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Adaptive Flocking of Multi-Agent Systems with Uncertain Nonlinear Dynamics and Unknown Disturbances Using Neural Networks*

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***Abstract*— Collective behavior of multi-agent systems brings some new problems in control theory and application. Especially, flocking problem of multi-agent systems with uncertain nonlinear dynamics and unknown external disturbances is a challenging problem. Some existing works assume that the intrinsic nonlinear dynamics of virtual leader is the same as those of the agents, which is unreasonable and impractical. To solve this issue, we consider an adaptive flocking problem of multi-agent systems with uncertain nonlinear dynamics and unknown external disturbances in this paper, where the intrinsic nonlinear dynamics of virtual leader is allowed to be different from the agents. Firstly, to approximate the uncertain nonlinear dynamics of each agent, an adaptive neural network is used, whose weights are updated online. Furthermore, an adaptive robust signal is designed to counteract the unknown external disturbances and neural network approximation errors, which is independent with the upper bound of the unknown external disturbances and neural network approximation errors. Moreover, an adaptive flocking control law is designed, which is proved that the flocking can be realized and the velocity errors converge to a small neighbor of the origin based on Lyapunov stability theory. Finally, the robustness and superiority of the proposed robust adaptive flocking control law are validated by two representative simulations.**

I. INTRODUCTION

Flocking is the collective behavior [1], which exists in the nature with various forms such as the flocking of birds, the swarming of bacteria, and the schooling of fishes [2], [3]. In recent years, flocking behavior has attracted a considerable amount of attention from many scientists in the fields of ecology, evolutionary biology, computer science, and control engineering [4]. These related researches focus on how a group of agents can form a uniform behavior by only using local interactions, which has promising applications in disaster rescue, reconnaissance, mobile sensor networks [5], etc.

In recent years, some works have focused on the flocking of multi-agent systems with nonlinear dynamics. In [1], adaptive controllers and update laws for heterogeneous multi-agent systems with nonlinear dynamics were presented by Zhang et al. Li et al. proposed a novel output feedback consensus algorithm for the flocking of disturbed nonlinear multi-agent systems in [6]. In [7], based on event-triggered control, the flocking problem of nonlinear multi-agent system with time varying delay was investigated by Sun et al. In some existing works, it is assumed that the nonlinear dynamics of all agents are the same with virtual leader. However, it is unreasonable and impractical in the target tracking problem where the target can be regarded as the virtual leader, since the dynamics of targets may not be the same with those of the agents. Besides, in real-world applications, unknown external disturbances coexist with uncertain nonlinear dynamics. Some unknown external disturbances are time-varying in the working process, which would deteriorate the control performance and even make systems break down [8], [9]. To reduce the impact of external disturbances, some attempts have been made in [10], [11]. Dong et al. proposed a dynamic position feedback control law to solve the problem of the flocking subject to external disturbances. It is assumed that the external disturbances are generated by linear exogenous system. In practice, the external disturbances are more complicated and cannot be known in advance.

Due to the property of universal approximation [12], neural networks have been employed in system identification, control, and uncertainty estimation [13]. In [14], the problem of tracking control with switching formation in constrained space for multi-agent systems based on neural network was researched by Liu et al. Xiong et al. investigated the time-varying formation tracking control problem with model uncertainties and presented a fixed-time observer based on adaptive neural network with minimal learning parameter approach for time-varying formation tracking problem in [15]. These works show the superiority performance of neural networks handling the uncertainty estimation.

To the best of our knowledge and based on the above-mentioned works, the research of the flocking of multi-agent systems with uncertain nonlinear dynamics and unknown external disturbances is still an open problem, which motivates our research in this paper. Firstly, to solve the problem that the intrinsic nonlinear dynamics of virtual leader is different from those of all agents, an adaptive neural network is used to approximate the uncertain nonlinear dynamics of each agent, whose weights are updated online, which is more practical compared with conventional methods. Secondly, an adaptive robust signal is designed to counteract the unknown external disturbances and neural network approximation errors, which is independent to the upper bound of the unknown external

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disturbances. Thirdly, an adaptive flocking control law is designed, and the Lyapunov stability analysis is conducted to guarantee that the velocity errors converge to a small neighbor of the origin. Finally, two representative simulations are given to verify the robustness and superiority of the proposed robust adaptive flocking control law.

The rest of this paper is organized as follows: the preliminaries are given in Section II. Section III provides the proposed robust adaptive flocking control law for the flocking of multi-agent systems with uncertain nonlinear dynamics and unknown external disturbances. The stability analysis is given in Section IV. Section V demonstrates the simulation results. The conclusions and future work are represented in Section VI.

II. PRELIMINARIES

A. Graph Theory

The information of communication among agents can be denoted by an undirected graph $G = (V, E, A)$, where $V = \{1, \cdots, N\}$ is the node set, $E \subseteq V \times V$ is the edge set, and $A = [\alpha_{ij}] \in R^{N\times N}$ is the adjacency matrix where element α_{ij} denotes the node i can receive the information of the node j. An edge from node i to node j is denoted as $(i, j) \in E$. The set of neighbors of the node i is defined as $\mathcal{N}_i = \{j|(j, i) \in E, i \neq j\}$. For adjacency matrix A, if $j \in \mathcal{N}_i$, then $\alpha_{ij} = 1$, otherwise $\alpha_{ij} = 0$. The degree of the node i is defined as $d_i = \sum_{j\in\mathcal{N}_i} \alpha_{ij}$, and the degree diagonal matrix of the graph G is represented as $D = diag(d_1, d_2, \cdots, d_n)$. The Laplacian matrix of the graph G is defined as $L = [l_{ij}] \in R^{N\times N}$, namely, $L = D - A$, where $l_{ii} = d_i$ and $l_{ij} = -\alpha_{ij}, \forall i \neq j$.

B. Problem Formulation

In this paper, N agents in a m-dimensional Euclidean space are taken into consideration. The dynamic model of each agent is described by

$$\begin{cases} \dot{q}_i(t) = p_i(t) \\ \dot{p}_i(t) = f_i(p_i(t)) + u_i(t) + d_i(t), i = 1, 2, \cdots N, \end{cases} \tag{1}$$

where, for agent i, $q_i(t) \in R^m$ is the position vector, $p_i(t) \in R^m$ is the velocity vector. $f_i(p_i(t)) \in R^m$ is the unknown nonlinear dynamics. $d_i(t) \in R^m$ represents the unknown external disturbances. $u_i(t) \in R^m$ is the control input.

Definition 1: When all the agents have the same velocity and the distances among the agents are stable, the behavior of the agents is said to be flocking.

Assumption 1: It is assumed that the communication capability of all agents is limited, and each agent has the same interaction range. The interaction range is denoted as r_c. Then the neighbor set of agent i is defined as

$$\mathcal{N}_i = \{j \in V : ||q_j - q_i|| < r_c\}, \tag{2}$$

where $|| \cdot ||$ is the Euclidean norm in R^m.

The goal of the flocking of the multi-agent systems is to design a decentralized control law for every agent by using local interactions such that *Definition 1* can be satisfied. To avoid regular fragmentation, a virtual leader is considered, and its dynamic model is characterized by

$$\begin{cases} \dot{q}_r(t) = p_r(t) \\ \dot{p}_r(t) = f_r(p_r(t)) \end{cases}, \tag{3}$$

where $q_r(t) \in R^m$ is the position vector of the virtual leader, $p_r(t) \in R^m$ is the velocity vector of the virtual leader. $f_r(p_r(t)) \in R^m$ is the internal dynamics of the virtual leader. In most current literature, it is assumed that the dynamic model of the virtual leader is the same as that of the agents, which is unreasonable and impractical. In this paper, the dynamic model of the virtual leader can be different from the agents. It is assumed that every agent in the group can obtain the velocity information of the virtual leader, and the virtual leader moves with a constant velocity.

Assumption 2: [16] The unknown external disturbances is bounded, i.e.,

$$||d_i(t)|| \leq d_{iM}. \tag{4}$$

The boundary is unknown for the controller design, and it will only be used in the stability analysis.

C. Neural Network and Function Approximator

The accurate model of agents with nonlinear dynamics may not be acquired in practice, which may cause the failure of the flocking. Therefore, the uncertain nonlinear dynamics of the agents need to be estimated. Due to satisfied nonlinear function approximating capability, neural networks are employed to approximate unknown functions. The radial basis function neural network (RBFNN) is a reliable method to realize the function approximator. Then, for any uncertain function $f(x)$, there exists an ideal constant weight matrix $W^* \in R^{l\times m}$ such that the following equation holds:

$$f(x) = W^{*T} S(x) + \varepsilon, \forall x \in \Omega_x, \tag{5}$$

where $S(x) \in R^{l\times 1}$ is the basic Gaussian function vector with l nodes, ε is the function approximation error satisfying $|\varepsilon| \leq \varepsilon_N$ with positive and bounded ε_N.

It is worth highlighting that the ideal weight matrix W^* is merely used for qualitative analysis, and can not be obtained directly. In engineering applications, $\hat{W}$ is used for actual function approximation. Therefore, the approximation of $f(x)$ denoted as $\hat{f}(x)$ can be selected by

$$\hat{f}(x) = \hat{W}^T S(x), \tag{6}$$

where $\hat{W}$ is current weight of RBFNN and will be updated online. In addition, the error of the weight matrix is represented as $\tilde{W} = W^* - \hat{W}$.

III. ADAPTIVE FLOCKING CONTROL LAW

In this section, a decentralized robust adaptive flocking control law is designed for the flocking of the multi-agent systems in the presence of uncertain nonlinear dynamics and unknown external disturbances. In particular, uncertain nonlinear dynamics are compensated by adaptive neural networks, which handle the flocking with uncertain nonlinear dynamics under the situation that the dynamics of the agents is different from virtual leader. An adaptive robust signal is designed to counteract the unknown external disturbances and neural network approximation errors, which does not rely on the upper bound of the unknown external disturbances. The specific robust adaptive flocking control law is designed as:

$$u_i = u_{1i} + u_{2i} + u_{3i}, \tag{7}$$

where u_{1i} embodies basic control rules of the Boild's model for each agent [17], u_{2i} is the adaptive compensative term, u_{3i} is an adaptive robust signal. The basic control rules of the Boild's model is composed of three components:

$$u_{1i} = f_i^g + f_i^d + f_i^r, \tag{8}$$

where f_i^g is a gradient term, which regulates the distance between agent i and its neighbor. The second term f_i^d is the velocity alignment term, which regulates the velocity of agent i to be consistent with its neighbor. A navigation term f_i^r guides agent i to follow the virtual leader. Therefore, u_{1i} is presented as follows:

$$\begin{aligned} u_{1i} = &- g_1 \sum_{j\in\mathcal{N}_i(t)} \nabla_{q_i}\psi_\alpha(||q_i - q_j||_\varrho) \\ &- \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(p_i - p_j) - c_1(q_i - q_r) - c_2(p_i - p_r), \end{aligned} \tag{9}$$

where g_1 is the gradient gain, c_1, c_2 are the tracking gains, $\mathcal{N}_i(t)$ satisfies Assumption 1, $||\cdot||_\varrho$ is the ϱ-norm and is differentiable everywhere, which is specially defined as

$$||y||_\varrho = \frac{1}{\iota}[\sqrt{1+\iota||y||^2} - 1], \iota > 0. \tag{10}$$

ψ_α is a positive potential function with minimum at $y = r_\alpha$ as follows:

$$\psi_\alpha(y) = \int_{d_\alpha}^{y} \phi_\alpha(s)ds, \tag{11}$$

where $r_\alpha = ||r_c||_\varrho$ and $d_\alpha = ||d||_\varrho$, d is the desired distance between its neighbor. $\phi_\alpha(y)$ is the function as following:

$$\phi_\alpha(y) = \rho_h(y/r_\alpha)\phi(y - d_\alpha), \tag{12}$$

where $\rho_h(y)$ is the following bump function:

$$\rho_h(y) = \begin{cases} 1 & , y \in [0,h) \\ \frac{1}{2}[1+\cos(\pi\frac{(y-h)}{(1-h)})] & , y\in[h,1], h\in(0,1]. \\ 0 & , \text{otherwise} \end{cases}$$

and $\phi(y)$ is the sigmoidal function defined as:

$$\phi(y) = \frac{1}{2}[(\iota_a + \iota_b)\sigma_1(y+\iota_c) + (\iota_a - \iota_b)], \tag{13}$$

where $\sigma_1(y+\iota_c) = (y+\iota_c)/\sqrt{1+(y+\iota_c)^2}$, $0 < \iota_a \le \iota_b$, $\iota_c = |\iota_a - \iota_b|\sqrt{4\iota_a\iota_b}$. In addition, $A(t) = [a_{ij}(t)] \in R^{N\times N}$ is the adjacent matrix defined as

$$a_{ij}(t) = \rho_h(||q_j - q_i||_\varrho / r_\alpha) \in [0,1], j \ne i \tag{14}$$

Based on the neural network approximation character presented in Section II-C, it is known that $f_i(p_i)$ can be expressed on prescribed compact set Ω_{p_i} by

$$f_i(p_i) = W_i^{*T} S_i(p_i) + \varepsilon_i, \tag{15}$$

where $W_i^* \in R^{l\times m}$ is the ideal weight matrix for agent i, and ε_i is the bounded approximation error satisfying $|\varepsilon_i| \le \varepsilon_{Ni}$ with positive and bounded ε_{Ni}. Therefore, to approximate the uncertain dynamics $f_i(p_i)$ in (1), the compensative term u_{i2} is designed as following:

$$u_{2i} = -\hat{W}_i^T S_i(p_i), \tag{16}$$

where $\hat{W}_i \in R^{l\times m}$ is current weight of RBFNN for agent i, which is updated online with the following adaptive updating law:

$$\dot{\hat{W}}_i = \beta S(p_i)\tilde{p}_i^T - \beta\alpha\hat{W}_i, \tag{17}$$

where $\tilde{p}_i \in R^m$ is the velocity error, which is defined as $\tilde{p}_i = p_i - p_r$. α, β are positive tunable parameters.

Considering the unknown external disturbances and neural network approximation error, an adaptive robust signal is additionally designed as follows:

$$u_{3i} = -\hat{\delta}_i\tilde{p}_i^{\gamma-1} sign(\tilde{p}_i^\gamma), \tag{18}$$

where $\hat{\delta}_i \in R^m$ is an adaptive robust gain. It is the estimation of the upper bound of the unknown external disturbances and the neural network approximation error denoted as $\delta_i > 0$, which means that $||d_{iM} + \varepsilon_{Ni}|| \le \delta_i$. γ is a parameter to tune the converging rate, which should satisfy the following constraints:

$$\begin{cases} 0 < \gamma < 1 & , |\tilde{p}| < 1 \\ \gamma \ge 1 & , |\tilde{p}| \ge 1. \end{cases}$$

It can induce that $\delta_i\tilde{p}_i - \delta_i\tilde{p}_i^\gamma \le 0$.

The adaptive robust gain $\hat{\delta}_i$ is designed as

$$\dot{\hat{\delta}}_i = \chi|\tilde{p}^\gamma| - \eta\hat{\delta}_i, \tag{19}$$

where χ, η are tunable parameters. In addition, the estimation error of the upper bound is denoted as $\tilde{\delta}_i = \delta_i - \hat{\delta}_i$.

Remark 1: It is noted that the adaptive robust signal can handle any bounded unknown external disturbances without extra constraints required like the linear exogenous system [11]. With the adaptive robust gain, it is more practical in the engineering applications.

IV. STABILITY ANALYSIS OF FLOCKING

In the above section, a decentralized robust adaptive flocking law has been put forward for each agent to realize the flocking in the presence of the uncertain nonlinear dynamics and unknown external disturbances. In this section, the stability analysis for our proposed control law is given by Lyapunov theorem.

Theorem 1 *For the multi-agent systems (1) and (3), under the robust adaptive flocking law (7), the velocity of all the agents will converge to the neighbor of the velocity of the virtual leader, and collision avoidance is always guaranteed.*

Proof: Construct the following Lyapunov function $Q(t) = Q_1(t) + Q_2(t) + Q_3(t) + Q_4(t)$ with

$$\begin{aligned} Q_1(t) = &\frac{1}{2}\sum_{i=1}^{N}[\sum_{j=1,j\ne i}^{N}\frac{1}{g_1}\psi_\alpha(||q_i - q_j||_\varrho) \\ &+ c_1(q_i - q_r)^T(q_i - q_r)], \end{aligned} \tag{20}$$

$$Q_2(t)=\frac{1}{2}\sum_{i=1}^{N}(p_i-p_r)^T(p_i-p_r), \tag{21}$$

$$Q_3(t)=\frac{1}{2\beta}\sum_{i=1}^{N}tr(\tilde{W}_i^T\tilde{W}_i), \tag{22}$$

$$Q_4(t)=\frac{1}{2\chi}\sum_{i=1}^{N}\tilde{\delta}_i^T\tilde{\delta}_i. \tag{23}$$

Let $\tilde{q}_i=q_i-q_r$, $\tilde{q}_{ij}=\tilde{q}_i-\tilde{q}_j$. Then (7), (20) and (21) are rewritten as the following form:

$$\begin{aligned}u_i=&-\sum_{j\in\mathcal{N}_i(t)}g_1\nabla\tilde{q}_i\psi_\alpha(||\tilde{q}_{ij}||_\varrho)-\hat{W}_i^TS(p_i)-c_1\tilde{q}_i\\&-\hat{\delta}_i\tilde{p}_i^{\gamma-1}sign(\tilde{p}_i^\gamma)-\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{p}_i-\tilde{p}_j)-c_2\tilde{p}_i,\end{aligned} \tag{24}$$

$$Q_1(t)=\frac{1}{2}\sum_{i=1}^{N}[\sum_{j=1,j\neq i}^{N}\frac{1}{g_1}\psi_\alpha(||\tilde{q}_{ij}||_\varrho)+c_1\tilde{q}_i^T\tilde{q}_i], \tag{25}$$

$$Q_2(t)=\frac{1}{2}\sum_{i=1}^{N}\tilde{p}_i^T\tilde{p}_i, \tag{26}$$

Considering the symmetry of the potential function ψ_α and the adjacent matrix $A(t)$, then the derivative of Q_1 with respect to time obtains

$$\dot{Q}_1(t)=\sum_{i=1}^{N}[\sum_{j=1,j\neq i}^{N}g_1\nabla\tilde{q}_i\psi_\alpha(||\tilde{q}_{ij}||_\varrho)\tilde{p}_i+c_1\tilde{q}_i^T\tilde{p}_i]. \tag{27}$$

Using (24), the derivative of Q_2+Q_3 with respect to time obtains

$$\begin{aligned}\dot{Q}_2(t)+\dot{Q}_3(t)=&\sum_{i=1}^{N}[-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}g_1\nabla\tilde{q}_i\psi_\alpha(||\tilde{q}_{ij}||_\varrho)\\&-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{p}_i-\tilde{p}_j)-c_1\tilde{p}_i^T\tilde{q}_i\\&-c_2\tilde{p}_i^T\tilde{p}_i+\tilde{p}_i^T\tilde{W}_i^TS(p_i)+\tilde{p}_i^T(\varepsilon_i+d_i)\\&-\tilde{p}_i^T\hat{\delta}_i\tilde{p}_i^{\gamma-1}sign(\tilde{p}_i^\gamma)+\frac{1}{\beta}tr(\tilde{W}_i^T\dot{\tilde{W}}_i)].\end{aligned} \tag{28}$$

Considering the fact that $\mathbf{y}^T\mathbf{z}=tr(\mathbf{z}\mathbf{y}^T)$, $\forall\mathbf{y}$, $\mathbf{z}\in R^n$ and $tr(AB)=tr(BA)$, (28) can be rewritten as

$$\begin{aligned}\dot{Q}_2(t)+\dot{Q}_3(t)=&\sum_{i=1}^{N}[-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}g_1\nabla\tilde{q}_i\psi_\alpha(||\tilde{q}_{ij}||_\varrho)\\&-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{p}_i-\tilde{p}_j)-c_1\tilde{p}_i^T\tilde{q}_i\\&-c_2\tilde{p}_i^T\tilde{p}_i+\frac{1}{\beta}tr((\beta S(p_i)\tilde{p}_i^T-\dot{\hat{W}}_i)\tilde{W}_i^T)\\&+\tilde{p}_i^T(\varepsilon_i+d_i)-\tilde{p}_i^T\hat{\delta}_i\tilde{p}_i^{\gamma-1}sign(\tilde{p}_i^\gamma)].\end{aligned} \tag{29}$$

Substituting the adaptive updating law into (29), it has

$$\begin{aligned}\dot{Q}_2(t)+\dot{Q}_3(t)=&\sum_{i=1}^{N}[-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}g_1\nabla\tilde{q}_i\psi_\alpha(||\tilde{q}_{ij}||_\varrho)\\&-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{p}_i-\tilde{p}_j)-c_1\tilde{p}_i^T\tilde{q}_i\\&-c_2\tilde{p}_i^T\tilde{p}_i+\alpha tr(\hat{W}_i\tilde{W}_i^T)+\tilde{p}_i^T(\varepsilon_i+d_i)\\&-\tilde{p}_i^T\hat{\delta}_i\tilde{p}_i^{\gamma-1}sign(\tilde{p}_i^\gamma)]\\=&\sum_{i=1}^{N}[-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}g_1\nabla\tilde{q}_i\psi_\alpha(||\tilde{q}_{ij}||_\varrho)\\&-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{p}_i-\tilde{p}_j)-c_1\tilde{p}_i^T\tilde{q}_i\\&-c_2\tilde{p}_i^T\tilde{p}_i+\alpha tr(W_i^*\tilde{W}_i^T)-\alpha tr(\tilde{W}_i\tilde{W}_i^T)\\&+\tilde{p}_i^T(\varepsilon_i+d_i)-\tilde{p}_i^T\hat{\delta}_i\tilde{p}_i^{\gamma-1}sign(\tilde{p}_i^\gamma)].\end{aligned} \tag{30}$$

The derivative of $Q_4(t)$ is

$$\dot{Q}_4(t)=\sum_{i=1}^{N}-\delta_i|\tilde{p}^\gamma|+\hat{\delta}_i|\tilde{p}^\gamma|-\frac{\eta}{\chi}\hat{\delta}_i^2+\frac{\eta}{\chi}\delta_i\hat{\delta}_i. \tag{31}$$

Using (27), (30), and (31), the derivative of Q with respect to t changes to

$$\begin{aligned}\dot{Q}(t)=&\sum_{i=1}^{N}[-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{p}_i-\tilde{p}_j)-c_2\tilde{p}_i^T\tilde{p}_i-\delta_i|\tilde{p}^\gamma|\\&+\alpha tr(W_i^*\tilde{W}_i^T)-\alpha tr(\tilde{W}_i\tilde{W}_i^T)+\tilde{p}_i^T(\varepsilon_i+d_i)\\&-\tilde{p}_i^T\hat{\delta}_i\tilde{p}_i^{\gamma-1}sign(\tilde{p}_i^\gamma)+\hat{\delta}_i|\tilde{p}^\gamma|-\frac{\eta}{\chi}\hat{\delta}_i^2+\frac{\eta}{\chi}\delta_i\hat{\delta}_i].\end{aligned} \tag{32}$$

According to Young's inequality, the derivative of Q with respect to t is

$$\begin{aligned}\dot{Q}(t)\leq&\sum_{i=1}^{N}[-\tilde{p}_i^T\sum_{j\in\mathcal{N}_i(t)}a_{ij}(t)(\tilde{p}_i-\tilde{p}_j)-c_2\tilde{p}_i^T\tilde{p}_i-\frac{\eta}{2\chi}\hat{\delta}_i^2\\&+\alpha||\tilde{W}_i||_F||W_i^*||_F-\alpha||\tilde{W}_i||_F^2+\frac{\eta}{2\chi}\delta_i^2].\end{aligned} \tag{33}$$

Define $\tilde{P}=[\tilde{p}_1^T,\tilde{p}_2^T,\cdots,\tilde{p}_N^T]^T$, $\hat{\delta}=[\hat{\delta}_1^T,\hat{\delta}_2^T,\cdots,\hat{\delta}_N^T]^T$, $\tilde{W}=\mathrm{diag}[\tilde{W}_1,\tilde{W}_2,\cdots,\tilde{W}_N]$, $W^*=\mathrm{diag}[W_1^*,W_2^*,\cdots,W_N^*]$, $\Delta=\sum_{i=1}^{N}\frac{\eta}{2\chi}\delta_i^2$. Then (33) is rewritten as

$$\begin{aligned}\dot{Q}(t)\leq&-\underline{\sigma}((L+C_2)\otimes I_2)||\tilde{P}||^2-\frac{\eta}{2\chi}||\hat{\delta}||\\&+\alpha||\tilde{W}||_F||W^*||_F-\alpha||\tilde{W}||_F^2+\Delta\\\leq&-\begin{bmatrix}||\tilde{P}||\\||\hat{\delta}||\\||\tilde{W}||_F\end{bmatrix}^T\begin{bmatrix}\underline{\sigma}((L+C_2)\otimes I_2)&0&0\\0&\frac{\eta}{2\chi}&0\\0&0&\alpha\end{bmatrix}\\&\cdot\begin{bmatrix}||\tilde{P}||\\|\hat{\delta}||\\||\tilde{W}||_F\end{bmatrix}+\begin{bmatrix}0\\0\\\alpha||W^*||_F\end{bmatrix}^T\cdot\begin{bmatrix}||\tilde{P}||\\||\hat{\delta}||\\||\tilde{W}||_F\end{bmatrix}+\Delta,\end{aligned} \tag{34}$$

where $\underline{\sigma}((L + C_2) \otimes I_2)$ denotes the min eigenvalue of the symmetric positive definite matrix $(L + C_2) \otimes I_2$.

According to Stone-Weierstrass approximation theorem [18], there exists positive real number W_M, such that $||W^*|| \leq W_M$. Define $\xi = [||\tilde{P}||, ||\hat{\delta}||, ||\tilde{W}||_F]^T$, $\omega = [0, 0, \alpha W_M]^T$, $M = \begin{bmatrix} \underline{\sigma}((L + C_2) \otimes I_2) & 0 & 0 \\ 0 & \frac{\eta}{2\chi} & 0 \\ 0 & 0 & \alpha \end{bmatrix}$. Then,

$$\begin{aligned} \dot{Q}(t) \leq & -\xi^T M \xi + \omega^T \xi + \Delta \\ = & -Q_\xi(\xi). \end{aligned} \tag{35}$$

It is clear that the matrix M is positive definite. Hence, $Q_\xi(\xi)$ is positive definite if

$$||\xi|| \geq \frac{-||\omega|| + \sqrt{||\omega||^2 + 4\underline{\sigma}(M)\Delta}}{2\underline{\sigma}(M)}. \tag{36}$$

Therefore, according to the Lyapunov theory, $\xi(t)$ is invariant to the following sets:

$$||\xi|| \leq \frac{-||\omega|| + \sqrt{||\omega||^2 + 4\underline{\sigma}(M)\Delta}}{2\underline{\sigma}(M)}. \tag{37}$$

Furthermore, it can be obtained that $\tilde{P}$, $\hat{\delta}$, and $\tilde{W}$ are bounded. Hence, the velocity of every agent can converge to the neighbor of the virtual leader under the robust adaptive flocking control law (7). On the basis of the property of the potential function in (11), it is obtained that the smaller the distance of the agent with its neighbor the bigger the repulsive force by choosing the parameters of the potential function appropriately. Therefore, collisions among the agents can be avoided. This completes the proof.

Remark 2: By choosing parameters η, χ, α appropriately, $||\xi||$ can reach to a small neighbor of the origin.

V. SIMULATION RESULTS

In this section, two representative simulations are given to verify the effectiveness of the proposed robust adaptive flocking control law (7).

Consider 40 agents described by (1) moving in a 2-dimensional space. The following uncertain nonlinear dynamic function $f_i(p_i)$ is used for simulations.

$$f_i(p_i) = \begin{pmatrix} 10\sin(k_{ix} \cdot p_x(t))p_y(t) \\ 10\cos(k_{iy} \cdot p_y(t))p_x(t) \end{pmatrix}, \tag{38}$$

where k_{ix} and k_{iy} are selected randomly in the range [-2,2]. The initial positions and velocities of the 40 agents are selected randomly in the range $[0, 25] \times [0, 25]$ and $[-2, 2] \times [-2, 2]$, respectively, as shown in Fig. 1, where, the black dots are the positions, and the red arrows indicate the velocities. The initial position and velocity of the virtual leader are set as $q_r(0) = [10, 9]^T$ and $p_r(0) = [1, 2]^T$. The sensing range r_c is set as 3.6, and the desired distance d is set as 3. $\iota = 0.1$ is chosen for the ϱ norm. $h = 0.9$ is selected for the function $\rho_h(\cdot)$. $\iota_a = 10$ and $\iota_b = 10$ are chosen for the function $\phi(\cdot)$. $c_1 = 3, c_2 = 4$ are chosen for the navigation feedback, $g_1 = 2$ is chosen for the gradient term. Besides, the same configurations of the RBFNN are chosen for all agents.

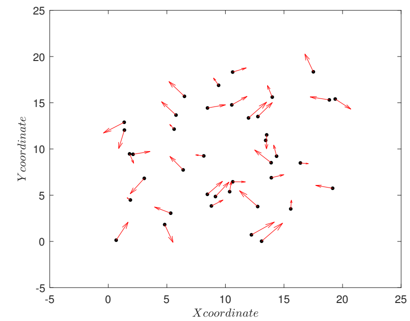


Fig. 1. Initial configuration.

The number of hidden neurons for each RBFNN l are chosen as 5. The centers of the RBFNN basic Gaussian functions are distributed uniformly in the range $[-5, 5] \times [-5, 5]$, and the variances are all set to be 10. The initial weight matrix $\hat{W}_i(0)$ is randomly chosen in the range $[-1, 1]$, and the gains α and β are selected as 0.5, 50 respectively. For the adaptive robust signal, χ, η are set as 2, 0.8 respectively. If $|\tilde{q}| < 1$, then γ is chosen as 0.8, otherwise γ is chosen as 2.

Scenario 1: In the first scenario, the simulation is conducted by applying the proposed robust adaptive flocking control law to the flocking of the multi-agent systems with uncertain nonlinear dynamics and unknown external disturbances. The unknown external disturbances d_i is assumed as

$$d_i(t) = \begin{pmatrix} w_i \exp(-2t) \\ w_i \exp(-t) \end{pmatrix}, \tag{39}$$

where w_i is selected randomly in the range [-3,3]. The uncertain nonlinear dynamics is the same as (38). The simulation results are shown in Figs. 1 - 3. Fig. 2 depicts the final configuration of the 40 agents and the virtual leader after 30 seconds with the robust adaptive flocking control law (7), where the green pentagram represents the virtual leader, and the black line denotes the communication link. From Fig. 2, it is obvious that the 40 agents have the same velocity with the virtual leader and the distance between nearby agents is stable. Fig. 3 is the velocity errors on the X-axis, Y-axis, respectively. It is illustrated that although there have small jumps at the beginning caused by the new neighbors joining, the velocity errors ultimately converge to zero after 10 seconds, which indicates the flocking can be realized by the robust adaptive flocking control law.

Scenario 2: In order to show the practicability and robustness of the robust adaptive flocking control law, a more general disturbance is considered. The unknown external disturbances d_i for every agent is modeled by

$$d_i(t) = \begin{pmatrix} w_i \sin(2t) \\ w_i \cos(-t) \end{pmatrix}. \tag{40}$$

where w_i is also selected randomly in the range[-3,3]. The simulation results are shown in Fig. 4. Fig. 4 is the velocity errors on the X-axis, Y-axis, respectively. It is illustrated that the velocity errors also finally converge to the small

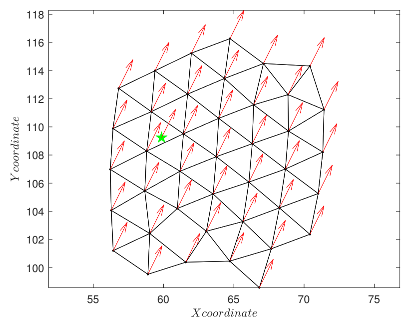
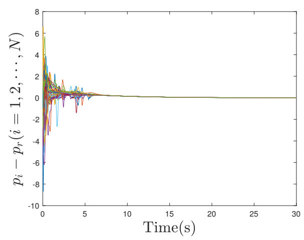
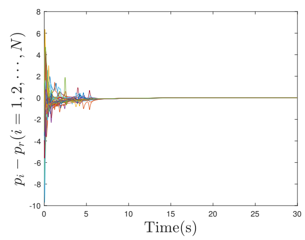


Fig. 2. Final configuration.



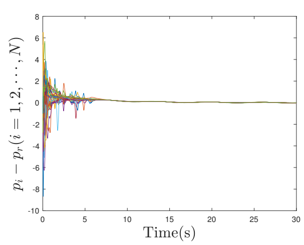
(a) Velocity errors on X-axis



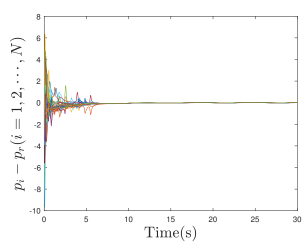
(b) Velocity errors on Y-axis

Fig. 3. Velocity errors on X-axis and Y-axis for the agents under the robust adaptive flocking control law in *Scenario 1*.

neighbour of the origin after 10 seconds, which further demonstrates the effectiveness and robustness of the robust adaptive flocking control law.



(a) Velocity errors on X-axis



(b) Velocity errors on Y-axis

Fig. 4. Velocity errors on X-axis and Y-axis for the agents under the robust adaptive flocking control law in *Scenario 2*.

VI. CONCLUSION

This paper investigates the flocking of the multi-agent systems with uncertain nonlinear dynamics and unknown external disturbances. A decentralized robust adaptive flocking control law is proposed, in which the intrinsic nonlinear dynamics of the agents is allowed to be different from the virtual leader. The uncertain nonlinear dynamics is approximated by the neural network with online updating. The unknown external disturbances and neural network approximation error are counteracted by the designed adaptive robust signal. The stability analysis is theoretically explored through Lyapunov function approach. Two representative simulation results are given to demonstrate the effectiveness of the proposed robust adaptive flocking control law. In our future work, the adaptive flocking with uncertain nonlinear dynamics and time delay will be considered for the flocking of multi-agent systems.

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