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A path planning and following algorithm of observing targets for the UVMS Robocutt-I

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***Abstract*—The paper investigates a path planning and following algorithm of observing targets for an underwater vehicle-manipulator system(UVMS). The algorithm including the operational task assignment and path following for the UVMS is proposed. The operational task assignment consists of path planning and task assignment. Path planning provides planned trajectory distance between any two targets for task assignment, and task assignment produces a suitable task order for the minimum of the UVMS sailing distance. In path planning, the shortest path considering UVMS nonholonomic constraints—Dubins curve, is employed. It is implemented by matrix transformation. Then a digraph is generated by marking the shortest distance between any two targets. Hence the task assignment problem is considered as a traveling salesman problem, which can be achieved by the genetic algorithm. Finally a path-following guidance method is employed. Simulation results show the effectiveness of the proposed method.**

***Index Terms*—task assignment, path planning, path following, path tracking, Dubins curves**

I. Introduction

Recently, the autonomous robotic system has developed rapidly. It has widely been used in many areas, from traditional industry to medical service, education, exploration and exploitation, etc. It is an inevitable trend that a robot can complete more complicated tasks autonomously in extreme environment. As for an underwater vehicle-manipulator system(UVMS), it is necessary to develop a path planning and following algorithm of observing targets algorithm. A reasonable task scheduling can be generated by this algorithm. Moreover it can guide the UVMS to realize each task autonomously.

The overall algorithm architecture is illustrated in Fig. 1. The operational task assignment is composed of path planning and task assignment. Path planning is to find an optimal and feasible trajectory between any two task, and provides the information about the distances of the planned paths for task assignment. Task assignment can produce an optimal scheduling scheme according to path lengths produced by path planning. Subsequently, the UVMS follows the generated path by the guidance law. In the process of UVMS task execution, the sensors such as cameras, sonar can perceive environment information, path planning and task assignment strategy can be adjusted based on environment information.

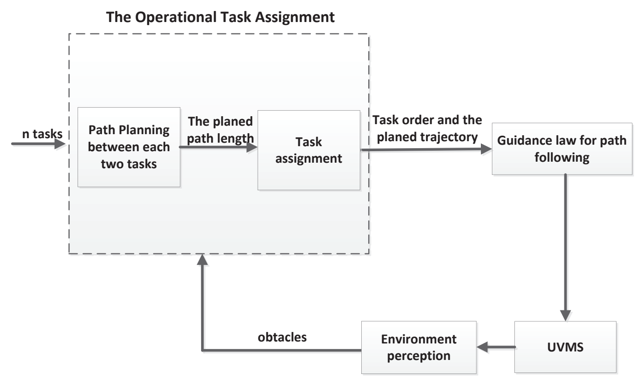

Fig. 1. The algorithm architecture

Many solutions have been employed for task assignment. Sivanandam et al. developed a Hybrid Particle Swarm Optimization method to solve the task assignment problem [1]. Choi et al. devised two methods including the consensus-based auction algorithm and the consensus-based bundle algorithm for task assignment [2]. Additionally, Bellingham et al. employed MILP method to realize task allocation [3]. These methods are mainly devoted to the targets assignment, but the actual trajectory is usually taken place by straight lines or other curves. Hence the operational task assignment including path planning and task assignment is valuable. Task assignment can produce a reasonable task order according to useful path information provided by path planning. Beard et al. presented path planning based on Voronoi diagram and target management using satisfying game theory [4]. Target management achieves suitable task allocation according to optimal paths produced by path planning. Analogously, Zhong et al. used the route planning algorithm based on dynamic programming to obtain available routes between any

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two targets and provide reasonable routing information for task assignment, then the ant colony algorithm is applied to solve the task assignment problem [5].

In the operational task assignment, path planning is a significant component, and many useful methods can be exploited for path planning. Jaillet et al. combined the exploratory strength of RRTs, with the efficiency of stochastic-optimization methods to obtain low-cost paths [6]. Dolgov et al. presented A^* algorithm and non-linear optimization to get a local optimal and kinematically feasible trajectory [7]. In addition, Hwang et al. proposed a fast path planning method by optimization of a path graph [8]. Pamosoaji et al. developed a path-planning method utilizing parameterized vector potential functions, which can produce a collision-free region-to-region path [9]. In 2-D space, it is proved that the optimal path with nonholonomic constraints is the Dubins curve [10]. Furthermore, Bezier and B-spline curves with continuous tangent vector and curvature have been used in path planning. Han et al. employed Bezier curve to achieve path planning for car-like autonomous vehicle systems [11]. Maekawa et al. applied cubic B-spline curves to generate collision-free paths for unmanned vehicles [12].

Path following for unmanned vehicles has been applied in critical applications. It can be realized by the line-of-sight(LOS) algorithm. The geometric assignment based on LOS algorithm to minimize the cross-track error was achieved [13]. Moreover 3-dimensional cross-track control based on LOS guidance law was considered in [14]. Path following for 3-degrees-of-freedom marine surface vessels was addressed, and the stability of the system was analyzed by Lyapunov techniques and nonlinear cascaded systems theory [15]. Some other schemes based on LOS algorithm could be found in [16]–[18].

II. Problem Statement

In this paper, we investigate the UVMS performs multiple tasks in the environment without obstacles, the component "environment perception" in the framework is neglected. It is supposed that the UVMS needs to reach a fixed pose for target observation or manipulation, moreover it should accomplish all the tasks and return to its initial pose. The UVMS should complete each task only once. As illustrated in Fig. 2, let $T = \{T_1, T_2..., T_n\}$ represents n tasks. Assume that $P_i(x_i, y_y, \theta_i)$ is the pose of the task T_i, where (x_i, y_i) is the position in a Cartesian coordinate frame, and θ_i is the heading angle. The UVMS initial pose is P_1. And no obstacles or threats need to be avoided. The UVMS should pass through all the poses, and finally return to the pose P_1.

In order to produce a feasible path for UVMS in path planning and provide useful information about the distances of the paths for task assignment, the UVMS nonholonomic constraints is considered in the operational task assignment.

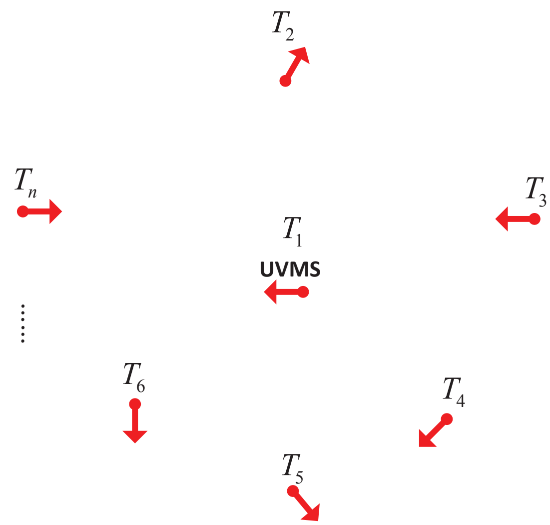


Fig. 2. Problem statement

The UVMS should meet the following kinematic equation:

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ -R_{\min} \leq \frac{\dot{\theta}}{v} \leq R_{\min} \end{cases} \tag{1}$$

where v is the speed of the UVMS, θ is the heading angle, and R_{min} is the minimum radius of the UVMS. Our purpose is to let the UVMS performs each task once, and minimum the total sailing distance of the UVMS.

III. The Algorithm of Observing Targets

Two components including the operational task assignment and path following are implemented in the paper. First we employ 2D Dubins curve to achieve path planning, which is the shortest and feasible path between any two poses. and the matrix transformation method is proposed to get the Dubins curve. Then the task assignment problem is converted into an asymmetric traveling salesman problem according to the distance of the planned path between the two tasks. The problem is solved by genetic algorithm. Therefore, the shortest UVMS sailing trajectory is obtained. Finally a nonlinear guidance is applied to achieve the path following of UVMS.

A. The Operational Task Assignment for the UVMS

1) 2-D Dubins path planning based on matrix transformation: It is supposed that the initial and final poses are $P_s(x_s, y_s, \theta_s)$ and $P_g(x_g, y_g, \theta_g)$. Path planning is to find a feasible and optimal path with the nonholonomic constraint between P_s and P_g. With the purpose of compactness of the total algorithm and easy realization in computer, the matrix transformation method is used in 2-D Dubins planning. As depicted in Fig. 3, there are four CSC paths in 2-D Dubins

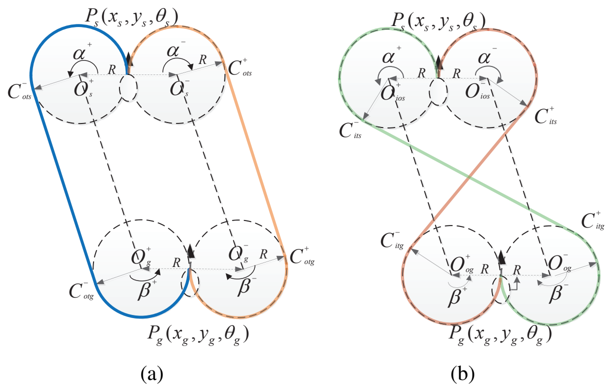


Fig. 3. 2-D Dubins generation.(a) External tangential trajectory on the x-y plane.(b) Internal tangential trajectory on the x-y plane.

curves when the distance between the two points is larger than 2*R, where R is the curvature radius of 2-D path. Taking one of four CSC paths which is the blue and highlighted path in Fig. 3(a) for an example, the matrix transformation method is described. As the calculations of the other three CSC paths are similar to the following procedures by the matrix transformation method, they will not be repeated.

(1) The circle centers O_s^+ and O_g^+

The center of the initial arc is $O_s^+(x_{os}^+,y_{os}^+)$ computed as

$$\begin{bmatrix} x_{os}^+ \\ y_{os}^+ \end{bmatrix} = \begin{bmatrix} x_s \\ y_s \end{bmatrix} + R \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} \cos\theta_s \\ \sin\theta_s \end{bmatrix} \quad (2)$$

The center of the final arc is $O_g^+(x_{og}^+,y_{og}^+)$ computed as

$$\begin{bmatrix} x_{og}^+ \\ y_{og}^+ \end{bmatrix} = \begin{bmatrix} x_g \\ y_g \end{bmatrix} + R \begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} \begin{bmatrix} \cos\theta_g \\ \sin\theta_g \end{bmatrix} \quad (3)$$

(2) The cut-off points C_{ots}^- and C_{otg}^-

The cut-off points of the arcs, $C_{ots}^-(x_{ots}^-,y_{ots}^-)$ and $C_{otg}^-(x_{otg}^-,x_{otg}^-)$ are achieved by

$$\begin{bmatrix} x_{ots}^- \\ y_{ots}^- \end{bmatrix} = \begin{bmatrix} x_{os}^+ \\ y_{os}^+ \end{bmatrix} + R \begin{bmatrix} \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) \\ \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) \end{bmatrix} \overrightarrow{p_o} \quad (4)$$

$$\begin{bmatrix} x_{otg}^- \\ y_{otg}^- \end{bmatrix} = \begin{bmatrix} x_{og}^+ \\ y_{og}^+ \end{bmatrix} + R \begin{bmatrix} \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) \\ \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) \end{bmatrix} \overrightarrow{p_o} \quad (5)$$

where, $\overrightarrow{p_o}$ is the unit vector of $\overrightarrow{O_s^+ O_g^+}$.

(3) The initial rotation angle α^+

Suppose α is the angle between $\overrightarrow{O_s^+ S}$ and $\overrightarrow{O_s^+ C_{ots}^-}$, and the initial rotation angle α^+ has the following relationship with α:

$$\alpha^+ = \begin{cases} \alpha & sin\theta_s[y_{ots}^- - y_s + cot\theta_s(x_{ots}^- - x_s)] > 0 \\ 2\pi - \alpha & sin\theta_s[y_{ots}^- - y_s + cot\theta_s(x_{ots}^- - x_s)] \leq 0 \end{cases} \quad (6)$$

(4) The final rotation angle β^+

Suppose that β is the angle between $\overrightarrow{O_g^+ G}$ and $\overrightarrow{O_g^+ C_{otg}^-}$, and the final rotation angle β^+ has the following relationship with β:

$$\beta^+ = \begin{cases} \beta & sin\theta_g[y_{otg}^- - y_g + cot\theta_g(x_{otg}^- - x_g)] < 0 \\ 2\pi - \beta & sin\theta_g[y_{otg}^- - y_g + cot\theta_g(x_{otg}^- - x_g)] \geq 0 \end{cases} \quad (7)$$

(5) The length of 2-D Dubins curve L_{Dubins}

L_{Dubins} is computed by

$$L_{Dubins} = \alpha^+ \cdot R + \left\| \overrightarrow{C_{ots}^- C_{otg}^-} \right\| + \beta^+ \cdot R \quad (8)$$

(6) Select the shortest one of the four CSC paths when $R = R_{min}$.

Hence we get the optimal path with nonholonomic constraints when the distance between the two positions is larger than $2R_{min}$. Several parameters need to be stored to describe a path:

1) The initial and final pose.
2) The centers of the initial and final arcs:O_s^+ and O_g^+.
3) The initial and final rotation angles:α^+ and β^+.
4) The minimum radius R_{min}.

2) Task assignment: Suppose that d_{ij} is the length of 2D Dubins path from P_i to P_j, thus a matrix (9) can be obtained, and the problem is transformed into an Asymmetric Traveling Salesman Problem, which is a NP hard problem. It can be solved by genetic algorithm. The genetic algorithm is summarized in Algorithm 1, where N_{Iter} is the maximum number of iterations. It has three basic components: selection, crossover and mutation. Roulette Wheel Selection is utilized in selection operator to find the individuals with higher fitness as parents, PMX Operator and SWAP Operator are employed in crossover and mutation respectively.

$$\begin{matrix} P_1 \\ \vdots \\ P_n \end{matrix} \begin{pmatrix} \infty & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{n1} & \cdots & \infty \end{pmatrix} \quad (9)$$

B. Path Following

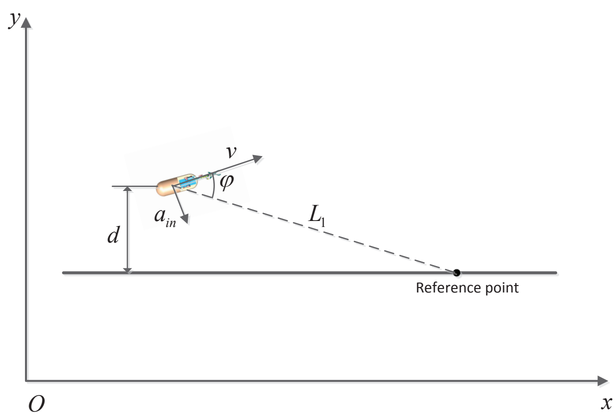
In path planning, the Dubins curve satisfying the radius constraint is utilized, which is a combination of a straight line and arcs. In order to achieve UVMS path tracking, we apply a nonlinear guidance, which can follow a straight line and a circle with any radius [19]. As depicted in Fig. 4, we set a reference point on the planed path, and the sight distance L_1 between the reference point and the UVMS position is constant. φ is the angle between the direction of the straight line segment L_1 and the UVMS attitude. A lateral acceleration a_{in} is produced by the nonlinear guidance to achieve path tracking, which is expressed as:

$$a_{in} = 2\frac{v^2}{L_1}\sin\varphi \quad (10)$$

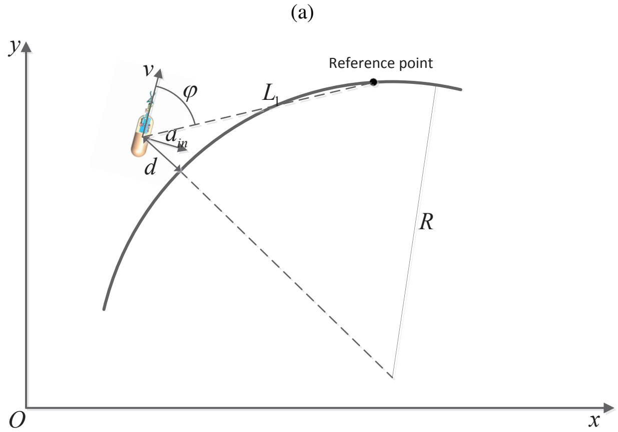
Algorithm 1 Genetic algorithm

Require:
The matrix (9);
Ensure:
The optimal task order;

genetic algorithm parameters initialization;
generate initial populations randomly;
while ($N_{Iter} \neq 0$) **do**
  compute each individual's fitness
  if the convergence is achieved **then**
    Output the optimal path;
    break;
  else
    Roulette Wheel Selection;
    Partially Matched Crossover;
    Swap Mutation;
    produce new populations;
  end if
  $N_{Iter} = N_{Iter} - 1$;
end while



(a)



(b)

Fig. 4. The nonlinear guidance.(a) follow a straight line.(b) follow an arc.

and the direction of the lateral acceleration a_{in} is perpendicular to the UVMS attitude.

It is assumed that pa is a symbol, which is used to distinguish the reference path is a straight line or an arc. When $pa = 0$, the reference path is a straight line, when $pa = 1$, the reference path is an arc. By linearizing the guidance equation, we can get:

$$\begin{cases} \frac{2v}{L_1}(\dot{d} + \frac{v}{L_1}d) & pa = 0 \\ \frac{2v^2(1-\frac{L_1^2}{4R^2})}{L_1}d + \frac{2v\sqrt{(1-\frac{L_1^2}{4R^2})}}{L_1}\dot{d} + \frac{v^2}{R} & pa = 1 \end{cases} \tag{11}$$

From (11), it is obvious that this method approximates a linear PD controller when the path is a straight line or an arc. When the speed of UVMS is stationary, a fixed and suitable sight distance L_1 is found to achieve path following. And the specific analysis and the Lyapunov stability of this method can be found in [20].

C. Summary

Algorithm 2 The algorithm of observing targets

Require:
The UVMS speed v;
The minimum radius of the UVMS: R_{min};
n targets:$P_1, P_2....P_n$;
Ensure:
n feasible path, $Path_1, ...Path_n$;

parameters initialization;
for $i = 1...n$ **do**
  for $j = 1...n$ **do**
    obtain the shortest path from P_i to P_j by 2-D Dubins path planning;
    store the parameters of every path;
    compute the distance of the generated path d_{ij};
  end for
end for
get the matrix and solve the Asymmetric Traveling Salesman Problem by genetic algorithm;
get n Dubins curves according to the stored path parameters;
guide the UVMS to follow the path by (10) .

The algorithm of observing targets is summarized as depicted in Algorithm 2. The shortest path between any two targets with vehicle nonholonomic constraints are obtained by 2-D Dubins path planning based on matrix transformation, the parameters of which are stored in the memory unit of the UVMS processor. Meanwhile, all the distances of the generated paths form a matrix like (9). Then the matrix as the input is sent into Algorithm 1, therefore the optimal path which can pass through all the tasks is attainable. The method can solve the problem where the targets are considered as the poses. If the targets are seen as the positions, we only need to

TABLE I
THE POSES OF THE TASKS

Tasks	Position	Heading angle(°)
P_1	$(0, 0)$	0
P_2	$(30, 0)$	135
P_3	$(0, -30)$	60
P_4	$(50, 50)$	45
P_5	$(50, -50)$	330
P_6	$(-40, -40)$	30
P_7	$(60, -20)$	60
P_8	$(-40, 30)$	300
P_9	$(-40, 0)$	330
P_{10}	$(0, 50)$	30

change path planning strategy correspondingly, the algorithm framework is similar.

Remark 1: As 2-D path planning between the two tasks only employs CSC paths of Dubins curves, the proposed algorithm of observing targets can be applied when the distance between any two tasks is larger than $2R_{min}$.

IV. SIMULATION RESULTS

A. The Operational Task Assignment Simulation

The algorithm is evaluated by numerical simulations. Suppose that when the vehicle speed is $3m/s$, the UVMS radius constraints are $|R| \geq 5m$. The poses of the tasks are listed in Table I, and the UVMS initial pose is the pose of the task P_1. Fig. 5(a) shows the poses of the tasks and the UVMS initial pose. The distance d_{ij} from the pose P_i to the pose P_j can be computed by 2-D Dubins path planning based on matrix transformation, hence we can get the matrix(12). The problem is converted into an asymmetric traveling salesman problem, and the optimal path is solved by the genetic algorithm, which is $P_1 \rightarrow P_5 \rightarrow P_7 \rightarrow P_2 \rightarrow P_4 \rightarrow P_{10} \rightarrow P_8 \rightarrow P_9 \rightarrow P_6 \rightarrow P_3 \rightarrow P_1$. The distance of the optimal path is $492.678m$. As illustrated in Fig. 5(b), the total trajectory of the UVMS is given, which is optimal and can pass though all the poses of the tasks. Fig. 5(c) shows the graph reflecting the relationship between the UVMS heading angle and the abscissa along the total trajectory.

B. Path Following Simulation

Taking the planned trajectory from P_4 to P_{10} for an example, the path following simulation is given by the nonlinear guidance. When the UVMS speed is $3m/s$, we set the sight distances L of a straight line and an arc are both $4m$. Moreover random disturbance is added to simulate the actual situations. As shown in Fig. 6, the blue trajectory is the planned path generated by 2D Dubins path generation, and the red one is the actual UVMS path obtained by the nonlinear guidance.

V. CONCLUSION AND FUTURE WORK

A path planning and following algorithm of observing targets for an underwater vehicle-manipulator system(UVMS)

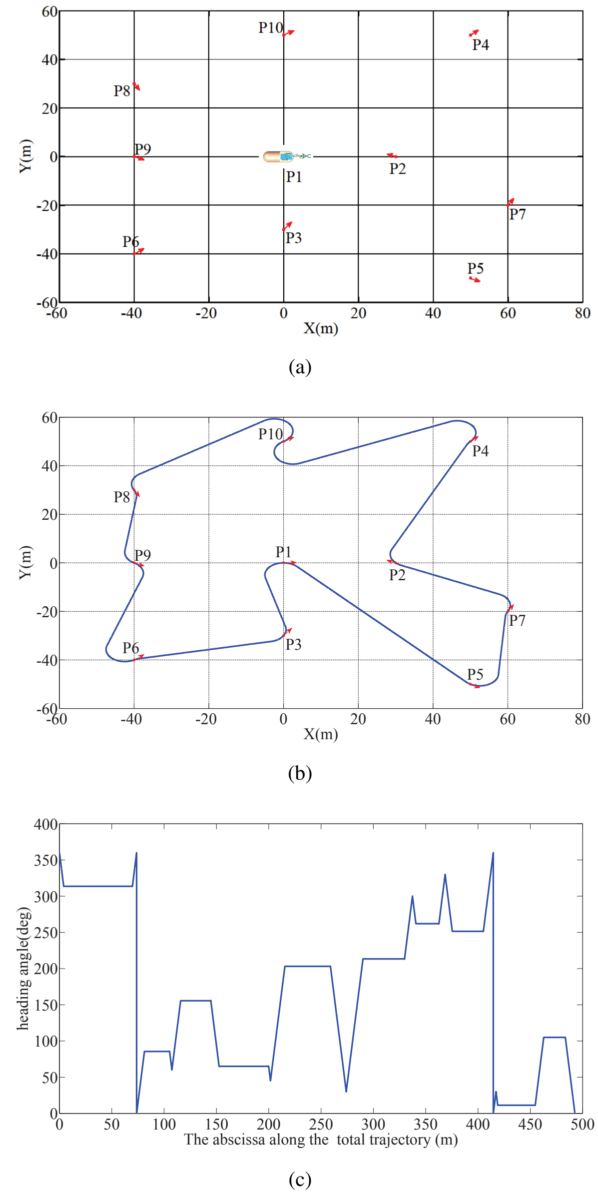


Fig. 5. The operational task assignment result. (a)The poses of 10 tasks and the UVMS initial pose. (b) The total trajectory of UVMS. (c) The UVMS heading angle along the total trajectory.

is developed in this paper. It is composed of the operational task assignment and path following. In the operational task assignment, 2-D Dubins path planning based on matrix transformation is employed to achieve smooth transition between any two tasks with UVMS nonholonomic constraints, then a traveling salesman problem is formed by marking the shortest distance between any two targets provided by the Dubins path generation. It can be solved by the genetic algorithm. Then we exploit a nonlinear guidance to realize path following. Simulation results further verify the effectiveness of the

$$
\begin{bmatrix}
0 & 39.64 & 47.98 & 71.12 & 71.14 & 81.15 & 65.38 & 71.22 & 66.30 & 54.46 \\
47.44 & 0 & 61.42 & 55.21 & 66.55 & 98.34 & 56.80 & 86.23 & 81.46 & 61.49 \\
33.70 & 45.67 & 0 & 94.35 & 56.23 & 66.81 & 62.05 & 88.64 & 69.12 & 81.09 \\
94.68 & 70.92 & 123.15 & 0 & 109.81 & 156.09 & 89.32 & 107.04 & 122.19 & 72.43 \\
94.61 & 63.93 & 71.94 & 106.97 & 0 & 111.77 & 36.33 & 146.63 & 134.37 & 125.33 \\
57.00 & 85.25 & 41.71 & 127.29 & 90.82 & 0 & 102.52 & 82.60 & 48.66 & 98.91 \\
81.91 & 38.99 & 78.21 & 71.66 & 51.01 & 123.84 & 0 & 125.07 & 119.26 & 98.02 \\
50.28 & 88.84 & 78.16 & 93.95 & 120.45 & 76.79 & 114.60 & 0 & 31.24 & 47.56 \\
40.12 & 78.85 & 53.85 & 103.87 & 102.96 & 48.66 & 103.55 & 52.69 & 0 & 66.41 \\
60.92 & 74.58 & 98.66 & 50.56 & 115.31 & 119.49 & 99.74 & 63.27 & 81.79 & 0
\end{bmatrix}
\tag{12}
$$

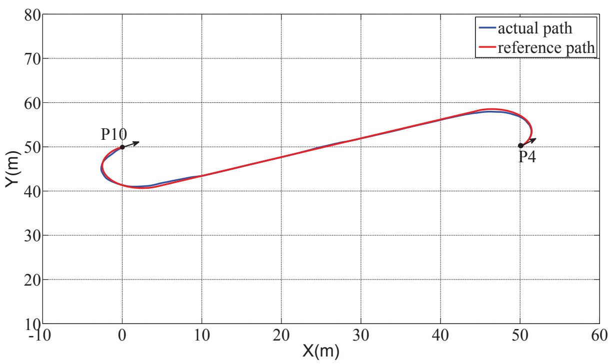


Fig. 6. Path following result

proposed system.

Recently the UVMS RobCutt-I is still in the development stage, future research will concentrate on two aspects: first, autonomous operation of the UVMS will be investigated; second, the algorithm proposed in this paper will be employed in our UVMS.

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