

# Evaluating the Impact of Vaccination on COVID-19 Pandemic Used a Hierarchical Weighted Contact Network Model

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**Abstract**—The 2019 Novel Coronavirus Disease (COVID-19) vaccines have been placed significant expectation to end the COVID-19 pandemic sooner. However, issues related to vaccines still need to be resolved urgently, including the vaccination number and range. In this paper, we proposed an epidemic spread model based on the hierarchical weighted network. This model fully considers the heterogeneity of the community social contact network and the epidemiological characteristics of COVID-19 in China, which enables to evaluate the potential impact of vaccine efficacy, vaccination schemes, and mixed interventions on the epidemic. The results show that a mass vaccination can effectively control the epidemic but cannot completely eliminate it. In the case of limited resources, giving vaccination priority to the individuals with high contact intensity in the community is necessary. Joint implementation with non-pharmacological interventions strengthening the control of virus transmission. The results provide insights for decision-makers with effective vaccination plans and prevention and control programs.

**Index Terms**—vaccination, COVID-19, hierarchical weighted network, strategy evaluation, transmission model

## I. INTRODUCTION

Severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) has rapidly spread globally due to high transmissibility and pathogenicity [1]. As of 26 August 2021, the disease has infected more than 200 million people across 216 countries and territories. In response to the global public health and economic crisis caused by the outbreak, governments across the world have implemented a variety of nonpharmaceutical interventions, including lockdown, limited travel, social distancing, and remote learning. Despite these efforts are critical for slowing transmission in the short term, a vaccine that protects against the SARS-CoV-2 and halts community transmission is the most effective way to successfully prevent and control the pandemic [2].

There are currently 135 vaccine candidates worldwide, six of which have been evaluated for safety and efficacy by the World Health Organization. But it is still unclear what levels of vaccine efficacies will be sufficient to curb the spread of

the virus. And it is critical that decision-making on vaccine distribution is well-motivated, particularly in the initial phases when vaccine availability is limited [3]. While most current studies have focused on the performance of clinical trials of vaccines, few articles have assessed the potential impact of vaccine efficacy, vaccination schemes, and mixed interventions on outbreaks. Clear answers to these questions surrounding vaccines are required urgently because they could inform decisions by national governments and thus effectively contain outbreaks and reduce losses.

Mathematical models have been widely used for evaluating the effectiveness of control strategies [4]. Through mathematical modeling, the epidemiological characteristics and transmission mechanism of infectious diseases can be reflected. And the scenarios considering distinct hypothetical conditions that are impossible to analyze in real circumstances can be tested [5]. In the past few decades, the design of mathematical models of disease transmission has attracted the attention of scholars. The classical SIS (Susceptible-Infect-Susceptible) and SIR (Susceptible-Infect-Recovered) epidemic models originally proposed by Kermack [6] and Bailey [7] laid the foundation for later development. With the deepening understanding of epidemic diseases, some extended models have been applied to predict and analyze the spread of disease, such as SEIR [8], [9], SIRD [10], and SIVS [11] compartmental models. These models, all based on assumptions of randomness and uniform mixing, are appropriate for explaining the global behavior of an epidemic on larger scales. Because populations have underlying structural properties and individuals tend to interact with each other, the interaction between individual behaviors at the micro-level is considered in the model. The complex network theory has been used in epidemiology [12]–[14]. In the latest study, Nande [13] built a stochastic epidemic model to examine the transmission network structure on the outcomes of social distancing interventions.

However, limited studies used mathematical models to assess the impact of COVID-19 vaccines on epidemic trends and do not take into account realistic population-based het-

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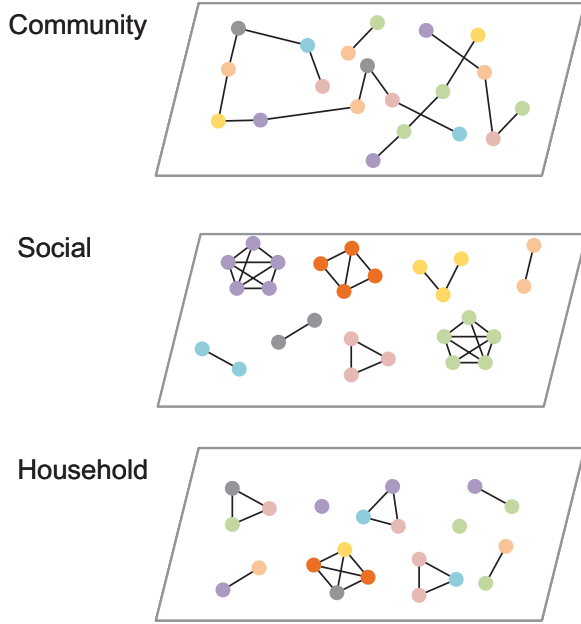


Fig. 1. Schematic of the hierarchical contact network.

erogeneous contact patterns. Evaluation of vaccine strategies is an important tool to assist authorities in making informed decisions. To quantify the effectiveness of the vaccine strategy, based on the patterns of community contact and the evolution of the COVID-19 outbreak in China, this paper proposed a SEINRHD (Susceptible, Exposed, Infectious, No symptoms, Hospitalized and reported, Recovered, Death) transmission model based on hierarchical weighted network. This network explicitly characterizes the heterogeneity of social contact networks, reflecting the differences in contact patterns of individuals at the family level, social level, and community level. The model was then used to conduct computational experiments to evaluate the effects of vaccination on the infection curve in various cases. We examined the impact of vaccination rates, protection rates, vaccination schemes, and mixed strategies of nonpharmaceutical control interventions on outbreaks, respectively. Finally, some constructive suggestions on the present stage of vaccination are provided according to the experimental results. This study is timely and significant to the understanding of vaccination scheme choices to pandemic evolution.

## II. METHODS

### A. Constructing the hierarchical contact network with edge weights

In the modeled network structure, we aimed to incorporate the pattern of transmission of COVID-19 in a Chinese community. We constructed a weighted three-layer network, consisting of a layer for within-household connections, social connections, and community connections (Fig. 1). Then, considering the strength of the connections that cause different

propagation possibilities, we set weights for the edges in the network. In our simulation, the number of individuals in the network was set to  $n=10000$ , which can represent a typical community size [13].

**Household layer:** Individuals were first assigned households using the distribution of household sizes in China (data obtained from the 2019 China National Population Sampling Survey). The distribution of family size is shown in Fig. 2. All individuals in a household were interconnected. Then, we set a network tie strength (edge weight), which represents the intensity of a possible transmission over household ties to be 1 as a default [15], [16].

**Social layer:** The social layer represents the social contacts of people. Friends, coworkers, and classmates who study, work, or are in close contact with this node. On this layer, the degree distribution of nodes follows the social contact survey by Sun [17]. These connections constitute total social contacts for the individuals and can be considered as close ties [18]. we set the network tie strength of close ties to be 0.5, as the secondary attack rate of close ties is around half of the tie strength of family ties [18].

**Community layer:** This layer constitutes the additional random contacts an individual has during the course of their day. For example, customers or employees who interact with this individual in restaurants, entertainment venues, or other service settings. On this layer, the degree distribution of nodes follows the community contact survey by Sun [17]. Since the frequency of encounters is much lower than in the previous two layers, the connections in this layer are considered as weak ties. The past literature [15] shows that the secondary attack rate or the possibility of transmission more broadly of family ties is 10 times as high as that of weak ties. The tie strength of weak ties is set as 0.1.

### B. Modeling the transmission and clinical progression of COVID-19

We extended the classic SEIR compartmental epidemiological model (susceptible (S), exposed (E), clinically ill and infectious (I), and recovered (R)) to describe the dynamics of COVID-19 infection in China. Considering the spread characteristics and treatment strategies of COVID-19 in the

TABLE I  
MODEL PARAMETERS

Variable	Meaning	Values
$\omega_1$	Proportion of no symptoms	27.3%
$\omega_2$	Proportion of mildly symptoms	55.9%
$\omega_3$	Proportion of severely symptoms	10.0%
$\omega_4$	Proportion of critically symptoms	6.8%
$T_e$	Incubation period (days)	N (5.1, 1)
$\mu_2$	Symptomatic infection period (days)	7
$\mu_1$	Asymptomatic infection period (days)	7.5
$\delta_c$	Mortality rate of critically symptoms	53.4%
$\beta$	Propagation rate	0.0665

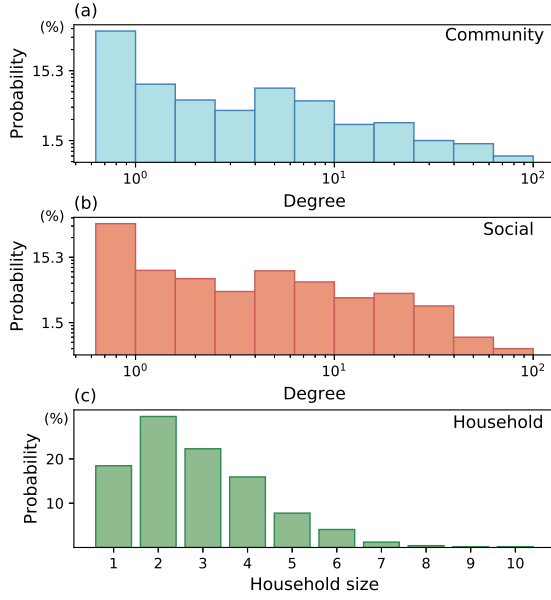


Fig. 2. (a)-(b) The distribution of the number of distinct contacts (degree distribution) of the individuals in community layer and social layer. (c) The distribution of household size.

Chinese environment, SEINRHD model added no symptoms status (N), hospitalized and reported (H), and death (D) status, and classified the symptomatic status according to the severity of the disease (Fig. 3). Table I presents the value of model parameters. After infection, individuals pass through an ~5-day incubation period before developing an infection. Infectious individuals are divided into either no symptomatic or different symptomatic groups: mild, severe, or critical symptoms. The proportion of Infectious individuals was summarized from the previous literature [15], [19]–[24], and <http://wjw.sz.gov.cn/>. The incubation period in our model followed a truncated normal distribution with a mean of 5.1 and a variance of 1 [25]–[27]. The symptomatic patients are hospitalized and diagnosed within an average delay of 7 days, and the asymptomatic patients become recovered after an average period of 7.5 days [27]. Both recovered and hospitalized individuals are no longer capable of infection. Among hospitalized individuals, only critically patients are likely to die [19], and the mortality rate of critical patients is 53.4% [19]–[21], [26], [28]. The remaining patients recover and become immune after a period of treatment. All parameters in the model are consistent with epidemiological studies of COVID-19.

#### C. Converting $R_0$ to $\beta$ for network-based propagation model

The reproduction number  $R_0$  is defined as the average number of new infections generated by one infected individual during the entire infectious period in a fully susceptible population. Propagation rate  $\beta$  means the probability of transmitting the virus per day per network tie.  $R_0$  for the network-based SEINRHD is given by [29]:

$$R_0 = \beta \times N \times \tau \quad (1)$$

Where,  $\beta$  represents the average duration of infectiousness and  $N$  means the number of network ties per infectious individual. is given as below when the model employs a structured social network as its framework [30]:

$$N = m + \frac{s^2}{m} \quad (2)$$

The “network  $N$ ” here incorporates the mean degree ( $m$ ) as well as the SD ( $s$ ) of the degree distributions. In our model, we obtained the network  $N$  of 6.55. Then, we calculated  $\beta = 0.0665$  using (1).

#### D. Setting up the parameters and simulation scenarios

In the vaccine strategy evaluation experiment, we assumed that the individuals protected by the vaccine would not become infected and would not be capable of transmission. People who were given the vaccine were already completed vaccination of all injections. Experimental parameters are shown in Table II, where  $\rho$  represents the proportion of vaccination.  $r$  represents the protection rate of the vaccine. There are two ways of vaccination: random and targeted. Random refers to the random vaccination in the population, and the targeted refers to the priority vaccination of individuals with high contact in the community-layer network.  $d_1$  represents the reduced contact intensity of the community layer.  $d_2$  represents the reduced contact intensity of the social layer. In all experiments, the default protection rate is set to 65.9% [18].

The experiment is measured in days. Experiment 1 studied the effect of vaccination rate on disease transmission. Experiment 2 studied the influence of vaccination way on the epidemic situation. Experiment 3 studied the effect of vaccine protection rate on disease transmission, and Experiment 4 studied the effect of mixed strategies on disease transmission. The results of each group of experiments are the statistical results after 1000 simulations under the setting of this parameter. The total duration of all experiments was 180 days.

### III. RESULTS

#### A. A mass vaccination can curb the outbreak and reduce the pressure of hospitalization

We first considered the impact of the COVID-19 vaccination rate on the scale of the epidemic and its peak. Fig. 4(a) shows the progression of the epidemic under different scenarios. We call the case of no vaccination as the benchmark scenario.

The 100<sup>th</sup> day, rising epidemic, relative to the benchmark, infection density after vaccination was reduced by 45.5% ( $\rho = 25\%$ ), 73.8% ( $\rho = 50\%$ ), 89.5% ( $\rho = 75\%$ ), 98.1% ( $\rho = 100\%$ ) (Fig. 4(a)). On day 150, the epidemic reached a plateau with almost no new cases. Compared with the benchmark, the overall scale of infection per 10,000 people decreased by 1027.4 ( $\rho = 25\%$ ), 1899.4 ( $\rho = 50\%$ ), 2833.5 ( $\rho = 75\%$ ) and 3805.3 ( $\rho = 100\%$ ) (Fig. 4(a)). Overall, a population vaccination rate of 50% would reduce the epidemic

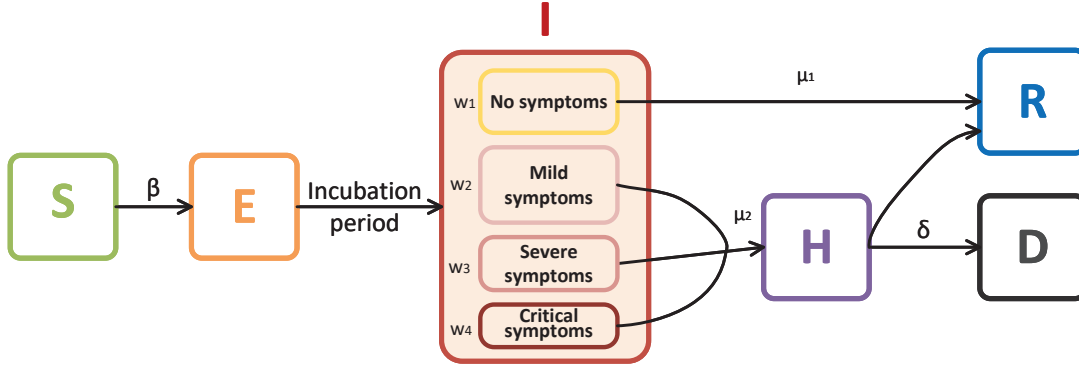


Fig. 3. The SEINRHD model of epidemic progress.

TABLE II  
MODEL PARAMETERS PER EXPERIMENT

Experiment	Schemes	Values
<b>Exp 1:</b> $r = 65.9\%$ $d_1 = d_2 = 0$ random	0	$\rho = 0$
	1	$\rho = 25\%$
	2	$\rho = 50\%$
	3	$\rho = 75\%$
	4	$\rho = 100\%$
<b>Exp 2:</b> $r = 65.9\%$ $d_1 = d_2 = 0$	1	$\rho = 10\%$ , random
	2	$\rho = 10\%$ , targeted
	3	$\rho = 25\%$ , random
	4	$\rho = 25\%$ , targeted
<b>Exp 3:</b> $\rho = 50\%$ $d_1 = d_2 = 0$ random	1	$r = 65\%$
	2	$r = 75\%$
	3	$r = 85\%$
	4	$r = 95\%$
<b>Exp 4:</b> $r = 65.9\%$ $d_1 = d_2 = 0$ random	1	$\rho = 50\%$
	2	$\rho = 75\%$ , $d_1 = 0, d_2 = 0$
	3	$\rho = 50\%$ , $d_1 = 25\%$ , $d_2 = 0$
	4	$\rho = 50\%$ , $d_1 = 50\%$ , $d_2 = 0$
	5	$\rho = 50\%$ , $d_1 = 50\%$ , $d_2 = 50\%$

by 43.6% compared with no vaccination at all. Full vaccination could reduce the number of cases by 87.2 % .

Compared with the benchmark, when the vaccine coverage rate reaches 50% in the population, the peak number of new infections will be reduced by 56.6% and the peak number of new hospitalizations will be reduced by 54.4% (Fig. 4(f)). When the vaccine is fully covered, the peak number of new infections will be reduced by 92.7%, and the peak number of new hospitalizations will be reduced by 91.9% (Fig. 4(f)).

*B. In the case of limited vaccines, priority should be given to the allocation of vaccines according to the role of the community*

Secondly, considering a very practical problem, in the initial stage of vaccine use, vaccines cannot be supplied on a large scale. Which groups of people should be given priority to allocate the small part of the vaccine to be more conducive to

the control of the epidemic? Therefore, we studied the impact of vaccination schemes on the development of the epidemic. In Experiment 2, we studied the progression of epidemics under both random and targeted vaccination modes at 10% and 25% vaccination rates (Fig. 4(b)). In random vaccination, when the vaccination rate is 10%, we can find that due to the low vaccination rate, compared with the benchmark ( $\rho = 0$ ), the final outbreak scale is only reduced by 7.4%. However, if targeted vaccination is adopted, community workers, public place attendants, etc. are given priority for vaccination, and the final scale will be reduced by 17.7%. When the vaccination rate is 25%, compared with random vaccination, targeted vaccination can reduce infections by 16.8% (Fig. 4(b)). When the vaccination rate is 10%, compared with random vaccination, targeted vaccination will reduce the peak number of new infections by 21.2%, and the peak number of new hospitalizations by 20.5%. When the vaccination rate is 25%, 23.4% and 23.2% are reduced accordingly. Analyzing from multi-dimensional indicators (Fig. 4(b)(e)(f)), when 10% targeted vaccination is carried out, the effect of random vaccination can be close to 25%.

*C. The vaccine protection rate should be further increased to reduce losses*

In the first set of experiments, we found that when the vaccination rate reached 100%, it still infected about 5.8% of the population, and those individuals who were vaccinated but not protected were still at risk of infection. According to empirical studies, the protection rate of current vaccines on the market is between 65% and 95%. We simulated epidemic trends under different vaccine protection rates. In Experiment 2, The vaccine protection rates were 65%, 75%, 85% and 95%, respectively. Other parameters were fixed. Fig. 4(c) shows the progression of the epidemic under different scenarios. The effect of the vaccine protection rate on the total scale of infection was uniform. A 10% increase in vaccine protection was associated with an average 5.4% reduction in morbidity per 10,000 population, an average 18.7 reduction in peak new infections and an average 12.7 reduction in peak new hospitalizations (Fig. 4(e)(f)).

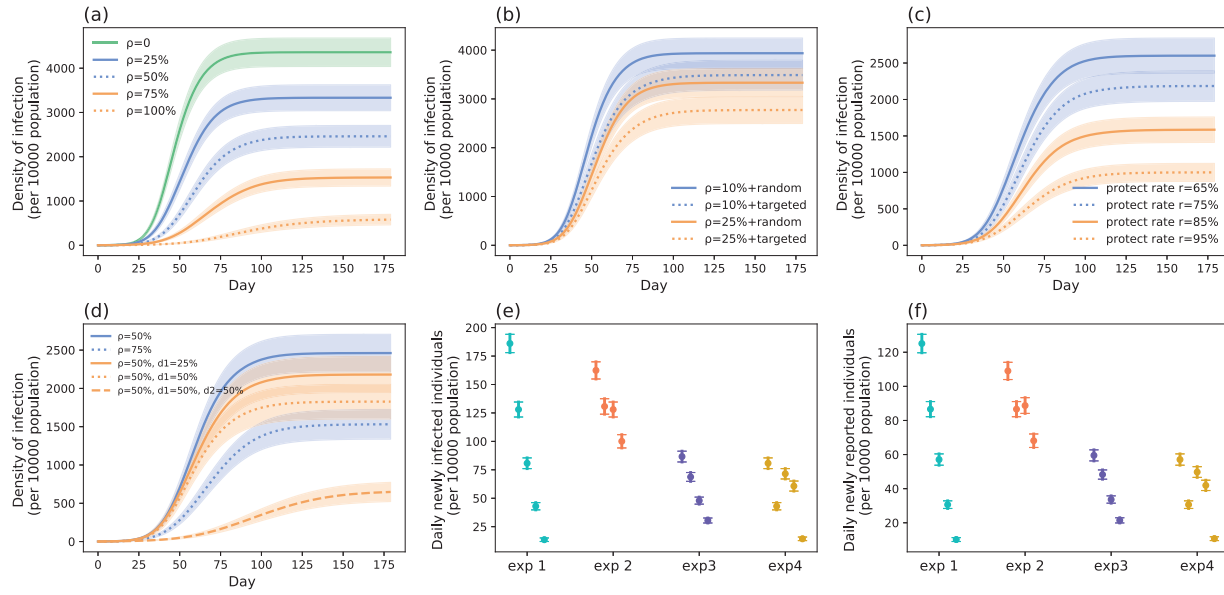


Fig. 4. (a)-(d) Infection status changes in density of infection. The lines represent the mean density of infection per 10,000 people, while the shaded areas represent the 95% reference range. (e)-(f) The peak number of daily newly infected individuals and reported individuals. Bars represent 5th and 95th percentile.

#### D. The mixed strategy of social distancing and vaccination can achieve the best prevention and control effect

As can be seen from the results of the first three experiments, if only vaccine intervention was implemented, there are very high requirements for vaccine production and vaccination scale to achieve good control effects, and it is a great challenge for human and material resources. So, we continue to study the impact of mixed strategies combined with social distancing on the epidemic curve. In the case of a 50% vaccination rate, a 50% reduction in community-layer contacts resulted in a 11.4% reduction in cumulative infection density, and a 50% reduction in social-layer contacts resulted in a 24.8% reduction in cumulative infection density. When 50% of the community layer and 50% of social layer contacts were reduced, the cumulative infection density will reduce by 82.2% (Fig. 4(d)). The scale of infection at this time was much lower than the 75% vaccination rate under the single vaccination strategy and close to the outbreak scale under the 100% vaccination strategy. Compared with the single strategy, the number of deaths under the three mixed strategies decreased by 8.3%, 23.3%, and 72.6%, respectively.

#### IV. DISCUSSION

As SARS-CoV-2 continues to spread globally, pharmaceutical companies are also racing to produce safe and efficient vaccines to combat the spread of disease. Pfizer Inc BioNTech, and Moderna have announced positive results from the first interim analyses of their Phase 3 vaccine trials. These results are more reflected at the individual level. In order to quantify the effectiveness of vaccination for the entire pandemic, our study used a SEINRHD model based on a hierarchical weighted network to simulate COVID-19 spreading in Chinese

communities and assessed the effect of vaccination on pandemic reduction. This research aims to provide evidence for future decisions. However, since the obtained data is collected in Chinese communities, the model might not be accurately applicable to other countries.

Experimental results show that the implementation of vaccination intervention can effectively control the epidemic. When a large-scale vaccination is carried out, the outbreak can be controlled within a small area. However, due to the limited protection rate of the vaccine, even the complete vaccination of the population will not eliminate the epidemic.

When the vaccine protection rate is further increased, the scale of the outbreak decreases linearly. The lower the effectiveness of the vaccine, the more people need to be vaccinated to eliminate the peak of infection. Under the same vaccination rate, the effect of targeted vaccination strategy is significantly better than random vaccination. Compared with the scheme when solely considering vaccination, the integrated measures mixed with social distancing are more conducive to the control of the epidemic. On the basis of vaccination, it is more effective to limit the social contact than to the community contact. At the same time, the effect of limit the contact of social layer and the community layer is higher than the sum of the two schemes implemented separately.

For public health guidance, regardless of the vaccine, the introduction of vaccination has reduced the scales of infections. A high protection rate can contain the epidemic more efficiently. It highlights the necessity of vaccination and improving the protection rate of vaccines. However, the government should consider that when the vaccine is first put into use, it often faces a decline in public confidence in the vaccine, which may lead to hesitation in vaccination. On the



other hand, due to the limited supply of vaccines, especially in the early stages of deployment, most authorities must choose between priority plans for vaccination. We can represent the coupling interaction of individuals in the family, social circles, and communities as a hierarchical weighted network based on case investigations. Accordingly, we can clearly understand the roles that individuals play in different layers. Individuals with larger degree in the community layer are mostly workers in departments such as medical services, food services, and accommodation. They have a higher risk of being infected as they contact more people in their work environment. The results of targeted vaccination show that this strategy can solve the shortage of vaccines and maximize the relative effect.

Due to the inability of vaccination to completely contain the epidemic and resource constraints, relevant departments can appropriately implement integrated measures combined with non-pharmaceutical interventions. In addition to those individuals reduce their contact with people at the community level, they should also avoid unnecessary contacts at the social level. When the scale of vaccination is insufficient, relevant departments can formulate implementation strategies after cost analysis. In summary, our model provides individuals, governments, and organizations with strategic insights into vaccination during a pandemic.

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