# Formation of Autonomous Underwater Vehicles by Impulsive Information with Quantized Effects and Transmission Delays

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Abstract—This paper investigates the formation cooperation problem of networked autonomous underwater vehicles (AUVs). By considering the underwater communication environments, the impulsive communication scheme with logarithmic quantization is developed. Moreover, the effects of time-varying transmission delays in the communication channels are taken into account. By means of impulsive control theory, distributed controllers are designed and sufficient conditions are derived for ensuring the desired formation. The simulation results are provided to demonstrate the effectiveness of our theoretical results.

## I. Introduction

Autonomous underwater vehicles (AUVs) are promising for accomplishing various underwater tasks [1], [2], [3]. Particularly, cooperation of multiple AUVs has become an active research area to meet the increasing demands that can not be done by a single AUV. Compared with traditional control on a single AUV, advantages can be obtained by cooperative control of multiple AUVs including improving efficiency and robustness, and reducing the costs [4], [5]. The formation problem of AUVs, as an important issue of cooperative control, has attracted much attention in recent years and has wide applications in underwater mapping, mine localization, sea inspection, etc. With this background, some effective formation control strategies have been developed for AUVs [6], [7].

It should be pointed out that most of the existing results for the AUVs rely on the restrictive continuous-time underwater communication model. Unfortunately, this assumption may be not practical for the underwater information exchanges due to the constraints on the underwater communications. So far, most effective underwater communications are based on the underwater acoustic communication or optical devices,

such that continuous-time information exchanges in the weak underwater communication environment are too expensive or unavailable [4], [7], [8]. Especially, transmission delays are inevitable in the underwater information exchanges. This is partly due to the congestion of the communication channels or the finite transmission speed by the underwater medium transmitting the information among the AUVs. As is well known, the delay effects may degrade or even destroy the formation performance. Thus, delay issues should be addressed in the early stage of formation design. Another practical problem of the formation control is the limit communication capabilities [9], [10], [11]. As a result, it is very important to consider quantized effects during the information exchanges. Up until now, to the best of the authors' knowledge, how to achieve the formation of multiple AUVs with considerations of real world underwater communications has seldom been discussed despite its significance in both theory and applications.

Motivated by the above discussions, we deal with the formation problem of multiple AUVs by adopting the impulsive communication strategy in this paper. Furthermore, the timevarying transmission delays in the communication channels are taken into account in the formation problem, which is more widely applicable for the real world applications. In particular, since the multiple AUVs communicate with each other with limited communication resources, the quantized effect of information exchanges is considered to model the real communication environment of underwater communication networks with limited bandwidth. It is noteworthy that our proposed scheme can be directly extended to other multi-agent systems with communication constraints and limited energy supplies.

The remainder of our paper is arranged as follows. Section

2 introduces some necessary preliminaries and formulates the formation problem. In Section 3, distributed controllers for AUVs with impulsive networking scheme are developed by considering the quantized effects and time-varying transmission delays. Moreover, sufficient criteria are established on the basis of impulsive control theory. Section 4 gives the simulation results for verifying the effectiveness of our proposed method and Section 5 concludes the paper.

**Notation:**  $\mathbb{R}^n$  and  $\mathbb{R}^{m \times n}$  represents the n dimensional Euclidean space and the space of  $m \times n$  matrices, respectively. A-B>0(A-B<0) means that A-B is positive definite (negative definite).  $A\otimes B$  stands for Kronecker product.  $s_\alpha$  and  $c_\alpha$  denote  $\sin\alpha$  and  $\cos\alpha$ , respectively.  $\lambda_{\min}(P)$  and  $\lambda_{\max}(P)$  denote the largest and the smallest eigenvalues of the symmetric matrix P.

#### II. PRELIMINARIES AND PROBLEM FORMULATION

## A. Autonomous underwater vehicle (AUV) dynamics

We are considering a group of N AUVs indexed by the set  $\mathcal{I} = \{1, 2, \cdots, N\}$ . By defining the global coordinate frame  $\{E\}$  and the body-fixed reference frame  $\{B\}$ , the translational dynamics with fixed attitude of the ith AUV,  $i \in \mathcal{I}$  can be described by [12]

$$\begin{cases}
\dot{p}_i = J_i(\Theta_i)v_i, \\
M_i\dot{v}_i = -D_i(v_i)v_i - g_i(\Theta_i) + \tau_i,
\end{cases}$$
(1)

where  $p_i = [x_i, y_i, z_i]^T$  and  $\Theta_i = [\phi_i, \theta_i, \psi_i]^T$  represent the generalized position and attitude (described by Euler angles, i.e., roll  $\phi_i$ , pitch  $\theta_i$ , and yaw  $\psi_i$  angles) in the global coordinate frame  $\{E\}$ , respectively;  $J_i(\Theta_i)$  denotes the kinematic transformation matrix from  $\{B\}$  to  $\{E\}$  with

$$\begin{split} J_i(\Theta_i) &= \left[ \begin{array}{cc} J_i(\Theta_i)_1 & J_i(\Theta_i)_2 \\ -s_{\theta_i}, & J_i(\Theta_i)_3 \end{array} \right], \\ J_i(\Theta_i)_1 &= \left[ \begin{array}{cc} c_{\varphi_i}c_{\theta_i} & -s_{\varphi_i}c_{\phi_i} + c_{\varphi_i}s_{\theta_i}s_{\phi_i} \\ s_{\varphi_i}c_{\theta_i} & c_{\varphi_i}c_{\phi_i} + s_{\phi_i}s_{\theta_i}s_{\varphi_i} \end{array} \right], \\ J_i(\Theta_i)_2 &= \left[ \begin{array}{cc} s_{\varphi_i}s_{\phi_i} + c_{\varphi_i}c_{\phi_i}s_{\theta_i} \\ -c_{\varphi_i}s_{\phi_i} + s_{\theta_i}s_{\varphi_i}c_{\phi_i} \end{array} \right], \\ J_i(\Theta_i)_3 &= \left[ \begin{array}{cc} c_{\theta_i}s_{\phi_i} & c_{\theta_i}c_{\phi_i} \end{array} \right], \end{split}$$

 $v=[u_i,v_i,\omega_i]^T$  represents the generalized linear velocity;  $\tau_i=[\tau_{i1},\tau_{i2},\tau_{i3}]^T\in\mathbb{R}^3$  denotes the control input;  $M_i$ ,  $D_i(v_i)$  and  $g_i(\Theta_i)$  are the inertia matrix, the damping matrix and the restoring force vector, respectively.

## B. Graph theory

An undirected graph  $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ ,  $\mathcal{I} = \{1, \cdots, N\}$  is adopted for describing the communication topology of AUVs, where  $\mathcal{V}(\mathcal{G}) = \{v_1, \cdots, v_N\}$  and  $\mathcal{E}$  denote the sets of nodes and edges, respectively.  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$  is the weighted adjacency matrix.  $a_{ij} > 0$  if  $(v_i, v_j) \in \mathcal{E}$  and  $a_{ij} = 0$  otherwise. The Laplacian  $L = [l_{ij}] \in \mathbb{R}^{N \times N}$  can be defined as  $l_{ii} = \sum_{j=1, j \neq i}^N a_{ij}$  and  $l_{ij} = -a_{ij}, i \neq j$ . An undirected graph is connected if and only if there exists an undirected path between any two vertices in it.

#### C. Formation objective

The formation is defined with a prescribed configuration in  $\{E\}$ , which is given by

$$\lim_{t \to \infty} p_i - p_j = d_{ij}, \ \forall i \in \mathcal{I}, j \in \mathcal{N}_i,$$
 (2)

$$\lim_{t \to \infty} v_i = v_j, \ \forall i \in \mathcal{I}. \tag{3}$$

where  $d_{ij} \in \mathbb{R}^3$  is a constant configuration vector.

Before proceeding, we introduce the following significant lemmas for the subsequent analysis.

**Lemma 1.** [13] Consider the following differential inequalities:

$$\dot{f}(t) \le -\alpha f(t) + \beta |f_t|, t \ne t_k, f(t_k) \le a_k f(t_k^-) + b_k |f_{t_-}^-|,$$

where  $f(t) \ge 0$ ,  $f_t(s) = f(t+s)$ ,  $s \in [-\tau, 0]$ ,  $|f_t| = \sup_{t-\tau \le s \le t} f(s)$ ,  $|f_{t-}| = \sup_{t-\tau \le s \le t^{-}} f(s)$  and  $f(t_0)$  is a continuous function.

Suppose that  $\alpha > \beta \geq 0$  and there exists a scalar  $\delta > 1$ , such that  $t_k - t_{k-1} > \delta \tau$ , then

$$f(t) \le \rho_1 \rho_2 \cdots \rho_{k+1} e^{k\lambda \tau} |f_{t_0}| e^{-\lambda(t-t_0)},$$

where  $t \in [t_k, t_{k+1}]$ ,  $\rho_i = \max\{1, a_i + b_i e^{\lambda \tau}\}$ , i = 1, 2, ..., k+1 and  $\lambda$  is the unique positive root of equation  $\lambda = \alpha - \beta e^{\lambda \tau}$ . In particular, if

$$v = \sup_{k=1,2,...} \{1, a_k + b_k e^{\lambda \tau}\},$$

then

$$f(t) \le v|f_{t_0}|e^{-(\lambda - \ln v e^{\lambda \tau})/\delta \tau (t - t_0)}$$

**Lemma 2.** [14] For any constant matrices with appropriate dimensions A and B and any positive matrix Q, the following inequality is satisfied:

$$A^T B + A B^T \le A^T Q A + B^T Q^{-1} B.$$

#### III. MAIN RESULTS

In this section, the impulsive communication scheme with quantized effects and time-varying transmission delays is developed. Based on this scheme, the design procedure of the corresponding distributed formation controllers for AUVs are given.

Let  $v_i := J_i(\Theta_i)v_i$ ,  $i \in \mathcal{I}$  and one can obtain that

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = -J_i(\Theta_i)M_i^{-1}D_i(J_i^{-1}(\Theta_i)v_i)J_i^{-1}(\Theta_i)v_i \\ -J_i(\Theta_i)M_i^{-1}g_i(\Theta_i) + J_i(\Theta_i)M_i^{-1}\tau_i. \end{cases}$$

Consequently, the distributed formation controller for each AUV is designed as follows:

$$\tau_{i} = D_{i}(J_{i}^{-1}(\Theta_{i})v_{i})J_{i}^{-1}(\Theta_{i})v_{i} + g_{i}(\Theta_{i}) - kM_{i}J_{i}^{-1}(\Theta_{i})v_{i}$$

$$-M_{i}J_{i}^{-1}(\Theta_{i})\sum_{k=1}^{\infty} \{\rho \sum_{j=1, j\neq i}^{N} a_{ij}[((p_{i} - d_{i}) - (p_{j} - d_{j})) + (v_{i} - v_{j})]\}\delta(t - t_{k}), i \in \mathcal{I},$$
(4)

where  $\rho>0, k>0, d_i>0$  denotes formation configuration of the ith AUV with  $d_{ij}=d_i-d_j$ , Dirac function  $\delta(t)$  denotes the impulsive effects at the time moment  $t=t_k$ , the time sequence  $\{t_k\}$  with  $0=t_0< t_1< \cdots < t_k< \cdots$ ,  $\lim_{k\to\infty} t_k=\infty$  forms a strictly increasing sequence in the time interval  $[0,\infty)$ . The impulsive distance is defined by  $\Delta t_k=t_k-t_{k-1}, k=1,2,\ldots$ 

It should be pointed out that in the underwater communication environments, the transmission delays always exist, which can affect the information communicated among the AUVs. By denoting  $\tau_k \leq \tau$ ,  $\tau > 0$  as the transmission delays at the time moment  $t = t_k$ , (4) can be rewritten as

$$\tau_{i} = D_{i}(J_{i}^{-1}(\Theta_{i})v_{i})J_{i}^{-1}(\Theta_{i})v_{i} + g_{i}(\Theta_{i}) - kM_{i}J_{i}^{-1}(\Theta_{i})v_{i},$$

$$-M_{i}J_{i}^{-1}(\Theta_{i})\sum_{k=1}^{\infty} \{\rho \sum_{j=1, j\neq i}^{N} a_{ij}[((p_{i}(t-\tau_{k}) - d_{i}) - (p_{j}(t-\tau_{k}) - d_{j})) + (v_{i}(t-\tau_{k}) - v_{j}(t-\tau_{k}))]\}\delta(t-t_{k}), i \in \mathcal{I}.$$
(5)

**Remark 1.** The transmission delays are important and practical issues for the underwater communications. Furthermore, the transmission delays can be time-varying such that time-varying transmission delays should be modeled.

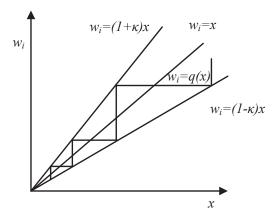


Fig. 1. The logarithmic quantizer

In addition, the quantized effects should be considered for the underwater communications, since underwater signals have to be quantized with an appropriate precision due to the limited bandwidth of communication network. In this paper, the logarithmic quantizer  $q(\cdot):\mathbb{R}\to\Gamma$  is utilized to model the quantized effects. The quantizer  $q(\cdot)$  is called logarithmic if it has the form

$$\Gamma = \{w_i = \mu^i w_0, i = 0, \pm 1, \pm 2, \ldots\} \cup \{0\}, w_0 > 0,$$

where  $\mu \in [0, 1]$ . The associated quantizer  $q(\cdot)$  shown in Fig. 1 is defined as follows

$$q(x) = \begin{cases} w_i, & \text{if } \frac{1}{1+\kappa} w_i < x \le \frac{1}{1-\kappa} w_i, \\ 0, & \text{if } x = 0, \\ -q(-x), & \text{if } x < 0, \end{cases}$$
 (6)

where  $\kappa = \frac{1-\mu}{1+\mu}$  is called sector bound [15]. The quantization density for quantizer (6) is defined as  $\frac{-2}{\ln \mu}$ . It can be found that a small  $\mu$  means coarse quantization and a large  $\mu$  implies dense quantization. Then, the quantization error satisfies the following sector bound condition:

$$q(x) - x = \Omega x, \exists \Omega \in [-\kappa, \kappa], \forall x \in \mathbb{R},$$

such that

$$(1 - \kappa)x \le q(x) \le (1 + \kappa)x.$$

Hence, the distributed formation control input is rewritten as

$$\tau_{i} = D_{i}(J_{i}^{-1}(\Theta_{i})v_{i})J_{i}^{-1}(\Theta_{i})v_{i} + g_{i}(\Theta_{i}) - kM_{i}J_{i}^{-1}(\Theta_{i})v_{i},$$

$$-M_{i}J_{i}^{-1}(\Theta_{i})\sum_{k=1}^{\infty} \{\rho \sum_{j=1,j\neq i}^{N} a_{ij}\sigma_{ij}[(q(p_{i}(t-\tau_{k})-d_{i}) - q(p_{j}(t-\tau_{k})-d_{j})) + (q(v_{i}(t-\tau_{k})) - q(v_{i}(t-\tau_{k})))]\}\delta(t-t_{k}), i \in \mathcal{I}.$$
(7)

Based on the above results, the following theorem is provided for solving the prescribed formation problem of AUVs.

**Theorem 1.** The formation can be achieved if the communication topology is connected and there exist constants  $\rho > 0$ ,  $\delta^* > 0$ ,  $\alpha > 0$ ,  $\varepsilon > 0$  and matrix P > 0 with appropriate dimensions, such that the following inequalities hold:

$$\inf_{k=1,2,...} \{t_k - t_{k-1}\} > \delta^* \tau,$$

$$P\mathcal{A} + \mathcal{A}^T P + \alpha P < 0,$$

$$\delta^* > \frac{\ln(\rho^* \exp(\alpha \tau))}{\alpha \tau},$$
where  $\rho^* = \max\{1, a + b \exp(\alpha \tau)\}$  and
$$a = (1 + \rho \varepsilon),$$

$$b = \rho(1 + \kappa)\lambda_{\max}(\mathcal{L}^T \mathcal{L})/\varepsilon,$$

$$\mathcal{A} = \begin{bmatrix} 0 & I_{3N} \\ 0 & -kI_{3N} \end{bmatrix},$$

$$\mathcal{L} = \begin{bmatrix} 0 & 0 \\ L \otimes I_3 & L \otimes I_3 \end{bmatrix}.$$

*Proof.* By denoting  $\tilde{p}_i := p_i - d_i$ , it can be obtained that  $q(\tilde{p}_i(t - \tau_k)) - \tilde{p}_i(t - \tau_k) = \Omega_i \tilde{p}_i(t - \tau_k)$ .

Then, the closed loop dynamics of the networked AUVs can be obtained as

$$\begin{cases}
\dot{\tilde{p}} = v, \\
\dot{v} = -kv, t \neq t_k, \\
\Delta v = -\rho(L \otimes I_3)q(\tilde{p}(t - \tau_k)) - \rho(L \otimes I_3)q(v(t - \tau_k)) \\
= -\rho((L \otimes I_3)(I_{3N} + \tilde{\Omega}))\tilde{p}(t - \tau_k) \\
- \rho((L \otimes I_3)(I_{3N} + \tilde{\Omega}))v(t - \tau_k) \\
= -\rho(L(I_N + \tilde{\Omega}) \otimes I_3)\tilde{p}(t - \tau_k) \\
- \rho(L(I_N + \tilde{\Omega}) \otimes I_3)v(t - \tau_k), t = t_k,
\end{cases}$$
(8)

where

$$\begin{split} \tilde{p} &:= [\tilde{p}_1^T, \tilde{p}_2^T, \dots, \tilde{p}_N^T]^T, \\ v &:= [v_1^T, v_2^T, \dots, v_N^T]^T. \end{split}$$

It can be found that the formation can be achieved asymptotically if for any initial value  $\tilde{p}(0), \ v(0) \in \mathbb{R}^{3N}, \ \tilde{p}_i \to \bar{p} := \frac{1}{N} \sum_{j=1}^N \tilde{p}_i(t), \ v_i \to \bar{v} := \frac{1}{N} \sum_{j=1}^N v_i(t) \ \text{as} \ t \to \infty.$  Consequently, it can be obtained that

$$\tilde{p} = \bar{p}\mathbf{1} + \varepsilon_{\tilde{p}},$$
 $v = \bar{v}\mathbf{1} + \varepsilon_{v},$ 

where  $\mathbf{1} = [1, 1, \dots, 1]^T \in \mathbb{R}^{3N}$ ,  $\varepsilon_{\tilde{p}}$  and  $\varepsilon_{\upsilon}$  are the disagreement vectors satisfying  $\mathbf{1}^T \varepsilon_{\tilde{p}} = 0$  and  $\mathbf{1}^T \varepsilon_{\upsilon} = 0$ . It follows

$$\begin{cases} \dot{\varepsilon}_{\tilde{p}} = \varepsilon_{\upsilon}, \\ \dot{\varepsilon}_{\upsilon} = -k\varepsilon_{\upsilon}, t \neq t_{k}, \\ \Delta\varepsilon_{\upsilon} = -\rho(L(I_{N} + \tilde{\Omega}) \otimes I_{3})\varepsilon_{\tilde{p}}(t - \tau_{k}) \\ -\rho(L(I_{N} + \tilde{\Omega}) \otimes I_{3})\varepsilon_{\upsilon}(t - \tau_{k}), t = t_{k}, \end{cases}$$

which can be further rewritten as

$$\begin{cases} \dot{\delta}(t) = \mathcal{A}\delta(t), t \neq t_k, \\ \Delta\delta(t) = \rho \mathcal{B}\delta(t - \tau_k), t = t_k, \end{cases}$$

where A is defined in Theorem 1 and

$$\begin{split} \delta(t) &:= \left[ \varepsilon_{\tilde{p}}^T, \varepsilon_v^T \right]^T, \\ \mathcal{B} &:= \left[ \begin{array}{cc} 0 & 0 \\ \left( -L(I_N + \tilde{\Omega}) \otimes I_3 \right) & \left( -L(I_N + \tilde{\Omega}) \otimes I_3 \right) \end{array} \right]. \end{split}$$

Choose the Lyapunov function as follows

$$V(t) = \delta^{T}(t)P\delta(t), \tag{9}$$

and define the Dini derivative of V(t) by

$$D^{+}V(t) = \lim_{h \to 0^{+}} \sup[V(t+h) - V(t)]/h.$$

As a result, for  $t \in [t_{k-1}, t_k)$ , it can be derived from Theorem 1 that

$$D^{+}V(t) = \dot{\delta}^{T}(t)\delta(t) + \delta^{T}(t)\dot{\delta}(t)$$

$$= \delta^{T}(t)(P\mathcal{A} + \mathcal{A}^{T}P)\delta(t)$$

$$\leq -\alpha\delta^{T}(t)P\delta(t). \tag{10}$$

In addition, for  $t = t_k$ , it can be derived by Lemma 2 that

$$V(t_{k}^{+}) = \delta^{T}(t_{k}^{+})\delta(t_{k}^{+})$$

$$= [\delta^{T}(t_{k}) + \rho\delta^{T}(t - \tau_{k})\mathcal{B}^{T}] \times$$

$$[\delta(t_{k}) + \rho\mathcal{B}\delta(t - \tau_{k})]$$

$$= \delta^{T}(t_{k})\delta(t_{k}) + 2\rho\mathcal{B}^{T}\delta^{T}(t - \tau_{k})\delta(t_{k})$$

$$+ \rho^{2}\delta^{T}(t - \tau_{k})\mathcal{B}^{T}\mathcal{B}\delta(t - \tau_{k})$$

$$\leq \delta^{T}(t_{k})\delta(t_{k}) + \rho\varepsilon\delta^{T}(t_{k})\delta(t_{k})$$

$$+ \frac{\rho\mathcal{B}^{T}\mathcal{B}}{\varepsilon}\delta^{T}(t - \tau_{k})\delta(t - \tau_{k})$$

$$\leq aV(t_{k}) + bV(t - \tau_{k}), \tag{11}$$

where  $\varepsilon > 0$  is a constant,  $a = (1 + \rho \varepsilon)$ ,  $b = \rho(1 + \kappa) \lambda_{\max}(\mathcal{L}^T \mathcal{L})/\varepsilon$ ,  $\mathcal{L} = \begin{bmatrix} 0 & 0 \\ L \otimes I_3 & L \otimes I_3 \end{bmatrix}$ .

By Lemma 1, it can be obtained that if the inequalities in Theorem 1 can be satisfied, then it follows that

$$V(t) \le \rho^* |V(t_0)| \exp\{-(\alpha - \ln \rho^* e^{\alpha \tau})/\delta^* \tau(t - t_0)\},$$

and

$$\|\delta(t)\| \le \sqrt{\frac{\rho^*}{\lambda_{\min}\{P\}}} |V(t_0)| \exp\{\frac{-\frac{1}{2}(\alpha - \ln \rho^* e^{\alpha \tau})}{\delta^* \tau(t - t_0)}\},$$

which implies that  $\delta(t) \to 0$  as  $t \to \infty$ . Therefore, it follows that  $\varepsilon_{\tilde{p}} \to 0$  and  $\varepsilon_v \to 0$  as  $t \to \infty$ , which completes the

**Remark 2.** It is worth mentioning that the so-called synchronization (consensus) problem can also be solved with the proposed approach by choosing  $d_i = 0$ .

#### IV. ILLUSTRATIVE EXAMPLE

In this section, the feasibility and effectiveness of our proposed scheme is demonstrated by the following simulation example.

Consider a group of 4 AUVs with identical dynamics, where the parameters are given by  $M_i = \text{diag}\{150, 120, 120\},\$  $D_i(v_i) = \text{diag}\{100 + 80|u_i|, +60|v_i|, 80 + 60|\omega_i|\}, \text{ and } \phi_i =$  $-\pi/8$ ,  $\theta_i = \pi/12$ ,  $\psi_i = \pi/4$ ,  $\forall i \in \mathcal{I}$ . Fig. 2 shows the communication topology of AUVs.

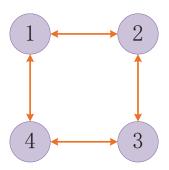


Fig. 2. The communication topology of AUVs

In the simulation, the impulsive interval is set as  $\Delta t_k =$ 0.5s. The time-varying delays are set as  $\tau_k = \{0.1, 0.2, 0.3\}$ s. The parameter of the logarithmic quantizer is chosen by  $\mu = 0.65$ . Moreover, set  $\rho = 50$  and k = 10. The initial conditions are assumed to be  $p_i = v_i = 0$  and the configuration parameters are given as  $d_1 = [0; 4; 0], d_2 = [4; 4; 0],$  $d_3 = [4;0;0]$  and  $d_4 = [0;0;0]$ . Figs. 3-4 show the simulation results of the formation procedure for the AUVs, which can support our theoretical results.

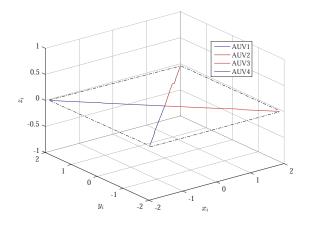


Fig. 3. The generalized position of AUVs

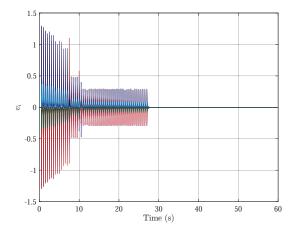


Fig. 4. The generalized linear velocity of AUVs

#### V. CONCLUSION

This paper addresses the formation problem of AUVs by the method of impulsive information exchanges with quantized effects and transmission delays. The impulsive information exchanges are adopted for more practical underwater communications. By means of impulsive control theory, we have developed sufficient conditions for ensuring the prescribed formation configuration. The simulation results are provided for illustrating the effectiveness of the proposed formation

design. An interesting further extension can be the cases with randomly transmission delays or data packet dropouts.

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