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Iterated Extended Kalman Filter for Time-delay Systems with Multi-sample-rate Measurements*

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Abstract— **Although optimal fusion algorithms of system state estimation have been proposed and studied for FAST in the past years, multi-sample-rate measurements and observation time-delay have always been restrictive factors to derive a glorious fusion results for state estimation. Based on the optimal fusion algorithm in the minimum mean square error sense, an iterated extended Kalman filter is investigated for discrete-time systems with multi-sample-rate measurements and delayed measurements in this paper, re-sampling observations from high sampling frequency channel and reducing the usage rate of observations with greater noises to investigate the estimation problem. The performance and improvement is clearly demonstrated through the numerical example. This study is manifestly advantageous for the feed supporting system of FAST.**

Index Terms— **multi-frequency measurement channels, time-delay measurement, iterated extended Kalman filter**

I. INTRODUCTION

Scientists in National Astronomical Observatories, Chinese Academy of Sciences have been committing to build Five hundred meter Aperture Spherical radio Telescope (FAST) [1], [2] since 1994. In this project, system state estimation according to measurements is one of crucial issues. GPS and total station are two decided measuring means for FAST. Due to the determination fundamentals restrictions, total station has greater delay but lower noise, and GPS is opposite. Meanwhile, the sampling frequencies of these two measuring means are unequal [3], [4], [5], [6].

A classical optimal state estimation is termed as Kalman filter [7], whose principle is the minimum mean square error estimation [8], [9], [10]. But the standard Kalman filter can only applicable to the system without multi-sample-rate measurements and time-delay [11]. To dispose time-delay systems, the common estimation tool is augmented matrix method [12], [13]. Due to the augmented matrix method costs large storage space and high-dimensional calculation, reference [14] studied the non-augmented matrix optimal filter. To reduce computational burden, references [9], [15] and [16] studied metabolic Kalman filter for discrete-time systems with observation delays via innovation re-organization.

Although system state estimation for FAST has attracted significant attention in the past years, multi-sample-rate measurements and observation time-delay have still been restrictive factors to derive a glorious fusion results. One of the reasons is the slack cognition to multi-sample-rate observation systems. A common restrictive condition in all above is that the sampling frequencies of different observation systems are equal. Another reason is an oversimplification in previous methods. Owing to the neglect of the restrictive correlation between time-delay and observation noise, the condition in [9] assumes that the observation errors of multiple observation systems are the equal.

In this paper, we establish a new iterated extended Kalman filter with the assumptions that the sampling frequencies of different systems are unequal, moreover the system delay and the observation noise are mutual suppression. Our aim is to deduce a new optimal state estimator to improve the state estimation accuracy for FAST.

The rest of this paper is organized as follows. Section II introduces the problem formulation. The iterated extended Kalman filtering formulation is presented in Section III. In Section IV, a set of numerical examples are presented to illustrate the main results and improvement effect. Finally, some conclusions are drawn in Section V.

II. PROBLEM STATEMENT

We consider the following simplified discrete-time linear system for our optimal filter problem:

$$x(t+1) = \Phi(t)x(t) + \Gamma(t)u(t) \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^r$, represent the state and process noise, $\Phi(t)$ and $\Gamma(t)$ are bounded time-varying matrices with appropriate dimensions. Assume that the state $x(t)$ is observed by two different systems with delays and multi-sample-rate measurements which are described by

$$y_0(t) = H_0(t)x(t) + v_0(t) \tag{2}$$

$$y_1(t) = H_1(t)x(t_1) + v_1(t) \quad (t_1 = t - \lambda_1, \lambda_1 > 0) \tag{3}$$

where $y_0(t) \in R^{m0}$ and $y_1(t) \in R^{m1}$ are respectively the current and the delayed output measurement with time-delay λ_1, $v_0(t) \in R^{m0}$ and $v_1(t) \in R^{m1}$ are the measurement noises. We make following assumptions for the above system.

Assumption 1: To simplify description, the observation

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systems $\{y_0\}$ and $\{y_1\}$ have unequal and known sampling frequencies. In view of our research background, we stipulate that

$$f_{y_1} = (1/n) \cdot f_{y_0} = 1 \tag{4}$$

where f_{y_i} is the sampling frequency of observation system $\{y_i\}(i = 0, 1)$, n is a positive integer, and integer 1 can be obtained from (1).

Assumption 2: The noises $u(t)$, $v_0(t)$ and $v_1(t)$ are mutually uncorrelated white noises with zero means and known covariance matrices as

$$Cov[u(t), u(j)] = Q_u(t)\delta_{tj} \tag{5}$$

$$Cov[v_0(t), v_0(j)] = Q_{v_0}(t)\delta_{tj} \tag{6}$$

$$Cov[v_1(t), v_1(j)] = Q_{v_1}(t)\delta_{tj} \tag{7}$$

where δ_{tj} is the Kronecker delta.

Assumption 3: The initial system state $x(0)$ is an independent sequence with known covariance matrix as

$$Cov[x(0), x(0)] = P_0. \tag{8}$$

Assumption 4: The covariance matrices satisfy that $Q_{v_1} < Q_{v_0}$, in view of our research background.

Considering assumptions $1 - 4$, the problem that we study is to find an optimal state estimator $\hat{x}(t|t)$ for $x(t)$ stemming from standard Kalman filter, given two multi-sample-rate observation sequences $\{y_0(\epsilon)|_{0\leq\epsilon\leq t}\}$ and $\{y_1(\tau)|_{0\leq\tau\leq t}\}$. Therein, ϵ and τ satisfy that

$$\begin{gathered} \epsilon = r_0 \cdot (1/f_{y_0}) \\ (r_0 = 0, 1, 2, ...) \end{gathered} \tag{9}$$

$$\begin{gathered} \tau = r_1 \cdot (1/f_{y_1}) \\ (r_1 = 0, 1, 2, ...). \end{gathered} \tag{10}$$

III. CONSTRUCTION OF THE STATE ESTIMATOR

In this section, the basic idea to deal with the computational problem resulting from unequal sampling frequencies is re-sampling. Meanwhile, we have to dispose the time-delay with the help of re-organizing the observations. Our improved Kalman filter is derived by using the re-organization innovation and projection theorem. The improvement in the process of final iterative computation could avoid those redundancy measurements of high frequency channel and take full account of noise difference for different observation systems. To simplify discussions, we suppose the time $t > \lambda_1$, the case of $0 \leq t \leq \lambda_1$ could be considered homoplastically.

A. Re-sample Observations

According to discrete-time linear system (1) and assumption 1, those observations of observation system (2) among state transition periods actually have no effect on state estimation. We could re-sample every time interval $r_1 \cdot (1/f_{y_1})$ from the high frequency observation sequence $\{y_0(\epsilon)|_{0\leq\epsilon\leq t}\}$ to meet the filter condition, where r_1 equals the value in (10). Then we can get re-sampling observation sequence and re-sampling noise sequence as

$$\begin{aligned} &\{y_0^*(\tau)|_{0\leq\tau\leq t}\} \\ =&\{y_0^*(0), y_0^*(1/f_{y_1}), y_0^*(2/f_{y_1}), ..., y_0^*(t)\} \\ =&\{y_0(0), y_0(n/f_{y_0}), y_0(2n/f_{y_0}), ..., y_0(t)\} \end{aligned}$$

$$\begin{aligned} &\{v_0^*(\tau)|_{0\leq\tau\leq t}\} \\ =&\{v_0^*(0), v_0^*(1/f_{y_1}), v_0^*(2/f_{y_1}), ..., v_0^*(t)\} \\ =&\{v_0(0), v_0(n/f_{y_0}), v_0(2n/f_{y_0}), ..., v_0(t)\}. \end{aligned}$$

Let $y(t)$ denote the consequential observations from system (2)-(3) at time t and $v(t)$ denote the related observation noises at time t, then we get

$$y(t) = \begin{cases} y_0^*(t) & 0 \leq t < \lambda_1 \\ col\{y_0^*(t), y_1(t)\} & \lambda_1 \leq t \end{cases} \tag{11}$$

$$v(t) = \begin{cases} v_0^*(t) & 0 \leq t < \lambda_1 \\ col\{v_0^*(t), v_1(t)\} & \lambda_1 \leq t \end{cases} \tag{12}$$

where $col\{\cdot\}$ means column structure.

Then the optimal state filter we study is based on the observation sequence $\{y(\tau)|_{0\leq\tau\leq t}\}$.

B. Re-organization Observations

In view of the fact that the optimal estimator $\hat{x}(t|t)$ is the projection of the state $x(t)$ onto the linear space $L\{y(\tau)|_{0\leq\tau\leq t}\}$. We could re-organize the observations to eliminate time-delay element.

The linear space $L\{y(\tau)|_{0\leq\tau\leq t}\}$ is equivalent to

$$L\{ry_2(\tau)|_{0\leq\tau\leq t_1}; ry_1(\tau)|_{t_1<\tau\leq t}\} \tag{13}$$

where $ry_1(\tau)$ and $ry_2(\tau)$ are the re-organization observations as

$$ry_1(\tau) \triangleq [y_0^*(\tau)] \tag{14}$$

$$ry_2(\tau) \triangleq \begin{bmatrix} y_0^*(\tau) \\ y_1(\tau + \lambda_1) \end{bmatrix}. \tag{15}$$

At the given time t, the re-organization observations satisfy that

$$\begin{gathered} ry_i(t) = rH_i(t)x(t) + rv_i(t) \\ (i = 1, 2) \end{gathered} \tag{16}$$

where

$$rH_1(t) \triangleq \left[H_0^*(t)\right]$$

$$rH_2(t) \triangleq \begin{bmatrix} H_0^*(t) \\ H_1(t+\lambda_1) \end{bmatrix}$$

$$rv_1(t) \triangleq \left[v_0^*(t)\right]$$

$$rv_2(t) \triangleq \begin{bmatrix} v_0^*(t) \\ v_1(t+\lambda_1) \end{bmatrix}.$$

Furthermore, $rv_1(t)$ and $rv_2(t)$ are mutually uncorrelated white noises with zero means and covariance matrices as

$$rQ\_v_1(t) \triangleq \left[Q_{v_0^*}(t)\right]$$

$$rQ\_v_2(t) \triangleq \begin{bmatrix} Q_{v_0^*}(t) & 0 \\ 0 & Q_{v_1^*}(t+\lambda_1) \end{bmatrix}.$$

It can be seen from (1) and (16) that the system representation has been the standard form. The re-organization observations (13)-(15) are the current measurements without multi-sample-rate measurements and delay.

C. Optimal Estimator

In view of the standard Kalman filter and the above re-organization observations, we derivate our iterated extended Kalman filter for system (1)-(3).

From (14) and (15), we get two re-organization observations $ry_1(t)$ and $ry_2(t_1)$ at the given time $t(t > \lambda_1)$. We define that $\hat{x}_i(\tau|\tau)$ is the intermediate variable based on re-organization observation $ry_i(\tau)$, and $\hat{x}(t|t)$ is the output optimal estimation, as shown in (17)-(20):

$$\hat{x}(t|t) = \hat{x}_1(t|t) \tag{17}$$

$$\begin{aligned} \hat{x}_i(\tau|\tau) =& \Phi(\tau-1)\hat{x}_i(\tau-1|\tau-1) + [\Phi(\tau-1)\cdot \\ & P_i(\tau-1|\tau-1)\Phi^T(\tau-1) + \Gamma(\tau-1)\cdot \\ & Q_u(\tau-1)\Gamma^T(\tau-1)] \cdot rH_i^T(\tau)M_i^{-1}(\tau)\cdot \\ & [ry_i(\tau) - rH_i(\tau)\Phi(\tau-1)\hat{x}_i(\tau-1|\tau-1)] \\ & (i = 1, 2) \end{aligned} \tag{18}$$

$$\begin{aligned} P_i(\tau|\tau) =& [\Phi(\tau-1)P_i(\tau-1|\tau-1)\Phi^T(\tau-1)+ \\ & \Gamma(\tau-1)Q_u(\tau-1)\Gamma^T(\tau-1)]\{I- \\ & rH_i^T(\tau)M_i^{-1}(\tau)rH_i(\tau)[\Phi(\tau-1)\cdot \\ & P_i(\tau-1|\tau-1)\Phi^T(\tau-1)+ \\ & \Gamma(\tau-1)Q_u(\tau-1)\Gamma^T(\tau-1)]\} \\ & (i = 1, 2) \end{aligned} \tag{19}$$

$$\begin{aligned} M_i(\tau) =& rH_i(\tau)[\Phi(\tau-1)P_i(\tau-1|\tau-1)\cdot \\ & \Phi^T(\tau-1) + \Gamma(\tau-1)Q_u(\tau-1)\cdot \\ & \Gamma^T(\tau-1)]rH_i^T(\tau) + rQ\_v_i(\tau) \\ & (i = 1, 2) \end{aligned} \tag{20}$$

where $P_i(\tau|\tau)$ is the covariance matrices of estimator errors as

$$P_i(\tau|\tau) = Cov[e_i(\tau), e_i(\tau)] \tag{21}$$

$$e_i(\tau) = x(\tau) - \hat{x}_i(\tau|\tau) \tag{22}$$

satisfy that $\tau = t_1$ or $\tau = t$ for $i = 2$ or $i = 1$, $\hat{x}_2(0|0) = \hat{x}(0|0)$, $P_2(0|0) = P_0$.

In order to calculate (18) and thus compute the output optimal estimation using (17), we use iterative algorithm to take recursive calculation of $\hat{x}_1(t-1|t-1)$. To optimize the state estimation result and take full account of the assumption 4, we improve the method in [9] to deduce our iterative formula, as shown in (23) and (24):

$$\hat{x}_1(t_1|t_1) = \hat{x}_2(t_1|t_1) \tag{23}$$

$$\begin{aligned} &\hat{x}_1(t_1+k|t_1+k) = \\ &\quad \{I - [\Phi(t_1+k-1)P_1(t_1+k-1|t_1+k-1)\cdot \\ &\quad \Phi^T(t_1+k-1) + \Gamma(t_1+k-1)Q_u(t_1+k-1)\cdot \\ &\quad \Gamma^T(t_1+k-1)]rH_1^T(t_1+k)M_1^{-1}(t_1+k)\cdot \\ &\quad rH_1(t_1+k)\}^{-1}\{\Phi(t_1+k-1)\cdot \\ &\quad \hat{x}_1(t_1+k-1|t_1+k-1) + [\Phi(t_1+k-1)\cdot \\ &\quad P_1(t_1+k-1|t_1+k-1)\Phi^T(t_1+k-1)+ \\ &\quad \Gamma(t_1+k-1)Q_u(t_1+k-1)\Gamma^T(t_1+k-1)] \\ &\quad rH_1^T(t_1+k)M_1^{-1}(t_1+k)[-rH_1(t_1+k)\cdot \\ &\quad \Phi(t_1+k-1)\hat{x}_1(t_1+k-1|t_1+k-1)]\} \\ &\quad (k = 1, 2, ..., \lambda_1 - 1). \end{aligned} \tag{24}$$

From the above, optimal state estimation $\hat{x}(t|t)$ is computed through the following steps:

Step 1) Calculating $\hat{x}_2(t_1|t_1)$ using (18), where $P_2(t_1 - 1|t_1 - 1)$ and $M_2(t_1 - 1)$ can be obtained from (19) and (20).

Step 2) Calculating $\hat{x}_1(t_1|t_1)$ using (23).

Step 3) Calculating $\hat{x}_1(t_1 + k|t_1 + k)$ using (24), where $P_1(t_1+k-1|t_1+k-1)$ and $M_1(t_1+k-1)$ can be obtained from (19) and (20).

Step 4) Calculating $\hat{x}_1(t|t)$ and $\hat{x}(t|t)$ using (18) and (17), where $P_1(t-1|t-1)$ and $M_1(t-1)$ can be obtained from (19) and (20).

IV. NUMERICAL EXAMPLE

In this section, we present a numerical example to illustrate the previous theoretical results and to compare the effect with standard Kalman filter whose state estimation can be calculated only from observation system (2).

Consider a discrete-time linear system described in (1)-(3) with the following specifications:

$$\Phi(t) = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.8 \end{bmatrix}, \Gamma(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$H_0(t) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}^T, H_1(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T.$$

The initial states are as follows: $x(0) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$, $\hat{x}(0|0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $P_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. We denote that $u(t)$, $v_0(t)$ and $v_1(t)$ are uncorrelated Gaussian noises with zero means and known covariance matrices, where $\lambda_1 = 5s$, $u \sim N(0,1)$, $v_0 \sim N(0,5)$, $v_1 \sim N(0,0.1)$, let sampling period $T_{y_0} = 0.2s$ and $T_{y_1} = 1s$.

Let Filter A denote our approach and Filter B denote the standard Kalman filter. The simulation results are drawn in Figs. 1-4. Fig. 1 and Fig. 2 represent the real states and their fusion estimations subject to delay λ_1 via Filter A. Fig. 3 and Fig. 4 represent the similar information via Filter B by single channel measurements (observation system (2)) without time-delay. It can be observed from the simulation results that the iterated extended Kalman filter produces very good performance. The RMS estimation errors of these two filters are shown in Table I and Table II. As $u(t)$, $v_0(t)$ and $v_1(t)$ can affect the estimated results randomly, we complete 5 simulation experiments with same system model and initial states to compare Filter A with Filter B.

TABLE I
RMS ERRORS OF STATE ESTIMATION $\hat{x}_1(t|t)$

	Filter A	Filter B	Upgrade Rate
Experiment 1	0.3838	0.4089	6.14%
Experiment 2	0.3846	0.4159	7.53%
Experiment 3	0.3425	0.3979	13.92%
Experiment 4	0.3764	0.4117	8.57%
Experiment 5	0.3681	0.3832	3.94%
Average	0.3711	0.4035	8.03%

TABLE II
RMS ERRORS OF STATE ESTIMATION $\hat{x}_2(t|t)$

	Filter A	Filter B	Upgrade Rate
Experiment 1	0.4041	0.4296	5.94%
Experiment 2	0.3915	0.4371	10.43%
Experiment 3	0.3536	0.4006	11.73%
Experiment 4	0.3807	0.4193	9.21%
Experiment 5	0.3740	0.3940	5.08%
Average	0.3808	0.4161	8.48%

The upgrade rate in Table I and Table II is defined as follow:

$$R_i = (RMS_{Bi} - RMS_{Ai})/RMS_{Bi} \times 100\% \qquad (25)$$

where R_i is the upgrade rate of state estimation result, RMS_{Ai} and RMS_{Bi} are the RMS errors of state estimations $\hat{x}_i(t)$ via Filter A and Filter B respectively.

V. CONCLUSION

This paper studied an iterated extended Kalman filter for discrete-time systems with multi-sample-rate measurements and delayed measurements. It solves the computational problem of observation systems with unequal sampling frequencies and reduces the adverse effect of observation channels with high noises in iteration process. Comparing with the previous method, a significant improvement of this approach is that it avoids those redundancy measurements of high frequency channel. Moreover, this presented filter gives a useful approach for disposing the time-delay engineering projects with multi-sample-rate measurements such as FAST project, as it takes full account of noise difference for different observation channels.

It should be pointed out that the effect of this filter depends on the value range of transfer matrix $\Phi(t)$. And differing from assumption 1, the sample frequencies of different observing systems are often non-integer multiple in realistic projects. Recognizing these, our future study is to work out the dependency between estimated results and transfer matrix, and the optimal filter via multiple observing systems with non-integer multiple sample frequencies measurements.

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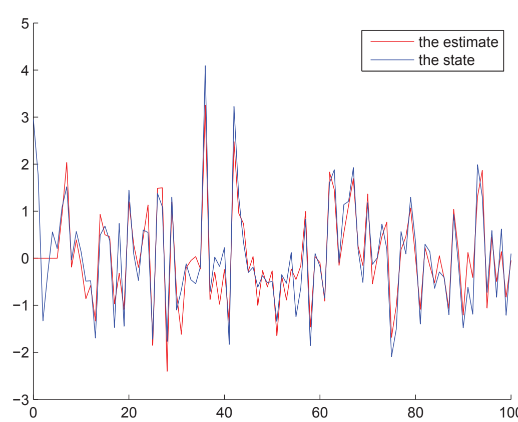

Fig. 1. Real state $x_1(t)$ and its estimation via Filter A.

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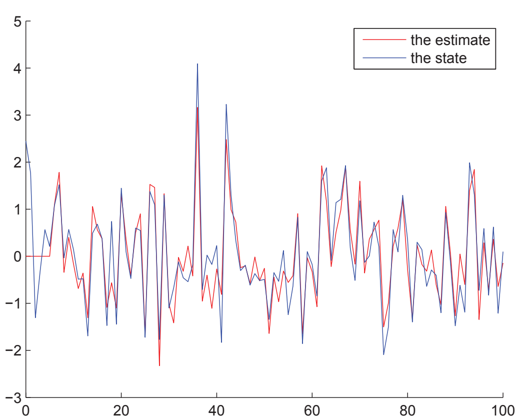

Fig. 2. Real state $x_2(t)$ and its estimation via Filter A.

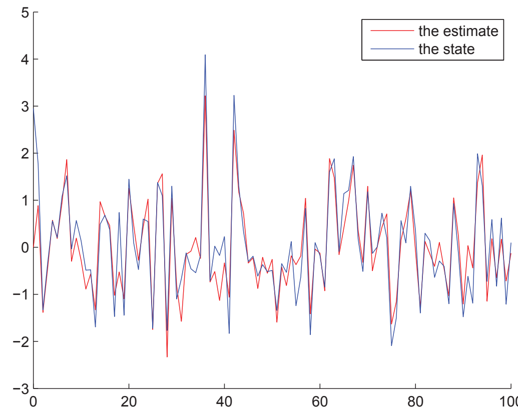

Fig. 3. Real state $x_1(t)$ and its estimation via Filter B.

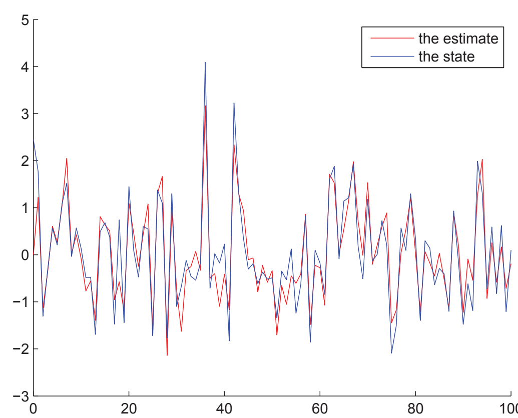

Fig. 4. Real state $x_2(t)$ and its estimation via Filter B.