(t)$ be a given square wave with the period of 300 steps and amplitude between 1 and –1. Four cases are illustrated here to compare the performance of the different control strategies: no coordination, coordination without communication delays, coordination without compensating for communication delays, and coordination with compensating for communication delays.

Case 1: No coordination

In the case of no coordination between the three agents, all the agents are independent. No communication delays need to be considered between the agents. From (12) and (13), when the coordinative parameters $K_{ij}^C = 0$, for $i, j = 1,2,3,\ \ i \neq j$, the controllers of the three agents are

$$\begin{cases} u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), v_i(t)) \\ v_i(t) = K_{i0}^P y_i(t) + K_i^I z_i(t) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \end{cases}$$

for i = 1, 2, 3. The output responses and control inputs of the three agents are shown in Figs. 2 and 3. They clearly show that the difference between the outputs of the three agents are large without coordination between the three agents.

Further, if the integral error term is not considered and is replaced by a proportional error term, the controllers of the three agents are changed to the following:

$$\begin{cases} u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1), v_i(t)) \\ v_i(t) = K_{i0}^P y_i(t) + K_i^I (y_i(t)-r(t)) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \end{cases}$$

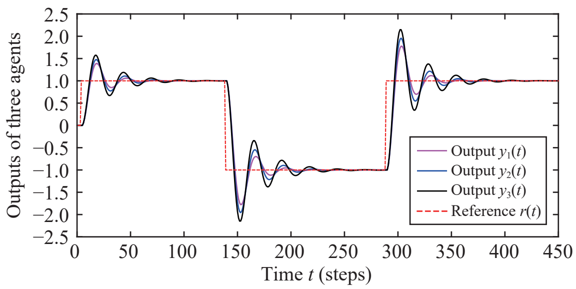

Fig. 2. The output responses of the three agents (Case 1).

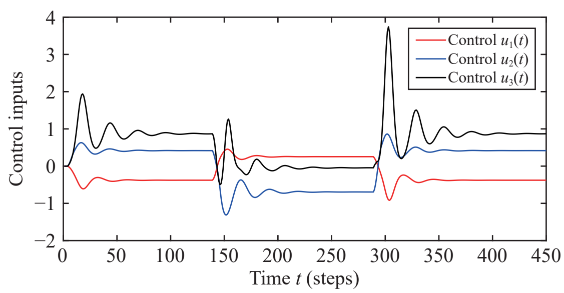

Fig. 3. The control inputs of the three agents (Case 1).

for i = 1, 2, 3. The output responses of the three agents shown in Fig. 4 illustrate that there exists not only the large steady-state error between the reference and the output of each agent but also the significant difference between three agent outputs.

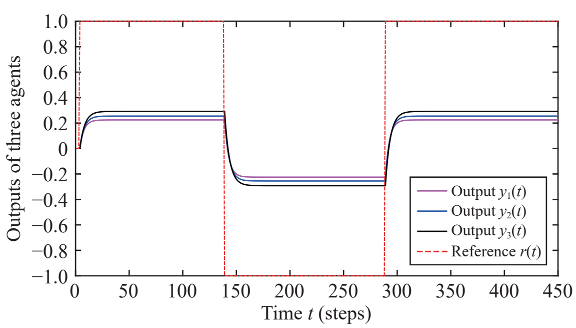

Fig. 4. The output responses of the three agents without the integrator (Case 1).

Case 2: Coordination without delays

This case assumes that there is no communication delay between networked three agents, i.e., the communication delays $s_{ij} = 0$, for i, j = 1, 2, 3, $i \neq j$. Then, from (12) and (13), the controllers of the three agents are

$$\begin{cases} u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t),\, u_i^{[m_i-1]}(t-1), v_i(t)) \\ v_i(t) = K_{i0}^P y_i(t) + K_i^I z_i(t) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \\ \qquad + \displaystyle\sum_{j=1, j\neq i}^{3} K_{ij}^C \big(y_i(t) - y_j(t)\big) \end{cases}$$

for i = 1, 2, 3. Using those controllers, the output responses and control inputs of the three agents are shown in Figs. 5 and 6. The simulation results demonstrate that all the outputs of the three agents are almost the same. So, the coordination of the three agents is achieved.

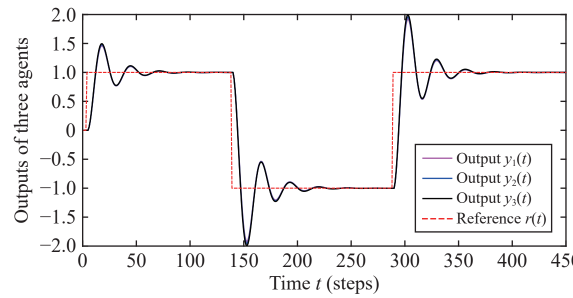

Fig. 5. The output responses of the three agents (Case 2).

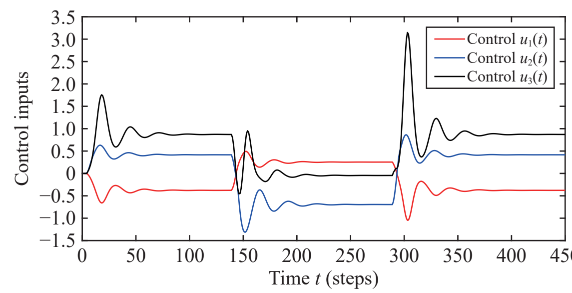

Fig. 6. The control inputs of the three agents (Case 2).

Case 3: Coordination without compensating for delays

There usually exist communication delays between networked multi-agents. Here, it is assumed that the communication delays between the three agents are below:

$$s_{12} = 2,\ s_{13} = 1,\ s_{21} = 3,\ s_{23} = 2,\ s_{31} = 1,\ s_{32} = 3.$$

If the compensation for the communication delays are not taken into account, following the conventional control strategy, the controllers of the networked three agents are

$$\begin{cases} u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t),\, u_i^{[m_i-1]}(t-1), v_i(t)) \\ v_i(t) = K_{i0}^P y_i(t) + K_i^I z_i(t) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \\ \qquad + \displaystyle\sum_{j=1, j\neq i}^{3} K_{ij}^C \big(y_i(t) - y_j(t - s_{ij})\big) \end{cases}$$

for i = 1, 2, 3. For this case, the output responses and control inputs of the networked three agents are shown in Figs. 7 and 8. The results indicate that all the three agents without compensating for the communication delays between the agents are unstable.

Case 4: Coordination with compensating for delays

The active compensation strategy for delays is applied when there exist communication delays between networked three agents. The proposed HOFA predictive coordination controllers (12) and (13) of the agents for this case are

$$\begin{cases} u_i(t) = g_i^{-1}(y_i^{[n_i-1]}(t),\, u_i^{[m_i-1]}(t-1), v_i(t)) \\ v_i(t) = K_{i0}^P y_i(t) + K_i^I z_i(t) - f_i(y_i^{[n_i-1]}(t), u_i^{[m_i-1]}(t-1)) \\ \qquad + \displaystyle\sum_{j=1, j\neq i}^{3} K_{ij}^C \big(y_i(t) - \hat{y}_j(t|t - s_{ij})\big) \end{cases}$$

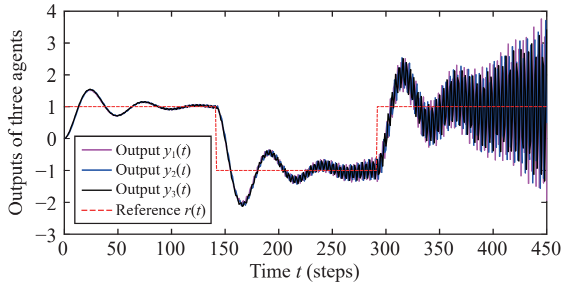

Fig. 7. The output responses of the three agents (Case 3).

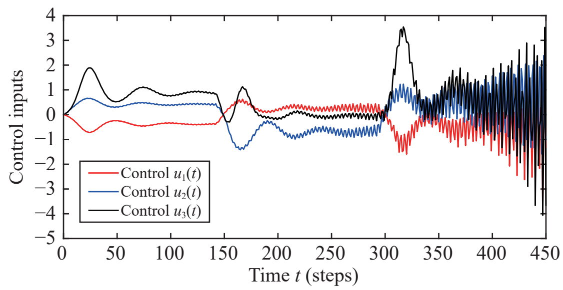

Fig. 8. The control inputs of the three agents (Case 3).

for i = 1, 2, 3. Employing the above controllers, the output responses and control inputs of the three agents with communication delays given in Case 3 are shown in Figs. 9 and 10. The simulation results demonstrate that all the three agents achieve simultaneous stability and consensus, and the control performance is exactly the same as the one of Case 2. It also shows that the communication delays between the three agents are completely compensated by the HOFA predictive coordination method presented in Section III.

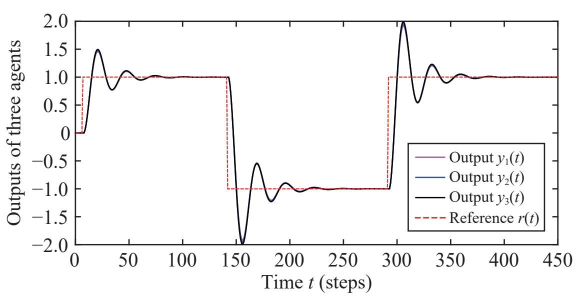

Fig. 9. The output responses of the three agents (Case 4).

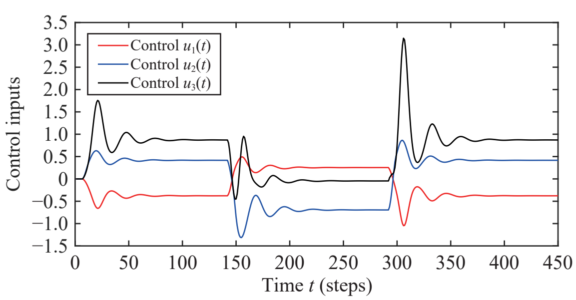

Fig. 10. The control inputs of the three agents (Case 4).

VII. Conclusions

This paper has addressed the coordinative control problem of a class of networked nonlinear multi-agents. To compensate for communication delays, a HOFA predictive coordination method has been proposed to make the closed-loop networked nonlinear multi-agent system achieve simultaneous stability and consensus. Compared with other existing coordination methods of networked nonlinear multi-agents, the HOFA predictive coordination method is simple, active and universal. Also, it has two important advantages: firstly, the control performance of the closed-loop networked multi-agents is the same in the two cases: with communication delays and without communication delays; Secondly, the necessary and sufficient conditions derived for the simultaneous stability and consensus of networked nonlinear multi-agents are independent of communication delays. The parameters of the HOFA predictive coordination controller are designed in two steps. The simulation results illustrated in this paper have confirmed the above advantages. In fact, there still exist various challenges on the HOFA predictive coordination of networked nonlinear multi-agents. They include the internal nonlinear uncertainties (such as modelling error), external uncertainties (such as random disturbances), and time-varying communication constraints in most practical multi-agent systems. A possible way to overcome those challenges will be to combine the proposed method in this paper with other existing control methods, for example, robust control methods, adaptive control methods, disturbance rejection methods, networked control methods and so on. The proposed HOFA predictive coordination method still needs further research to deal with those challenging issues.

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