Proceedings of the 10th
World Congress on Intelligent Control and Automation
July 6-8, 2012, Beijing, China

A Vison-based Grasping Strategy for the Mineral Sorting*

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***Abstract*—In industry application, e.g, mineral sorting, various mineral should be picked from the conveyer. The problem can be regarded as grasping prior unknown objects. In this paper, we aim to present a method to grasp the objects with a three-finger gripper guided by vision. Based on our previous works on attractive region, the way to locate a stable grasp is illustrated in the rotation space of the object. More specifically, the down-hill direction of the attractive region is used to guide the motion of the camera. And then, a grasp planar is proposed to generate hypothetic grasps from the contour of object. The vision and the grasping detection are coupled to reconstruct only few points for contacting, such that, the method can be conducted effectively. At last, the efficiency of the method is illustrated by a simulation.**

***Index Terms*—three-finger grasp, Vision-guided grasp, inscribed equilateral triangle**

I. Introduction

The ability of effectively grasping various objects is essential for robots to interact with the environment, such as the mineral sorting in the mining industry. Grasp synthesis and analysis for prior known objects have been extensively investigated in the last twenty years. However, grasp planning strategy on practical objects has always been a challenge for the difficulties in modeling real-life objects, errors in sensing and end effecter's positioning, and high computational complexity.

In this paper we discuss the grasp of prior unknown 3D objects with a three-finger gripper. We aim to give a vision-based grasping strategy with real-time performance.

Most works in the area consider model-based grasp synthesis, i.e., supposing that the object's geometric model is a priori known. Grasp synthesis has been identified as a geometrical problem and algorithms were developed to construct stable contact regions for polygonal objects [1]. Algorithms have also been proposed to construct force- or form-closure grasp for 2D polygonal object [3] [4], 3D polyhedral [5] [6] and curved objects [7] [8] [9]. Some approaches take the object's discrete point cloud model as input with no specific constraints on them [10].

However in practical applications, e.g. mineral sorting, the position, orientation and the accurate model of mineral are hard to obtain. Vision improves the flexibility of the grasping. In most applications, vision has been used to reconstruct the geometrical model the objects. For 2D object, the boundary is extracted and feasible grasp configurations are found satisfying several geometrical constrains in [11]. A fast algorithm has been proposed in [12] by minimizing a cost function. Moreover, to make it more applicable grippers' constraints should be also considered [13] [14]. These works have relatively low computational effort with planar or prismatic assumption on object's shape. For 3D objects, the target object was reconstructed with vision [22] and laser scanner [15]. These approaches typically suffer from the problem of high computational complexity in both reconstruction and grasp synthesis. Their time consumption are unacceptable for the industry. To tackle this problem, the process of reconstruction and grasp planning were merged [16]. The camera rotates around the target object to approximate its geometric model with a sphere and fingers floats on the model to find a local optimal grasp configuration. Reference [17] provided an interesting strategy integrating grasp planning with active vision control. Active contour was fit to the object and relative depth of contact points was estimated through motion. However, only antipodal grasps were derived rather than stable ones.

This paper contributes through two aspects. Firstly, we constructed "attractive regions" on the grasping with three-finger grippers to find a stable grasp. Properties on the each part of the "attractive regions" are also described. Secondly, we introduce this theory's usage in vision guided grasp planning process with algorithm proposed for both 2D grasp planning and camera's motion planning. In our work, we also move camera through a sequence of viewpoints, and a stable grasps are finally achieved, guided by the information provided by last images towards the stable grasp configuration.

The rest of this paper is arranged as follows. Section II describes the problem and the general framework to solve it. The definition of attractive region based on the observation of several non-linear rigid contact dynamic systems is formally introduced in Section III. Illustration and properties of attractive regions together with practical algorithms are given in 2D (Section IV) and 3D (Section V) condition respectively. Experimental result is shown in Section VI. Section VII draws the conclusion of this paper.

* The work was supported in part by the National Natural Science Foundation (NNSF) of China under Grants 61033011, 61101221, 61105085, by Chinese Academy Of Sciences Visiting Professorship For Senior International Scientists under Grant 2011T1G91, by the 863 Program of China under Grant 2007AA04Z228.

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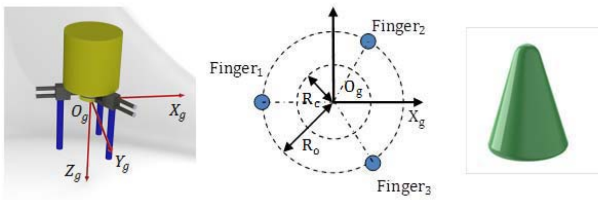
II. PROBLEM DESCRIPTION AND BASIC APPROACH

In our discussion, three cylindrical fingers are mounted on three guiding trails of the gripper. The trails are fixed on the palm at 120 degrees angles from each other. The coordinate is



(a) The three-finger gripper (b) Bottom View (c) A cone like mineral

Fig. 1 The gripper and a mineral to be grasped.

built as shown in Fig.1. The origin of the gripper coordinate, denoted as O_g, lies in the center of the palm's bottom end. X_g points to the opposite direction of F_1 in the plane perpendicular to the fingers, Y_g is perpendicular to X_g and points to F_2, and Z_g points downwards along the fingers. The three fingers move simultaneously, i.e., the distances D between each finger F_i to the center of the gripper O_g is always the same value. As an articulated gripper, nonbackdrivable mechanism is also utilized. The radius indicating the limits for the gripper when it is fully open or closed are denoted as R_o and R_c.

In this paper, we consider the mineral with smooth surface (Fig.1(c)). The projection to a plane is a simple closed curve. And we only consider the apparent contour of the object, the internal boundaries are ignored.

In the following, we mainly discuss the grasping of smooth object with the given three-finger gripper. We regard a finger as a line in 3D case and a point in 2D case. And also, we will detail on the situation where a single contact points appears on one finger. As to the situation when multiple contact points appear on one finger, same method can be used by traversing all possible combinations.

The basic idea is to conduct sensing, grasp planning and control on the viewpoint simultaneously (Fig.2). The contour is obtained by simple thresholding method, and then a cubic B-spline is fit to the contour. And the partial 3D reconstruction can be done with existing technique. We focus on 2D grasp planning and next-best-view determination in this paper.

III. ATTRACTIVE REGION AND RIGID CONTACT CONSTRAINT

In this section, we will provide the definition of the attractive region first, and then the gripper is described. The rigid body contact constraint on three-finger grasping is briefly reviewed. We will use these constraints as tools for further analysis.

A. *Attractive Region*

The term "attractive region" is defined as follows [18]: Assume the nonlinear dynamic system we are studying can be presented as $dX/dt = f(X,u,t)$, where X is the state of the system and u is the external input. If there exists a function $g(X)$ and for some positive real number ε satisfying: for all state X in the region $||X - X_0|| < \varepsilon$, where X_0 is a state of the system:

(1) $g(X) > g(X_0), X \neq X_0$
(2) $g(X) = g(X_0), X = X_0$
(3) $g(X)$ has continuous partial derivatives with respect to all components of X
(4) $dg(X)/dt < 0$

B. Then the region $||X - X_0|| < \varepsilon$ is called the "attractive region" formed by the environment, and X_0 is called the stable state of the attractive region. Rigid body contact constraints

Assuming both the object and fingers are rigid body. The motion of the object constrained by the fingers can be expressed as follows [2]. For D dimensional space, we denote $\boldsymbol{p}_i \in \mathbb{R}^D$ as the vector representing the contact points, $\boldsymbol{n}_i \in \mathbb{R}^D$ as the contact normal pointing into the object, where $(i = 1, \ldots, n_c)$ and n_c is the number of contact points. Then the grasp matrix is:

$$G = \begin{bmatrix} \boldsymbol{I}_D & \cdots & \boldsymbol{I}_D \\ C(\boldsymbol{p}_1) & \cdots & C(\boldsymbol{p}_{n_c}) \end{bmatrix} \tag{1}$$

where $C(*)$ is the cross-product matrix of matrix $*$. We denote the instantaneous speed and angular speed of the object as $\dot{\boldsymbol{u}} = (\boldsymbol{v}^T, \boldsymbol{\omega}^T)^T$, where $\boldsymbol{v} \in \mathbb{R}^D$ and $\boldsymbol{\omega} \in \mathbb{R}^{\frac{1}{2}D\,(D-1)}$. Then the instantaneous speed of the ith contact point is $\hat{\boldsymbol{v}}_i = \boldsymbol{v} + \boldsymbol{p}_i \times \boldsymbol{\omega}$. According to the rigid body assumption, we have

$$\hat{\boldsymbol{v}}_i \cdot \boldsymbol{n}_i \geq 0 \tag{2}$$

It is easy to see that $\begin{bmatrix} \hat{\boldsymbol{v}}_1 \\ \vdots \\ \hat{\boldsymbol{v}}_{n_c} \end{bmatrix} = G^T \dot{\boldsymbol{u}}$, hence

$$N^T G^T \dot{\boldsymbol{u}} \geq 0 \tag{3}$$

Where $N = \begin{bmatrix} \boldsymbol{n}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \boldsymbol{n}_{n_c} \end{bmatrix}$. It should be noted that all the contact normal have zero z component during to the parallel structure of the gripper.

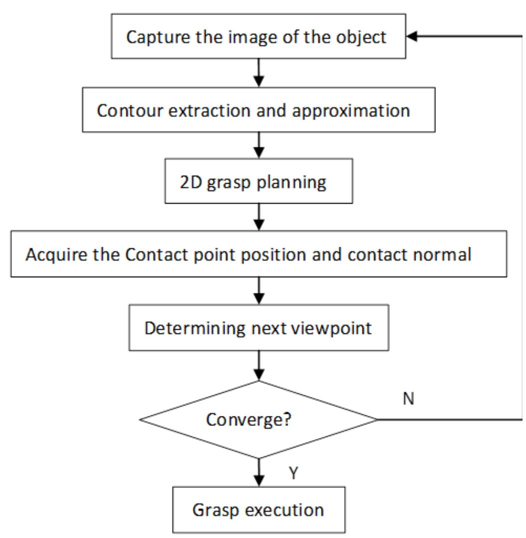


Fig. 2 The process in which sensing and grasp planning are conducted alternatively.

IV. Attractive Region Used in 2-D Grasping

In this section, attractive region formed when grasping a simple closed curve is illustrated. And, the mathematical explanation on the relation between grasp stability and each part of the attractive is presented.

Every equilateral triangle on the plane can be represented with four parameters (x, y, θ, δ), where (x, y) is the center of the triangle and θ is the angle between the triangle and a fixed frame, δ is the radius. In the 2-D grasping context, these parameters can also be taken as the relative position, orientation and opening radius of the gripper. Davidson et al. [19] pointed out that, when grasping a smooth contour, all the equilateral triangle grasp with all fingers touching the contour form a one-dimensional manifold.

As a matter of fact, it has been proven that: all but at most two points of any closed curve are vertex points of an equilateral triangle inscribed in a simple closed curve $\mathcal{C}$ [20]. Hence there is a dense point set D on the contour, on which we can build a piecewise continuous mapping $(x, y, \theta, \delta) = f(t)$, where t is used to parameterize the curve. Since it forms a one-dimensional manifold, we can find a mapping $\delta = \delta(\theta)$. If we denote θ as the state and δ as the energy function, an attractive region can be easily found on the mapping curve (Fig.3).

The continuous parts of the curve can be divided into several "bowls", each of them being a *feasible attractive region* or a *trap region*. A feasible attractive region contains grasp configurations where the object cannot escape by pure translation constrained by the fingers (Fig.3 (b, c)), while it is not true in the trap regions. We now provide the mathematical analysis on different parts of the attractive region.

1) Points at the bottom of the feasible attractive region

First of all, we propose two conditions that combine contact restraint with attractive region. The first condition is call *first order partial stable condition*. It is defined as the sufficient and necessary condition of: $N^T G^T \dot{\boldsymbol{u}} \geq \boldsymbol{0}$ ***shares the same solution space with*** $N^T G^T \dot{\boldsymbol{u}} = \boldsymbol{0}$.

For the points at the bottom of the attractive regions, the first order partial stable condition must suffice. Since if there exists a $\tilde{\boldsymbol{u}}$ such that $N^T G^T \tilde{\boldsymbol{u}} \geq \boldsymbol{0}$ and $N^T G^T \tilde{\boldsymbol{u}} \neq \boldsymbol{0}$, then with an infinite small motion $\tilde{\boldsymbol{u}}dt$ of the object, a "gap" will arise between the object and the fingers. This permits the contraction of three fingers towards the center.

The second condition is *non-translational escape condition*. Considering the following inequality

$$N^T [I_2 \quad \dots \quad I_2] \boldsymbol{v} \geq 0. \tag{4}$$

The non-translational escape condition is equivalent to (4) having a null solution space.

The grasp which satisfies both conditions achieves stable. Since any rotation of the object will cause an increment of δ, the freedom of rotation is restrained. And since the non-translational escape condition is satisfied, any pure translational motion is forbidden by the fingers.

As a matter of fact, there is an explicit expression of the first order partial stable condition. By abusing the usage of denotation, we call lines with direction $\boldsymbol{n}_i$ passing through $\boldsymbol{p}_i$ as the contact line.

Proposition 1: The first order partial stable condition is achieved when the three contact lines intersect in one point.

Proof: The contact normal vector is denoted by $\boldsymbol{n}_i = (n_{xi}, n_{yi})$, contact points are denoted by $\boldsymbol{p}_i = (p_{xi}, p_{yi}), i = 1,2,3$ where

$$\begin{cases} p_{x1} = -\delta,\ p_{yi} = 0 \\ p_{x2} = \delta/2,\ p_{y2} = \sqrt{3}\delta/2 \\ p_{x1} = \delta/2,\ p_{yi} = -\sqrt{3}\delta/2 \end{cases}$$

Then

$$Z = N^T G^T = \begin{bmatrix} n_{x1} & n_{y1} & -n_{x1}p_{y1} + n_{y1}p_{x1} \\ n_{x2} & n_{y2} & -n_{x2}p_{y2} + n_{y2}p_{x2} \\ n_{x3} & n_{y3} & -n_{x3}p_{y3} + n_{y3}p_{x3} \end{bmatrix}.$$

It should be noted that $Z\dot{\boldsymbol{u}} \geq 0$ always have solution with three contact points. The linear programming based method to solve this inequality is provided in [2].

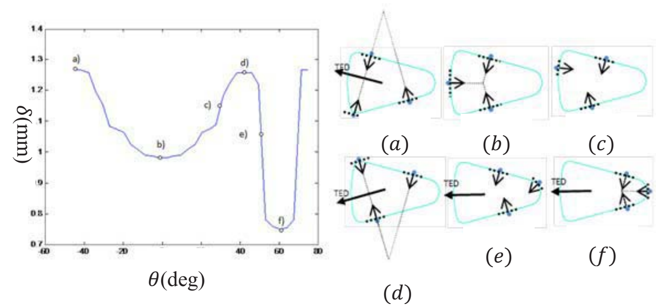


Fig. 3 Several typical points in attractive regions. (a, d) lie on the boundary of the attractive region; (b) lies at the bottom of a feasible attractive region, which corresponds to a stable grasp; (c) lies in the interior of a feasible attractive region; (e) lies in the interior of the trap region, the object can escape from fingers by pure translation; (f) is the bottom of the trap region, which is particularly unstable. The arrow with TED is the possible translational escape direction.

Obviously $\boldsymbol{rank}(Z)$=3-h, $h = 0,1,2$. The vector space satisfying $Z\dot{\boldsymbol{u}} \geq 0$ is composed of two parts: the null space W of Z and vectors $\boldsymbol{x}$ satisfying $Z\boldsymbol{x} \geq 0$ in null space W_i of the $\binom{3}{h+1}$ matrixes M_i formed by $3 - (h+1)$ rows chosen from Z.

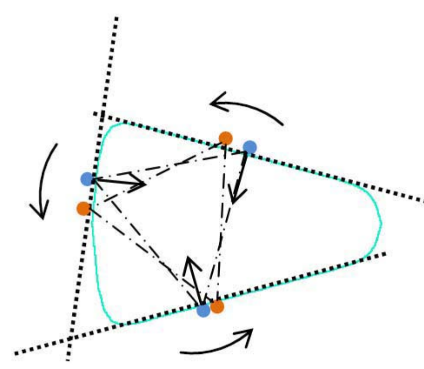


Fig. 4 Calculate the fingers' movement direction by tangent line approximation. The original grasp is marked with blue circles, and the red circles form the minimum inscribed equilateral triangle.

a) *When $h = 0$, the solution space of $Z\dot{\boldsymbol{u}} \geq 0$ is always larger than $Z\dot{\boldsymbol{u}} = 0$.*

Suppose w_i is a vector belonging to the null space of the matrix M_i formed by the first two rows of Z. Then we have $Zw_i = \begin{bmatrix} 0 \\ 0 \\ \epsilon \end{bmatrix}$, $\epsilon \neq 0$. Obviously, w_i or $-w_i$ is another basis satisfying $Z\dot{\boldsymbol{u}} \geq 0$ other than W. Hence the first order partial stable condition can be satisfied when rank(Z)=3.

b) *When $h = 1$, three contact lines intersect in one point.*

When $h = 1$, we have

$$|Z| = \begin{vmatrix} n_{x1} & n_{y1} & n_{y1}p_{x1} - n_{x1}p_{y1} \\ n_{x2} & n_{y2} & n_{y2}p_{x2} - n_{x2}p_{y2} \\ n_{x3} & n_{y3} & n_{y3}p_{x3} - n_{x3}p_{y3} \end{vmatrix} = 0.$$

And $\boldsymbol{rank}(Z) > 1$ is as long as $\delta \neq 0$. It should be noticed this is exactly the necessary and sufficient condition for the existence of the intersecting point of the three contact lines when they are not parallel to each other. When $\boldsymbol{n}_1 \parallel \boldsymbol{n}_2 \parallel \boldsymbol{n}_3$ the above equation is achieved, which can be understood as an intersection of three line with a intersect point at infinity.

c) *When $h = 2$, δ must be zero.*

It is easy to prove that $\boldsymbol{rank}(Z) = 1$ if and only if $\delta = 0$. □

2) Points at the boundary of attractive regions

As for these points (Fig.3 (a, d)), first order stable condition is also satisfied, since the contact normal vector also intersect. While the non-translational escape condition is not satisfied at these points. The state corresponding to these points may fall into a feasible attractive region of a trap region with a slight disturbance。

3) Points in the interior of the feasible attractive region, except for the bottom point

A motion combining rotation and translation may cause the gripper to contract at these points (Fig.3 (c)). Hence, the first order stable condition cannot be satisfied.

4) Points in the trap region

The non-translational escape condition is not fullfilled at these points (Fig.3 (e, f)). This makes the criterion for distinguishing trap regions and attractive regions.

Based on above analysis, we propose an iterative algorithm to find the bottom of attractive region on a simple close curve. It should be noted that since only first order information, i.e. the contact points' coordinate and contact normal vector, were used in the algorithm, we can replace the curve with the tangential lines at these three contact points to derive the same instantaneous rotating direction. Then the rotating direction can be easily calculated by considering the problem of finding the minimum inscribed equilateral triangle in these three lines (Fig.4). With this calculation, we propose an iterative algorithm to find the exact grasp configuration at the bottom of a feasible attractive region while avoiding the trap regions.

Algorithm. 1 *Finding the bottom of a feasible attractive region*

The algorithm aims to find the exact contact points on the 2D contour corresponding to the bottom of a feasible attractive region. The contour is represented as a cubic spline Pt which is parameterized with $x \in [0,1)$ and $Pt(x) = Pt(x - [x])$. Given a x, the grasp can be uniquely determined. $FD(x)$ returns the movement direction of the finger when it is the vertex of the inscribed equilateral triangle, -1 representing the decreasing direction of x and 1 representing the increasing direction. $TE(x)$ determines whether this grasp is translational escapable.

1. Set $\lambda_1 = 0, \mu_1 = 1$, the threshold $\varepsilon > 0$,k=1;
2. $c_k = (\lambda_k + \mu_k)/2$;
3. **if** $FD(\lambda_k) = FD(\mu_k)$ **then**
4. **if** $TE(c_k)$ or $FD(c_k) * FD(\lambda_k) = -1$ **then**
5. **if** $FD(\lambda_k) = 1$ **then**
6. $\lambda_{k+1} = \lambda_k, \mu_{k+1} = c_k$;
7. **else**
8. $\lambda_{k+1} = \mu_k, \mu_{k+1} = c_k + 1$;
9. **endif**
10. **elseif** $FD(c_k) = FD(\lambda_k)$
11. $\lambda_{k+1} = c_k, \mu_{k+1} = \mu_k$;
12. **endif**
13. **goto** 20
14. **endif**
15. **if** $FD(c_k) * FD(\lambda_k) = 1$ **then**
16. λ_{k+1}=$c_k, \mu_{k+1} = \mu_k$;
17. **else**
18. λ_{k+1}=$\lambda_k, \mu_{k+1} = c_k$;
19. **endif**
20. $k = k + 1$;
21. **if** $|\lambda_k - \mu_k|$<ε **then end**;
22. **else goto** 2;
23. **endif**

This algorithm involves the calculation of an equilateral triangle inscribe in a simple closed curve given one vertex. The method can be found in [21].

V. Attractive Region Used in 3-D Grasping

A. A demonstration of attractive region on 3-D grasping

As for 3-D objects, attractive regions can be constructed in a similar way with 2-D objects. The configuration space of the object can be represented as $\mathcal{G} = \mathbb{R}^3 \times SO(3)$, where $\boldsymbol{R} \in SO(3)$ represents the orientation and $(x, y, z) \in \mathbb{R}^3$ is the object's position in gripper's frame. The opening radius is denoted by δ as in 2-D case. We use the composition of X-Y-Z extrinsic rotations to parameterize $SO(3)$ as $(\psi, \Theta, \varphi) \in \mathbb{R}^3$, representing the rotation angle around X_g, Y_g and Z_g.

With a pair of (ψ, Θ), the object's apparent contour $\mathcal{C}$ projected on plane $X_g O_g Y_g$ is determined. Similar to the 2-D situation, we can construct a piecewise smooth function $\delta(\psi, \Theta, \varphi)$ on an open set $\mathcal{E} \subset \mathbb{R}^3$ in which attractive regions can be found.

An example of attractive region on grasping an object is shown in Fig.5. As we can see, even though rigid body dynamic was not taken into consideration when building the attractive regions, it partially illustrates the object's movement patterns.

Considering the feasible attractive regions, when squeezed by the fingers, the object's rotation follows the descent direction of the attractive regions. This ensures that its state reaches the bottom eventually. When it reaches the bottom, any rotation of the object will cause an increment of δ, and since the non-translational escape condition is satisfied in feasible attractive regions all freedoms except for the translation along Z_g are restrained. Since translation along Z_g can be prevented by friction, we come to the conclusion that the bottom of the feasible attractive region corresponds to a stable grasp.

B. Finding the path to the bottom

In order to achieve stable grasp, we want to control the gripper to the orientation corresponding to the bottom of the feasible attractive region. As defined in Section II.A, the derivative of δ should satisfy $d\delta/dt \le 0$, then $d\delta/dt = 0$ is achieved at the bottom. Hence our goal is to find the path $R(t) = R_0 + \int_0^t \boldsymbol{\omega}(t)\mathbf{d}t$ in the rotation space such that:

$$\frac{d\delta}{dt} = \frac{d\delta}{dR}\cdot\frac{dR}{dt} = \frac{d\delta}{dR}\cdot\boldsymbol{\omega}(t) \le 0 \qquad (5)$$

where $\boldsymbol{\omega}$ is the angular velocity to be calculated.

When considering the object's rotation direction under contact constraints of the fingers in 3-D, it becomes more complicated than that in the 2-D circumstance. An optimal rotation direction must be determined. Hence we formulate the problem as a programming problem.

Solving the inequality (1), we can derive the object's free motion space $\mathcal{F}$ which is proved to be a cone in $\mathbb{R}^6$. Our goal is to find a single vector $\dot{\boldsymbol{u}}^*$ such that:

1) The velocity of three fingers' contraction is non-negative;

2) The velocity of three fingers' contraction equal with each other to form a new equilateral triangle afterwards;

3) The velocity of three fingers' contraction should be maximized.

We denote $\boldsymbol{dir_i}$ as the direction pointing from the ith finger to the gripper's center, then to suffice the first condition, the contact point's instantaneous speed projected onto the contraction direction should satisfy:

$$\hat{\boldsymbol{v}}_i \cdot \boldsymbol{dir}_i \ge 0 \qquad (6)$$

It can be represented with

$$\widehat{N}^T G^T \dot{\boldsymbol{u}} \ge 0 \qquad (7)$$

where $\widehat{N} = \begin{bmatrix} \boldsymbol{dir}_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \boldsymbol{dir}_{n_c} \end{bmatrix}$.

To satisfy the second condition, we have

$$\hat{\boldsymbol{v}}_1 \cdot \boldsymbol{dir_1} = \hat{\boldsymbol{v}}_2 \cdot \boldsymbol{dir_2} = \hat{\boldsymbol{v}}_3 \cdot \boldsymbol{dir_3} \qquad (8)$$

Hence, from (3),(6) and(7), we obtain the following programming problem:

$$maximize\ f = \hat{\boldsymbol{v}}_1 \cdot \boldsymbol{dir_1}$$
$$\text{s.t.}$$
$$\hat{\boldsymbol{v}}_1 \cdot \boldsymbol{dir_1} = \hat{\boldsymbol{v}}_2 \cdot \boldsymbol{dir_2} = \hat{\boldsymbol{v}}_3 \cdot \boldsymbol{dir_3}$$
$$\begin{bmatrix} N \\ \widehat{N} \end{bmatrix}^T G^T \dot{\boldsymbol{u}} \ge 0$$
$$\|A_\omega \dot{\boldsymbol{u}}\| = 1$$

A_ω is a positive definite diagonal matrix.

With a little transformation, we get

$$minimize\ g = -\frac{\dot{\boldsymbol{u}}^T \hat{Z}^T \hat{Z} \dot{\boldsymbol{u}}}{\dot{\boldsymbol{u}}^T {A_\omega}^T A_\omega \dot{\boldsymbol{u}}}$$
$$\text{s.t.}$$
$$-Z\dot{\boldsymbol{u}} \le 0$$
$$Q\dot{\boldsymbol{u}} = 0$$

$$\hat{Z} = \widehat{N}^T G^T,\ Q = \begin{bmatrix} \boldsymbol{dir_1}^T & -\boldsymbol{dir_2}^T & \boldsymbol{0} \\ \boldsymbol{dir_1}^T & \boldsymbol{0} & -\boldsymbol{dir_3}^T \end{bmatrix} G^T.$$

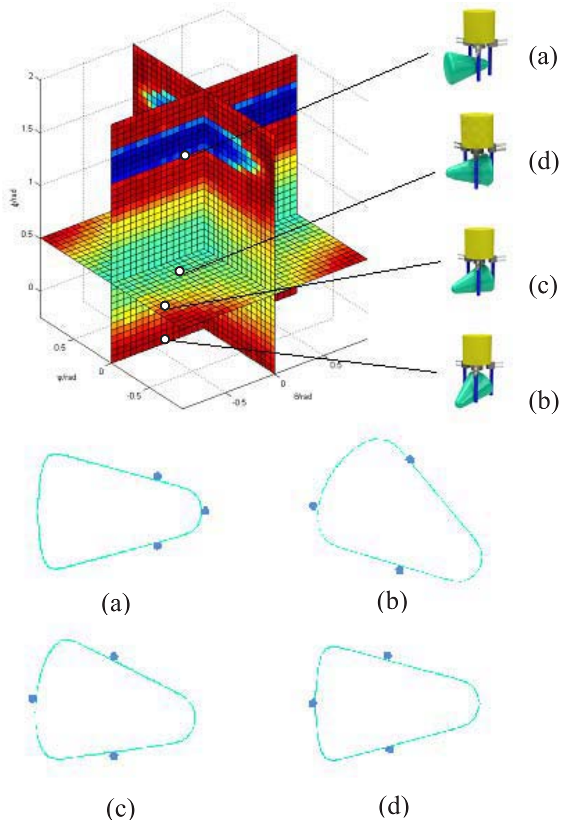


Fig .5 The attractive region formed in three-dimensional rotation space. (a) corresponds to a point in a trap region; (b, c, d) represent points in a feasible attractive region following the descent direction.

The optimal motion $\dot{\boldsymbol{u}}^*$ contains both velocity and angular velocity part, and the three fingers can form a new equilateral triangle under infinite small motion $\dot{\boldsymbol{u}}^* dt$. However in real-world applications, with a motion $\dot{\boldsymbol{u}}^* \Delta t$ this cannot be guaranteed. Hence only the $(\dot{\psi}, \dot{\Theta})$ part is used to control the motion of the camera, while $\dot{\varphi}$ and $\boldsymbol{v}$ are only simply referred. The 2-D grasp planning and 3-D motion planning are conducted alternatively.

VI. SIMULATION

We illustrate the ability of our method with a simulation. The cone like object is fixed in the working coordinate with an arbitrary orientation, and the camera is initially placed at two positions pointing to the center of the object. Based on the theoretical analysis and method provided above, the rotation axis and the rotation direction are calculated first. Then a rotation is conducted with a fixed angle (9 degrees is used in the discussion).

We can see from Fig.6 that after 6 and 7 steps of iteration, the bottom of a feasible attractive region is achieved where the opening radius is the minimum.

Even though different passes are taken different end points are achieved, they are both bottom of feasible attractive regions. This is during to the fact that with a rotation around the axis of the cone, same projection is derived and calculated grasps are the same essentially.

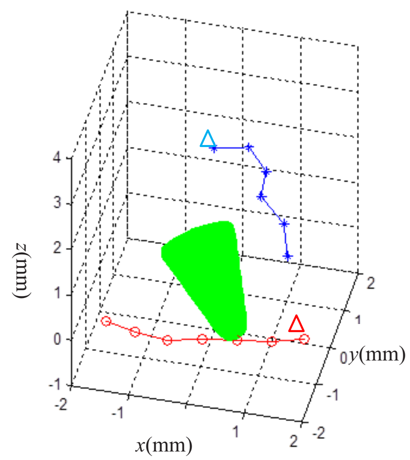


Fig .6 The trajectory of the virtual camera rotating around the object. The starting points are marked with triangle.

VII. Conclusion

It has been shown that attractive regions can be used to derive stable grasps for parallel three-finger gripper. The mathematical properties of the attractive regions have been revealed, and the method for calculating both stable 2-D grasp and optimal moving direction of the camera in 3-D has been proposed. It has been proved that these methods can be used to find a stable grasp.

In the future work, we expect to extend the theory of attractive region to more complicated hands, such as the underactuated hand or even dexterous hands.

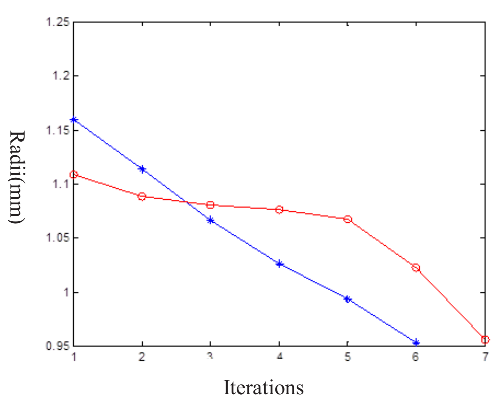


Fig .7 The opening radius δ decreases in each iteration.

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