Letter

Fixed-Time Neural Control of a Quadrotor UAV With Input and Attitude Constraints

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Dear Editor,

This letter is concerned with the attitude control of a quadrotor unmanned aerial vehicle (UAV) subject to the input constraint, attitude constraint and model uncertainty. Firstly, we construct an auxiliary system to eliminate the adverse impact of the input saturation. Secondly, we introduce the nonlinear state-dependent function to deal with the attitude constraint directly. Thirdly, the neural network is utilized to identify the unknown terms in the system. Finally, with the help of the backstepping technology, a fixed-time control scheme is presented, which guarantees that the desired signal is followed with the fixed-time convergent rate. The effectiveness of the proposed scheme is validated by a simulation verification.

In the past decade, the quadrotor UAV has been extensively used in various fields, such as pesticide spraying, aerial photography, surveying and mapping and security inspection [1]. With the widespread application of the quadrotor UAV, the increasing flight quality is required to carry out different tasks. As we all know, the quadrotor UAV reaches the desired positon by adjusting its attitude. Thus, it is essential to design a robust and accurate control law for the attitude system. The main factors affecting the flight quality include input saturation, attitude constraint and inaccurate system model. Therefore, this letter concentrates on solving these problems.

When the quadrotor UAV flies in a narrow area, the flight attitude must be restricted to avoid collision. Besides, when using the quadrotor UAV to convey liquids, large attitude changing will lead to liquid sloshing. Therefore, it has practical meaning to research the attitude constraint problem. The barrier Lyapunov function (BLF), as a common means to handle the state constraint, has been widely studied by numerous scholars [2], [3]. In [4], to solve the problem of full states constraint, an adaptive neural fault-tolerance control strategy by using the logarithmic BLF was developed for a quadrotor UAV. Further, the time-varying logarithmic BLF was utilized to solve the attitude constraint problem for multi-rotor UAV in [5].

In a real system, the physical characteristics of the actuator determine that its input cannot exceed a certain threshold. Of cause, the quadrotor UAV is no exception and the input constraint often effects its flight quality. Thus, many works are devoted to solve the input saturation, and abundant results have been achieved so far. In [6], the Nussbaum function was introduced to solve the input saturation. In [7], with the help of the Gaussian function, a continuous differentiable saturation model was built so as to employ the recursive control algorithm to design controller. In [8], an auxiliary system was constructed to deal with input saturation for the medium-scale unmanned autonomous helicopter.

In recent decades, the combination of advanced control theory and neural network (NN) [9] or fuzzy logic system (FLS) [10], [11], commonly as function approximator, has promoted the development of intelligent control theory. Further, the NN or FLS is applied to the practical system to tackle the system uncertainty by numerous scholars. A composite learning control scheme with finite-time convergence was proposed for the quadrotor UAV by using the terminal sliding mode in [12]. In [13], an adaptive neural backstepping control strategy in presence of input saturation was presented for quadrotor crafts and obtained finite-time convergence. However, the inherent defect of the finite-time control is that the convergent time is related to system initial values. To remove the limits, this letter will study a fixed-time control scheme combined with the NN for a quadrotor UVA.

Motivated by the analysis mentioned above, a fixed-time neural control scheme with input and attitude constraints is designed to improve the flight quality of a quadrotor UAV. The main contributions are summarized as: 1) The NN and anti-saturation auxiliary system are combined with the fixed-time control theory to design the attitude controller, which ensures that the convergent time is independent of initial conditions. 2) The nonlinear state-dependent function (NSDF) is introduced to guarantee that the attitude constraint bound is not violated while the fixed-time convergent rate is achieved.

Notations: $\lambda_{\min}(\bullet)$ denotes the smallest eigenvalue of a square matrix $\bullet$. $r\in\mathbb{R}^n$ and $s\in\mathbb{R}^n$ are n-dimension vectors, and $r\circ s$, $|r|^\lambda$ and $\text{sign}(s)$ are defined as $r\circ s=[r_1s_1,\ldots r_ns_n]^T$, $|r|^\lambda=[|r_1|^\lambda,\ldots,|r_n|^\lambda]^T$, $\text{sign}(s)=[\text{sign}(s_1),\ldots,\text{sign}(s_n)]^T$.

Problem formulation: The attitude model is given by [14]

$$\begin{cases}\ddot\phi=\dot\theta\dot\psi\dfrac{I_y-I_z}{I_x}+\dfrac{u_1(w_1)}{I_x}+d_1\\ \ddot\theta=\dot\phi\dot\psi\dfrac{I_z-I_x}{I_y}+\dfrac{u_2(w_2)}{I_y}+d_2\\ \ddot\psi=\dot\phi\dot\theta\dfrac{I_x-I_y}{I_z}+\dfrac{u_3(w_3)}{I_z}+d_3\end{cases}\tag{1}$$

where ϕ, θ and ψ are the roll angle, pitch angle and yaw angle respectively, I_x, $I_y$and I_z denote moment of inertia, d_1, d_2 and d_3 indicate external disturbances, w_1, w_2 and w_3 are desired control torque inputs to be designed, u_1, u_2 and u_3 represent actual control torque inputs, and

$$u_k(w_k)=sat(w_k)=\begin{cases}\text{sign}(w_k)u_{Mk}, & |w_k|\ge u_{Mk}\\ w_k, & |w_k|<u_{Mk}\end{cases}\tag{2}$$

for $k=1,2,3$, where $u_{Mk}>0$ is the constraint bound of u_k.

By defining $x_1=[x_{11},x_{12},x_{13}]^T=[\phi,\theta,\psi]^T$ and $x_2=[x_{21},x_{22},x_{23}]^T=[\dot\phi,\dot\theta,\dot\psi]^T$, system (1) is written as

$$\begin{cases}\dot x_1=x_2\\ \dot x_2=f(x_2)+bu(w)+d\end{cases}\tag{3}$$

where $b=\text{diag}\left\{1/I_x,1/I_y,1/I_z\right\}$, $d=[d_1,d_2,d_3]^T$, $u(w)=[u_1(w_1),u_2(w_2),u_3(w_3)]^T$, $w=[w_1,w_2,w_3]^T$, $f(x_2)=[\dot\theta\dot\psi\left(I_y-I_z\right)/I_x, \dot\phi\dot\psi(I_z-I_x)/I_y,\dot\phi\dot\theta\left(I_x-I_y\right)/I_z]^T$.

Assumption 1 (see [8]): For control torque constraint (2), the difference $\Delta u=u-w$ is deemed to be bounded, namely, $\|\Delta u\|\le\tau$ with τ being a positive constant.

Assumption 2 (see [8]): The disturbance d is deemed to be bounded, namely, $\|d\|\le\bar d$ with $\bar d$ being a positive constant.

In this letter, the attitude angles are subject to the following constraint:

$$x_{1k}\in D_k:=\{x_{1k}\in\mathbb{R}:-g_{Lk}(t)<x_{1k}(t)<g_{Hk}(t)\}\tag{4}$$

where $g_{Lk}(t):\mathbb{R}_+\to\mathbb{R}_+$ and $g_{Hk}(t):\mathbb{R}_+\to\mathbb{R}_+$ are such that $g_{Lk}(t)<\underline{g}_k$ and $g_{Hk}(t)<\overline{g}_k$ with $\underline{g}_k$ and $\overline{g}_k$ being positive constants. Next, for writing convenience, $g_{Lk}(t)$ and $g_{Hk}(t)$ are abbreviated to g_{Lk} and g_{Hk}.

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In order to constrain the attitude angle, the following NSDF are conducted [15]:

$$T_{1k} = \frac{x_{1k}}{(g_{Lk}+x_{1k})(g_{Hk}-x_{1k})} \tag{5}$$

$$T_{dk} = \frac{x_{dk}}{(g_{Lk}+x_{dk})(g_{Hk}-x_{dk})} \tag{6}$$

where $x_d = [x_{d1}, x_{d2}, x_{d3}]^T$ is desired attitude angle.

From (5), we deduce that for any initial value $x_{1k}(0) \in D_k$, T_{1k} tends to infinity when x_{1k} goes to the boundary of D_k, that is,

$$T_{1k} \to \infty, \text{ if and only if } x_{1k} \to -g_{Lk} \text{ or } x_{1k} \to g_{Hk}. \tag{7}$$

Furthermore, we obtain that for any initial state $x_{1k}(0) \in D_k$, if $T_{1k}(t) \in L_\infty$, $\forall t > 0$, it is naturally ensured $x_{1k}(t) \in D_k$. Hence, we conclude that as long as T_{1k} is bounded, the attitude constraint is strictly maintained.

Differentiating T_{1k} and T_{dk}, it yields

$$\dot{T}_{1k} = \beta_{1k}\dot{x}_{1k} + \gamma_{1k} \tag{8}$$

$$\dot{T}_{dk} = \beta_{dk}\dot{x}_{dk} + \gamma_{dk} \tag{9}$$

with

$$\beta_{1k} = \frac{g_{Lk}g_{Hk} + x_{1k}^2}{(g_{Lk}+x_{1k})^2(g_{Hk}-x_{1k})^2} \tag{10}$$

$$\beta_{dk} = \frac{g_{Lk}g_{Hk} + x_{dk}^2}{(g_{Lk}+x_{dk})^2(g_{Hk}-x_{dk})^2} \tag{11}$$

$$\gamma_{1k} = \frac{(\dot{g}_{Lk}-\dot{g}_{Hk})x_{1k}^2 - (\dot{g}_{Lk}g_{Hk}+g_{Lk}\dot{g}_{Hk})x_{1k}}{(g_{Lk}+x_{1k})^2(g_{Hk}-x_{1k})^2} \tag{12}$$

$$\gamma_{dk} = \frac{(\dot{g}_{Lk}-\dot{g}_{Hk})x_{dk}^2 - (\dot{g}_{Lk}g_{Hk}+g_{Lk}\dot{g}_{Hk})x_{dk}}{(g_{Lk}+x_{dk})^2(g_{Hk}-x_{dk})^2}. \tag{13}$$

Define $T_1 = [T_{11}, T_{12}, T_{13}]^T$ and $T_d = [T_{d1}, T_{d2}, T_{d3}]^T$. Then, we obtain

$$\dot{T}_1 = \beta_1\dot{x}_1 + \gamma_1 \tag{14}$$

$$\dot{T}_d = \beta_d\dot{x}_d + \gamma_d \tag{15}$$

where $\beta_1 = \mathrm{diag}\{\beta_{11}, \beta_{12}, \beta_{13}\}$, $\beta_d = \mathrm{diag}\{\beta_{d1}, \beta_{d2}, \beta_{d3}\}$ $\gamma_1 = [\gamma_{11}, \gamma_{12}, \gamma_{13}]^T$ and $\gamma_d = [\gamma_{d1}, \gamma_{d2}, \gamma_{d3}]^T$.

Lemma 1: For $r \in \mathbb{R}^n$, $s \in \mathbb{R}^n$ and $P \in \mathbb{R}^{n\times n}$ the following inequality holds:

$$r^T P s \le \frac{l}{2} r^T r tr\left(P^T P\right) s^T s + \frac{1}{2l} \tag{16}$$

where l is a positive constant.

Proof:

$$\begin{aligned} r^T P s &\le \frac{l}{2}\left(r^T P s\right)^2 + \frac{1}{2l} \\ &= \frac{l}{2}\left(r^T (p_1, \dots p_n) s\right)^2 + \frac{1}{2l} \\ &= \frac{l}{2}\left(\left(r^T p_1, \dots, r^T p_n\right) s\right)^2 + \frac{1}{2l} \\ &\le \frac{l}{2}\left(\left(r^T p_1\right)^2 + \dots + \left(r^T p_n\right)^2\right) s^T s + \frac{1}{2l} \\ &\le \frac{l}{2} r^T r \left(p_1^T p_1 + \dots + p_n^T p_n\right) s^T s + \frac{1}{2l} \\ &= \frac{l}{2} r^T r tr\left(P^T P\right) s^T s + \frac{1}{2l} \end{aligned}$$

■

The control objective in this letter is to design a fixed-time neural control law for a quadrotor UAV with input and attitude constraints such that the desired trajectory x_d can be followed with fixed-time convergent rate and all closed-loop signals are uniformly ultimately bound.

Main results: In this section, the detailed design process of fixed-time control law combined with the NN and anti-saturation auxiliary system will be presented for the attitude system.

For the control torque constraint problem, the following auxiliary system is introduced [8]:

$$\begin{cases} \dot{\zeta}_1 = -\left(\frac{1}{2}\right)^{\frac{3}{4}} K_1|\zeta_1|^{\frac{3}{4}} \circ \mathrm{sign}(\zeta_1) - \left(\frac{1}{2}\right)^2 \zeta_1^T K_2\zeta_1\zeta_1 + \zeta_2 \\ \dot{\zeta}_2 = -\left(\frac{1}{2}\right)^{\frac{3}{4}} K_3|\zeta_2|^{\frac{3}{4}} \circ \mathrm{sign}(\zeta_1) - \left(\frac{1}{2}\right)^2 \zeta_2^T K_4\zeta_2\zeta_1 - \zeta_1 \\ \quad - \frac{3}{2}\zeta_2 + b\Delta u \end{cases} \tag{17}$$

with $K_1, K_2, K_3, K_4 \in \mathbb{R}^{3\times3}$ being the positive definite diagonal matrices, $\zeta_1, \zeta_2 \in \mathbb{R}^3$ being the auxiliary system internal states.

Remark 1: In the existing researches, smoothing function, dynamic auxiliary system and NN are used to deal with input saturation, but there are few relevant studies that consider input saturation in fixed-time control. Compared with literature [8], the dynamic auxiliary system is designed to deal with input saturation with fixed-time convergence in this letter.

Step 1: The virtual control law is designed as

$$\begin{aligned} \alpha = &-\left(\frac{1}{2}\right)^{\frac{3}{4}} \beta_1^{-1}\left(Q_1|e_1|^{\frac{1}{2}} \circ \mathrm{sign}(e_1) + K_1|\zeta_1|^{\frac{1}{2}} \circ \mathrm{sign}(\zeta_1)\right) \\ &-\beta_1^{-1}\left(\gamma_1 - \beta_d\dot{x}_d - \gamma_d + \frac{1}{2}e_1 + \frac{1}{2}\beta_1\beta_1^T e_1\right) \\ &-\left(\frac{1}{2}\right)^2 \beta_1^{-1}\left(e_1^T Q_2 e_1 e_1 + \zeta_1^T K_2\zeta_1\zeta_1\right) \end{aligned} \tag{18}$$

where $Q_1 \in \mathbb{R}^{3\times3}$ and $Q_2 \in \mathbb{R}^{3\times3}$ are positive definite diagonal matrices to be designed. Details are given in Section II of the Supplementary Material.

Step 2: The actual control law is designed as

$$\begin{aligned} w = &-\left(\frac{1}{2}\right)^{\frac{3}{4}} b^{-1}\left(Q_3|e_2|^{\frac{1}{2}} \circ \mathrm{sign}(e_2) + K_3|\zeta_2|^{\frac{1}{2}} \circ \mathrm{sign}(\zeta_2)\right) \\ &-b^{-1}\left(\beta_1 e_1 + e_2 + \zeta_1 + \frac{3}{2}\zeta_2 + \frac{\hat{\xi}S^T S e_2}{2\delta^2}\right) \\ &-\left(\frac{1}{2}\right)^2 b^{-1}\left(e_2^T Q_4 e_2 e_2 + \zeta_2^T K_4\zeta_2\zeta_2\right) \end{aligned} \tag{19}$$

where $Q_3 \in \mathbb{R}^{3\times3}$ and $Q_4 \in \mathbb{R}^{3\times3}$ are positive definite diagonal matrices, $\hat{\xi}$ is the estimation of ξ, $\tilde{\xi} = \hat{\xi} - \xi$. Details are given in Section II of the Supplementary Material.

Step 3: The update law for $\hat{\xi}$ is designed as

$$\dot{\hat{\xi}} = \frac{e_2^T e_2 S^T S}{2\delta^2} - \lambda_1\hat{\xi} - \lambda_2\hat{\xi}^3 \tag{20}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$ are constants to be designed.

Some lemmas in [16] and [17] are used in these three steps. Details are given in Section II of the Supplementary Material.

Theorem 1: Under the auxiliary system (17), the virtual control law (18), the actual control law (19) and the update law (20), the system (3) obtains: 1) The whole signals of the system are bounded, and the tracking error converges into a small set around zero within fixed-time T. 2) It is ensured that the state constraint is not violated, that is, $g_{Lk}(t) < x_{1k}(t) < g_{Hk}(t)$, $t > 0$.

Proof: 1) According to (33) in the Supplementary Material, we easily obtain that V_3 is bounded. Hence, all signals are bounded. And using Lemma 1 in [1], we get that the system (3) is practical fixed-time stable. Further, we get the fixed-time $T \le \frac{4}{\mu_1\iota} + \frac{10}{\mu_2\iota}$, where $0 < \iota < 1$. 2) Since all signals are bounded, $e_{1k} = T_{1k} - T_{dk} - \zeta_{1k}$ and ζ_{1k} are bounded. Note that $T_{dk} \in L_\infty$ in the compact set D_{dk}, it is ensured $T_{1k} \in L_\infty$. Then, in the light of analysis below (7), we conclude that for any $x_{1k}(0) \in D_k$, the state x_{1k} remains the predefined set D_k. ■

Numerical example: In this section, numerical simulation experiment will be implemented to validate the feasibility of the proposed fixed-time control method. In order to present the superiority of the proposed scheme, we compare it with the control method presented in [13].

An example with practical parameters of a quadrotor UAV is given by $I_x = 0.045\ \mathrm{kg}\cdot\mathrm{m}^2$, $I_y = 0.045\ \mathrm{kg}\cdot\mathrm{m}^2$ and $I_z = 0.083\ \mathrm{kg}\cdot\mathrm{m}^2$.

The quadrotor UAV is required to track the time-varying signals with external disturbances. The desired signals are given by

$$\begin{cases} x_{d1} = 0.1 + 0.1\sin(0.5t) \\ x_{d2} = 0.1 + 0.1\sin(0.2t) \\ x_{d3} = 0.1 + 0.1\sin(0.1t) - 0.1\cos(0.1t). \end{cases} \tag{21}$$

The external disturbances are $d_1 = 0.1\sin(t)$, $d_2 = 0.2\sin(0.2t)$ and $d_3 = 0.1\sin(0.1t)$.

The constraint bounds of three channels are

$$\begin{cases} g_{L1} = 0.1 + 0.01\cos(t), g_{H1} = 0.3 + 0.02\sin(t) \\ g_{L2} = 0.1 + 0.01\cos(t), g_{H2} = 0.3 + 0.02\sin(t) \\ g_{L3} = 0.1 + 0.01\cos(t), g_{H3} = 0.3 + 0.02\sin(t). \end{cases} \tag{22}$$

The design matrices are

$$\begin{cases} K_1 = \mathrm{diag}\{5,5,5\}, K_2 = \mathrm{diag}\{5,5,5\} \\ K_3 = \mathrm{diag}\{1,1,1\}, K_4 = \mathrm{diag}\{2,2,2\} \\ Q_1 = \mathrm{diag}\{10,10,10\}, Q_2 = \mathrm{diag}\{10,10,10\} \\ Q_3 = \mathrm{diag}\{2,2,2\}, Q_4 = \mathrm{diag}\{50,50,50\}. \end{cases} \tag{23}$$

The design parameters are $\delta = 50$, $\lambda_1 = 50$ and $\lambda_2 = 50$.

The constraint bounds of actual controller are assumed as $u_{M1} = 0.0022$, $u_{M2} = 0.0022$ and $u_{M3} = 0.0022$.

Figs. 1−3 present the tracking results of the attitude system of a quadrotor UAV in the presence of external disturbance and input constraint. In the view of tracking results, both proposed control scheme and control scheme presented in [13] achieve the desired signal with a very small tracking error. However, it is noteworthy that the proposed control scheme has faster convergent rate than control scheme presented in [13]. Besides, the proposed control scheme can track the desired signal within fixed-time which means that the convergent time is independent of the initial conditions. Therefore, our proposed method is superior to the method presented in [13] in terms of convergent time. We can also find that the attitude angles are kept within the specified range with the help of the NSDF.

Fig. 4 shows the trajectory of $\hat{\xi}$. We can find that $\hat{\xi}$ is convergent, which means that the NN we constructed is feasible. By constructing the NN, we do not need to know the exact system model when design the attitude controller. Therefore, our proposed method can achieve precise control without knowing the exact model of system. The smooth and bounded curves of virtual control law is shown in Fig. 5, Fig. 6 gives the curves of actual control input with input saturation. It is noteworthy that the control inputs are confined within prescribed range and it can be seen that the inputs are continually changing to resist the time-varying disturbances and track the time-varying desired signals. All figures are given in Section III of the Supplementary Material.

Conclusion: In this letter, the fixed-time neural attitude control under input and attitude constraints has been investigated. First, by introducing the anti-saturation auxiliary system, the adverse effect of the input saturation is overcome. Besides, with the help of the NSDF, the attitude angles are constrained within prescribed sets. Finally, the proposed method based on backstepping technique can guarantee that the desired signal is followed with fixed-time convergent rate.

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References

[1] G. Cui, W. Yang, J. Yu, Z. Li, and C. Tao, "Fixed-time prescribed performance adaptive trajectory tracking control for a quadrotor UAV," *IEEE Trans. Circuits Syst. II*, vol. 69, no. 2, pp. 494–498, 2022.

[2] Y.-J. Liu, L. Ma, S. Liu, S. Tong, and C. P. Chen, "Adaptive neural network learning controller design for a class of nonlinear systems with time-varying state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 31, no. 1, pp. 66–75, 2019.

[3] L. Liu, T. Gao, Y.-J. Liu, and S. Tong, "Time-varying asymmetrical BLFs based adaptive finite-time neural control of nonlinear systems with full state constraints," *IEEE/CAA J. Autom. Sinica*, vol. 7, no. 5, pp. 1335–1343, 2020.

[4] Z. Shen, L. Tan, S. Yu, and Y. Song, "Fault-tolerant adaptive learning control for quadrotor UAVs with the time-varying CoG and full-state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 12, pp. 5610–5622, 2021.

[5] C. Fu, W. Hong, and H. Lu, "Adaptive robust backstepping attitude control for a multi-rotor unmanned aerial vehicle with time-varying output constraints," *Aerosp. Sci. Technol.*, vol. 78, pp. 593–603, 2018.

[6] Y. Li, S. Tong, and T. Li, "Direct adaptive fuzzy backstepping control of uncertain nonlinear systems in the presence of input saturation," *Neural Comput Appl*, vol. 23, no. 5, pp. 1207–1216, 2013.

[7] J. Ma, S. S. Ge, and Z. Zheng, "Adaptive NN control of a class of nonlinear systems with asymmetric saturation actuators," *Neurocomput.*, vol. 275, pp. 1674–1683, 2018.

[8] M. Chen, K. Yan, and Q. Wu, "Multiapproximator-based fault-tolerant tracking control for unmanned autonomous helicopter with input saturation," *IEEE Trans. Syst. Man Cybern. Syst.*, vol. 52, no. 9, pp. 5710–5722, 2022.

[9] H. Yang and J. Liu, "An adaptive RBF neural network control method for a class of nonlinear systems," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 2, pp. 457–462, 2018.

[10] S. Tong. X. Min, and Y. Li, "Observer-based adaptive fuzzy tracking control for strict-feedback nonlinear systems with unknown control gain functions," *IEEE Trans. Cybern.*, vol. 50, no. 9, pp. 3903–3913, 2020.

[11] Y. Pan, P. Du, and H. Xue, "Singularity-free fixed-time fuzzy control for robotic systems with user-defined performance," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 8, pp. 2388–2398, 2020.

[12] B. Xu, "Composite learning finite-time control with application to quadrotors," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 48, no. 10, pp. 1806–1815, 2017.

[13] Q. Xu, Z. Wang, and Z. Zhen, "Adaptive neural network finite time control for quadrotor UAV with unknown input saturation," *Nonlinear Dynam*, vol. 98, no. 3, pp. 1973–1998, 2019.

[14] K. Eliker and W. Zhang, "Finite-time adaptive integral backstepping fast terminal sliding mode control application on quadrotor UAV," *Int. J. Control Autom.*, vol. 18, no. 2, pp. 415–430, 2020.

[15] K. Zhao and Y. Song, "Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems," *IEEE Trans. Autom. Control*, vol. 64, no. 3, pp. 1265–1272, 2018.

[16] X. Jin, "Adaptive fixed-time control for MIMO nonlinear systems with asymmetric output constraints using universal barrier functions," *IEEE Trans. Autom. Control*, vol. 64, no. 7, pp. 3046–3053, 2018.

[17] M. Chen, H. Wang, and X. Liu, "Adaptive fuzzy practical fixed-time tracking control of nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 98, no. 3, pp. 1973–1998, 2019.