

Letter

A Sandwich Control System with Dual Stochastic Impulses

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Dear editor,

Along with the progress of science and technology and the development of social civilization, control system brings an increasingly significant function in daily life. The application field of control system is very wide, for instance, in mobile technology [1], artificial earth satellite [2], pest control [3], etc.

Ribeiro [4] first put forward the concept of random pulse in 1967. At present, impulsive control is used in networked control [5], secure communication [6], etc. In the 21st century, the impulsive control has been used in synchronization of coupled system, intelligent fault identification, image encryption, etc. [7]–[9]. Meanwhile, alternate control is often applied to traffic, irrigation of crop and current switching control [10]–[12], etc. Adaptive control is often used in robots [13], neural networks [14], etc.

At present, there are lots of means to stabilize a nonlinear system. For example, impulsive control [1]–[9], [15]–[17], alternate control [10]–[12], adaptive control [13], [14], [18], etc. In this letter, the goal is to devise some superior control systems after researching the general means of system control being used currently.

In some cases, the state is complicated, so the exact state of the system is not known. In this letter, in order to stabilize the current system, we propose a new method, which is called sandwich control system with dual stochastic impulses (SCSDSI). By derivation, it can be obtained that the exponential stability criterion of the system is in terms of a set of linear matrix inequalities (LMIs). Fig.1 reveals the operating principle of SCSDSI. In the first portion of the cycle, there exists a continuous input $C_1x(t)$, it ends stochastically, and an impulse H_1 is imposed after $C_1x(t)$. The second portion also has $C_2x(t)$ and H_2 in a similar way to the first one. In the third portion of the cycle, there also exists a continuous input $C_3x(t)$, but no impulse anymore. At the end of this letter, the use of this method controls the Chua's oscillator [19].

Problem statement: A classical nonlinear system is of the form

$$\begin{cases} \dot{x} = Gx(t) + f(x(t)) + \mu(t) \\ x(t_0) = x_0 \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous nonlinear function which satisfies $f(0) = 0$. Suppose that there is a

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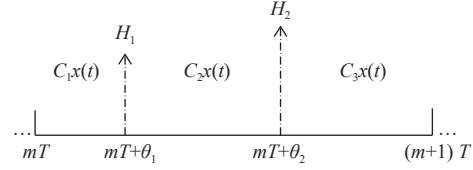


Fig. 1. Operating principle of SCSDSI within one control cycle T .

diagonal matrix $L = \text{diag}(v_1, v_2, \dots, v_n) \geq 0$ so that $\|f(x)\|^2 \leq x^T L x$ for any $x \in \mathbb{R}^n$. Concurrently, $Gx(t)$ is the linear part of the system, $f(x(t))$ is the nonlinear disturbance, and $\mu(t)$ is the outside input of system (1). The initial condition of system (1) is $x(t_0) = x_0$.

Let T be the control cycle of the system. m is a nonnegative integer. In order to stabilize the origin of system (1) by SCSDSI, T is divided into three parts:

- From mT to $mT + \theta_1$, $\mu(t) = C_1x(t)$ is installed, where $C_1 \in \mathbb{R}^{n \times n}$ is a constant matrix, and at time $mT + \theta_1$, a stochastic impulse H_1 occurs;
- From $mT + \theta_1$ to $mT + \theta_2$, $\mu(t) = C_2x(t)$ is installed, where $C_2 \in \mathbb{R}^{n \times n}$ is a constant matrix, and at time $mT + \theta_2$, a stochastic impulse H_2 occurs;
- From $mT + \theta_2$ to $(m+1)T$, $\mu(t) = C_3x(t)$ is installed, where $C_3 \in \mathbb{R}^{n \times n}$ is a constant matrix.

Through the above settings, system (1) can be described as

$$\begin{cases} \dot{x} = Gx(t) + f(x(t)) + C_1x(t), & mT < t < mT + \theta_1 \\ x(t) = x(t^-) + H_1x(t^-), & t = mT + \theta_1 \\ \dot{x} = Gx(t) + f(x(t)) + C_2x(t), & mT + \theta_1 < t < mT + \theta_2 \\ x(t) = x(t^-) + H_2x(t^-), & t = mT + \theta_2 \\ \dot{x} = Gx(t) + f(x(t)) + C_3x(t), & mT + \theta_2 < t \leq (m+1)T \end{cases} \quad (2)$$

where $C_1, C_2, C_3, H_1, H_2 \in \mathbb{R}^{n \times n}$ are constant matrices and $T > 0$ is the control cycle. Our purpose is to find conditions to make system (2) stable. So, it is of great necessity to determine $C_1, C_2, C_3, H_1, H_2, \theta_1, \theta_2$ and T .

In this letter, the maximum eigenvalue, the minimum eigenvalue and the transpose of a square matrix P_1 are denoted respectively by $\lambda_M(P_1)$, $\lambda_m(P_1)$ and P_1^T . The Euclidean norm of vector $x \in \mathbb{R}^n$ is denoted as $\|x\|$. When $P_1^T = P_1 > 0$ it is considered that the matrix P_1 is symmetric positive-definite while, if $P_1^T = P_1 < 0$, it is regarded that the matrix is symmetric negative-definite. Furthermore, $P_1^T = P_1 \geq 0$ is regarded that the matrix is symmetric positive semi-definite and $P_1^T = P_1 \leq 0$ is regarded that the matrix is symmetric negative semi-definite. $f(x(b^-))$ is defined as

$$f(x(b^-)) = \lim_{t \rightarrow b^-} f(x(t)).$$

Theorem 1: If there is a symmetric and positive definite matrix $P_1 \in \mathbb{R}^{n \times n}$ and real numbers $T > 0$, $g_1 > 0$, $g_2 > 0$, $g_3 > 0$, $\theta_1 > 0$, and $\theta_2 > \theta_1$ such that the following inequalities hold:

$$P_1G + G^T P_1 + P_1C_1 + C_1^T P_1 + \theta_1 P_1^2 + \theta_1^{-1} L + g_1 P_1 \leq 0$$

$$P_1G + G^T P_1 + P_1C_2 + C_2^T P_1 + \theta_2 P_1^2 + \theta_2^{-1} L + g_2 P_1 \leq 0$$

$$P_1G + G^T P_1 + P_1C_3 + C_3^T P_1 + \theta_3 P_1^2 + \theta_3^{-1} L + g_3 P_1 \leq 0$$

$$g_1 \theta_1 + g_2(\theta_2 - \theta_1) + g_3(T - \theta_2) - \ln \lambda_1 - \ln \lambda_2 > 0$$

then, system (2) has global exponential stability.

Proof: The Lyapunov function constructed is shown as follow:

$$V_1(x(t)) = x^T(t)P_1x(t) \quad (3)$$

thus, it can be obtained that

Since $P_1G + G^TP_1 + P_1C_2 + C_2^TP_1 + \vartheta_2P_1^2 + \vartheta_2^{-1}L + g_2P_1 \leq 0$, the following is obtained

$$\lambda_m(P_1)\|x(t)\|^2 \leq V(x(t)) \leq \lambda_M(P_1)\|x(t)\|^2. \quad (4)$$

Suppose $mT < t < mT + \theta_1$, then by (2) and (4), the following is obtained

$$\begin{aligned} \dot{V}_1(x) &= 2x^TP_1\dot{x} \\ &= x^T[P_1G + G^TP_1 + P_1C_1 + C_1^TP_1]x + 2x^TP_1f(x) \\ &\leq x^T[P_1G + G^TP_1 + P_1C_1 + C_1^TP_1]x \\ &\quad + \vartheta_1x^TP_1^2x + \vartheta_1^{-1}x^TLx \\ &= -g_1V_1(x) + x^T[P_1G + G^TP_1 + P_1C_1 + C_1^TP_1 \\ &\quad + \vartheta_1P_1^2 + \vartheta_1^{-1}L + g_1P_1]x. \end{aligned}$$

Since $P_1G + G^TP_1 + P_1C_1 + C_1^TP_1 + \vartheta_1P_1^2 + \vartheta_1^{-1}L + g_1P_1 \leq 0$, the following formula can be derived

$$\dot{V}_1(x) \leq -g_1V(x).$$

Thus,

$$V_1(x(t)) \leq V_1(x(mT)^+)\exp(-g_1(t-mT)) \quad (5)$$

where $mT < t < mT + \theta_1$.

Suppose $t = mT + \theta_1$, then

$$\begin{aligned} V_1(x)|_{t=mT+\theta_1} &= (x(t^-) + H_1x(t^-))^TP_1(x(t^-) + H_1x(t^-)) \\ &= x(t^-)^T(I + H_1)^TP_1(I + H_1)x(t^-) \\ &\leq \lambda_1V_1(x(t^-)) \end{aligned} \quad (6)$$

where $\lambda_1 = \lambda_M((I + H_1)^TP_1(I + H_1))/\lambda_m(P_1)$.

Suppose $mT + \theta_1 < t < mT + \theta_2$, then

$$\begin{aligned} \dot{V}_1(x) &\leq v x^T[P_1G + G^TP_1 + P_1C_2 + C_2^TP_1]x \\ &\quad + \vartheta_2x^TP_1^2x + \vartheta_2^{-1}x^TLx \\ &= -g_2V_1(x) + x^T[P_1G + G^TP_1 + P_1C_2 + C_2^TP_1 \\ &\quad + \vartheta_2P_1^2 + \vartheta_2^{-1}L + g_2P_1]x. \end{aligned}$$

$$\dot{V}_1(x) \leq -g_2V_1(x).$$

Thus,

$$V_1(x(t)) \leq \lambda_1V_1(x(mT + \theta_1)^-)\exp(-g_2(t-mT - \theta_1)) \quad (7)$$

where $mT + \theta_1 < t < mT + \theta_2$.

Suppose $t = mT + \theta_2$, then

$$V_1(x)|_{t=mT+\theta_2} \leq \lambda_2V_1(x(t^-)) \quad (8)$$

where $\lambda_2 = \lambda_M((I + H_2)^TP_1(I + H_2))/\lambda_m(P_1)$.

Suppose $mT + \theta_2 < t \leq (m+1)T$, then,

$$\begin{aligned} \dot{V}_1(x) &\leq x^T[P_1G + G^TP_1 + P_1C_3 + C_3^TP_1]x \\ &\quad + \vartheta_3x^TP_1^2x + \vartheta_3^{-1}x^TLx \\ &= -g_3V_1(x) + x^T[P_1G + G^TP_1 + P_1C_2 + C_3^TP_1 \\ &\quad + \vartheta_3P_1^2 + \vartheta_3^{-1}L + g_3P_1]x. \end{aligned}$$

Since $P_1G + G^TP_1 + P_1C_3 + C_3^TP_1 + \vartheta_3P_1^2 + \vartheta_3^{-1}L + g_3P_1 \leq 0$, the following is obtained

$$\dot{V}_1(x) \leq -g_3V_1(x).$$

Thus,

$$V_1(x(t)) \leq \lambda_2V_1(x(mT + \theta_2)^-)\exp(-g_3(t-mT - \theta_2)) \quad (9)$$

where $mT + \theta_1 < t \leq (m+1)T$.

Similar to the method used in [17], we get.

Case 1: $m = 0$

Subcase 1: Suppose $0 < t < \theta_1$, then

$$V_1(x(t)) \leq V_1(x(0))\exp(-g_1t)$$

hence,

$$V_1(x(\theta_1)^-) \leq V_1(x(0))\exp(-g_1\theta_1).$$

Subcase 2: Suppose $t = \theta_1$, it can be obtained that

$$V_1(x(\theta_1)) \leq \lambda_1V_1(x(0))\exp(-g_1\theta_1).$$

Subcase 3: Suppose $\theta_1 < t < \theta_2$, then

$$V_1(x(t)) \leq \lambda_1V_1(x(0))\exp(-g_1\theta_1 - g_2(t-\theta_1))$$

and

$$V_1(x(\theta_2)^-) \leq \lambda_1V_1(x(0))\exp(-g_1\theta_1 - g_2(\theta_2 - \theta_1)).$$

Subcase 4: Suppose $t = \theta_2$, it can be obtained that

$$V_1(x(\theta_2)) \leq \lambda_1\lambda_2V_1(x(0))\exp(-g_1\theta_1 - g_2(\theta_2 - \theta_1)).$$

Subcase 5: Suppose $\theta_2 < t \leq T$, then

$$\begin{aligned} V_1(x(t)) &\leq \lambda_2V_1(x(\theta_2)^-)\exp(-g_3(t-\theta_2)) \\ &\leq \lambda_1\lambda_2V(x(0))\exp(-g_1\theta_1 \\ &\quad - g_2(\theta_2 - \theta_1) - g_3(t-\theta_2)) \end{aligned}$$

and

$$V_1(x(T)) \leq \lambda_1\lambda_2V_1(x(0))\exp(-g_1\theta_1 - g_2(\theta_2 - \theta_1) - g_3(T - \theta_2)).$$

Case $n+1$: $m = n$.

Subcase 1: Suppose $nT < t < nT + \theta_1$, then

$$V_1(x(t)) \leq \lambda_1^n\lambda_2^nV_1(x(0))\exp(-ng_1\theta_1 - ng_2(\theta_2 - \theta_1) - ng_3(T - \theta_2) - g_1(t-nT)). \quad (10)$$

Subcase 2: Suppose $t = nT + \theta_1$, then

$$V_1(x(t)) \leq \lambda_1^{n+1}\lambda_2^nV_1(x(0))\exp(-(n+1)g_1\theta_1 - ng_2(\theta_2 - \theta_1) - ng_3(T - \theta_2)). \quad (11)$$

Subcase 3: Suppose $nT + \theta_1 < t \leq nT + \theta_2$, then

$$V_1(x(t)) \leq \lambda_1^{n+1}\lambda_2^nV_1(x(0))\exp(-(n+1)g_1\theta_1 - ng_2(\theta_2 - \theta_1) - ng_3(T - \theta_2) - g_2(t-nT - \theta_1)). \quad (12)$$

Subcase 4: Suppose $t = nT + \theta_2$, then

$$V_1(x(t)) \leq \lambda_1^{n+1}\lambda_2^{n+1}V_1(x(0))\exp(-(n+1)g_1\theta_1 - (n+1)g_2(\theta_2 - \theta_1) - ng_3(T - \theta_2)). \quad (13)$$

Subcase 5: Suppose $nT + \theta_2 < t < (n+1)T$, then

$$V_1(x(t)) \leq \lambda_1^{n+1}\lambda_2^{n+1}V_1(x(0))\exp(-(n+1)g_1\theta_1 - (n+1)g_2(\theta_2 - \theta_1) - ng_3(T - \theta_2) - g_3(t-nT - \theta_2)). \quad (14)$$

From (10) it can be obtained that if $nT < t \leq nT + \theta_1$, and $t = nT + \theta_1$, then

$$\begin{aligned} V_1(x(t)) &\leq \lambda_1^{n+1}\lambda_2^nV_1(x(0))\exp(-(n+1)g_1\theta_1 \\ &\quad - ng_2(\theta_2 - \theta_1) - ng_3(T - \theta_2)) \\ &\leq V_1(x(0))\exp(-(g_1\theta_1 + g_2(\theta_2 - \theta_1) \\ &\quad + g_3(T - \theta_2) - \ln\lambda_1 - \ln\lambda_2)n \\ &\quad + \ln\lambda_1 - g_1\theta_1)). \end{aligned}$$

From (14) it is obtained that if $nT + \theta_1 < t \leq (n+1)T$, and $t = (n+1)T$, then

$$\begin{aligned} V_1(x(t)) &\leq \lambda_1^{n+1}\lambda_2^{n+1}V_1(x(0))\exp(-(n+1)g_1\theta_1 \\ &\quad - (n+1)g_2(\theta_2 - \theta_1) - ng_3(T - \theta_2) \\ &\quad - g_3(t-nT - \theta_2)) \\ &\leq V_1(x(0))\exp(-(g_1\theta_1 + g_2(\theta_2 - \theta_1) \\ &\quad + g_3(T - \theta_2) - \ln\lambda_1 - \ln\lambda_2)(n+1)). \end{aligned}$$

By the above inequalities, and the conditions of Theorem 1, it is obtained that $V_1(x(t)) \rightarrow 0$ as $t \rightarrow \infty$. Therefore, $\lim_{t \rightarrow \infty} V_1(x(t)) = 0$. ■

Numerical results: The Chua's oscillator [19] is described as follow:

$$\begin{cases} \dot{x}_1 = \varpi(x_2 - x_1 - g_1(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = \nu x_2 \end{cases} \quad (15)$$

where ϖ and ν are two parameters, $g_1(x)$ is a piecewise linear characteristic function of Chua's diode. It can be defined as

$$g_1(x) = ax + 0.5(b-a)(|x+1| - |x-1|). \quad (16)$$

Furthermore, a and b are two constants and $b < a \leq 0$.

Then, we select the parameters $\varpi = 9.1156$, $\nu = 15.9946$, $b = -1.24905$, and $a = -0.75735$ which makes the Chua's oscillator chaotic at $x(0) = (2, -1, -2)^T$, as shown in Fig.2.

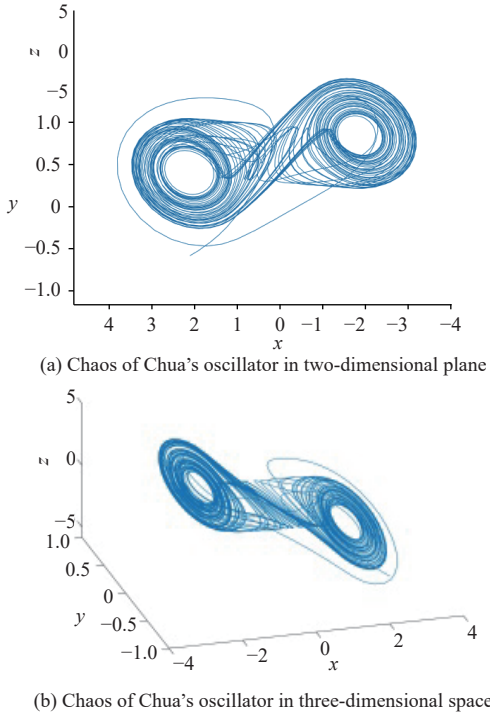


Fig. 2. The chaotic phenomena of Chua's oscillator with $x(0) = (2, -1, -2)^T$.

Setting $L = \text{diag}(\varpi^2(b-a)^2, 0, 0)$, and selecting

$$C_1 = \text{diag}(-7, -8, -5)$$

$$C_2 = \text{diag}(-8, -7, -5)$$

$$C_3 = \text{diag}(-9, -8, -5)$$

$$H_1 = \begin{bmatrix} 0.08 & 0 & 0 \\ 0 & -0.25 & 0 \\ 0 & 0 & -0.4 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.31 & 0 & -0.2 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.92 \end{bmatrix}$$

with $T = 1$, solving the LMIs listed in Theorem 1, we obtain $\vartheta_1 = 7$, $\vartheta_2 = 8$, $\vartheta_3 = 9$, $\theta_1 = 0.0357$, $\theta_2 = 0.2321$, $g_1 = 4$, $g_2 = 5$, $g_3 = 6$, and

$$P_1 = \begin{bmatrix} 0.6120 & -0.1267 & 0.1338 \\ -0.1267 & 0.9317 & -0.0032 \\ 0.1338 & -0.0032 & 0.0793 \end{bmatrix}.$$

Therefore, the Chua's system (15) is exponentially stable by Theorem 1. The time response curves of Chua's oscillator, using the proposed method, are shown in Fig. 3.

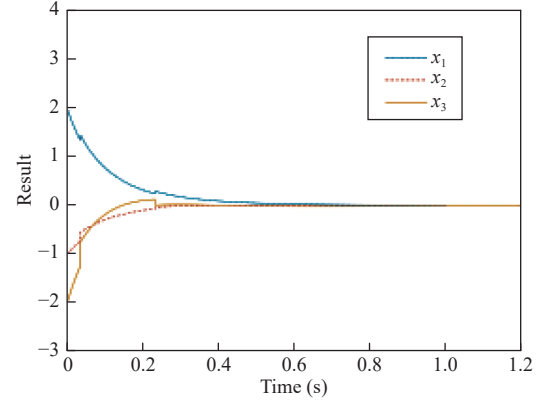


Fig. 3. The time response curves of Chua's oscillator by using SCSDSI.

Conclusions: In this letter, a new model of Sandwich control system with dual stochastic impulses (SCSDSI), where the exact time of occurrence of impulses is uncertain, was proposed. It established that the chaotic Chua's circuit can be controlled by this new method. In fact, by this method, a host of other nonlinear systems in robotics, electronics and other fields can be controlled.

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