Correction to "Global Practical Stabilization of Discrete-time Switched Affine Systems via a General Quadratic Lyapunov Function and a Decentralized Ellipsoid"

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In [1], some corrections should be made in the statements of Lemmas 2 and 4, and proofs of Lemma 2 and Prop. 3 as follows. In what follows, all the equations, Lemmas and Propositions numbers are referred to the original article [1].

Lemma 2. Ellipsoid $\mathcal{E}(Q,c,\rho)$ with $Q=Q^T\succ 0$ and (8) is an attractive set for the system (2) according to Def. 2 and using switching function (12) if $\forall e(k)\notin\mathcal{E}(Q,c,\rho)$, $\exists i\in\mathbb{K}$ and $\exists\gamma>0$ such that

$$\begin{bmatrix} A_ie(k)+l_i \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & c \\ c^T & \rho \end{bmatrix} \begin{bmatrix} A_ie(k)+l_i \\ 1 \end{bmatrix} - \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & c \\ c^T & \rho \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \le -\gamma \quad (25)$$

Lemma 4. Let the conditions (6)-(8) be held. Ellipsoid $\mathcal{E}(Q,e_c,1)$ in (5) with $Q=Q^T\succ 0$ is an attractive set for the system (2) according to Def. 2 and using switching function (12) if $\forall e(k)\notin\mathcal{E}(Q,e_c,1)$, $\exists i\in\mathbb{K}$ and $\exists\gamma>0$ such that

$$(A_ie(k)+l_i-e_c)^TQ(A_ie(k)+l_i-e_c) - (e(k)-e_c)^TQ(e(k)-e_c)\le-\gamma, \forall e(k)\notin\mathcal{E}(Q,e_c,1)$$

The proof of Prop. 3. is modified as follows. We show that the conditions of Prop. 3 imply the condition of Lemma 2. Using the similar arguments in the proof of Prop. 2, from (27) one can obtain

$$\begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & * \\ c^T & \rho \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} > 0 \Rightarrow \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \tilde{M}_1' & * \\ \tilde{M}_2' & \tilde{M}_3' \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \le -\gamma, \quad (32)$$

where $\gamma>0$ is a positive constant. By substituting $\tilde{M}_1'$, $\tilde{M}_2'$, and $\tilde{M}_3'$ from (29)-(31) into (32) and after some algebra, one can reach

$$e(k)\notin\mathcal{E}(Q,c,\rho)\Rightarrow \sum_{i\in\mathbb{K}}\lambda_{2i}'\Big[\begin{bmatrix} A_ie(k)+l_i \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & * \\ c^T & \rho \end{bmatrix} \begin{bmatrix} A_ie(k)+l_i \\ 1 \end{bmatrix} - \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & * \\ c^T & \rho \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix}\Big] \le -\gamma \quad (33)$$

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Now using (28) and (33), we have

$$e(k)\notin\mathcal{E}(Q,c,\rho)\Rightarrow\exists i\in\mathbb{K}\ \text{ such that } \begin{bmatrix} A_ie(k)+l_i \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & * \\ c^T & \rho \end{bmatrix} \begin{bmatrix} A_ie(k)+l_i \\ 1 \end{bmatrix} - \begin{bmatrix} e(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} Q & * \\ c^T & \rho \end{bmatrix} \begin{bmatrix} e(k) \\ 1 \end{bmatrix} \le -\gamma \quad (34)$$

The expression in (34) is identical to the condition of Lemma 2, and as a result, the attractiveness of the ellipsoid $\mathcal{E}(Q,c,\rho)$ is concluded under switching rule (12). The proof of Prop. 3 is completed.

Within the proof of Lemma 2, the relation (60) must be modified as

$$\begin{aligned} &e(k)\notin\mathcal{E}(Q,c,\rho)\Rightarrow\exists i\in\mathbb{K}\text{ such that}\\ &v(A_{\sigma(e(k))}e(k)+l_{\sigma(e(k))})-v(e(k))=\Delta v(e(k))\\ &\le v(A_ie(k)+l_i)-v(e(k))\le-\gamma<0 \end{aligned} \quad (60)$$

The proof is completed just after the forgoing modified relation (60) by the following statement:

Therefore, conditions (b) and (c) in Lemma 1 are fulfilled, and the attractive property of the ellipsoid $\mathcal{E}(Q,c,\rho)$ is concluded.

Indeed, the expressions just after (60) up to (66) in the proof of Lemma 2 in [1] must be omitted. The problem returns to the definition of function $\phi(s)$ in (62) where $\|e(k)\|$ must be replaced by the distance function $\|e(k)\|_{\mathcal{E}}=\inf_{y\in\mathcal{E}}\|e(k)-y\|$ denoting the distance from point $e(k)$ to the ellipsoid $\mathcal{E}$. Therefore, using (60) in [1] and Lyapunov set asymptotic stability arguments [2], we can only conclude that $\phi(\|e(k)\|_{\mathcal{E}})\to 0$ as $k\to\infty$, and the existence of a positive constant γ is not proved.

References

[1] M. Hejri, "Global practical stabilization of discrete-time switched affine systems via a general quadratic Lyapunov function and a decentralized ellipsoid," *IEEE/CAA Journal of Automatica Sinica*, vol. 8, no. 11, pp. 1837–1851, November 2021.

[2] R. Goebel, R. G. Sanfelice, and A. R. Teel, *Hybrid Dynamical Systems: Modeling, Stability and Robustness*. Princeton University Press, 2012.