A novel matrix used in regularization term for model-based photoacoustic reconstructions

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ABSTRACT

In this paper, a novel matrix used in regularization term is proposed to acquire high quality reconstructed image. The use of Central-Enhancement Laplace (CEL) matrix is essentially a improvement of the conventional L2-norm regularization algorithm. By adding this matrix into the regularization term, we can obtain the reconstructed images with higher quality. The use of CEL matrix can enhance the edge information of images while reducing the reconstruction artifacts. Combined with the results of the in-vivo reconstructed images, we can confirm the above properties of this matrix. More importantly, the reconstructed images of L2-norm regularization algorithm using CEL matrix can achieve better reconstruction quality than some of the complex L1-norm regularization algorithms. Due to the flexibility of the matrix center element settings, we can also weigh the smoothness and fineness of the reconstructed image as needed.

Keywords: Photoacoustic tomography, Image reconstruction, Regularization, Central-Enhancement Laplace matrix

1. INTRODUCTION

Photoacoustic tomography (PAT) is a hybrid and noninvasive imaging method which combines high optical contrast with high acoustic resolution. In photoacoustic, biological tissues absorb light energy and lead to the effect of thermal expansion. Due to the thermal expansion, the ultrasonic waves are generated and propogate out of the biological tissue. The ultrasonic signal is collected by the ultrasonic transducer for image reconstruction. PAT has been widely used for multi-scale anatomical, functional, and molecular imaging of biological tissues.[1–3]

At present, many reconstruction algorithms have been proposed. Compared with the filtered back-projection algorithms, the model-based algorithms can acquire better reconstruction image quality.[4] These model-based reconstruction are performed by minimizing the mean square error between the theoretical pressure and the measured pressure. In general, for obtaining stable and accurate reconstruction, it is essential to regularize the inversion procedure with a high-pass spatial filter kernel to reduce the artifacts within the reconstruction. In order to implement this operation, regularization matrix is adopted. However, conventional regularization matrices based on high-pass filter such as Laplace high-pass filter will lead to the loss of edge information in the reconstructed images. Therefor, a novel regularization matrix named Central-Enhancement Laplace (CEL) matrix was designed. By adding an appropriate weight factor to the central element of the Laplace high-pass filter kernel, the details of the reconstructed image can be better preserved.since the original high-pass filtering effect still exists in CEL matrix, it will only bring a very small amount of noise to image.

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2. THEORETICAL BACKGROUND

The model-based reconstruction algorithm is based on the following simplified photoacoustic equation:

$$\frac{\partial^2 p(\boldsymbol{r},t)}{\partial t^2} - c_0^2 \nabla^2 p(\boldsymbol{r},t) = \Gamma H(\boldsymbol{r}) \frac{\partial \delta(t)}{\partial t} \,, \tag{1}$$

where $H(\boldsymbol{r})$ denotes the absorption heat density at position $\boldsymbol{r}$, $p(\boldsymbol{r},t)$ denotes the ultrasonic pressure in time t at position $\boldsymbol{r}$, c_0 is the speed of ultrasonic and Γ is dimensionless Gruneisen parameters. In order to solve the numerical solution based on the photoacoustic equation, we need to consider the forward problem in Equation (1) as a solution for a initial value problem of the wave equation. In this way, the numerical solution of Equation (1) can be expressed in the following Poisson integral (ignore the constant coefficient $\frac{\Gamma}{4\pi c_0}$)

$$p(\boldsymbol{r}_0,t) = \frac{\partial}{\partial t} \int_{S'(t)} \frac{H(\boldsymbol{r}')}{|\boldsymbol{r}_0 - \boldsymbol{r}'|} \, \mathrm{d}S'(t) \,, \tag{2}$$

where $\boldsymbol{r}_0$ is the position of transducer, $\boldsymbol{r}'$ denotes the points on the integral curve $S'(t)$ and $|\boldsymbol{r}_0 - \boldsymbol{r}'| = c_0 t$. Then, we discretize Equation (2) into matrix representation according to the interpolation method in Ref. 4, which is

$$\boldsymbol{P} = \boldsymbol{M}\boldsymbol{Z} \,, \tag{3}$$

where $\boldsymbol{P}$ is the signal received by transducers, $\boldsymbol{M}$ is the model matrix determined by the discrete method and $\boldsymbol{Z}$ denotes the corresponding gray level of the reconstructed image.

Since Equation (3) is a ill-conditioned equation, we can not solve it directly. Thus, we usually use the regularization method to solve it. The basic form of the regularization problem based on L2-norm is to solve the following optimization problem:

$$\min_{\boldsymbol{Z}} \left\{ \|\boldsymbol{M}\boldsymbol{Z} - \boldsymbol{P}\|_2^2 + \lambda^2 \|\boldsymbol{Z}\|_2^2 \right\} \,. \tag{4}$$

In addition to the L2-norm regularization algorithm, there are also some other regularization optimization methods based on L1-norm, Total-Variation (TV) norm and so on.[5] Among them, L2-norm problem is simplest and there are so many optimization methods can solve it. In the field of photoacoustic tomography, the sparsity for the reconstructed image pixels is much lower than the fluorescence imaging. Therefore, the regularization algorithm based on L2-norm has often been able to obtain better image reconstruction results than some L1-norm algorithms. However, there is no free lunch in the world. Due to the low image sparsity reconstructed by the L2-norm algorithm, the artifact and noise in the reconstructed images are increased. Fortunately, this shortcoming can be overcome by adding a matrix in the regularization term of Equation (4).

3. REGULARIZATION MATRIX

In general , we can reduce the noise artifacts in reconstructed images by multiplying the regularization term in Equation (4) by a matrix[6]

$$\min_{\boldsymbol{Z}} \left\{ \|\boldsymbol{M}\boldsymbol{Z} - \boldsymbol{P}\|_2^2 + \lambda^2 \|\boldsymbol{L}\boldsymbol{Z}\|_2^2 \right\} \,, \tag{5}$$

where $\boldsymbol{L}$ is the regularization matrix. In order to punish the noise, the regularization matrix is usually based on some high-pass filter kernel. For the field of image processing, common hign-pass kernels include first-order differential kernels (such as Sobel kernel) and second-order differential kernels (such as Laplace kernel). The first-order differential kernels are usually anisotropic, which can lead to unnecessary directional ripples in reconstructed images. Therefore, we usually choose second-order high-pass filter kernels. Among them, the most commonly used is the Laplace high-pass kernel

$$\boldsymbol{K}_L = \frac{1}{9} \begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{pmatrix} \,. \tag{6}$$

Compared with the traditional L2-norm algorithm without regularization matrix, this method can achieve the purpose of improving image contrast and eliminating noise artifacts. However, many details in reconstructed images will be lost due to the isotropy property of the high-pass Laplace kernel. Therefore, if we can combine the original optimization problem (4) with the optimization problem including the Laplace regularizization matrix at a certain ratio, we can eliminate more noise artifacts while preserving more details. Based on this idea, a new regularization matrix is proposed in this paper. By adding a certain weight to the central element of kernel (6), a novel kernel named Central-Enhancement Laplace (CEL) kernel can be obtained, that is

$$\boldsymbol{K}_{CEL} = \frac{1}{9}\begin{pmatrix} -1 & -1 & -1 \\ -1 & 8+w & -1 \\ -1 & -1 & -1 \end{pmatrix}, \tag{7}$$

where w is a weight value ($w > 0$), abbreviated as the enhancement factor or weight. When $w = 0$, CEL kernel degenerates to Laplace High-pass filter kernel. In fact, it can be expressed in the following formation

$$\begin{aligned} \boldsymbol{K}_{CEL} &= \frac{1}{9}\left[\begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{pmatrix} + w\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] \\ &= \boldsymbol{K}_L + \frac{w}{9}\boldsymbol{K}_{\boldsymbol{I}}, \end{aligned} \tag{8}$$

Where $\boldsymbol{K}_L$ denotes the unit kernel, which is corresponding to the original L2-norm problem (4).

Let $\boldsymbol{L}_{CEL}$ and $\boldsymbol{L}$ be the regularization matrix based on CEL kernel and Laplace high-pass kernel, and rewrite the original L2-norm problem as

$$\min_{\boldsymbol{Z}}\left\{\|\boldsymbol{MZ}-\boldsymbol{P}\|_2^2 + \lambda^2\|\boldsymbol{L}_{CEL}\boldsymbol{Z}\|_2^2\right\}. \tag{9}$$

After taking Equation (8) into Equation (9), a new optimization problem can be obtained:

$$\min_{\boldsymbol{Z}}\left\{\|\boldsymbol{MZ}-\boldsymbol{P}\|_2^2 + \lambda^2\frac{w^2}{81}\|\boldsymbol{Z}\|_2^2 + \lambda^2\|\boldsymbol{LZ}\|_2^2 + \lambda^2\frac{w}{9}\boldsymbol{Z}^T(\boldsymbol{LZ}) + \lambda^2\frac{w}{9}(\boldsymbol{LZ})^T\boldsymbol{Z}\right\}. \tag{10}$$

From Equation (10), we can obtain four regularization terms, which are $\|\boldsymbol{Z}\|_2^2$, $\|\boldsymbol{LZ}\|_2^2$, $\boldsymbol{Z}^T(\boldsymbol{LZ})$ and $(\boldsymbol{LZ})^T\boldsymbol{Z}$. $\|\boldsymbol{Z}\|_2^2$ and $\|\boldsymbol{LZ}\|_2^2$ are just the regularization terms in Equation (4) and Equation (5). $\boldsymbol{Z}^T(\boldsymbol{LZ})$ and $(\boldsymbol{LZ})^T\boldsymbol{Z}$. $\|\boldsymbol{Z}\|_2^2$ are two new regularization terms, which are equivalent to the point-multiplication operation between the images before and after high-pass filtering. In order to facilitate the analysis later, we divide these regularization terms into three categories according to the penalty intensity for noise. Naturally, $\|\boldsymbol{Z}\|_2^2$ belongs to the weak penalty term, $\|\boldsymbol{LZ}\|_2^2$ belongs to the strong penalty term, and the remaining two terms belong to moderate penalty term. The increase in penalty intensity results in higher image smoothness, loss of more detailed information, less noise and artifacts, and this can indirectly improve the contrast of the image. On the contrary, the decrease in penalty intensity will lead to image sharpening, loss of less detailed information, more noise and artifacts. Because the optimization problem contains three kinds of regularization terms with different penalty intensity, the ratio of smoothness and fineness for the reconstructed images can be freely adjusted by changing the weight w. This illustrates that by using the CEL matrix, we can remove the artifacts while preserving details, and the parameter w can be adjusted for different images, for different devices and for different requirement. As w increases, so does the weight of weak penalty term and moderate penalty terms, which leads to the overall weakening of noise penalty intensity. Thus, the noise level in the reconstructed images increase and more details can be restored. Similarly, when w decreases, the penalty of noise increases, the reconstructed image becomes smoother, and more detailed information is lost. Because the selection of parameter w is very flexible, how to choose the most suitable w for each image is a question for further research.

4. PERFORMANCE EVALUATION OF CEL MATRIX

In this section, we demonstrate the performance of CEL matrix in vivo studies. We took a nude mouse with a tumor on its liver as an experimental object and utilized a small animal PAT system (MSOT inVision 128) to acquire the raw data.[7] In this experiment, we used the interpolation algorithm used in Ref. 4 to generate the model matrix and used LSQR algorithm to solve the minimum optimization problem. In order to control variables, the number of discrete points was set to 300.

By using the MSOT system and taking different weight w, we can obtain the reconstruction images of the nude mouse, which are shown in Fig. 1. From Fig. 1(a)-(e), we can intuitively feel that the use of regularization matrix can indeed suppress noise and artifacts. Although the reconstructed images using CEL matrix had more artifacts than Laplace matrix, most of artifacts were concentrated on the edge of each image. In contrast, the noise and artifacts inside this nude mouse were similar with the reconstructed images using Laplace matrix. Thus, the images obtained by using CEL matrix were smooth without losing the detailed information inside the mouse. By adjusting the weight w, different reconstruction effect appeared. When w was becoming larger, the detailed information, especially the boundary information was becoming sharper at the cost of more artifacts appearing outside the mouse. As is shown in Fig. 1(f), compared to the conventional L2-norm method, the edge contrast of reconstructed images obtained by using Laplace matrix was severely reduced. However, through changing the weight of the central element in Laplace kernel, the edge information can be restored and even can exceed the conventional L2-norm regularization method.

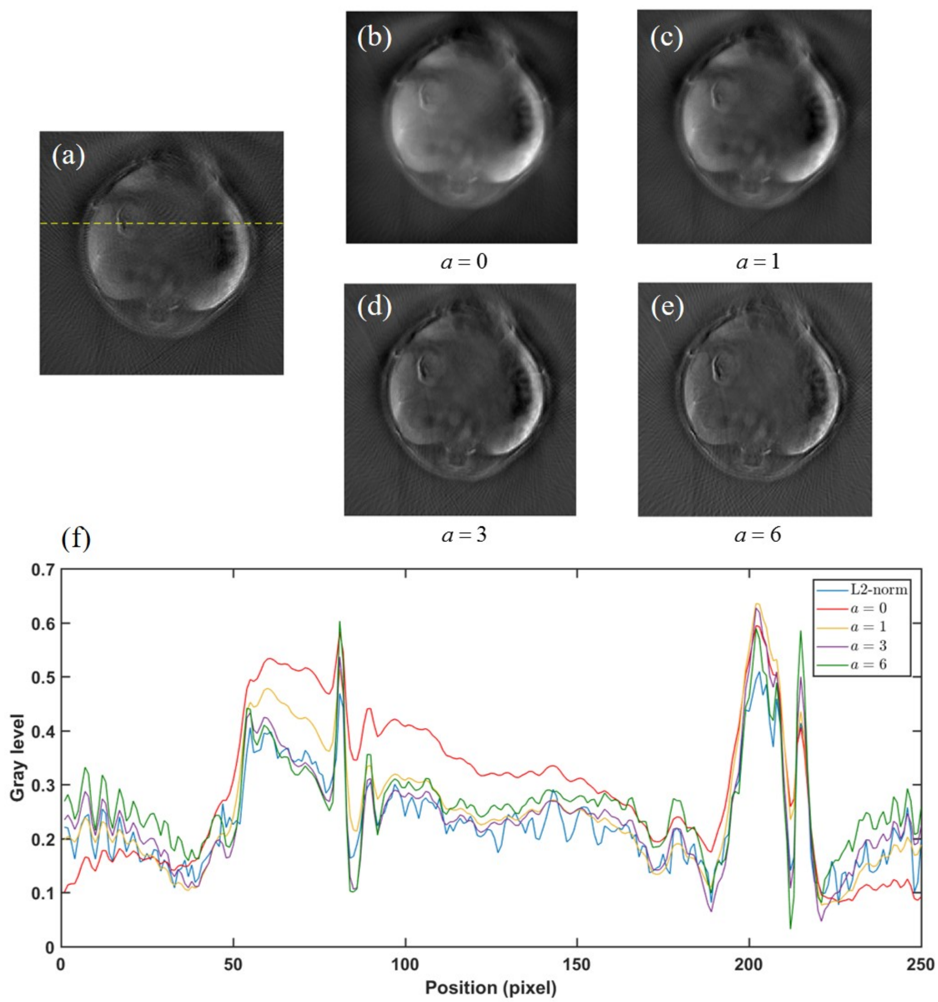
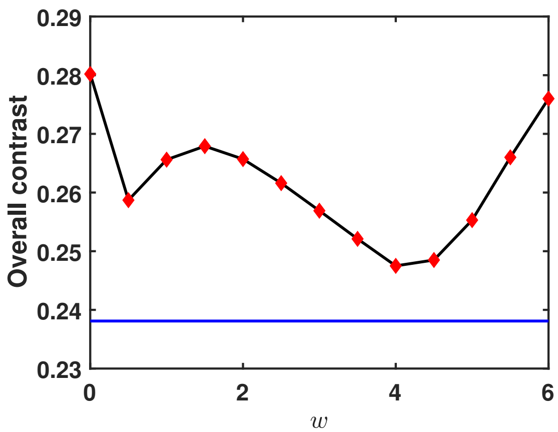
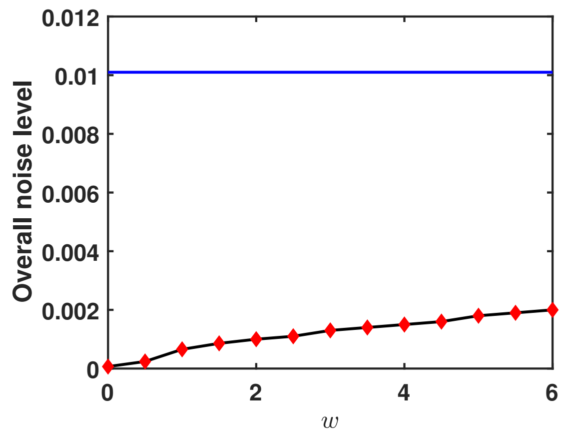


Figure 1. Different tomographic reconstructions of a nude mouse in the cross-section of a tumor on its liver. (a) Reconstruction result of conventional L2-norm method without using any regularization matrix. (b) Reconstruction result of L2-norm method using Laplace regularization matrix. (c)-(e) Reconstruction results of L2-norm method using CEL regularization matrix with different parameter w. (f) Intensity profiles extracted from the reconstructed images using different methods. The extracted position was marked by the yellow dashed line in (a).

Next, we sampled the weight w more fine, and made a quantitative analysis of the contrast and noise level in these reconstructed images. The result of overall contrast and noise level are shown in Fig. 2. The noise level was quantized by the weak textured patches method[8,9] and the contrast was quantized by the root mean square method.[10] The experimental results illustrate that with the increase of w, the image contrast is irregularly changed, but the overall contrast was higher than L2-norm regularization method without using regularization matrix. The overall noise level of reconstructed images was also consistent with the visual perception. With the increase of w, the overall noise level of images was almost linearly increased, but the rate of increase was very slow. Even when w was 6, the noise level was far less than the conventional L2-norm method. In general, the overall noise level in reconstructed images based on CEL matrix was higher than that of L2-norm regularization algorithm based on Laplace matrix and lower than that of conventional L2-norm regularization algorithm.



(a)



(b)

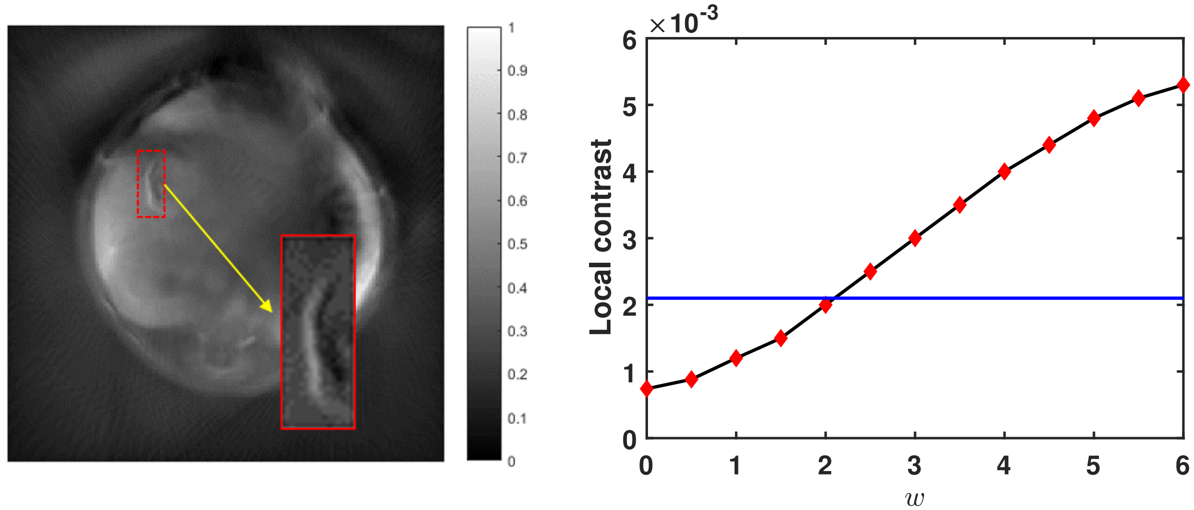
Figure 2. Results of quantitative analysis about reconstructed images using different weight w. (a) The relationship between the weight w and overall contrast. (b) The relationship between the weight w and overall noise level.

In order to analyze the performance of CEL matrix more carefully, we further tested the local contrast for the effective region (the tumor) in reconstructed images. The results about this experiment is shown in Fig. 3. It is not difficult to find that when Laplace matrix was added to the regularization term, the contrast and sharpness of tumor's edge was less than conventional L2-norm regularization method. Therefore, this was bound to be accompanied by the loss of image details. By using CEL matrix and increasing the weight w, the contrast of the reconstructed image was gradually increasing. When w was greater than 2, image contrast was higher than the conventional L2-norm algorithm.

Finally, we compared the reconstruction effect of CEL matrix with the L1-norm regularization algorithm and Fig. 4 is the reconstruction results. For the L1-norm regularization optimization problem in this experiment, we used the method in Ref. 11 to solve it. Although the reconstructed image of L1-norm regularization method had little noise and artifacts, the critical information inside the mouse was seriously lost. This is due to the sparsity property of L1-norm regularization problem and is unavoidable. In the reconstructed image using CEL matrix, the information inside the mouse was preserved very well, which was only at the expense of a small increase in noise. At the same time, we also see that, compared to the L1- norm regularization method, the reconstruction algorithm based on L2-norm regularization is more suitable for PAT.

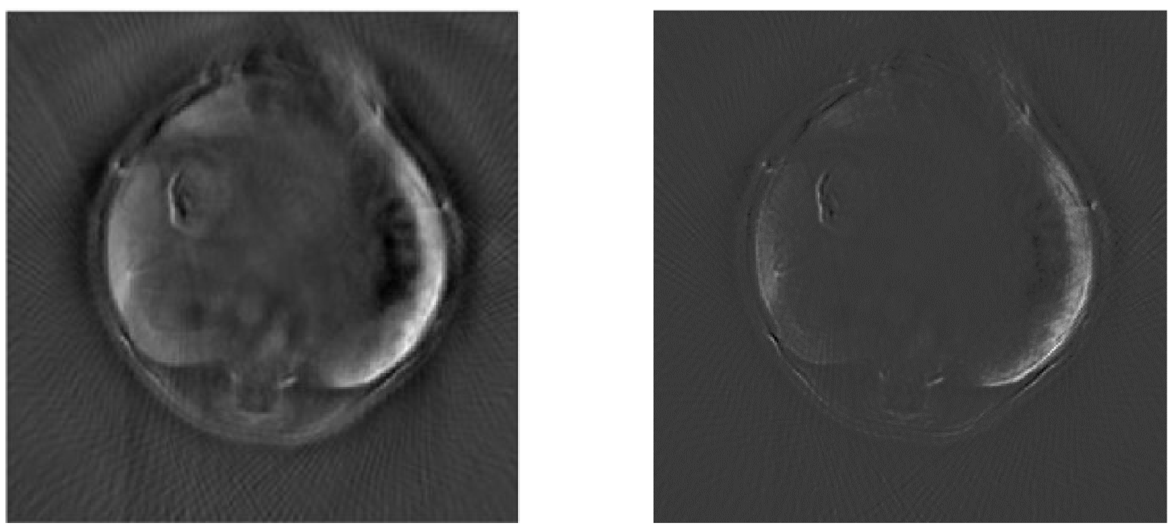
5. CONCLUSION

In this work, we present a novel regularization matrix, the CEL matrix, for model-based reconstruction in photoacoustic tomography. It is an improvement of the conventional L2-norm regularization algorithm. We conducted a series of experiments to demonstrate the performance of CEL matrix. Using CEL matrix can



(a) (b)

Figure 3. Local quantitative analysis about reconstructed images using different weight w. (a) Schematic of the local region. One edge of the tumor on the liver is in the middle of this region. (b) The relation between weight w and local contrast in the area shown in (a).



(a) (b)

Figure 4. The comparison between L2-norm regularization method based on CEL matrix and L1-norm regularization method. (a) Reconstructed image with the use of CEL matrix ($w = 3$). (b) Reconstructed image using L1-norm regularization method.

obtain the reconstructed images with better quality than conventional L2-norm regularization method. By quantitative analysis of contrast and noise level, the performance of CEL matrix is better than conventional L2-norm method. Compared with the reconstructed image obtained by the Laplace regularization matrix and the L1-norm regularization method, the reconstruction quality of CEL matrix was also significantly improved. This is mainly due to the smooth effect of the Laplace matrix, and a lot of details in images are lost. The CEL matrix can retain more detail information at the cost of external additional noise.

On the basis of the above discussion, we can summarize the properties of CEL matrix as follows: (1) The use of CEL matrix doesn't affect the overall contrast of the reconstructed image. (2) An increase in weight w will result in an increase in image noise. However, the increase in noise is mainly concentrated on the lateral side of the mouse. Therefore, this doesn't lead to the loss of the detail information. (3) The increase in weight w will greatly improve the detail contrast of the reconstructed images and this is beneficial to the extraction of tumor boundaries. (4) The setting of weight w needs to consider the actual factors and larger w doesn't mean that we can get a better reconstructed image. Thus, both subjective qualitative and objective quantitative aspects should be taken into account.

Although CEL matrix has great flexibility, the process of manually adjusting the parameters is very cumbersome. It is very important to further study a adaptive algorithm for CEL matrix, that is, different suitable weight w can be given for different reconstructed images and even the different regions of the same image can be set with different weights.

In conclusion, the use of CEL matrix can enhance the image details and suppress the noise synchronously. Compared with the conventional L2-norm regularization method, the addition of CEX regularization matrix can obtain the reconstructed images with higher contrast and lower noise level. By adjusting the weight w, the reconstructed images with different effects can be obtained, which can meet the needs of different situations.

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