Continuous-Time Linear Parallel Output Regulation

Hongyang Li, Qinglai Wei, and Fei-Yue Wang
The State Key Laboratory of Management and Control for Complex Systems
Institute of Automation, Chinese Academy of Sciences
Beijing, China
emails: lihongyang2019@ia.ac.cn, qinglai.wei@ia.ac.cn, feiyue.wang@ia.ac.cn

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***Abstract*—A novel parallel output regulation method is presented in this paper. The framework of the parallel output regulation problem is first introduced. Then, the existence of parallel controller is analyzed detailly in the situation that only the measurable information is obtained. Finally, the simulation example is given to show the performance of the presented method.**

***Index Terms*—Parallel controller, output regulation, parallel system theory.**

I. Introduction

Parallel system theory [1]–[9] has gained widespread attention because of the superiority of improving the system performance in recent years. Inspired by parallel system theory, a novel parallel control structure was proposed in [10], where the derivative of control law was modeled as shown in Fig. 1. This parallel control structure can alleviate the curves of system control laws and improve the dynamic characteristics of control systems. Therefore, it arouses wide concern of researchers. In [11], a novel method to prove the existence of parallel controller was proposed for continuous-time linear systems. In [12], the parallel optimal tracking problem was studied based on adaptive dynamic programming, which showed that the parallel controller can smooth the control signal for the step tracking signal comparing with traditional feedback control. In [13], an event-trigger based parallel control method was proposed. It can be seen that the parallel controllers can realize better system performance than traditional feedback controller. Therefore, it is a promising research direction.

On the other hand, interference is ubiquitous in control systems. In tracking control problems, interference suppression is necessary, which leads to the popularity of output regulation techniques [14]–[18]. The main target of output regulation is to track given reference signals while cope with the effects of external interference. Since an exhaustive study were made for the linear output regulation problems in 1970s [19]–[22], many researcher devoted themselves to find various output regulation methods, such as sliding-mode methods [23], [24], optimal control methods [25], [26], and so on [27]–[29]. However, it should be noted that all the above methods are based on feedback control theory, which has some disadvantages in improving the system performance [10]. It is the starting point of this paper.

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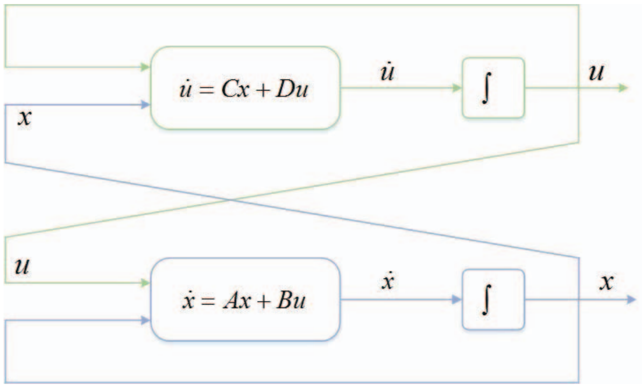

Fig. 1. The structure of parallel controller

Inspired by the parallel control theory proposed in [10], a novel output regulation method is presented for continuous-time linear systems in this paper. The basic structure of parallel output regulator is first introduced. Then, the existence of parallel output regulator is analyzed detailly. Finally, numerical analysis is provided to show the performance of the presented parallel output regulation method.

II. Problem Formulation

Consider the following continuous-time linear systems:

$$\begin{aligned} \dot{x} &= Ax + Bu + Sw, \\ \dot{w} &= Pw, \\ e &= C_e x + D_e w \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is system state, $u \in \mathbb{R}^m$ is control input, $w \in \mathbb{R}^l$ is external signal, and $e \in \mathbb{R}^k$ is error. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $S \in \mathbb{R}^{n\times l}$, $P \in \mathbb{R}^{l\times l}$, $C_e \in \mathbb{R}^{k\times n}$ and $D_e \in \mathbb{R}^{k\times l}$ are system matrices. For the system (1), Sw, $-D_e w$ and $C_e x$ represent system disturbance, tracking signal, and output signal, respectively. To further investigate the output regulation problem, the following assumptions are provided.

Assumption 1. *For the matrix P, we have $\mathrm{Re}(\lambda_i) \geq 0, i = 1, 2, \ldots, l$, where $\lambda_i, i = 1, 2, \ldots, l$ are the eigenvalues of P and $\mathrm{Re}(\cdot)$ represents real part.*

Assumption 2. *For the system (1), we can only obtain the measurable information, which can be expressed as follows:*

$$y = C_y x + D_y w. \tag{2}$$

Assumption 3. *The system (A, B) is controllable, and the system $\left(\begin{pmatrix} A & S \\ 0 & P \end{pmatrix}, \begin{pmatrix} C_y & D_y \end{pmatrix}\right)$ is detectable.*

We can design parallel controller as follows:

$$\dot{u} = \begin{pmatrix} K_x & K_w \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{w} \end{pmatrix} + K_u u \tag{3}$$

where $\hat{x}$ and $\hat{w}$ are the estimation values of x and w with the following expression:

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{\hat{w}} \end{pmatrix} = \left(\begin{pmatrix} A & S \\ 0 & P \end{pmatrix} - \begin{pmatrix} H_x \\ H_w \end{pmatrix} \begin{pmatrix} C_y & D_y \end{pmatrix}\right) \begin{pmatrix} \hat{x} \\ \hat{w} \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + \begin{pmatrix} H_x \\ H_w \end{pmatrix} y. \tag{4}$$

The main target of this paper is to design the parallel controller (3) which can make the system (1) realize output regulation properties [14].

III. Main Results

The existence of parallel controller (3) will be analyzed in this section. If there exists no external signal, i.e., $w = 0$, we can rewrite (1)–(2) as

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ e &= C_e x \end{aligned} \tag{5}$$

and

$$y = C_y x. \tag{6}$$

Based on (3)–(6), we have

$$\begin{pmatrix} \dot{x} \\ \dot{u} \\ \dot{\hat{x}} \\ \dot{\hat{w}} \end{pmatrix} = G \begin{pmatrix} x \\ u \\ \hat{x} \\ \hat{w} \end{pmatrix} \tag{7}$$

where

$$G = \begin{pmatrix} A & B & 0 & 0 \\ 0 & K_u & K_x & K_w \\ H_x C_y & B & A - H_x C_y & S - H_x D_y \\ H_w C_y & 0 & -H_w C_y & P - H_w D_y \end{pmatrix}.$$

Define $z = \begin{pmatrix} x^{\mathrm{T}} & u^{\mathrm{T}} & \hat{x}^{\mathrm{T}} & \hat{w}^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$, we can take coordinate transformation as

$$z' = \begin{pmatrix} I & & & \\ & I & & \\ I & & I & \\ & & & I \end{pmatrix} z.$$

We can obtain that

$$\dot{z}' = \begin{pmatrix} A & B & 0 & 0 \\ K_x & K_u & K_x & K_w \\ 0 & 0 & A - H_x C_y & S - H_x D_y \\ 0 & 0 & -H_w C_y & P - H_w D_y \end{pmatrix} z. \tag{8}$$

For the matrix $\begin{pmatrix} A & B \\ K_x & K_u \end{pmatrix}$, we can assign the poles according to [10]. The poles of the matrix $\begin{pmatrix} A - H_x C_y & S - H_x D_y \\ -H_w C_y & P - H_w D_y \end{pmatrix}$ can be assigned according to [30]. Then, we can provide the following theorem to analyze the existence of parallel controller (3).

Theorem 1. *Give the matrices K_x, K_u, H_x and H_u which make the system (7) stable. For the system (1), there exists parallel controller (3) which can realize output regulation properties, if and only if there exist matrices K_w, Γ_1, Γ_2 and Γ_3, which satisfies the following equations:*

$$A\Gamma_1 + B\Gamma_2 + S = \Gamma_1 P, \tag{9}$$

$$\begin{pmatrix} K_x & K_w \end{pmatrix} \Gamma_3 + K_u \Gamma_2 = \Gamma_2 P, \tag{10}$$

$$\begin{aligned} &\begin{pmatrix} A - H_x C_y & S - H_x D_y \\ -H_w C_y & P - H_w D_y \end{pmatrix} \Gamma_3 + \begin{pmatrix} B \\ 0 \end{pmatrix} \Gamma_2 \\ &+ \begin{pmatrix} H_x \\ H_w \end{pmatrix} C_y \Gamma_1 + \begin{pmatrix} H_x \\ H_w \end{pmatrix} D_y = \Gamma_3 P, \end{aligned} \tag{11}$$

$$C_e \Gamma_1 + D_e = 0. \tag{12}$$

Proof. For (1)–(4), we can take coordinate transformations as

$$\begin{aligned} x' &= x - \Gamma_1 w, \\ w' &= w, \\ u' &= u - \Gamma_2 w, \\ \varsigma' &= \varsigma - \Gamma_3 w \end{aligned} \tag{13}$$

where $\varsigma = \begin{bmatrix} \hat{x}^{\mathrm{T}} & \hat{w}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$. It can be obtained that

$$\begin{aligned} \dot{x}' =& \dot{x} - \Gamma_1 \dot{w} \\ =& A(x' + \Gamma_1 w') + B(u' + \Gamma_2 w') + Sw' - \Gamma_1 P w' \\ =& Ax' + Bu' + Sw' + (A\Gamma_1 + B\Gamma_2 + S - \Gamma_1 P) w', \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{u}' =& \begin{pmatrix} K_x & K_w \end{pmatrix} (\varsigma' + \Gamma_3 w') \\ &+ K_u (u' + \Gamma_2 w') - \Gamma_2 P w' \\ =& \begin{pmatrix} K_x & K_w \end{pmatrix} \varsigma' + K_u u' \\ &+ \left(\begin{pmatrix} K_x & K_w \end{pmatrix} \Gamma_3 + K_u \Gamma_2 - \Gamma_2 P\right) w', \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{\varsigma}' =& \begin{pmatrix} A - H_x C_y & S - H_x D_y \\ -H_w C_y & P - H_w D_y \end{pmatrix} (\varsigma' + \Gamma_3 w') \\ &+ \begin{pmatrix} B \\ 0 \end{pmatrix} (u' + \Gamma_2 w') - \Gamma_3 P w' \\ &+ \begin{pmatrix} H_x \\ H_w \end{pmatrix} (C_y (x' + \Gamma_1 w') + D_y w') \\ =& \begin{pmatrix} A - H_x C_y & S - H_x D_y \\ -H_w C_y & P - H_w D_y \end{pmatrix} \varsigma' + \begin{pmatrix} B \\ 0 \end{pmatrix} u' \\ &+ \begin{pmatrix} H_x \\ H_w \end{pmatrix} C_y x' + \begin{pmatrix} A - H_x C_y & S - H_x D_y \\ -H_w C_y & P - H_w D_y \end{pmatrix} \Gamma_3 w' \\ &+ \begin{pmatrix} B \\ 0 \end{pmatrix} \Gamma_2 w' + \begin{pmatrix} H_x \\ H_w \end{pmatrix} C_y \Gamma_1 w' \\ &+ \begin{pmatrix} H_x \\ H_w \end{pmatrix} D_y w' - \Gamma_3 P w'. \end{aligned} \tag{16}$$

We can obtain that

$$\begin{aligned} e =& C_e \left(x' + \Gamma_1 w' \right) + D_e w' \\ =& C_e x' + \left(C_e \Gamma_1 + D_e \right) w'. \end{aligned} \tag{17}$$

For (9)–(11), we have

$$G \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} - \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} P = - \begin{pmatrix} S \\ 0 \\ H_x D_y \\ H_w D_y \end{pmatrix}. \tag{18}$$

Give the matrix H_w, the Sylvester equation (18) has unique solution $\begin{pmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix}$. Therefore, we can always obtain matrices K_w, Γ_1, Γ_2 and Γ_3, which satisfy the equations (9)–(11). We have

$$\begin{pmatrix} \dot{x}' \\ \dot{u}' \\ \dot{\varsigma}' \end{pmatrix} = G \begin{pmatrix} x' \\ u' \\ \varsigma' \end{pmatrix}. \tag{19}$$

If Γ_1 is the solution of (12), we have

$$\lim_{t\to\infty} e = \lim_{t\to\infty} C_e x' = 0. \tag{20}$$

The necessity is completed. If parallel controller (3) can realize output regulation of the system (1), we have

$$\lim_{t\to\infty} e = \lim_{t\to\infty} \left(C_e x' + \left(C_e \Gamma_1 + D_e \right) w' \right) = 0. \tag{21}$$

For (21), w' cannot decay to zero because of Assumption 1. Therefore, (12) is satisfied.

The proof is completed. □

IV. Numerical Analysis

Consider linear system and external system of the form (1) with the following system matrices:

$$\begin{aligned} A &= \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ S &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, P = \begin{pmatrix} 0 & 50 \\ -50 & 0 \end{pmatrix}, \\ C_e &= \begin{pmatrix} 0.5 & 1 \end{pmatrix}, D_e = \begin{pmatrix} 1 & 0 \end{pmatrix}. \end{aligned} \tag{22}$$

The measurable output matrices C_y and D_y can be given as

$$C_y = \begin{pmatrix} 0 & 1 \end{pmatrix}, D_y = \begin{pmatrix} 1 & 0 \end{pmatrix}. \tag{23}$$

We can design parallel controller as

$$\dot{u} = \begin{pmatrix} 4 & -12 & 2548 & -155 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{w} \end{pmatrix} - 5u \tag{24}$$

where

$$\begin{aligned} \begin{pmatrix} \dot{\hat{x}} \\ \dot{\hat{w}} \end{pmatrix} =& \begin{pmatrix} 0 & 0.0064 & 0.0064 & 0 \\ -1 & 1.7195 & -0.2805 & 1 \\ 0 & -9.7195 & -9.7195 & 50 \\ 0 & 48.1680 & -1.8320 & 0 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{w} \end{pmatrix} \\ &+ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} u + \begin{pmatrix} 0.9936 \\ 0.2805 \\ 9.7195 \\ -48.1680 \end{pmatrix} y. \end{aligned} \tag{25}$$

The initial conditions are provided as $x(0) = \begin{pmatrix} 2 & -4 \end{pmatrix}^{\mathrm{T}}$, $w(0) = \begin{pmatrix} -1 & 1 \end{pmatrix}^{\mathrm{T}}$, $u(0) = 2$, $\hat{x}(0) = \begin{pmatrix} 0 & 0 \end{pmatrix}^{\mathrm{T}}$ and $\hat{w}(0) = \begin{pmatrix} 0 & 0 \end{pmatrix}^{\mathrm{T}}$. We can obtain the simulation results as shown in Figs. 2 and 3. The control curve is provided in Fig. 2, and the error curve is given in Fig. 3. The performance of the presented parallel control method can be verified.

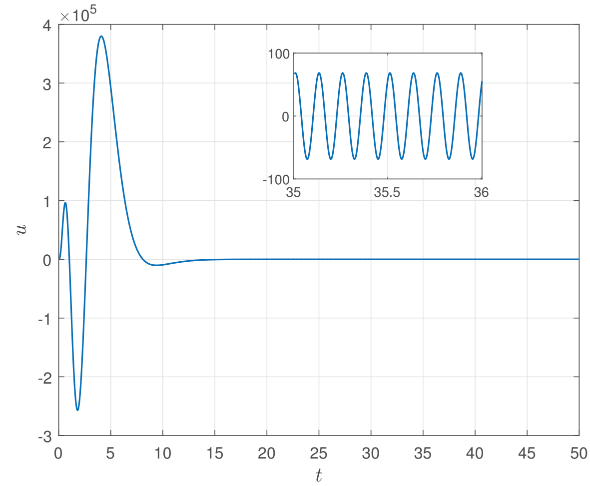

Fig. 2. Control curve

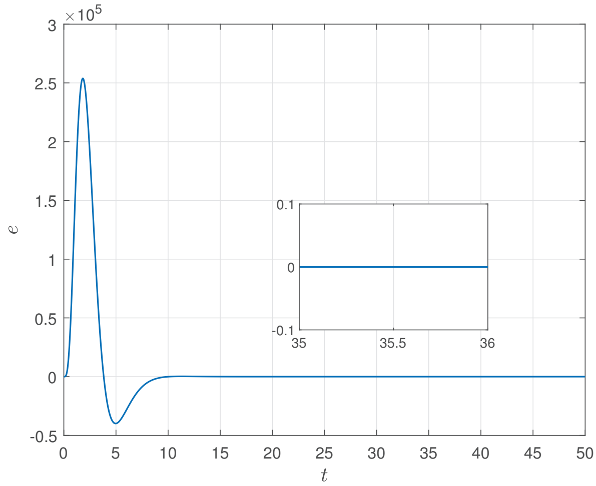

Fig. 3. Error curve

V. Conclusion

The parallel control method for continuous-time linear output regulation problem is studied in this paper. The existence of parallel controller is analyzed detailly. The simulation example is provided to show the good performance of the presented method.

References

[1] F.-Y. Wang, *Shadow Systems: A New Concept for Nested and Embedded Co-Simulation for Intelligent Systems*, University of Arizona, Tucson, Arizona State, USA, 1994.

[2] F.-Y. Wang, "On the modeling, analysis, control and management of complex systems," *Complex Systems and Complexity Science*, vol. 3, no. 2, pp. 26–34, Jun. 2006.

[3] F.-Y. Wang, "Parallel control and digital twins: Control theory revisited," *Chinese Journal of Intelligent Science and Technology*, vol. 2, no. 3, pp. 213–222, 2020.

[4] Q. Wei, L. Wang, J. Lu, and F.-Y. Wang, "Discrete-time self-learning parallel control," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, in press 2020.

[5] F.-Y. Wang, "Parallel system methods for management and control of complex systems," *Control and Decision*, vol. 19, no. 5, pp. 485–489, 514, May. 2004.

[6] F.-Y. Wang, "Computational experiments for behavior analysis and decision evaluation of complex systems," *Journal of System Simulation*, vol. 16, no. 5, pp. 893–897, May. 2004.

[7] F.-Y. Wang, D. Liu, G. Xiong, C. Cheng, and D. Zhao, "Parallel control theory of complex systems and applications," *Complex Systems and Complexity Science*, vol. 9, no. 3, pp. 1–12, Sep. 2012.

[8] F.-Y. Wang, "Parallel control: A method for data-driven and computation control," *Acta Automatica Sinica*, vol. 39, no. 4, pp. 293–302, Apr. 2013.

[9] F.-Y. Wang and Q. Wei, "Intelligent control: from learning control to parallel control," *Control Theory & Applications*, vol. 35, no. 7, pp. 939–948, Jul. 2018.

[10] Q. Wei, H. Li, and F.-Y. Wang, "Parallel control for continuous-time linear systems: A case study," *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 4, pp. 919–928, Jun. 2020.

[11] Q. Wei, H. Li, and F.-Y. Wang, "Continuous-Time Linear Parallel Control," *In Proceedings of 2020 Chinese Automation Congress (CAC)*, pp. 1605–1608, Nov. 2020.

[12] J. Lu, Q. Wei, and F.-Y. Wang, "Parallel control for optimal tracking via adaptive dynamic programming," *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 6, pp. 1662–1674, Nov. 2020.

[13] J. Lu, Q. Wei, Y. Liu, T. Zhou, and F.-Y. Wang, "Event-triggered optimal parallel tracking control for discrete-time nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, early access, doi: 10.1109/TSMC.2021.3073429.

[14] J. Huang, *Nonlinear output regulation: Theory and applications*, Phildelphia, PA: SIAM, 2004.

[15] L. Li, J. Fu, Y. Zhang, T. Chai, L. Song, and P. Albertos, "Output regulation for networked switched systems with alternate event-triggered control under transmission delays and packet losses," *Automatica*, vol. 131, Sep. 2021.

[16] C. Deng, "Cooperative fault-tolerant output regulation of linear heterogeneous multiagent systems under directed network topology," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 8, pp. 4773–4781, Aug. 2021.

[17] R. Marino and P. Tomei, "Adaptive output regulation for minimum-phase systems with unknown relative degree," *Automatica*, vol. 130, Aug. 2021.

[18] H. Zhang, H. Liang, Z. Wang, and T. Feng, "Optimal output regulation for heterogeneous multiagent systems via adaptive dynamic programming," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 1, pp. 18–29, Jan. 2017.

[19] W. Liu and J. Huang, "Output regulation of linear systems via sampled-data control," *Automatica*, vol. 113, Mar. 2020.

[20] B. A. Francis, "The linear multivariable regulator problem," *SIAM Journal on Control and Optimization*, vol. 15, no. 3, pp. 486–505, 1977.

[21] B. A. Francis and W. M. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, no. 5, pp. 457–469, 1976.

[22] E. J. Davison, "The robust control of a servomechanism problem for linear time-invariant multivariable systems," *IEEE Transactions on Automatic Control*, vol. 21, no. 1, pp. 25–34, Feb. 1976.

[23] A. Chalanga, M. Patil, B. Bandyopadhyay, and H. Arya, "Output regulation using new sliding surface with an implementation on inverted pendulum system," *European Journal of Control*, vol. 45, pp. 85–91, Jan. 2019.

[24] S. Seshagiri and H. K. Khalil, "Robust output regulation of minimum phase nonlinear systems using conditional servocompensators," *International Journal of Robust and Nonlinear Control*, vol. 15, no. 2, pp. 83–102, Jan. 2005.

[25] W. Gao and Z.-P, Jiang, "Adaptive dynamic programming and adaptive optimal output regulation of linear systems," *IEEE Transactions on Automatic Control*, vol. 61, no. 12, pp. 4164–4169, Dec. 2016.

[26] J.-W. Lee and P. P. Khargonekar, "Optimal output regulation for discrete-time switched and Markovian jump linear systems," *SIAM Journal on Control and Optimization*, vol. 47, no. 1, pp. 40–72, 2008.

[27] X. Wang, Y. Hong, J. Huang, and Z.-P, Jiang, "A distributed control approach to a robust output regulation problem for multi-agent linear systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 12, pp. 2891–2895, Dec. 2010.

[28] Y. Su and J. Huang, "Cooperative semi-global robust output regulation for a class of nonlinear uncertain multi-agent systems," *Automatica*, vol. 50, no. 4, pp. 1053–1065, Apr. 2014.

[29] D. Ma, T. Wang, H. Zhang, and X. Xie, "Adaptive fault-tolerant output regulation of linear systems with unknown dynamics and actuator faults," *Applied Mathematics and Computation*, vol. 402, Aug. 2021.

[30] C.-T. Chen, *Linear system theory and design*, Oxford University Press, USA, 1998.