Modeling and Simulation of Folding-Boom Aerial Platform Vehicle Based on the Flexible Multi-body Dynamics

Haidong Hu, En Li, Xiaoguang Zhao, Zize Liang and Wensheng Yu

Abstract—Folding-boom aerial platform vehicle is a type of construction vehicle used to hoist personnel to the appointed location in the aerial. As for aerial platform vehicle, the flexible deformations of the arm system cannot be neglected. Therefore, the flexible multi-body dynamics equations of the arm system of folding-boom aerial platform vehicle are established based on flexible multi-body dynamics theory and Lagrange's equation. Following, the simulation is carried out. The simulation results show that the moving beams exist high frequency vibrations, which caused by the elastic deformations of them. As a result, the vibrations cause the work platform of aerial platform vehicle to shake, and at the same time, the deflections of the beams lead to small deviations of the trajectory of the work platform. Therefore, the establishment of the equations lays the basis of vibration controlling and accurately controlling the trajectory of the work platform of aerial platform vehicle.

I. INTRODUCTION

A ERIAL platform vehicle is a kind of construction vehicle which can hoist personnel to the appointed location in the aerial for installation or maintenance.

There are several types of aerial platform vehicle, including telescopic-boom, folding-boom and mixing-boom styles.

Presently, some researches were carried out on the telescopic-boom aerial platform vehicle [1, 2]. But for folding-boom aerial platform vehicle, few published literatures are found for its design and control. The research of folding-boom aerial platform vehicle could lay the basis for the aerial platform vehicle with the more complex arm system, such as mixing-boom style, the beams of which have the function of folding and extension. And what's more, the amount of folding-boom aerial platform vehicle occupies 80% of aerial platform vehicle used in China. Therefore, the research of it has its practical significance. In this paper, we focus on the folding-boom aerial platform vehicle, which could cross barriers easily. As is shown in the Fig.1, Folding-boom aerial platform vehicle has two beams, each of which is driven by a hydraulic cylinder.

As an apparatus with the human working on the platform,

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aerial platform vehicle requires very high safety. Therefore, the steady movement and accurately positioning of work platform should be ensured. As for the long beam of aerial platform vehicle, elastic deformations of them are not be neglected, so the dynamics of the arm system of aerial platform vehicle cannot be depicted accurately by the rigid model without considering the flexible deformations. In this paper, the two beams are seen as flexible and the flexible multi-body dynamics equations of the arm system are established based on flexible multi-body dynamics theory and Lagrange's equation. The theory of flexible multi-body dynamics is introduced in [3], and the theory has been used for the bridge detection vehicle and concrete pump truck in [4 5 6]

Based on the flexible dynamic model set up, the experimental study is carried on. By analysis of the simulation results, the model derived by the method of flexible multi-body dynamics seems more plausible by contrast with the rigid model.

The equations established by flexible multi-body dynamics theory lay the foundation of controlling vibration and accurately positioning the work platform of aerial platform vehicle.

This paper is organized as follows. In section II, the arm coordinate system is created and the kinetic energy and potential energy of the arm system are calculated based on the coordinate system. Following, the flexible multi-body dynamics equation of folding-boom aerial platform vehicle is derived. Then, the experimental study is carried on in section III. Finally, the concluding remarks are provided.

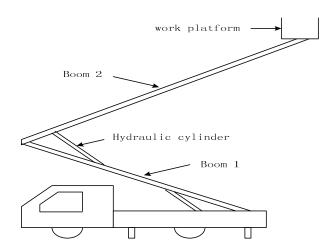


Fig.1 Scheme of folding-boom aerial platform vehicle

II. MODELING OF ARM SYSTEM BASED ON FLEXIBLE MULTI-BODY DYNAMICS

Based on the physical model of folding-boom aerial platform vehicle, we create the schematic diagram of arm coordinate system, as is shown in Fig.2.

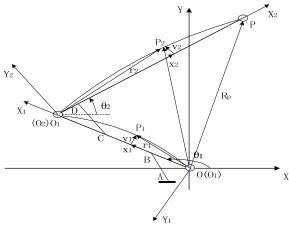


Fig.2 Schematic diagram of arm coordinate system

A. Calculation of Kinetic Energy of Arm System

In Fig.2, XOY is inertial coordinate system and $X_kO_kY_k$ is moving coordinate system of beam k (k=1,2). The rotating transformation matrix from $X_kO_kY_k$ to XOY is described by T_k , which can be defined as:

$$T_{k} = \begin{bmatrix} \cos \theta_{k} & -\sin \theta_{k} \\ \sin \theta_{k} & \cos \theta_{k} \end{bmatrix}, \tag{1}$$

where θ_k is the included angle between the axis X_k and axis X.

Suppose P_k is any point of beam k and \vec{r}_k is the position vector of point P_k in the moving coordinate system $X_k O_k Y_k$, then \vec{r}_k can be described as:

$$\vec{r}_k = \begin{bmatrix} x_k & v_k \end{bmatrix}^T \tag{2}$$

where x_k is the x-coordinate of \vec{r}_k in $X_k O_k Y_k$ and v_k is the deformation of the point P_k of beam k.

The position vector $\vec{r_k}$ of point P_k in $X_k O_k Y_k$ can be transformed into inertial coordinate system XOY by rotating transformation matrix T_k .

 \vec{R}_k is used to denote the position vector of point P_k in XOY, which can be written as

$$\vec{R}_1 = T_1 \vec{r}_1
\vec{R}_2 = T_1 \vec{r}_{01} + T_2 \vec{r}_2$$
(3)

Therefore, the velocity of \vec{R}_1 and \vec{R}_2 are given by

$$\dot{\vec{R}}_{1} = T_{1}\dot{\vec{r}}_{1} + ST_{1}\ddot{r}_{1}\dot{\theta}_{1}
\dot{\vec{R}}_{2} = ST_{1}\ddot{r}_{01}\dot{\theta}_{1} + T_{2}\dot{\vec{r}}_{2} + ST_{2}\ddot{r}_{2}\dot{\theta}_{2}$$
(4)

where
$$\dot{T}_k = ST_k \dot{\theta}_k$$
 and $S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

In addition, P denotes the ending point of beam 2 and \vec{R}_p is used to describe the position vector of point P in XOY, which is written as $\vec{R}_p = T_1 \vec{r}_{01} + T_2 \vec{r}_{02}$, so the corresponding velocity of \vec{R}_p is given by

$$\dot{\vec{R}}_{p} = ST_{1}\vec{r}_{01}\dot{\theta}_{1} + ST_{2}\vec{r}_{02}\dot{\theta}_{2} \tag{5}$$

where, $\vec{r}_{0k} = [l_k \quad 0]^T$ represented the position vector of the ending point of beam k in $X_k O_k Y_k$ when there is no deformations

The coordinates of the ending point P in XOY can be expressed as:

$$R_{px} = l_1 \cos \theta_1 + l_2 \cos \theta_2$$

$$R_{py} = l_1 \sin \theta_1 + l_2 \sin \theta_2$$
(6)

Assume that the two beams are homogeneous rods and the mass length density of the beam k is defined by $\rho_{lk} = \frac{m_k}{l}$,

where m_k and l_k denote the mass and the length of beam k respectively.

Therefore, the kinetic energy of beam system of folding-beam aerial platform vehicle is given by the following equation

$$K = K_1 + K_2 + K_m \,, \tag{7}$$

where, K_k is the kinetic energy of beam k, and can be described as $K_k = \frac{1}{2} \int_0^{l_k} \rho_{lk} \dot{\vec{R}}_k^T \dot{\vec{R}}_k dx_k$. K_m is the kinetic energy of work platform, and can be describe as $K_m = \frac{1}{2} m \dot{\vec{R}}_p^T \dot{\vec{R}}_p$.

B. Calculation of the Potential Energy of the Arm System v_k can be expressed as

$$v_k(x_k, t) = \sum_{p=1}^{n_k} q_{kp} \varphi_p(x_k),$$
 (8)

where $\varphi_p(x_k)$ is the p-order primary function of beam k and q_{kp} is the generalized coordinate corresponding to $\varphi_p(x_k)$. n_k is the order number of the Ritz function of beam k, and generally is taken as $n_k=2$. Therefore, v_k is written as

$$v_k(x_k,t) = q_{k1}\varphi_1(x_k) + q_{k2}\varphi_2(x_k)$$

where $\varphi_1(x_k)$ and $\varphi_2(x_k)$ are the two former model function, which is defined as

$$\varphi_1(x_k) = \sin \frac{\pi x_k}{l_k} , \quad \varphi_2(x_k) = \sin \frac{2\pi x_k}{l_k}$$

Therefore, considering the deformation potential energy, the potential energy of the beam system is given by the following equation

$$P = (\frac{1}{2}m_1 + m_2 + m)gl_1 \sin\theta_1 + (\frac{1}{2}m_2 + m)gl_2 \sin\theta_2 + \frac{1}{2}\sum_{k=1}^{2} EI_k \int_0^{l_k} (\frac{\partial^2 v_k}{\partial x_k})^2 dx_k$$
(9)

where the last term is deformation potential energy and the former two terms are gravitational potential energy. In equation (9), m is the mass of load, g is acceleration of gravity, E is the modulus of elasticity of the beam material, and I_k is the moment of inertia of the beam k cross-section.

C. Calculation of the Generalized forces

In Fig.2, a hydraulic cylinder is mounted between A and B where A is a fixed point in the vehicle body, and B is fixed at beam 1. C and D represent the fixed point of beam 1 and beam 2 respectively, and another hydraulic cylinder is mounted between C and D.

In inertial coordinate system XOY, assume that the coordinates of A are $(-a_{11},-a_{01})$. $OB=a_{12}$, $O_2C=a_{21}$ and $O_2D=a_{22}$.

Therefore, piston displacement y_1 and y_2 of the hydraulic cylinder can be expressed as

$$y_1 = [(a_{12}\cos\theta_1 + v_B\sin\theta_1 + a_{11})^2]$$

$$+ (a_{12}\sin\theta_1 - v_B\cos\theta_1 + a_{01})^2]^{\frac{1}{2}}$$

$$y_2 = [(-v_C\sin\theta_1 + a_{21}\cos\theta_1 + a_{22}\cos\theta_2 - v_D\sin\theta_2)^2]$$

+
$$(v_C \cos \theta_1 + a_{21} \sin \theta_1 + a_{22} \sin \theta_2 + v_D \cos \theta_2)^2$$
]^{\frac{1}{2}},

where $v_B \times v_C$ and v_D are the deformations of the points $B \times C$ and D, respectively.

Hydraulic cylinder force F is given by

$$F_k = m_{yk} \ddot{y}_k + d\dot{y}_k + cy_k + f_{0k}$$
 (10)

where d is damping factor, c is spring stiffness coefficient, f_{0k} is the initial value of elastic force and m_{yk} is the mass of piston rod of hydraulic cylinder.

Therefore, generalized force $Q = \begin{bmatrix} Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \end{bmatrix}^T$ is given by the following equation:

$$Q = J^T F \tag{11}$$

where, Jacobian matrix $J = \begin{bmatrix} \frac{\partial y_1}{\partial \theta_1} & \frac{\partial y_1}{\partial \theta_2} & \frac{\partial y_1}{\partial q_{11}} & \frac{\partial y_1}{\partial q_{12}} & \frac{\partial y_1}{\partial q_{21}} & \frac{\partial y_1}{\partial q_{22}} \\ \frac{\partial y_2}{\partial \theta_1} & \frac{\partial y_2}{\partial \theta_2} & \frac{\partial y_2}{\partial q_{11}} & \frac{\partial y_2}{\partial q_{12}} & \frac{\partial y_2}{\partial q_{21}} & \frac{\partial y_2}{\partial q_{22}} \end{bmatrix},$ $= \begin{bmatrix} J_{11} & 0 & J_{13} & J_{14} & 0 & 0 \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \end{bmatrix}$

and
$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$
.

D. Establishment of the flexible multi-body dynamics Equations of aerial platform vehicle

Lagrange equation is described by

$$\frac{d}{dt}(\frac{\partial L}{\partial z_j}) - \frac{\partial L}{\partial z_j} = Q_j \quad (j = 1, 2 \cdots 6),$$
 (12)

where L = K - P. Generalized coordinates are $z = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \end{bmatrix}^T$, which can also be expressed as $z = \begin{bmatrix} \theta & q \end{bmatrix}^T$, where $\theta = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T$, $q = \begin{bmatrix} q_{11} & q_{12} & q_{21} & q_{22} \end{bmatrix}^T$.

Taking (7)、 (9) and (11) into (12), the flexible multi-body dynamics Equations of folding-beam aerial platform vehicle become as follows:

$$\begin{cases}
G\ddot{\theta} + U\dot{\theta}^2 + H\ddot{q} + R = Q_{\theta} \\
M\ddot{q} + Nq + H^T\ddot{\theta} + V^T\dot{\theta}^2 = Q_q
\end{cases}$$
(13)

In equations (13), $Q_{\theta} = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix}^T$, $Q_{q} = \begin{bmatrix} Q_3 & Q_4 & Q_5 & Q_6 \end{bmatrix}^T$, $\ddot{\theta} = \begin{bmatrix} \ddot{\theta}_1 & \ddot{\theta}_2 \end{bmatrix}^T$, $\ddot{q} = \begin{bmatrix} \ddot{q}_{11} & \ddot{q}_{12} & \ddot{q}_{21} & \ddot{q}_{22} \end{bmatrix}^T$, $\dot{\theta}^2 = \begin{bmatrix} \dot{\theta}_1^2 & \dot{\theta}_2^2 \end{bmatrix}^T$.

G , M , H are mass matrix, which are expressed as follows:

$$G = \begin{bmatrix} (\frac{1}{3}m_1 + m_2 + m)l_1^2 & (\frac{1}{2}m_2 + m)l_1l_2\cos(\theta_1 - \theta_2) \\ (\frac{1}{2}m_2 + m)l_1l_2\cos(\theta_1 - \theta_2) & (\frac{1}{3}m_2 + m)l_2^2 \end{bmatrix}$$

$$M = \begin{bmatrix} \frac{1}{2}m_1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}m_1 & 0 & 0 \\ 0 & 0 & \frac{1}{2}m_2 & 0 \\ 0 & 0 & 0 & \frac{1}{2}m_2 \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{m_1l_1}{\pi} & -\frac{1}{2}\frac{m_1l_1}{\pi} & \frac{2m_2l_1}{\pi}\cos(\theta_1 - \theta_2) & 0 \\ 0 & 0 & \frac{m_2l_2}{\pi} & -\frac{1}{2}\frac{m_2l_2}{\pi} \end{bmatrix}$$

N is the coefficient matrix of generalized coordinate q, which is described by

$$N = \begin{bmatrix} \frac{EI_1\pi^4}{2I_1^3} & 0 & 0 & 0 \\ 0 & \frac{8EI_1\pi^4}{I_1^3} & 0 & 0 \\ 0 & 0 & \frac{EI_2\pi^4}{2I_2^3} & 0 \\ 0 & 0 & 0 & \frac{8EI_2\pi^4}{I_2^3} \end{bmatrix}$$

In addition, U and V are coefficient matrix of $\dot{\theta}^2$, and R is the matrix including the terms which are not related with $\ddot{\theta}$, \ddot{q} , $\dot{\theta}^2$ and q.

In the process of deriving equations (13), the terms with respect to the generalized coordinates q are neglected considering the deformations are negligibly small.

The equations of rigid model can be obtained without considering the deformation v_k .

III. SIMULATION RESULTS

Equations (13) can be written as

$$\overline{M}(y)\frac{dy}{dt} = f(y,t), \qquad (14)$$

where mass matrix $\overline{M}(y) = \begin{bmatrix} I_{6\times6} & 0 & 0 \\ 0 & G & H \\ 0 & H^T & M \end{bmatrix}$,

$$y = \begin{bmatrix} \theta & q & \dot{\theta} & \dot{q} \end{bmatrix}^T$$
.

The equations (14) are differential-algebraic equations (DAEs) of index 1[7], ode15s solver in matlab can solve these problems.

Parameters used for the simulation are selected as follows: $l_1 = 7.5m$, $l_2 = 8.5m$, $m_1 = 650kg$, $m_2 = 550kg$, m = 150kg , d = 0.05 , k = 0.03 , $m_{yk} = 20kg$, $EI_1 = 6 \times 10^8 \, N \cdot m^2$, $EI_2 = 5 \times 10^8 \, N \cdot m^2$, $a_{10} = 0.9m$, $a_{11} = 0.9m$, $a_{12} = 1.8m$, $a_{21} = 1.8m$, $a_{22} = 0.9m$, the initial angles of the two beams are $\theta_1 = 2.09 \, rad$ and $\theta_2 = 0.52 \, rad$, the initial angular-velocity $\dot{\theta}$ is zero.

The numerical solutions of equations (13) are shown in Fig.3~ Fig.6.

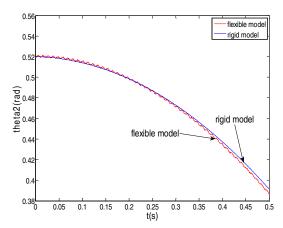


Fig. 3 the comparison of angular rotation θ_2 between flexible and rigid model

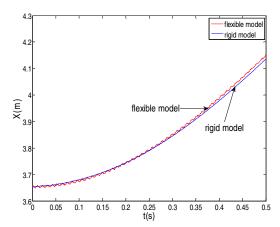


Fig.4 the comparison of the *X* -coordinate of endpoint *P* between flexible and rigid model

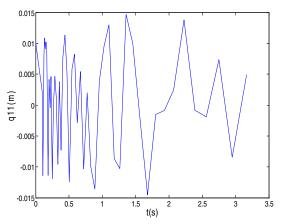


Fig. 5 the component q_{11} of the deformation v_1 varying with time

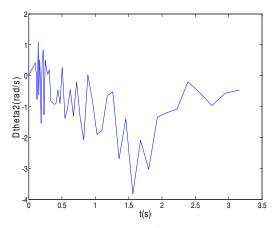


Fig.6 the angular velocity $\dot{\theta}_2$ changed over time

As can be seen from Fig.3, the angular rotation of flexible model has small vibrations, which shows that elastic deformations have an effect on the rigid angular rotation. Therefore, vibration control should be studied.

Fig.4 shows that there exist small deviations of the trajectory of the work platform due to the deflections of the beams in the flexible model. Similarly, the accurate control of the trajectory need be studied further.

Fig.5 and Fig.6 show that the moving beams exist high frequency vibrations, which are caused by the elastic deformations of the beams. In order to realize the steady movements of the work platform, the vibrations should be suppressed.

IV. CONCLUSIONS

Flexible multi-body dynamics equations of the arm system of folding-boom aerial platform vehicle are established. Based on the dynamics equations, the experimental study is carried on. Simulation results show that the vibrations associated with the flexible beams are significant and there exist small deviations of the trajectory of the work platform which caused by the beam deflection. Therefore, the control of vibrations will be carried on in order to attain the steady movements, and at the same time the control of reducing the trajectory deviation will be studied further in order to realize accurate positioning.

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