An Observer-Based Missile Guidance Law

Zheng Zhu, Yuanqing Xia, Mengyin Fu and Shuo Wang

Abstract— **Nonlinear sliding mode guidance laws with finite time convergence are derived for the missile interception in this paper. The proposed guidance laws guarantee that the missile can intercept a maneuverable target with unknown target acceleration. Specially, the control design consists of the estimation of the target acceleration by extended state observer (ESO) and thus it achieves the decrease of undesired chattering effectively. Also, simulation results are presented to illustrate the effectiveness of the control strategies.**

Index Terms— **Guidance law, Sliding mode control, ESO, Finite-time control**

I. INTRODUCTION

Because of its inherent simplicity and ease of implementation, proportional navigation (PN) guidance law and its variants are the most widely known and used guidance schemes in the practical missile interception engagements [1]-[4]. However, the PN guidance law is more applicable for the task of intercepting a non-maneuvering target or a weakly maneuvering target. In practice, target acceleration can change rapidly. For intercepting a target with powerful maneuvering capability, the performance of PN is degraded and it may be ineffective for some orientations between missile and target.

In order to achieve the interception when considering target maneuver capability, many control theories have been employed in the design of guidance laws. In [5], a robust H_∞ guidance law is proposed for homing missiles. By regarding the target acceleration as disturbances, the missile guidance problem is transformed into a nonlinear disturbance attenuation control problem. In [6], a missile guidance law utilizing variable structure control is proposed. The target acceleration is considered as an uncertainty which can be suppressed by the target acceleration bound. Therefore, the precise information of target acceleration during the maneuver is not required. In [7], adaptive fuzzy sliding-mode control guidance laws are presented to force the missile to move along the instantaneous line of sight. The fuzzy rules can be learned online by an adaptive law, which adjusts the parameters of the fuzzy rules and the target acceleration bound. In [8], the integrated guidance and control are considered, and an adaptive nonlinear guidance law is proposed to achieve the interception by compensating for the uncertainties in both target acceleration and control loop dynamics. In [9], a sliding-mode controller is derived for an integrated missile autopilot and guidance loop using the concept of zero-effort miss distance. In [10], a precision guidance law is presented based on the principle of following a circular arc toward the target. Specially, with this guidance scheme, interception can be accomplished under certain conditions without the knowledge of the range to the target. In [11], a stochastic optimal control guidance law for a missile is proposed based on the Markov chain approximation method.

Actually, the missile guidance problem considered in most literatures are solved by the asymptotic stability analysis which implies that the system trajectories converge to the equilibrium with infinite settling time. It is well known that finite-time stabilization of dynamical systems may give rise to a better disturbance attenuation besides fast convergence to the origin. So far, to the best of author's knowledge, there are fewer finite-time control results applied in the missile guidance problem. In [12], smooth second-order sliding mode control with finite time convergence is developed to enforce hit-to-kill guidance strategy in the presence of target maneuvers and dynamic uncertainty of airframe-actuator. It is the application of second-order sliding-mode control to guidance law design. However, the algorithm proposed can only deal with such uncertainty and disturbance that are assumed to be sufficiently smooth, which is not possible at all in reality. In [13], guidance laws based on sufficient conditions for the finite time convergence of the line-of-sight angular rate are proposed. The line-of-sight angular rate will converge to zero before the final time of the guidance process. However, the research work gives the control design based on the assumption that the target acceleration is bounded by a known upper bound. Actually, in practical spacecraft systems, the upper bound of the target acceleration may not be easily obtained due to the complexity of the target maneuvering capability. Even if the bound can be obtained sometimes, it is usually very conservative. Thus, the proposed controllers which rely on the known bound of target acceleration may not work well in practical situation.

Therefore, in this paper, we will further consider a more interesting missile guidance problem based on finite-time control approaches. The main contribution of this paper is that by estimating the target acceleration via ESO, the proposed control scheme is designed with requiring no information on the target acceleration. The rest of paper is organized as follows. The intercept strategy is formulated in Section II. Design methods for guidance laws with finite time convergence are presented in Section III. The main results are obtained in Section IV, where a sliding mode controller combining ESO is developed to achieve the interception

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in the presence of target maneuvers. Simulation results are presented in Section V and the paper ends with the conclusion remarks in Section VI.

II. INTERCEPT STRATEGY

Consider a standard two-dimensional geometry of planar interception shown in Fig.1. The corresponding kinematic equations in polar form are given by

$$\begin{aligned} \dot{r} &= V_T\cos(q-\varphi_T) - V_M\cos(q-\varphi_M) \quad (1)\\ r\dot{q} &= -V_T\sin(q-\varphi_T) + V_M\sin(q-\varphi_M) \quad (2)\\ \dot{\varphi}_M &= \frac{A_M}{V_M} \quad (3)\\ \dot{\varphi}_T &= \frac{A_T}{V_T} \quad (4) \end{aligned}$$

where q is the LOS angle, r is the range along LOS, A_M and A_T are normal acceleration of missile and target respectively, V_M and V_T are tangential velocity of missile and target respectively, φ_M and φ_T are flight path angle of missile and target respectively.

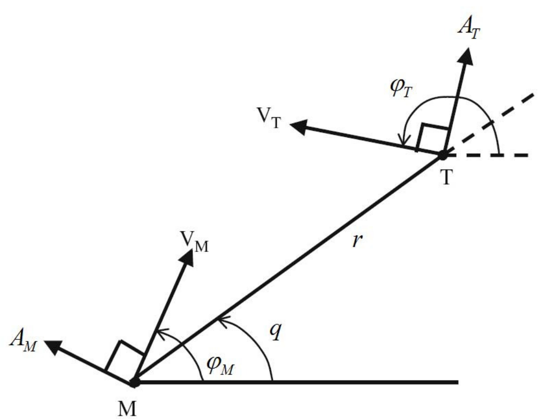

Fig.1. Missile target engagement geometry.

To simplify the transformation from the missile dynamics, we assume that the magnitudes of velocities of the missile and the target are constant. Then, differentiating (1) and (2) with respect to time yields [12]

$$\begin{aligned} \dot{r} &= V_r\\ \dot{V}_r &= \frac{V_\lambda^2}{r} + A_{Tr} - A_M\sin(q-\varphi_M) \quad (5)\\ \dot{q} &= \frac{V_\lambda}{r}\\ \dot{V}_\lambda &= -\frac{V_rV_\lambda}{r} + A_{T\lambda} - A_M\cos(q-\varphi_M) \end{aligned}$$

where $V_r = V_T\cos(q-\varphi_T) - V_M\cos(q-\varphi_M)$, $V_\lambda = -V_T\sin(q-\varphi_T)+V_M\sin(q-\varphi_M)$, $A_{Tr} = A_T\sin(q-\varphi_T)$, $A_{T\lambda} = A_T\cos(q-\varphi_T)$. A_{Tr} and $A_{T\lambda}$ are projections of target acceleration along and orthogonal to LOS. In practical applications, the target acceleration A_T is unknown and is usually difficult to estimate, thus, A_{Tr} and $A_{T\lambda}$ are considered as unknown bounded disturbances.

Assumption 2.1: The projections of target acceleration A_{Tr} and $A_{T\lambda}$ are assumed to be bounded and satisfy the following condition [12].

$$|A_{Tr}| \le A_{Tr}^{max} \quad |\dot{A}_{Tr}| \le \dot{A}_{Tr}^{max} \quad (6)$$
$$|A_{T\lambda}| \le A_{T\lambda}^{max} \quad |\dot{A}_{T\lambda}| \le \dot{A}_{T\lambda}^{max} \quad (7)$$

where A_{Tr}^{max}, $A_{T\lambda}^{max}$, $\dot{A}_{Tr}^{max}$ and $\dot{A}_{T\lambda}^{max}$ are unknown bounds which are not easily obtained due to the uncertainty of target acceleration in practical control systems.

It is well known that a direct interception can be achieved by zeroing the LOS angular rate $\dot{q} = 0$ [13]. Another less aggressive hit-to-kill guidance strategy is known [12]

$$V_\lambda = c_0\sqrt{r} \quad (8)$$

where $c_0 > 0$ is some constant.

In this paper, we aim at guidance law design in the presence of disturbances with bounded energy. The objective is to design a feedback controller such that the states of the closed-loop system (5) track the given desired motion (8).

III. BASIC CONTROL DESIGN

It is well known that sliding mode control (SMC) is a robust method to control nonlinear and uncertain systems which has attractive features to keep the systems insensitive to the uncertainties on the sliding surface. The conventional SMC design approach consists of two steps. First, a sliding manifold is designed such that the system trajectory along the manifold acquires certain desired properties. Then, a discontinuous control is designed such that the system trajectories reach the manifold in finite time. Sliding mode control as a general design tool for control systems has been well established, the primary advantages of sliding model control are: i) fast response and good transient performance; ii) its robustness against a large class of perturbations or model uncertainties; and iii) the possibility of stabilizing some complex nonlinear systems which are difficult to stabilize by continuous state feedback laws.

Simultaneously, the most important feature of variable structure systems with sliding modes is the finite-time convergence to a sliding surface or manifold, which can be applied to the finite-time control scheme in missile systems with external disturbance. And It is clear that finite-time stabilization of dynamical systems may give rise to a better disturbance attenuation besides fast convergence to the desired motion. Therefore, in this section, we will consider the missile guidance problem based on finite-time control approaches. Before giving the control design, we recalled some lemmas which will be utilized in the subsequent control development and analysis.

Lemma 3.1: [15] Suppose $V(x)$ is a C^1 smooth positive definite function (defined on $U \subset R^n$) and $\dot{V}(x)+\lambda V^\alpha(x)$ is a negative semi-definite function on $U \subset R^n$ for $\alpha \in (0,1)$ and $\lambda \in R^+$, then there exists an area $U_0 \subset R^n$ such that any $V(x)$ which starts from $U_0 \subset R^n$ can reach $V(x) \equiv 0$ in finite time. Moreover, if T_{reach} is the time needed to reach

$V(x) \equiv 0$, then

$$T_{reach} \leq \frac{V^{1-\alpha}(x_0)}{\lambda(1-\alpha)} \tag{9}$$

where $V(x_0)$ is the initial value of $V(x)$.

As usual in the sliding mode technique, the control forces the system evolution on a certain surface which guarantees the achievement of the control requirements. Based on the objective (8), the sliding surface is selected as

$$S = V_\lambda - c_0\sqrt{r} = 0 \tag{10}$$

Now, it is clear that if a dynamic state feedback control law is designed such that the trajectories of the closed-loop system (5) can be driven on the sliding surface (10) and evolve along it, then the guidance strategy (8) can be achieved.

Now consider the following reaching law

$$\dot{S} = -\tau S - \sigma|S|^\gamma \operatorname{sgn}(S) \tag{11}$$

where $\tau > 0$, $\sigma > 0$, $0 < \gamma < 1$.

In [16], it has been shown that the reaching control law can guarantee the convergence of the trajectory of the closed-loop system since it is driven onto the sliding surface in finite time, and the chattering is reduced by tuning the parameters τ and σ properly. With this choice, the guidance law can be obtained in the following propositions.

Proposition 1. With the sliding surface given by (10), the trajectory of the closed-loop system (5) can be driven onto the sliding surface $S(t) = 0$ in finite time with the control law (12).

$$\begin{aligned} A_M = \frac{1}{\cos(q-\varphi_M)}(&\tau S + \sigma|S|^\gamma \operatorname{sgn}(S) - \frac{V_r V_\lambda}{r} \\ &+ A_{T\lambda} - \frac{c_0}{2}V_r/\sqrt{r}) \end{aligned} \tag{12}$$

The proof is given in Appendix A.

Note that the control law (12) consists of the target acceleration $A_{T\lambda}$ which is not completely known to us, it could not be applied to the practice systems. In order to solve the problem, the target acceleration needs to be estimated and compensated. The most common method of disturbance attenuation is to obtain the upper bound of target acceleration [13], by which the target acceleration is assumed to be satisfied

$$|A_{T\lambda}| \leq d \tag{13}$$

where d is the upper bound of the target acceleration which can be estimated by the priori. Then, by appropriate modification of the guidance law (12), we obtain the following result.

Proposition 2. With the sliding surface given by (10), the trajectory of the closed-loop system (5) can be driven onto the sliding surface $S(t) = 0$ in finite time with the control law (14).

$$\begin{aligned} A_M = \frac{1}{\cos(q-\varphi_M)}(&\tau S + \sigma|S|^\gamma \operatorname{sgn}(S) - \frac{V_r V_\lambda}{r} \\ &+ d sign(S) - \frac{c_0}{2}V_r/\sqrt{r}) \end{aligned} \tag{14}$$

The proof is given in Appendix B.

Under the control law (14), the states of system (5) will be driven to the sliding surface $S(t) = 0$ in finite time in the presence of disturbance. However, using the method [13] to deal with disturbance, three drawbacks will be appeared in the controller.

(1) The upper bound of target acceleration d, in practical systems, may not be easily obtained due to the complexity of the target structure.
(2) In order to suppress the disturbance existing in the system, the upper bound needs to be selected large enough when the bound is not exactly known, which implies that the control input may lead to violent chattering which is normally undesirable.
(3) The large bound will result in a very high control input power, which may break the physical limitations of the control capacity.

IV. Sliding Mode Control with Extended State Observer

In order to suppress the disturbance with removing above three disadvantages, observer can be used here to make the disturbance estimated and compensated in the control input, which implies the decrease of the chattering and control power.

The ESO views the system model uncertainties and external disturbances as the extended state to be estimated. Here, the observer can be designed for estimating the target acceleration $A_{T\lambda}$ existing in the control law (14). We treat the target acceleration as an extended state, and the subsystem in (5) can be written as

$$\begin{aligned} \dot{V}_\lambda &= -\frac{V_r V_\lambda}{r} + A_{T\lambda} - A_M \cos(q-\varphi_M) \\ \dot{A}_{T\lambda} &= g(t) \end{aligned} \tag{15}$$

where the function $g(t)$ is the derivative of the target acceleration $A_{T\lambda}$, which is uncertain as well. Then the second-order ESO for systems (5) is proposed in the following

$$\begin{aligned} &E_1 = Z_1 - V_\lambda \\ &\dot{Z}_1 = Z_2 - \beta_{01}E_1 - \frac{V_r V_\lambda}{r} - A_M \cos(q-\varphi_M) \\ &\dot{Z}_2 = -\beta_{02} fal(E_1, \alpha_1, \delta) \end{aligned} \tag{16}$$

where E_1 is the estimation error of the ESO, Z_1 and Z_2 are the observer outputs, and β_{01}, β_{02} are the observer gains. The function $fal(\cdot)$ is defined as

$$fal(E_1, \alpha_1, \delta) = \begin{cases} |E_1|^{\alpha_1}\operatorname{sgn}(E_1), & |E_1| > \delta \\ E_1/\delta^{1-\alpha_1}, & otherwise \end{cases} \tag{17}$$

For appropriate values of β_{01}, β_{02}, α_1, δ, the observer output Z_2 approaches to $A_{T\lambda}$ and Z_1 approaches to V_λ. With the target acceleration $A_{T\lambda}$ estimated by the ESO, the control law (14) is modified as

$$\begin{aligned} A_{ESO} = \frac{1}{\cos(q-\varphi_M)}(&\tau S + \sigma|S|^\gamma \operatorname{sgn}(S) - \frac{V_r V_\lambda}{r} \\ &+ Z_2 - \frac{c_0}{2}V_r/\sqrt{r}) \end{aligned} \tag{18}$$

Then, we obtain the results presented in following theorem.

Theorem 1. Consider plant (5) and extended state observer (16), there exist observer gains β_{01}, β_{02}, α_1 and δ such that the estimated states Z_1, Z_2 converge into a residual set of the actual states V_λ, $A_{T\lambda}$ respectively. Then, the trajectory of the closed-loop system (5) can be driven into a neighborhood of the sliding surface (10) in finite time with the control law (18).

The proof is given in Appendix C.

Remark 4.1: Note that the third formula Z_2 in (16) is most important. It shows that Z_2 can estimate the target acceleration which is regarded as the disturbance. As Z_2 is the estimation for the total action of the unknown disturbances, in the feedback, Z_2 is used to compensate for the disturbances.

Remark 4.2: Since the observer cannot track the signal completely in any practical systems, it can only guarantee the bounded motion around the sliding surface. Therefore, we cannot analyze the stability of the dynamics of the sliding mode that is restricted on the sliding surface. In (34), the boundary layer of sliding surface is determined by the estimation error of the ESO. Thus the parameter selecting of the ESO is more important, since it not only determines the performance of ESO observing the uncertainties, but also impacts the behavior of sliding surface.

V. Simulation results

Numerical simulations are performed to investigate the performance of the proposed guidance law (18). It is assumed that the guidance command is not limited. The initial positions of the missile are $X_M(0) = 0m$, $y_M(0) = 0m$. Its initial velocity is $V_M = 800m/s$ and its initial flight-path is $\varphi_M = \frac{\pi}{2}rad$. The target's initial positions are $X_T(0) = 20000m$, $y_T(0) = 20000m$. Its initial velocity is $V_T = 450m/s$ and its initial flight-path is $\varphi_T = \pi rad$. Missile seeker that measures the LOS is taken as a first-order lag system with a time constant 30 ms, and the measurement noise is gaussian noise with standard deviation of 10 mrad. It is assumed that the target acceleration is chosen as

$$A_T = 100\sin(t)m^2/s \tag{19}$$

For comparison, the following finite time convergent (FTC) guidance law is also considered [13].

$$A_{MC} = -N\dot{r}\dot{q} + fsgn(\dot{q}) + \beta|\dot{q}|^{\eta}sgn(\dot{q}) \tag{20}$$
$$N = const. > 2, \qquad 0 \le \eta < 1$$

and the parameters are selected as $N = 3$, $\beta = 10$, $\eta = 0.5$. The upper bound of the target acceleration is chosen as $f = 100$.

The intercept geometry is shown in Fig. 2, which shows that the sliding mode controller (18) achieves the interception with shorter time than the FTC guidance law (20).

The parameters τ, σ, c_0 and γ can be used to regulate the convergence rate of the state trajectory and tuned to reduce the chattering on the sliding surface. Fig.3 are simulation results with $\tau = 10$, $\sigma = 1$, $\gamma = 0.5$, $c_0 = 0.1$. Obviously the sliding mode is stable in spite of the unknown target acceleration, which implies the achievement of objective (8).

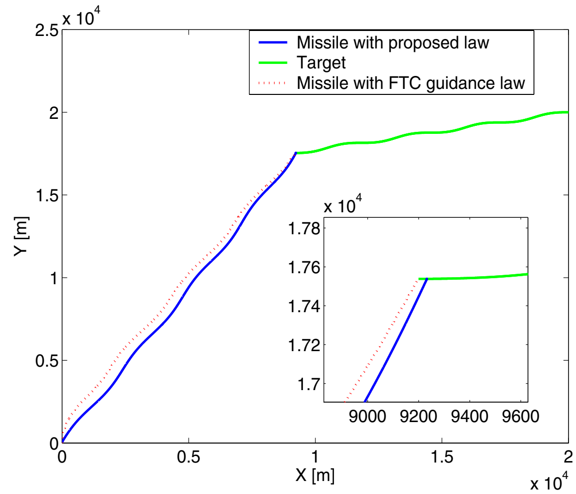

Fig.2. The trajectory

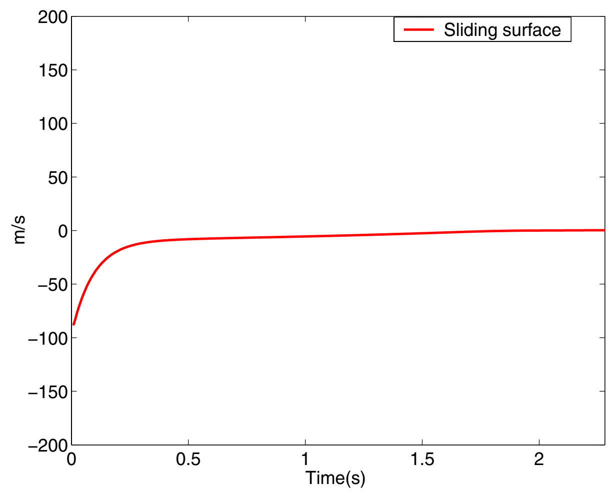

Fig.3. The sliding surface

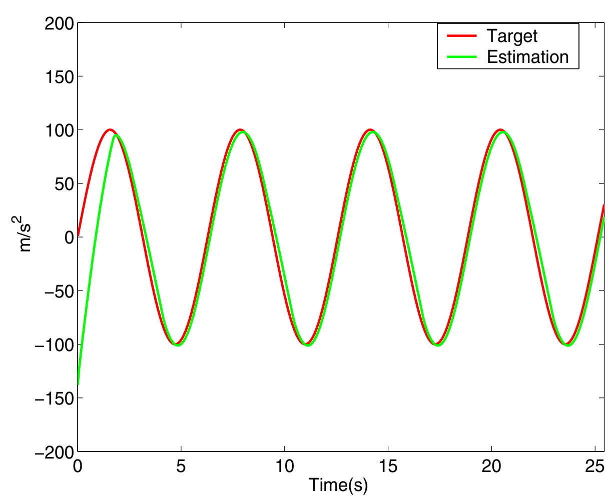

Fig.4. Target acceleration and the estimation

The performances of ESO observing the disturbance A_T are given in Fig.4. By selecting appropriate values of $\beta_{01} =$

50, $\beta_{02} = 100$, $\alpha_1 = 0.2$ and $\delta = 0.15$, the estimated states $Z_2(t)$ converges to the actual disturbance A_T in finite time.

Fig.5 depicts the input signal of the proposed guidance law (18). It is clear that when state trajectories cross the sliding surface, the undesired chattering can also be reduced effectively with the estimation of the disturbance by ESO.

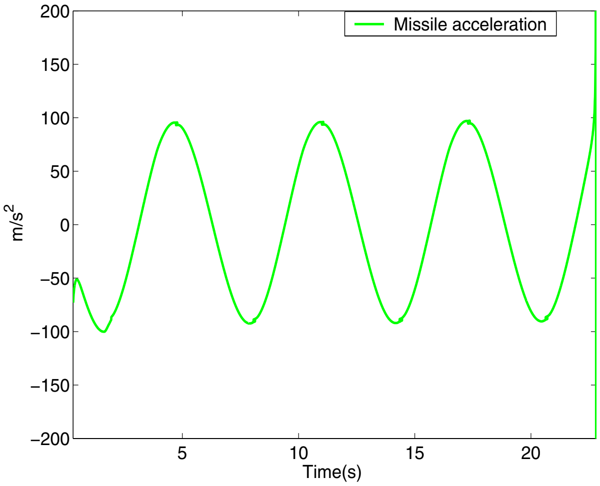

Fig.5. Control input of proposed guidance law

Based on above simulations, we can conclude that the parameters τ, σ in (18) are very important, they are the main parameters determining the bounded layer when state trajectories of (5) evolve around the sliding surface and also it guarantees the convergence precision of the system state. This is clear that the system states can not converge to the sliding surface $S = 0$, but larger τ and σ will force the state errors small enough even though there exist the estimation errors of ESO. Thus, the guidance accuracy is determined in a great degree by the parameters τ and σ, while ESO plays an auxiliary role in guaranteeing the tracking precision in the presence of disturbance. However, in practice, a compromise is made between the tracking accuracy and control input. Since too big τ and σ will require a very high control input, which is always bounded in reality. Thus, the parameters τ and σ can not be selected too large.

VI. Conclusion

In this paper, the missile guidance law based on ESO has been investigated. Sliding mode controllers are proposed to achieve the missile interception with finite-time convergent property. With the help of ESO by estimating the target acceleration, the controller is designed to robustly accomplish hit-to-kill guidance strategy in the presence of target maneuvers. Detailed simulation results have been presented to illustrate the developed method.

Appendix

A. Proof of Proposition 1

Proof: Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}S^2 \tag{21}$$

The derivative of V_1 along the sliding surface (10) and system trajectory (5) satisfies

$$\begin{aligned}\dot{V}_1 &= S(\dot{V}_\lambda - \frac{c_0}{2}V_r/\sqrt{r}) \\ &= S(-\frac{V_r V_\lambda}{r} + A_{T\lambda} - A_M\cos(q-\varphi_M) - \frac{c_0}{2}V_r/\sqrt{r})\end{aligned} \tag{22}$$

Making use of the control law (12) gives

$$\begin{aligned}\dot{V}_1 &= S(-\tau S - \sigma|S|^\gamma \operatorname{sgn}(S)) \\ &\le -2^{\frac{\gamma+1}{2}}\sigma V_1^{\frac{\gamma+1}{2}}\end{aligned} \tag{23}$$

According to the condition $0 < \gamma < 1$, there exists $0 < \frac{\gamma+1}{2} < 1$. Now, by Lemma 3.1, the system states can be driven to the sliding surface $S(t) = 0$ in finite time, and the settling time is given by

$$T_{r1} \le \frac{V_1^{\frac{1-\gamma}{2}}(S_0)}{2^{\frac{\gamma+1}{2}}\sigma(1-\gamma)} \tag{24}$$

where $V(S_0)$ is the initial value of $V_1(S)$.

B. Proof of Proposition 2

Proof: Consider the Lyapunov function candidate

$$V_2 = \frac{1}{2}S^2 \tag{25}$$

Taking the derivative of (25) and making use of the control law (14) gives

$$\begin{aligned}\dot{V}_2 &= S(-\tau S - \sigma|S|^\gamma \operatorname{sgn}(S) + A_{T\lambda} - dsign(S)) \\ &\le -2^{\frac{\gamma+1}{2}}\sigma V_2^{\frac{\gamma+1}{2}} + |S|(A_{T\lambda} - d) \\ &\le -2^{\frac{\gamma+1}{2}}\sigma V_2^{\frac{\gamma+1}{2}}\end{aligned} \tag{26}$$

By Lemma 3.1, the system states can be driven to the sliding surface $S(t) = 0$ in finite time, and the settling time is given by

$$T_{r2} \le \frac{V_2^{\frac{1-\gamma}{2}}(S_0)}{2^{\frac{\gamma+1}{2}}\sigma(1-\gamma)} \tag{27}$$

C. Proof of Theorem 1

Proof: The proof of Theorem 1 consists of two main steps. The overall objective is to demonstrate that the motion of the closed-loop system starting from any initial condition can be driven onto a neighborhood of the sliding surface $S(t) = 0$ in finite time.

Step 1: The objective in this step is to prove that the estimated states Z_1, Z_2 converge into a residual set of the actual states V_λ, $A_{T\lambda}$ respectively.

Defining $E_2 = Z_2 - A_{T\lambda}$ and differentiating E_1, E_2 with respect to time, the observer error dynamics are expressed as

$$\begin{cases}\dot{E}_1 = E_2 - \beta_{01}E_1 \\ \dot{E}_2 = -g(t) - \beta_{02}fal(E_1, \alpha_1, \delta)\end{cases} \tag{28}$$

The stability of ESO has been obtained by selecting appropriate parameters β_{01} and β_{02} [17][18]. When the observer is stable, the derivative of vector is obtained $\dot{E} = [\dot{E}_1 \ \dot{E}_2]^T = 0$.

Noting (17), if $|E_1| > \delta$, the errors of estimation are

$$\begin{cases} |E_1| = |g(t)/\beta_{02}|^{1/\alpha_1} \\ |E_2| = \beta_{01}|g(t)/\beta_{02}|^{1/\alpha_1} \end{cases} \quad (29)$$

and if $|E_1| \le \delta$, the errors of estimation can be expressed as

$$\begin{cases} |E_1| = |g(t)\delta^{1-\alpha_1}|/\beta_{02} \\ |E_2| = \beta_{01}|g(t)\delta^{1-\alpha_1}|/\beta_{02} \end{cases} \quad (30)$$

From (29) and (30), it is clear that the estimation errors E_1 and E_2 are determined by the parameters β_{01}, β_{02}, α_1 and δ. Via tuning these parameters appropriately, the estimation errors of the observer can be forced small enough such that the system state V_λ and extended state $A_{T\lambda}$ can be observed by the ESO effectively, which means that Z_1, Z_2 will converge into a neighborhood of the actual states V_λ, $A_{T\lambda}$ respectively. The fundamental selection of the parameters can be chosen as $\beta_{01} > 0$, $\beta_{02} > 0$, $0 < \alpha_1 < 1$, $0 < \delta < 1$. Furthermore, an appropriate β_{02} can be selected large enough such that $|g(t)/\beta_{02}|$ is small enough although $g(t)$ is unknown to us. Of course, β_{01} should be small enough to make the estimation error E_2 as small as possible. In addition, the smaller the α_1 is, the smaller the steady estimation errors will be.

Step 2: The objective in this step is to prove that the trajectory of the closed-loop system (5) can be driven onto a neighborhood of the sliding surface $S(t) = 0$ in finite time. Consider the Lyapunov function candidate

$$V_3 = \frac{1}{2}S^2 \quad (31)$$

Taking the derivative of (31) and making use of the control law (18) gives

$$\begin{aligned} \dot{V}_3 &= S(-\tau S - \sigma|S|^\gamma \text{sgn}(S) + A_{T\lambda} - Z_2) \\ &\le -2^{\frac{\gamma+1}{2}}\sigma V_3^{\frac{\gamma+1}{2}} + |SE_2| \end{aligned} \quad (32)$$

Suppose there exists a scalar $0 < \theta \le 1$ such that inequality (32) can be expressed as

$$\dot{V}_3 \le -2^{\frac{\gamma+1}{2}}\theta\sigma V_3^{\frac{\gamma+1}{2}} - 2^{\frac{\gamma+1}{2}}(1-\theta)\sigma V_3^{\frac{\gamma+1}{2}} + |SE_2| \quad (33)$$

Clearly, $\dot{V}_3(x) \le -2^{\frac{\gamma+1}{2}}\theta\sigma V_3^{\frac{\gamma+1}{2}}(x)$ if $V_3^{\frac{\gamma+1}{2}}(x) > \frac{|SE_2|}{2^{\frac{\gamma+1}{2}}(1-\theta)\sigma}$. According to Lemma 3.1, the decrease of $V(x)$ in finite time drives the the trajectories of the closed-loop system into $V_3^{\frac{\gamma+1}{2}}(x) \le \frac{|SE_2|}{2^{\frac{\gamma+1}{2}}(1-\theta)\sigma}$, which means that the trajectories of the closed-loop system is bounded in finite time as

$$\lim_{\theta \to \theta_0} S \in \left(S^\gamma \le \frac{|E_2|}{(1-\theta)\sigma} \right) \quad (34)$$

where $0 < \theta_0 < 1$. And the time needed to reach (34) is bounded as

$$\lim_{\theta \to \theta_0} T_{r3} \le \frac{V_3^{\frac{1-\gamma}{2}}(S_0)}{2^{\frac{\gamma+1}{2}}\sigma\theta_0(1-\gamma)} \quad (35)$$

Therefore, the trajectory of the closed-loop system (5) will be driven into the neighborhood of the sliding surface (10) in finite time when the observer is stable. From Theorem 1, E_2 can converge into a residual set of zero. Since τ and σ are positive parameters to be tuned, appropriate τ and σ can be selected large enough such that $\dot{V}_3 < 0$ when $V_3(t)$ is out of a certain bounded region which contains equilibrium point. From this, system states can be guaranteed to reach a close neighborhood of the sliding surface $S = 0$ in finite time and then evolve in it.

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