

Backstepping Controller Design for the trajectory tracking control of work platform of Folding-Boom Aerial Platform Vehicle

Haidong Hu, En Li, Xiaoguang Zhao, Min Tan and Wensheng Yu

Abstract—In this paper, a backstepping controller is presented for the tracking control of work platform of folding-boom aerial platform vehicle. The control objective is to suppress the vibration and drive the work platform to follow a desired reference trajectory. According to the Lyapunov stability theorem, the derived control law guarantees that the trajectory tracking system is exponentially asymptotically stable. In other words, the trajectory of work platform can follow the desired trajectory without vibration. Furthermore, the simulation results show that the performance of the proposed controller is excellent. Not only can the controller suppress the vibration, but also it can eliminate the trajectory tracking error of work platform. Therefore, the work platform can keep the steady movement following the desired trajectory.

I. INTRODUCTION

As a kind of construction vehicle which can hoist personnel to the appointed location in the aerial for installation or maintenance, aerial platform vehicle requires very high safety. Therefore, it is important to ensure the steady movement and accurate positioning of the work platform.

As far as the rigid model of aerial platform vehicle is concerned, vibration and trajectory deviation of the work platform cannot be controlled effectively because the model neglects the elastic deformation of long beam. Therefore, the flexible nature should be considered in order to set up an appropriate mathematical model. As a result, the flexible multi-body dynamics equations of the arm system of folding-boom aerial platform vehicle have been established based on flexible multi-body dynamics theory and Lagrange's equation in the literature[1], which has been accepted by ICICIP2010.

The numerical solutions of the flexible model show that the vibration associated with the flexible beam is significant and

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there exist small deviation of the trajectory of the work platform which is caused by the beam deflection. Therefore, the control objects are to suppress vibration in order to attain steady movement and drive trajectory of the work platform to track the reference trajectory quickly.

Because the two beams of folding-boom aerial platform vehicle are seen as flexible, the control methods used for the flexible link can be adopted for the control of the trajectory of the work platform.

There are several control schemes used to regulate the motion of two-link flexible manipulators, including proportional derivative control [2], adaptive control [3], computed torque control [4], optimal control [5], sliding control [6], etc.

In recent years, the backstepping control scheme introduced in [7,8,9,10] has been applied to many systems successfully. For example, the excellent performance of multiple-link rigid robotic manipulator is obtained by backstepping design approach [11]. And the approach is used for the tip-position trajectory tracking control of a single link flexible manipulator [12].

In this paper, the motion control of the work platform of folding-boom aerial platform vehicle by backstepping control scheme is presented. With the proposed controller, the vibration of the work platform is suppressed and the trajectory of the work platform tracks the reference trajectory quickly.

This paper is organized as follows. In Section II, flexible multi-body dynamic model of folding-boom aerial platform vehicle is introduced and analyzed. Then a backstepping controller is derived to attain the control objective in Section III. Finally, the numerical simulation results for the trajectory tracking control of the work platform are demonstrated in Section IV, and some concluding remarks are given in Section V.

II. FLEXIBLE MULTI-BODY DYNAMICS EQUATIONS OF ARM SYSTEM

The flexible multi-body dynamics Equations of folding-beam aerial platform vehicle can be written as[1]:

$$\begin{cases} G\ddot{\theta} + U\dot{\theta}^2 + H\ddot{q} + R = Q_\theta \\ M\ddot{q} + Nq + H^T\ddot{\theta} + V^T\dot{\theta}^2 = Q_q \end{cases} \quad (1)$$

In equations(1), $\ddot{\theta} = [\dot{\theta}_1 \quad \dot{\theta}_2]^T$, $\dot{\theta}^2 = [\dot{\theta}_1^2 \quad \dot{\theta}_2^2]^T$ and

$\ddot{q} = [\ddot{q}_{11} \quad \ddot{q}_{12} \quad \ddot{q}_{21} \quad \ddot{q}_{22}]^T$, in which, θ_1 and θ_2 are the angle of beam 1,2 with respect to the horizontal plane, and q_{11}, q_{12} and q_{21}, q_{22} are the deflection variables associated with the two former model function of beam 1,2, respectively.

In addition, $\mathcal{Q}_\theta = [\mathcal{Q}_1 \quad \mathcal{Q}_2]^T$ and $\mathcal{Q}_q = [\mathcal{Q}_3 \quad \mathcal{Q}_4 \quad \mathcal{Q}_5 \quad \mathcal{Q}_6]^T$ are the generalized forces vector corresponding to $\theta = [\theta_1 \quad \theta_2]^T$ and $q = [q_{11} \quad q_{12} \quad q_{21} \quad q_{22}]^T$.

For the convenience of the design of backstepping controller, assume that

$$\begin{aligned} x_1 &= [\theta_1 \quad \theta_2 \quad q_{11} \quad q_{12} \quad q_{21} \quad q_{22}]^T, \\ x_2 &= [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{q}_{11} \quad \dot{q}_{12} \quad \dot{q}_{21} \quad \dot{q}_{22}]^T. \end{aligned}$$

Then the state equations of (1) can be described as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \bar{M}^{-1}(\mathcal{Q} - \bar{V}x_2^2 - \bar{N}x_1 - \bar{R}) \end{cases} \quad (2)$$

where, $\bar{M} = \begin{bmatrix} G & H \\ H^T & M \end{bmatrix}$, $\bar{V} = \begin{bmatrix} U & 0 \\ V^T & 0 \end{bmatrix}$, $\bar{N} = \begin{bmatrix} 0 & 0 \\ 0 & N \end{bmatrix}$, $\bar{R} = [R^T \quad 0]^T$.

Let m_1 , m_2 and m denote the mass of the beam 1,2 and load, respectively; l_1 , l_2 represent the length of beam 1,2; g is acceleration of gravity; E is the modulus of elasticity of the beam material and I_1, I_2 are the moment of inertia of the cross-section of beam 1,2.

Therefore, the matrix G , H , M , U , V , N , R can be expressed as follows:

$$G = \begin{bmatrix} \frac{1}{3}m_1 + m_2 + m)l_1^2 & (\frac{1}{2}m_2 + m)l_1l_2 \cos(\theta_1 - \theta_2) \\ (\frac{1}{2}m_2 + m)l_1l_2 \cos(\theta_1 - \theta_2) & (\frac{1}{3}m_2 + m)l_2^2 \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{m_1l_1}{\pi} & -\frac{1}{2}\frac{m_1l_1}{\pi} & \frac{2m_2l_1}{\pi} \cos(\theta_1 - \theta_2) & 0 \\ 0 & 0 & \frac{m_2l_2}{\pi} & -\frac{1}{2}\frac{m_2l_2}{\pi} \end{bmatrix}$$

$$M = \begin{bmatrix} \frac{1}{2}m_1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}m_1 & 0 & 0 \\ 0 & 0 & \frac{1}{2}m_2 & 0 \\ 0 & 0 & 0 & \frac{1}{2}m_2 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & (\frac{1}{2}m_2 + m)l_1l_2 \sin(\theta_1 - \theta_2) \\ -\frac{1}{2}(m_2 + m)l_1l_2 \sin(\theta_1 - \theta_2) & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 0 & 0 & \frac{-2m_2l_1 \sin(\theta_1 - \theta_2)}{\pi} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{EI_1\pi^4}{2l_1^3} & 0 & 0 & 0 \\ 0 & \frac{8EI_1\pi^4}{l_1^3} & 0 & 0 \\ 0 & 0 & \frac{EI_2\pi^4}{2l_2^3} & 0 \\ 0 & 0 & 0 & \frac{8EI_2\pi^4}{l_2^3} \end{bmatrix}$$

$$R = \begin{bmatrix} \frac{2}{\pi}m_2l_1 \sin(\theta_1 - \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)\dot{q}_{21} + (\frac{1}{2}m_1 + m_2 + m)gl_1 \cos\theta_1 \\ (\frac{1}{2}m_2 + m)gl_2 \cos\theta_2 \end{bmatrix}^T$$

III. THE DESIGN OF BACKSTEPPING CONTROLLER

The backstepping control scheme is commonly used in nonlinear control systems. The main idea of this scheme is to design a controller recursively by regarding some of the state variables as virtual controls.

In this paper, the control objective for folding-boom aerial platform vehicle is to make the work platform follow a reference trajectory quickly. In addition, the vibration of work platform should be eliminated simultaneously. As a result, a backstepping controller is designed in order to achieve the control goal.

Assume that reference trajectory is given by $r(t) = [r_1(t) \quad r_2(t) \quad r_3(t) \quad r_4(t) \quad r_5(t) \quad r_6(t)]^T$.

Then,

$$\dot{r}(t) = [\dot{r}_1(t) \quad \dot{r}_2(t) \quad \dot{r}_3(t) \quad \dot{r}_4(t) \quad \dot{r}_5(t) \quad \dot{r}_6(t)]^T,$$

$$\ddot{r}(t) = [\ddot{r}_1(t) \quad \ddot{r}_2(t) \quad \ddot{r}_3(t) \quad \ddot{r}_4(t) \quad \ddot{r}_5(t) \quad \ddot{r}_6(t)]^T.$$

According to the backstepping design approach applied to the flexible joint robot system[1], the procedure of backstepping design consists of the following two steps:

Step1: Firstly, the position tracking error is defined as $z_1 = x_1 - r(t)$, where, x_1 is viewed as the virtual control variable. Following, define $\alpha_1 = -c_1 z_1$ as the stabilizing function, where c_1 is a positive constant. Then the error state variable corresponding to α_1 is given by

$$z_2 = x_2 - \alpha_1 - \dot{r}(t).$$

Therefore, the derivative of z_1 can be written as

$$\dot{z}_1 = \dot{x}_1 - \dot{r}(t) = x_2 - \dot{r}(t) = z_2 + \alpha_1.$$

Choosing a Lyapunov function candidate

$$V_1 = \frac{1}{2}z_1^T z_1,$$

the derivative of V_1 with respect to time is

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T (z_2 + \alpha_1) = z_1^T (z_2 - c_1 z_1) = -c_1 z_1^T z_1 + z_1^T z_2.$$

Apparently, $\dot{V}_1 \leq 0$ when $z_2 = 0$, but z_2 cannot always be

zero. As a result, z_2 should be controlled in the next step in order to make $\dot{V}_1 \leq 0$.

Step2: The derivative of α_1 is computed as

$$\dot{\alpha}_1 = -c_1 \dot{z}_1 = -c_1 (\dot{x}_1 - \dot{r}(t)).$$

Therefore, the derivative of z_2 can be expressed as

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - (\dot{\alpha}_1 + \ddot{r}(t)) \\ &= \bar{M}^{-1} (Q - \bar{V}x_2^2 - \bar{N}x_1 - \bar{R}) + c_1(x_2 - \dot{r}(t)) - \ddot{r}(t), \end{aligned} \quad (3)$$

where

$$\begin{aligned} x_1 &= [\theta_1 \quad \theta_2 \quad q_{11} \quad q_{12} \quad q_{21} \quad q_{22}]^T \\ x_2 &= [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{q}_{11} \quad \dot{q}_{12} \quad \dot{q}_{21} \quad \dot{q}_{22}]^T \\ x_2^2 &= [\dot{\theta}_1^2 \quad \dot{\theta}_2^2 \quad \dot{q}_{11}^2 \quad \dot{q}_{12}^2 \quad \dot{q}_{21}^2 \quad \dot{q}_{22}^2]^T. \end{aligned}$$

Lyapunov candidate function is chosen as

$$V_2 = V_1 + \frac{1}{2} z_2^T z_2$$

the time derivative of V_2 can be obtained as

$$\dot{V}_2 = -c_1 z_1^T z_1 + z_1^T z_2 + z_2^T \dot{z}_2 \quad (4)$$

Substitute (3) into (4) gives

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^T z_1 + z_1^T z_2 \\ &\quad + z_2^T [\bar{M}^{-1} (Q - \bar{V}x_2^2 - \bar{N}x_1 - \bar{R}) + c_1(x_2 - \dot{r}(t)) - \ddot{r}(t)] \end{aligned} \quad (5)$$

therefore, the control law is given by

$$Q = \bar{M}[-z_1 - c_2 z_2 - c_1(x_2 - \dot{r}(t)) + \ddot{r}(t)] + \bar{V}x_2^2 + \bar{N}x_1 + \bar{R}, \quad (6)$$

where $c_2 > 0$.

Substitute (6) into (5) leads to

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^T z_1 - c_2 z_2^T z_2 \\ &\leq 0 \end{aligned} \quad (7)$$

It is demonstrated from (7) that the derived control law (6) can make z_1 and z_2 converge to zero exponentially asymptotically according to Lyapunov stability theorem. That is to say, the trajectory tracking error will converge to zero by choosing suitable design constant c_1 and c_2 . Therefore, the trajectory of work platform can follow the desired trajectory without vibration.

IV. SIMULATION RESULTS

In (2), the parameters and the initial conditions for the simulation are selected as follows:

$l_1 = 7.5m$, $l_2 = 8.5m$, $m_1 = 650kg$, $m_2 = 550kg$, $m = 150kg$, $g = 9.8m/s^2$, $EI_1 = 6 \times 10^8 N \cdot m^2$, $EI_2 = 5 \times 10^8 N \cdot m^2$, the initial angles are $\theta_1 = \frac{2\pi}{3} rad$ and $\theta_2 = 0.52rad$, the initial angular velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ are zero.

The reference trajectory is given by

$$r(t) = \left[\frac{2\pi}{3} \quad \frac{\pi t}{180} \quad 0 \quad 0 \quad 0 \quad 0 \right]^T,$$

then

$$\dot{r}(t) = [0 \quad \frac{\pi}{180} \quad 0 \quad 0 \quad 0 \quad 0]^T,$$

$$\ddot{r}(t) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T.$$

The parameter values of c_1 and c_2 are set to $c_1 = c_2 = 20$.

The backstepping controller is realized in the MATLAB-Simulink environment.

The simulation results of the proposed controller are shown in Fig.1~Fig.6.

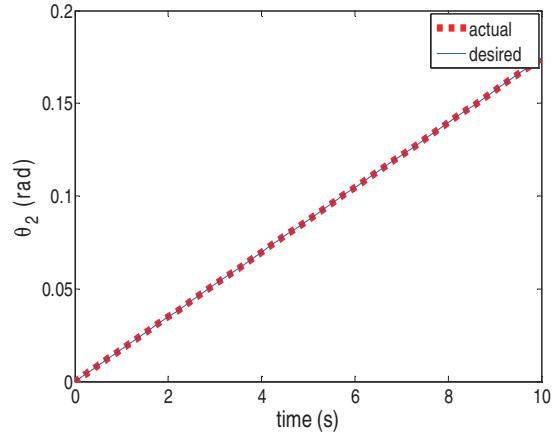


Fig.1 The tracking of θ_2

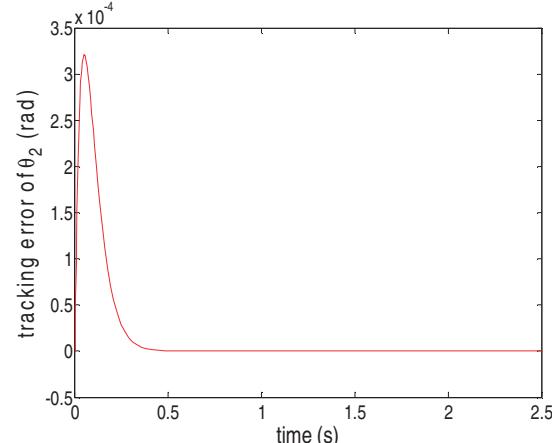


Fig.2 The tracking error of θ_2

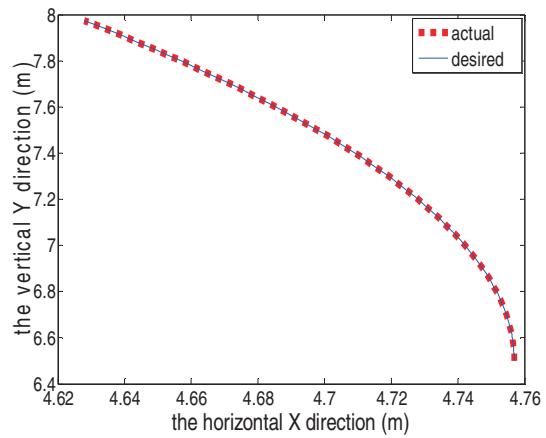


Fig.3 The trajectory tracking of the work platform

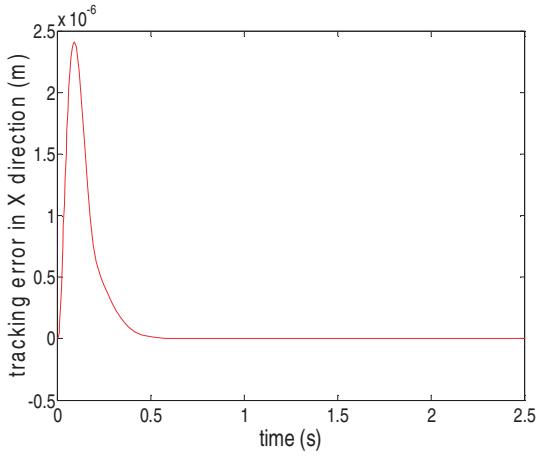


Fig.4 The trajectory tracking error of the work platform in the horizontal X direction

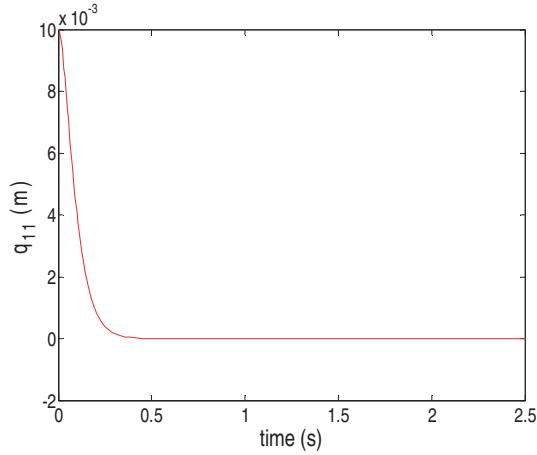


Fig.5 Deflection variable q_{11} varying with time

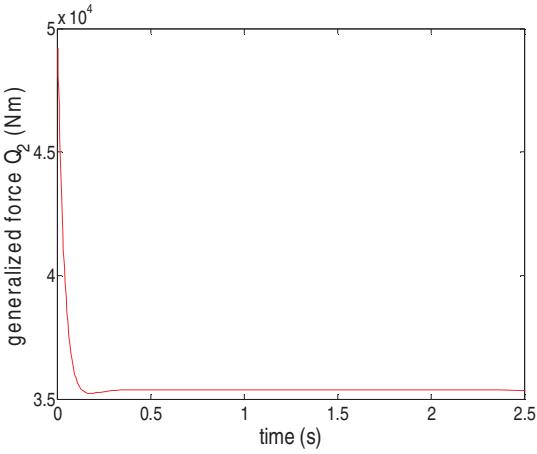


Fig.6 The adjustment of generalized force Q_2

As can be seen from Fig.1~ Fig.6, the backstepping controller developed for the folding-boom aerial platform vehicle performs reasonably well for both the trajectory tracking and vibration suppression of the work platform.

On one hand, it is evident from Fig.1 and Fig.3 that the proposed controller with the suitable design constant c_1 and c_2 can achieve excellent tracking performance for the

trajectory tracking of both θ_2 and work platform. Moreover, the vibration caused by the deflection of beam is restrained at the same time. On the other hand, as shown in Fig. 2 and Fig.4, the trajectory tracking errors of θ_2 and work platform converge to zero very quickly, which also reflects that the tracking performance is perfect.

Fig. 5 shows that the deflection variable q_{11} converges to zero with a transient, indicating that the vibration is eliminated completely. As a result, the work platform can track the desired trajectory steadily.

The adjustment of generalized force Q_2 for realizing the trajectory tracking control of work platform is shown in Fig.6.

V. CONCLUSIONS

In this paper, a backstepping controller for folding-boom aerial platform vehicle is presented to suppress the vibration and reduce the trajectory tracking error of the work platform. The simulation results demonstrate that the proposed controller is so effective that both the vibration and the tracking error are eliminated. As a result, the trajectory of the work platform can track the desired trajectory steadily.

In the case of system uncertainties, robust control scheme of nonlinear systems should be used in order to attain the control objectives. This topic will be discussed in the future research.

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