A Probabilistic Spectral Graph Matching Algorithm for Robust Correspondence between Lunar Surface Images*

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Abstract— The correspondence between key points is an important problem in lunar surface image processing, and further lays the foundation for the navigation of a rover and the terrain reconstruction of the lunar surface. However, the problem is still challenging due to the existence of large scale and rotation transformations, reflected view of the same scenery, and different illumination conditions between acquired images as the lunar rover moves forward. Traditional appearance matching algorithms, like SIFT, often fail in handling the above situations. By utilizing the structural cues between points, in this paper we propose a probabilistic spectral graph matching method to tackle the point correspondence problem in lunar surface images acquired by Yutu lunar rover which has been recently transmitted to the moon by China's Chang'e-3 lunar probe. Compared with traditional methods, the proposed method has three advantages. First, the incorporation of the structural information makes the matching more robust with respect to geometric transformations and illumination changes. Second, the assignment between points is interpreted in a probabilistic manner, and thus the best assignments can be easily figured out by ranking the probabilities. Third, the optimization problem can be efficiently approximately solved by spectral decomposition. Simulations on real lunar surface images witness the effectiveness of the proposed method.

Index Terms— graph matching, lunar surface image, spectral graph theory, point correspondence

I. INTRODUCTION

Key point correspondence finds applications in many lunar surface image processing problems, such as scenery recognition, lunar surface reconstruction. And it also plays an important role in some lunar rover operations, such as self-location, rover navigation. In the past four decades, various types of approaches have been proposed to tackle the point correspondence problems, including approaches for scenery matching. However, there are rare approaches dedicated for the correspondence of lunar surface images or images acquired on other celestial bodies. Due to some specific characters of these images, the correspondence is still a challenging problem. Taking China's first lunar rover

Yutu for instance, which forms part of Chang'e 3 mission to the moon, the cameras which it is equipped can shoot forward and backward alternately by pan-tilt rotation. As the rover moves on, two images acquired in these two situations may include the same lunar surface scenery. Matching these two images usually suffers from large scale and rotation transformations, reflected view, and different illumination conditions. How to get a robust correspondence between key points in the two images is an important problem to be settled.

Though traditional appearance matching algorithms, like SIFT [17], have been widely used in various correspondence problems [21], they often fail in matching the lunar surface images. One important reason is that the appearance descriptors around the same point may vary significantly in the two lunar surface images. These algorithms only take into account the appearance similarity, but ignore other useful information in the image, like the structural relations between points.

In this paper, by incorporating structural information we propose a probabilistic spectral graph matching method to tackle the point correspondence problem in lunar surface images acquired by Yutu lunar rover. Specifically, graph vertices are utilized to represent the points detected in the images, and graph edges are utilized to indicate the adjacency between points, with structural relation measures, like distances, orientations as their weights. The point correspondence between images is then formulated as a graph matching problem, and effectively optimized by the proposed graph matching algorithm. Note traditional appearance matching could also be incorporated in the pairwise matching by treating point descriptor as the vertex label.

Generally, compared with traditional algorithms, the proposed method has three advantages. First, the incorporation of the structural information makes the matching more robust with respect to geometric transformations and illumination changes. Second, the assignment between points is interpreted in a probabilistic manner, and thus the best assignments can be easily figured out by ranking the probabilities. Third, the optimization problem can be efficiently approximately solved by spectral decomposition. We compare the

proposed method with the state-of-the-art algorithms and validate its effectiveness on real lunar surface images.

II. Previous Works

Establishing correspondence between points has long be a fundamental problem in computer vision, and a large number of approaches have been proposed to tackle the problem in the past forty years. These approaches can be divided into three categories according the information types, which are respectively appearance information, pairwise information, and high-order information. And the latter two types are structural information.

Appearance matching, which explores the appearance similarity between points, has been widely used in the correspondence problem. Typical methods include SIFT [17], shape context [1], and bag-of-words [13]. These methods typically form an appearance descriptor around every point and construct the correspondence by minimizing the dissimilarity between these descriptors. Such a problem could also be treated as a bi-partite graph matching problem which is equivalent to minimizing a unary term, and can be efficiently solved by linear programming techniques, such as the Hungarian algorithm, and the interior point method.

Graph matching which incorporates pairwise constraints demands to preserve the edge-to-edge similarity. Taking distance constraints for instance, if two points are close in one image, then their corresponding points in the other image should also be kept close. Pairwise graph matching is usually divided into exact graph matching and inexact graph matching. The exact graph matching generally works on small scale correspondence problems or specific graph types like trees, and demands strict structure preservation between edges. By contrast, the inexact graph matching can tolerate certain differences between corresponding edges, and is more popular in computer vision. However, the inexact graph matching is still an NP-hard problem, and the approximate methods are necessary to make the problem tractable. The famous graduated assignment [8] is one of the earliest approximate graph matching algorithms, which is still considered as state-of-the-art due to its excellent performance and extensibility. This method first relaxes the discrete domain to its convex hull, and then gradually projects the continuous solution to be a discrete one by adjusting a deterministic annealing parameter. Similar idea appears in the recently proposed path following algorithms [23], [16], [25], also known as the convex-concave-relaxation-procedure (CCRP) which, by constructing the convex and concave relaxations of the original objective function, guarantees that a final discrete solution could be obtained. The graduated non-convexity and concavity procedure (GNCCP) [15], [22] further simplifies CCRP by the implicit construction of the convex and concave relaxations. The spectral decomposition based algorithms form another important group of approx-

imate methods. Umeyama's algorithm [20] is regarded as the first spectral graph matching algorithm. The method proposed in [11] formulates the objective fundtion based on an affinity matrix, and seeks its rank one approximation by spectral decomposition. Based on [11], many methods [5], [4], [12], [7] have been further proposed with considerable improvements. The method proposed in this paper is also a generalization of [11], which interprets the assignment in a probabilistic manner. The spectral methods also involve relaxing the discrete domain to be a continuous one, but there exist no effective ways to project the continuous solution back to be a discrete one.

Recently, a few high-order graph matching algorithms [18], [6], [3], [10], [24] have been proposed to incorporate high-order structural information, which preserve the similarity between triplets beyond pairwise constraints. These methods typically extend the pairwise spectral decomposition algorithm [11] and seek the rank one approximation of an affinity tensor. Though these methods improve the matching accuracy to some extent, the costs are great time and storage expenses.

III. Problem Formulation

Supposing a point set $G = \{g_i\}_{i=1}^{M}$ has been abstracted from every lunar surface image, a labeled weighted graph $\mathcal{G} = \{V, E, L, W\}$ can be used to represent the point sets, where $V = \{1, 2, \cdots, M\}$, $E \in V \times V$, $L = \mathbb{R}^{M \times d_l}$, $W = \mathbb{R}^{\|E\|_0 \times d_w}$ denote the sets of vertices, edges, vertex labels, and edge weights respectively, and d_l, and d_w are respectively the dimensions of each vertex label, and each edge weight. A vertex $i \in V$ is used to represent the point g_i, and an edge $\{i, j\} \in E$ is to denote the adjacency between vertices i and j. The appearance descriptor $l_i \in \mathbb{R}^{1 \times d_l}$ around the point g_i can be assigned as a label to vertex i. For instance, by utilizing a 128-dimensional SIFT histogram abstracted from the patches round each key point as the appearance descriptor, there is $d_l = 128$. The pairwise descriptor $w_{ij} \in \mathbb{R}^{1 \times d_w}$ between points i and j is assigned as a weight to edge $\{i, j\}$. For instance, by utilizing distances and orientations between points as relation descriptors, there is $d_w = 2$. Hereafter by the term *graph* we mean such a labeled weighted graph.

Given two graphs $\mathcal{G}^1 = \{V^1, E^1, L^1, W^1\}$ of size M and $\mathcal{G}^2 = \{V^2, E^2, L^2, W^2\}$ of size N, where without loss of generality it is assumed $M \leq N$, the associated graph $\mathcal{A}$, also known as the modular product of graphs, will be constructed below. Each assignment (i, a), where $i \in V^1$, $a \in V^2$, is treated as a vertex of the $\mathcal{A}$. The vertices (i, a) and (j, b) are considered to be adjacent, if $\{i, j\}$ and $\{a, b\}$ are respectively adjacent, where *adjacent* means an edge exists between the two vertices. The similarity measure $l_{(i,a)}$ between l_i and l_a is assigned as a label to vertex (i, a). And the consistency measure $w_{(i,a)(j,b)}$ between edges $\{i, j\}$ and $\{a, b\}$ is assigned as a weight to edge $\{(i, a), (j, b)\}$. Note such the definition of associated graph is a little different

from its traditional definition, which further considers (i, a) and (j, b) to be adjacent, if *neither* of $\{i, j\}$ and $\{a, b\}$ is adjacent. By contrast, in our definition, it is not considered to be adjacent, or say, the weight of edge $\{(i, a), (j, b)\}$ is zero. The advantage of such definition is sparsity, which results in more efficient computation and storage saving, as shown in section V.

The explorations and studies of graphs are usually through the manipulations of the matrices naturally associated with the graphs. Weighted adjacency matrix is one most common type of matrix associated with the weighted graph, which records the adjacency relations and the weights in the graph. In previous works [11], [25], [7], the adjacency matrix for the associated graph is often called *affinity matrix* $\mathbf{A}$. Since the dimension of $l_{(i,a)}$ is always 1, it is located in the diagonal of $\mathbf{A}$. Particularly, $\mathbf{A}$ can be defined by

$$
\begin{aligned}
\mathbf{A}_{ij} &= \mathbf{A}_{[(a-1)M+i][(b-1)M+j]} \\
&= \begin{cases}
(1-\alpha)\, l_{(i,a)}, & \text{if } i = j, a = b, \\
\alpha\, w_{(i,a)(j,b)}, & \text{if } i \neq j, a \neq b, \text{ and } \{i,j\}, \\
& \quad \{a,b\} \text{ are both adjacent} \\
0 & \text{otherwise}
\end{cases}
\end{aligned}
\tag{1}
$$

where α is used to balance the appearance similarity and pairwise consistency.

The problem to be solved is to find the assignments between V^1 and V^2, which can be represented by an assignment matrix $\mathbf{X} \in \{0, 1\}^{M \times N}$. $\mathbf{X}_{ia} = 1$ means vertex i in $\mathcal{G}^1$ is assigned to a in $\mathcal{G}^2$. If the one-to-one mapping constraints are further considered as follows,

$$
\sum_{i=1}^{M} \mathbf{X}_{ia} \leq 1, \sum_{a=1}^{N} \mathbf{X}_{ia} = 1, \mathbf{X}_{ia} = \{0, 1\}
\tag{2}
$$

$\mathbf{X}$ becomes a partial permutation matrix.

Based on the above notations and definitions, the objective function is given by

$$
\begin{aligned}
\mathbf{x} = \arg\max\ &\mathbf{x}^T \mathbf{A} \mathbf{x}, \\
\text{s.t. } &(\mathbf{I}_N \otimes \mathbf{1}_M^T) \mathbf{x} \leq \mathbf{1}_N, (\mathbf{1}_N^T \otimes \mathbf{I}_M) \mathbf{x} = \mathbf{1}_M, \\
&\mathbf{x} \in \{0, 1\}^{MN},
\end{aligned}
\tag{3}
$$

where $\mathbf{x}$ is the column replica of $\mathbf{X}$, and $\otimes$ is the Kronecker product between matrices. The constraints in (3) are the one-to-one mapping constraints, the same with (2). Actually (3) is a weighted sum of the corresponding vertex similarity measures and corresponding edge consistency measures after a permutation. However, solving (3) is an NP-hard problem, where some approximations are necessary, as will be shown in the next section.

IV. PROBABILISTIC SPECTRAL GRAPH MATCHING

An intuitive scheme to approximately solve the optimization problem (3), which is an NP-hard combinatorial optimization problem, is first to relax the discrete domain to be continuous, and then to project the continuous solution back to be a discrete one [19]. A representative algorithm of such scheme is proposed in [11]. [11] abandons all mapping constraints in (3), and adds the constraints that

$$
\mathbf{x}^T \mathbf{x} = 1.
\tag{4}
$$

The objective is then transformed to be Rayleigh quotient as

$$
\mathbf{x} = \arg\max \frac{\mathbf{x}^T \mathbf{A} \mathbf{x}}{\mathbf{x}^T \mathbf{x}}.
\tag{5}
$$

It is well known that (5) can be solved by finding the principal eigenvalue λ_1 and its corresponding eigenvector $\mathbf{x}^{\lambda_1}$. That is $\mathbf{x}^{\lambda_1}$ is the solution of (5). An effective and efficient method to obtain λ_1 and $\mathbf{x}^{\lambda_1}$ is the power method [9]. Since the vertex similarity measure $l_{(i,a)}$ and edged consistency $w_{(i,a)(j,b)}$, which forms $\mathbf{A}$, are usually symmetric and nonnegative, based on Peron-Frobenius theorem, we have λ_1 and $\mathbf{x}^{\lambda_1}$ both exist and $\mathbf{x}^{\lambda_1}$ is nonnegative. Finally, the continuous solution of (5) is projected to be a discrete vectorized assignment matrix based on the mapping constraints.

[11] actually solves the rank-one approximation problem of $\mathbf{A}$. The major drawback of [11] is that the mapping constraints are totally ignored in the optimization process, making the continuous solution less distinctive over certain vertices. To tackle the problem, next a spectral method is proposed, which incorporates the mapping constraints in each iteration by introducing the probabilistic interpretation of the assignments.

Before proceeding to the method, $\mathbf{A}$ is preprocessed as follows,

$$
\bar{\mathbf{A}} = \mathbf{A} \mathbf{D}^{-1},
\tag{6}
$$

where $\mathbf{D}$ is the diagonal degree matrix defined by

$$
D_{ii} = \begin{cases}
\sum_{i=1}^{MN} \mathbf{A}_{ij} & \text{if } i = j, \\
0 & \text{if } i \neq j.
\end{cases}
\tag{7}
$$

By such a column stochastic processing, each term $\bar{\mathbf{A}}_{(i,a)(j,b)}$ represents the belief of vertex (i, a) from the perspective of (j, b), i.e. the conditional probability $P((i, a)|(j, b))$. On the other hand, the preprocessing can be explained following the way in [5], which is to make the affinity measure more balanced. In [5], a doubly stochastic processing is adopted, and significant improvement in matching accuracy is observed.

Accordingly, each assignment is interpreted in a probabilistic manner, which is realized by setting

$$
\mathbf{x}^T \mathbf{1} = 1, 0 \leq \mathbf{x}_i \leq 1.
\tag{8}
$$

Thus each item in $\mathbf{x}$ represents a probability $P(i, a)$ of the corresponding assignment. If applying $\bar{\mathbf{A}}$ to $\mathbf{x}$ as

$$
\mathbf{x}^{t+1} = \bar{\mathbf{A}} \mathbf{x}^t,
\tag{9}
$$

then the probability for each assignment is updated by

$$P^{t+1}(i,a) = \sum_{(j,b)} = P^t((i,a)|(j,b))P^t((j,b)). \quad (10)$$

If iterating (9) until the convergence of $\mathbf{x}$, the stable distribution over all the assignments will be obtained. However, the mapping constraints in (2) are still not considered in the above scheme. Since in (2) $\sum_i \sum_j \mathbf{X}_{ij} = M$ is always a constant, beyond (8) the relaxed continuous domain is then

$$(\mathbf{I}_N \otimes \mathbf{1}_M^T)\mathbf{x} \le \frac{\mathbf{1}_N}{M}, (\mathbf{1}_N^T \otimes \mathbf{I}_M)\mathbf{x} = \frac{\mathbf{1}_M}{M}, 0 \le \mathbf{x}_i \le 1. \quad (11)$$

Note $\mathbf{x}^T \mathbf{1} = 1$ is implicitly realized in (11). Thus after each update in (9), $\mathbf{x}^{t+1}$ must be normalized to fulfill (11).

To further avoid the low distinctiveness encountered in the traditional spectral method [11] discussed above, in each update iteration, we also update $\bar{\mathbf{A}}$ to make it more distinctive over the beliefs between assignments. The basic idea is that if the probability $P((i,a))$ of an assignment (i,a) increases in the $t+1$th update given the $\bar{\mathbf{A}}^t$, then the conditional probability $P((i,a)|(j,b)), \forall (j,b)$ should also be increased, and vice versa. This idea can be realized by

$$\bar{\mathbf{A}}^{t+1} = \mathbf{E}\bar{\mathbf{A}}^t, \quad (12)$$

where $\mathbf{E}$ is a diagonal matrix with diagonal items defined by

$$\mathbf{E}_{ii} = \frac{\mathbf{x}_i^{t+1}}{\mathbf{x}_i^t}. \quad (13)$$

Generally, the probabilistic spectral graph matching can be summarized in Algorithm 1.

Algorithm 1: Probabilistic spectral graph matching

Input: Two graphs $\mathcal{G}^1$ and $\mathcal{G}^2$

  Construct the affinity matrix $\mathbf{A}$ by (1)

  Compute $\bar{\mathbf{A}}^0$ by column stochastic processing (6)

  Initialize $\mathbf{x}^0 = \frac{\mathbf{1}_{MN}}{MN}$

  repeat

    $\mathbf{x}^{t+1} = \bar{\mathbf{A}}^t \mathbf{x}^t$

    Transform $\mathbf{x}^{t+1}$ to be a matrix $\mathbf{X}$

    repeat (To make $\mathbf{x}^{t+1}$ fulfill (11))

      $\mathbf{X} = \frac{\mathbf{C}^{-1}\mathbf{X}}{M}$,

      where $\mathbf{C} \in \mathbb{R}^{M \times M}$ is diagonal with $\mathbf{C}_{ii} = \sum_j \mathbf{X}_{ij}$

      $\mathbf{X} = \frac{\mathbf{X}\mathbf{R}^{-1}}{M}$,

      where $\mathbf{R} \in \mathbb{R}^{N \times N}$ is diagonal with $\mathbf{R}_{jj} = \sum_i \mathbf{X}_{ij}$

    until $\mathbf{X}$ converges

    Transform $\mathbf{X}$ back to be a vector $\mathbf{x}^{t+1}$

    $\bar{\mathbf{A}}^{t+1} = \mathbf{E}\bar{\mathbf{A}}^t$ where $\mathbf{E}$ is given by (13)

  until $\mathbf{x}$ converges

Output: A probabilistic assignment vector $\mathbf{x}$

The proposed method shares some similarities with another probabilistic graph matching algorithm proposed in [7], such as the probabilistic interpretation and refinement of the affinity matrix. The difference is that the proposed method focuses on the associated graph.

V. LUNAR SURFACE CORRESPONDENCE

In this section, the overall scheme for the correspondence between lunar surface images will be given, together with postprocessing and some discussions.

There are various ways to extract key points in the lunar surface images. One most common way is by corner point extraction algorithms, such as the Harris operator, the Laplacian of Gaussian (LOG) operator. The points can also be uniformly sampled from the edges extracted from the images by such as Canny operator. Another way is to manually label the key points in some special applications. Then the appearance descriptor around each key point is built, such as SIFT descriptor, shape context descriptor. Also the pairwise descriptors like distances and orientations between points are computed. And the graphs representing point sets and further the associated graph are constructed accordingly. Then following the way described in Section IV, a probabilistic assignment $\mathbf{x}$ is obtained. To get the final correspondence between the two point sets, postprocessing to discretize $\mathbf{x}$ is necessary. Thanks to the probabilistic interpretation of the assignments, the best S assignments can be selected by ranking the probabilities following the winner-take-all strategy. Alternative methods include threshold method, RANSAC, or solving the linear programming problem

$$\mathbf{x}* = \arg\max_{\bar{\mathbf{x}}} \bar{\mathbf{x}}^T \mathbf{x} \quad (14)$$

$$\begin{bmatrix} \mathbf{I}_N \otimes \mathbf{1}_M^T \\ \mathbf{1}_N^T \otimes \mathbf{I}_M \end{bmatrix} \bar{\mathbf{x}} \le \mathbf{1}_{N+M}, \mathbf{1}^T\bar{\mathbf{x}} = S, 0 \le \bar{\mathbf{x}}_i \le 1,$$

by for example interior point method [2].

The overall lunar surface correspondence scheme is summarized in Algorithms 2.

Algorithm 2: Lunar surface correspondence

Input: Two lunar surface images I^1 and I^2

  Extract the point sets G^1 and G^2 from I^1 and I^2

  Build the appearance descriptor l_i for each key point

  Build the pairwise descriptor w_{ij} for each edge

  Build two graphs $\mathcal{G}^1$ and $\mathcal{G}^2$

  Get the probabilistic assignment vector $\mathbf{x}$ by Algorithm 1

  Select S best assignment by ranking the probabilities

Output: An assignment vector $\mathbf{x}^*$ with S '1's

The proposed method incorporates the pairwise structural cues into the traditional appearance matching. Since the pairwise relations are invariant to geometric transformations and illumination changes, the proposed method becomes more robust compared with traditional methods.

By interpreting the assignment in a probabilistic manner, the belief of each assignment can be easily figured out by its probability. Sometimes, a fixed number S of best assignments are needed to get the best matching accuracy rather than the largest matching point number. This can be easily achieved by ranking the probabilities.

The optimization problem can be efficiently approximately solved by spectral decomposition, and the proposed method can be applied to sparse graphs. The computational complexity is $\mathcal{O}(\|E^1\|_0\|E^2\|_0)$, where $\|E^1\|_0$ and $\|E^2\|_0$ respectively denote the edge number in $\mathcal{G}^1$ and $\mathcal{G}^2$. More sparse graph structure leads to more efficient optimization.

VI. SIMULATIONS

In the section, the proposed method is applied to a real lunar surface image dataset acquired by Yutu rover. Some image samples in the dataset are illustrated in Figure 1. According to point extraction methods, the simulations are divided into two parts. The first part manually labels all the points, and compares the proposed method with the state-of-the-art algorithms to assess its performance. The second part automatically extracts the points by LOG [14] operator to evaluate the real performance of the proposed method.

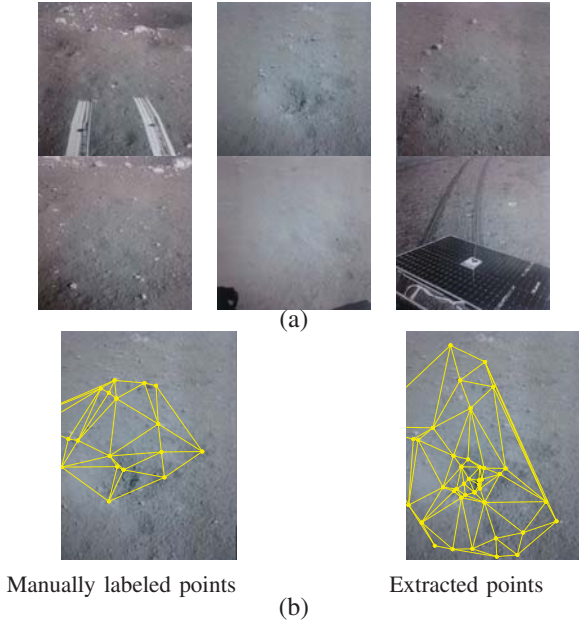Manually labeled points   Extracted points

(b)

Fig. 1. Dataset acquired by Yutu rover. (a) Some lunar surface image samples. (b) Labeled points. On the left the points are manually labeled, and on the right the points are extracted by LOG operator.

In the following simulations, the similarity measure between the appearance descriptors is defined by

$$l_{i,a} = \exp(-\frac{\|l_i - l_a\|_2^2}{\sigma_l}). \tag{15}$$

The pairwise consistency measure is defined based on normalized distances and orientations between points by

$$w_{(i,a)(j,b)} = \exp(-\frac{\|d_{ij} - d_{ab}\|_2^2 + \|o_{ij} - o_{ab}\|_2^2}{\sigma_w}), \tag{16}$$

where d_{ij} and o_{ij} respectively denote the normalized length and orientation of edge ij. The orientation is measured by the acute angle between ij and a horizontal line, and thus

$\mathbf{A}$ defined in (1) is a symmetric matrix. σ_l and σ_w are the kernel width parameters.

The algorithm included for comparison are spectral matching (SM) [11], graduated assignment (GA) [8], probabilistic graph matching [7]. Shape context is adopted as the appearance descriptor. The correspondence with only appearance descriptors is transformed into a bipartite graph matching problem which is solved by Hungarian algorithm (HUN). The proposed method is denoted by OUR.

All the simulations are carried out in Matlab 2011a on 3.07 GHz CPU (two core) and 2.00 GB RAM. SM, GA, and PGM are re-implemented by us, and HUN are implemented by a publicly available mex file.
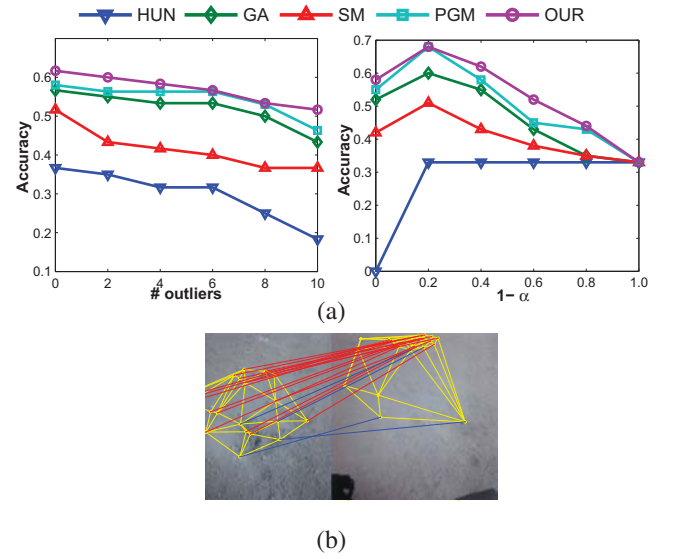

(a)

(b)

Fig. 2. Results on manually labeled points. (a) Matching accuracy comparison w.r.t outlier number and parameter α. (b) Matching samples achieved by OUR. The red line denotes the right assignment

A. On manually labeled points

In this simulation, the proposed method is compared with the state-of-the-art algorithms on manually labeled points where ground truth is available. 10 pairs of images are manually labeled with 30 ground truth points, as illustrated in Figure 1. The graph is built in a sparse manner by Delaunay triangulation. The similarity between vertices and edges are computed by (15) and (16), and we set $\sigma_l = 0.5$ and $\sigma_w = 0.15$.

We first compare the matching accuracy with respect to outlier number which is increased from 0 to 10 with a step size 2. α is set to be 1 which means only the structural information is utilized. The result is depicted in Figure 2, from which we can see that, first, the performances of all the algorithms deteriorate as the outlier number increases; second, OUR outperforms all the other algorithms; third, utilizing only the appearance similarity, HUN achieves the

lowest matching accuracy, compared with the other pairwise algorithms.

To evaluate the dependency of matching accuracy from parameter α, we also make a comparison as α is decreased from 1 to 0 with a step size -0.2. The results is given in Figure 2. We can observe that the best performance are usually achieved when α locates in between 0 and 1, where $\alpha = 1$ implies that only the pairwise consistency is utilized, and $\alpha = 0$ implies that only the appearance similarity is utilized.

One matching sample achieved by OUR is illustrated in Figure 2.

B. On extracted points

In the simulation, the proposed method is applied to points automatically extracted from images by LOG operator to evaluate its real performance. The graph structure is also constructed by Delaunay triangulation. One labeled sample is shown in Figure 1. We also set $\sigma_l = 0.5$ and $\sigma_w = 0.15$. Since the ground truth is not available, we choose to compare the optimal objective values achieved by different algorithms. The results are depicted in Figure 3, which witnesses OUR achieves the best performance among all the algorithms.

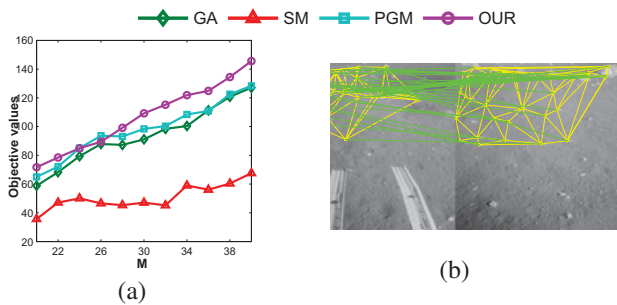One matching sample achieved by OUR is illustrated in Figure 3.

Fig. 3.   Results on extracted points. (a) Matching accuracy comparison w.r.t problem size M. (b) Matching samples achieved by OUR.

VII. CONCLUSION

In this paper a probabilistic spectral graph matching method is proposed to deal with the correspondence problem in lunar surface images. Compared with traditional methods, the proposed method has three advantages. First, the incorporation of the structural information makes the matching more robust with respect to geometric transformations and illumination changes. Second, the assignment between points is interpreted in a probabilistic manner, and thus the best assignments can be easily figured out by ranking the probabilities. Third, the optimization problem can be efficiently approximately solved by spectral decomposition. The proposed method is applied to the real lunar surface images acquired by China's first lunar rover Yutu, which witnesses its effectiveness.

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