

# *Causal Inference in Social Media Using Convergent Cross Mapping*

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**Abstract**—Revealing underlying causal structure in social media is critical to understanding how users interact, on which a lot of security intelligence applications can be built. Existing causal inference methods for social media usually rely on limited explicit causal context, pre-assume certain user interaction model, or neglect the nonlinear nature of social interaction, which could lead to bias estimations of causality. Inspired from recent advance in causality detection in complex ecosystems, we propose to take advantage of a novel nonlinear state space reconstruction based approach, namely Convergent Cross Mapping, to perform causal inference in social media. Experimental results on real world social media datasets show the effectiveness of the proposed method in causal inference and user behavior prediction in social media.

**Keywords**—causal inference; user influence; nonlinear dynamic system; social media

## I. INTRODUCTION

Recent years have witnessed an explosive growth of various social media sites such as online social networks, blogs, microblogs, social news websites and virtual social worlds. This gives researchers a great opportunity to study social interactions on an unprecedented scale [1]. Since individual behavior have influence on the decisions of friends in a social network, the knowledge of who-influences-whom and most-influential-person has enormous implications in security informatics [2]. For example, if we know the causal structure of people, we are able to predict people's action based on actions of her friends. Unfortunately, the causal structure among users is usually unknown and unobserved. For instance, it is very common that we only observe the times when particular users adopted a new emerging meme but we do not observe who infected them.

Although some methods were proposed to uncover the hidden causal structure in observational data among users by taking advantage of both user dynamical behavior data [3]–[5] and structural features [6], they perform causal inference based on either pre-assumption of certain particular user interaction model, a linear view of social interaction, or a purely stochastic view of social system. However, the pre-assumption of particular user interaction model usually fails to capture the complexity of human behavior. Besides, nonlinearity is ubiquitous in nature [7], [8] and user interaction may have nonlinear property. This means that interaction between two users is state-dependent, which is a defining signal of complex nonlinear systems [7]. In addition,

recent studies on online social interactions show that user's interaction sequences have strong deterministic components [9], which support the nonlinear dynamical system view of social system instead of a purely stochastic view.

Recently, a new causality detection method, called Convergent Cross Mapping (CCM), is developed by Sugihara et al. [8] for detecting causal relations in time series data from weakly interacting nonlinear dynamical systems. This method can deal with the mentioned observations and they have successfully applied this method to complex ecological systems for causality detection. Inspired from this advance in ecology research, we propose to adopt CCM to perform causal inference in social media in this paper. CCM is based on nonlinear state space reconstruction. Its key idea is that two time series variables have causal relationship if they belong to the same nonlinear dynamic system and then we can extract the “signature” left by influencers embedded in user activity level time series data using CCM. To the best of our knowledge, we are the first to take advantage of the nonlinear dynamic system based approach to tackle the causal inference problem in social media.

The rest of this paper is organized as follows. Section II surveys the related work on causal inference in social media. The problem definition is given in Section III. Section IV introduces the background definitions, basic idea and algorithm of CCM. Experimental results on real world datasets will be presented in Section V. Finally, we will give a summary in Section VI.

## II. RELATED WORK

Traditional approaches concerning influence measurement mainly focus on different topological structure measures such as in/out-degree [10], PageRank [11] and other centrality based measures [12]. Recent work has highlighted that it is not sufficient to use structural measures alone to measure user influence. For instance, researchers found that the ranking of the most influential users differ depending on the influence measures used [6], [13]. Besides, edges in social networks may reveal little information about the actual social dynamics. What's worse, the structural measures need to be recomputed from scratch when the social network changes.

To improve the topological structure based influence measurement, more recent work tried to involve dynamic information into influence model. For example, Romero et al. [3] proposed a heuristic which measure influence taking into

account the fact that many users are passive and will not retweet under any circumstances. The limitation inherent in these approaches is that they often require explicit causal knowledge such as who responded whom, which is usually not available in many scenarios. Our proposed method differs from them in that it does not need any explicit causal knowledge.

To infer the underlying causal structure, current studies [4], [5] are built on the a priori assumptions of certain information diffusion model to describe the user interaction mechanism such as Independent Cascade Model and Linear Threshold model [14], [15]. However, these models may fail to capture the complexity of user interaction. More recently, some researchers start to take advantage of information theory measures [16], such as Granger Causality [17] or its variants like Transfer Entropy [18], [19], to infer the causal structure among users. The drawback of such work is that they usually adopt a linear view of social interaction and a purely stochastic view of social system, which does not fit for the real world. The presented CCM differs from them in that it can deal with the nonlinear interaction between users based on the nonlinear dynamic system perspective.

### III. PROBLEM DEFINITION

We formulate the causal inference problem in social media as follows. Given a set of users  $U = \{u_1, u_2, \dots, u_n\}$ , for each user  $u$  we have the historical activity level time series  $\{X\} = [X(1), X(2), \dots, X(t), \dots, X(L)]$ , meaning that the activity level (i.e., the number of posts) of user  $u$  at time  $t$  is  $X(t)$ . The hypothesis here is that users interacted in an underlying influence network. The goal of causal inference problem is to infer the underlying causal structure among users, which is represented as a weighted directed graph  $G$ . The directed edge  $(u_i, u_j)$  indicates that user  $u_i$  has influence on user  $u_j$ , and the weight of this edge  $w_{ij}$  indicates the strength of this causal effect.

### IV. PROPOSED APPROACH

CCM is built on nonlinear state space reconstruction. In this section, we first introduce some background definitions in nonlinear dynamic systems and then demonstrate the basic idea and algorithm of the CCM method introduced in [8].

#### A. Background Definitions

Consider a dynamic process  $\phi$  describing the temporal evolution of points in an  $E$ -dimensional state space (e.g., the activity level of  $E$  users during time). Its trajectories converge to some  $d$ -dimensional ( $d \leq E$ ) manifold  $M$  such that  $\phi: M \rightarrow M$ . That is, if  $m(t)$  is a point on  $M$  then  $m(t+1) = \phi(m(t))$ . Let  $X$  be an observation function of  $\phi$  such that  $X: M \rightarrow \mathbb{R}$ . Here,  $X$  is a Cartesian coordinate (e.g., a certain user among the  $E$  users) of the actual  $E$ -dimensional state space containing  $M$ . For each  $X$ , there is a corresponding time series of length  $L$ ,  $\{X\} = [X(1), X(2), \dots, X(L)]$ , that tracks the trajectory of points in  $M$  mapped to a sequence of real numbers (e.g., the activity level time series of user  $X$ ).

A lagged-coordinate embedding uses  $E$  time-lagged values of  $\{X\}$  as coordinate axes or dimensions to reconstruct a shadow attractor manifold  $M_X$  as shown in Fig 1. The points in this manifold, denoted by  $x(t)$ , consist of the set of  $E$ -dimensional vectors  $x(t) = \langle X(t), X(t-\tau), X(t-2\tau), \dots, X(t-(E-1)\tau) \rangle$  where the time lag  $\tau$  is positive. According to Takens' theorem [20], points  $x(t)$  on  $M_X$  map 1:1 to points  $m(t)$  on  $M$  so that  $M_X$  is a diffeomorphic reconstruction of the original attractor manifold  $M$ .

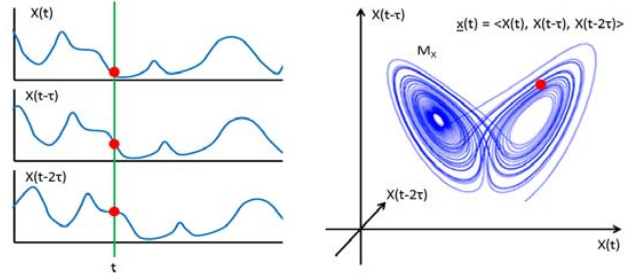


Figure 1. Construction of a shadow manifold  $M_X$ .

In dynamical systems theory, time-series variables (i.e.  $X$  and  $Y$ ) are causally linked if they belong to the same dynamic system [8], [20]. In other words, they share a common attractor manifold  $M$ .

#### B. Basic Idea of Convergent Cross Mapping

A general property of lagged-coordinate embedding is that points  $x(t)$  on  $M_X$  map 1:1 to points  $m(t)$  on  $M$  and local neighborhoods on  $M_X$  map to local neighborhoods on  $M$  [20]. As a result, for two variables  $X$  and  $Y$  that are dynamically coupled, local neighborhoods on their respective lagged reconstructions,  $M_X$  and  $M_Y$ , will map to each other since  $X$  and  $Y$  are essentially alternative observations of the common attractor manifold  $M$ .

Convergent cross mapping determines how well local neighborhoods on  $M_X$  correspond to local neighborhoods on  $M_Y$ . To do so, a manifold  $M_X$  is constructed from lags of variable  $X$  and used to estimate contemporaneous values of  $Y$ . Because  $M_X$  is diffeomorphic to  $M$ , estimates of  $Y$  converge as  $L$  goes to infinity. In practical application,  $M_X$  is an approximation that will display convergence up to the level set by observational error and process noise. CCM is therefore demonstrated by estimation precision (or correlation) that rises with  $L$  and reaches a plateau.

#### C. Algorithm of Convergent Cross Mapping

We will follow the presentation of Sugihara et al. [8] to introduce the algorithm of CCM. Let the two time series  $\{X\} = [X(1), X(2), \dots, X(L)]$  and  $\{Y\} = [Y(1), Y(2), \dots, Y(L)]$  be activity level of user  $X$  and  $Y$  over time. For  $r = S$  to  $r = L$  ( $S < L$ ), we get the partial time series  $[X(1), X(2), \dots, X(L_p)]$  and  $[Y(1), Y(2), \dots, Y(L_p)]$ . Initially, both partial time series are normalized to zero mean and unit variance. The shadow manifold  $M_X$  is reconstructed from  $\{X\}$ , which is the set of lagged-coordinate vectors  $x(t) = \langle X(t), X(t-\tau), X(t-2\tau), \dots, X(t-(E-1)\tau) \rangle$  for  $t = 1+(E-1)\tau$  to  $t = r$ . To generate a cross-

mapped estimate of  $Y(t)$ , denoted by  $\hat{Y}(t)|_{M_X}$ , we begin by locating the contemporaneous lagged-coordinate vector on  $M_X$ ,  $x(t)$ , and find its  $E+1$  nearest neighbors. Note that  $E+1$  is the minimum number of points needed for a bounding simplex in an  $E$ -dimensional space [7]. Next, denote the time indices (from closest to farthest) of the  $E+1$  nearest neighbors of  $x(t)$  by  $t_1, t_2, \dots, t_{E+1}$ . These time indices are used to identify neighbor points in  $Y$  (a putative neighborhood) to estimate  $Y(t)$  from a locally weighted mean of the  $E+1$   $Y(t_i)$  values, as shown in Equation 1.

$$\hat{Y}(t)|_{M_X} = \sum_{i=1}^{E+1} w_i Y(t_i) \quad (1)$$

In Equation 1,  $w_i$  is a weighting based on the distance between  $x(t)$  and its  $i^{th}$  nearest neighbor on  $M_X$  and  $Y(t_i)$  are the contemporaneous values of  $Y$ . The weights are determined as shown in Equation 2.

$$w_i = u_i / \sum_{j=1}^{E+1} u_j \quad (2)$$

In Equation 2,  $u_i = e^{-d[x(t), x(t_i)]/d[x(t), x(t_1)]}$ . And  $d[x(t), x(t_i)]$  is the Euclidean distance between two vectors. This kind of estimation is called simplex projection [7]. The estimation process is shown in Fig. 2.

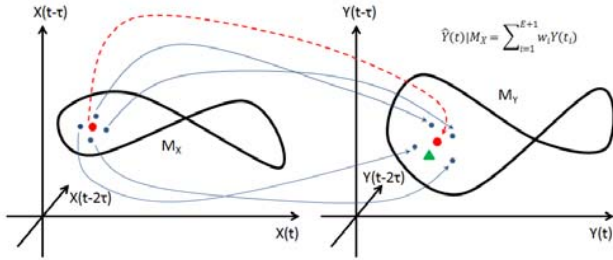


Figure 2. Estimating  $Y(t)$  from the shadow manifold  $M_X$ . The red circle on  $M_Y$  is the estimate result, which the green triangle is the actual value.

If  $X$  is influenced by  $Y$ , the nearest neighbors of  $M_X$  should identify the time indices of corresponding nearest neighbors on  $M_Y$ . As  $r$  increases, the attractor manifold fills in and the distances among the  $E+1$  nearest neighbors shrinks. As a result,  $\hat{Y}(t)|_{M_X}$  should converge to  $Y(t)$ .

For each  $r = S$  to  $r = L$ , we calculate the correlation coefficient  $\rho_r$  between the estimated and actual value of  $Y(t^*)$  for  $t^* = 1+(E-1)\tau$  to  $t^* = r$ . If the series  $\rho_S, \rho_{S+1}, \dots, \rho_L$  has an increasing tendency (i.e., the slope of the linear regression line is positive), the influence from  $Y$  to  $X$  is

$$CCM_{Y \rightarrow X} = \max\{\rho_S, \rho_{S+1}, \dots, \rho_L\} \quad (3)$$

Otherwise,  $CCM_{Y \rightarrow X} = 0$ .

## V. EXPERIMENT

We evaluate the validity of CCM by predicting user behavior based on the inferred causal strength in real-work datasets on three different social media websites, including Twitter, Sina Weibo (a popular microblog website in China)

and Digg. For Twitter, we use the 15M dataset [21], in which Twitter messages were collected in the period April-May 2011, related to the political events (15M movement) occurred at that time in Spain. The action in 15M dataset is defined as a user posts a tweet which contains a hashtag related to 15M movement at a certain timestamp. For Sina Weibo, we use the Diffusion dataset analyzed in [22], in which 300,000 popular microblog diffusion episodes were collected. The action in Diffusion dataset is defined as a user posts or reposts a microblog at a certain timestamp. For Digg, we use the Voting dataset studied in [23], in which the voting records for 3553 stories promoted to the front page over a period of a month in 2009 were collected. The action in Voting dataset is defined as a user digs a story at a certain timestamp. In order to focus on the active users, we further filter our datasets according to the action number and active time threshold conditions. For each dataset, we also have the given social network represented as a directed graph, in which if user  $u$  follows  $v$ , there is a directed edge from  $v$  to  $u$ . The filtering threshold and filtered dataset size are shown in Table I.

TABLE I. Dataset filtering threshold and size.

	Action Number Threshold	Active Time Threshold	User Number	Time Series Length
Twitter	0	80%	115	20
Weibo	150	80%	716	35
Digg	1000	0	162	36

We take a linear threshold classifier for user behavior prediction in social media. Intuitively, the more friends of a user perform a behavior, the larger probability the user will also perform it. Specifically, for a user-behavior pair  $(u, b)$ , we calculate a score  $P(u, b)$  as shown in Equation 4, in which  $I(u)$  is the set of users who have influence on  $u$ ,  $w_{vu}$  is the influence of user  $v$  on  $u$ . If user  $v$  performed behavior  $b$ , then  $\delta(v, b) = 1$ , else  $\delta(v, b) = 0$ . We predict user  $u$  will perform behavior  $b$  if  $P(u, b) > p_0$ . The value of threshold  $p_0 = 0.1$  is chosen to maximize the precision prediction.

$$P(u, b) = \frac{\sum_{v \in I(u)} w_{vu} \cdot \delta(v, b)}{\sum_{v \in I(u)} w_{vu}} \quad (4)$$

In order to calculate the value of parameter  $w_{vu}$ , we have three influence measures in total. The first one is Influence-Oblivious, meaning that if there is an edge from  $v$  to  $u$  in the given social network, then  $w_{vu} = 1$ , otherwise  $w_{vu} = 0$ . The second one is GC influence based on Granger Causality [17]. The third one is CCM influence based on our proposed approach. According to the adopted influence measurements, we have three kinds of classifiers to predict user behavior.

We select all the user-behavior tuples containing the 100 most popular hashtags or microblogs from Twitter 15M dataset and Weibo Diffusion dataset, and all the user-behavior tuples from Digg Vote dataset as testing samples. Each tuple represents that a user adopts a hashtag, or reposts a microblog, or digs a story. Fig. 3 shows the precision of

user behavior prediction in three datasets. The results demonstrate that influence inferred by CCM can significantly improve the user behavior prediction precision on three datasets. In addition, CCM based influence outperforms GC based influence in terms of the precision improvement.

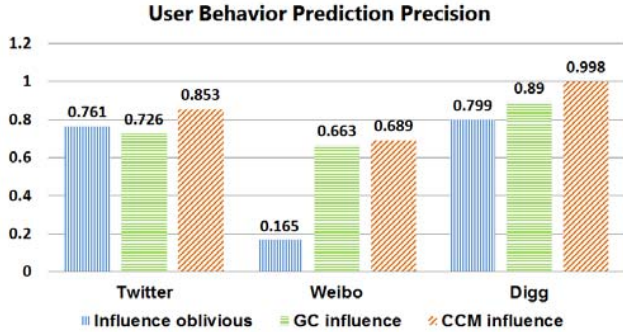


Figure 3. Precision of user behavior prediction in three datasets with three kinds of influence measurement.

## VI. CONCLUSIONS

In this paper, we have presented Convergent Cross Mapping, a novel causal inference approach in online social networks, from a nonlinear dynamic system perspective. The approach allows us to infer the causal relationship for any pair of users based on the user activity level time series data alone. This approach for measuring strength of causal effect does not require any explicit causal knowledge like retweeting or other content information. Furthermore, it is a model free approach which does not require the pre-assumption of any particular user activity model. We evaluated the proposed approach with real-world social media datasets. The experimental results show that this approach can successfully uncover the underlying causal structure among users and the causal effect strength inferred by CCM can benefit the user behavior prediction in social media greatly.

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## REFERENCES

- [1] C. Castellano, S. Fortunato, and V. Loreto, "Statistical physics of social dynamics," *Rev. Mod. Phys.*, vol. 81, no. 2, pp. 591–646, May 2009.
- [2] K. Glass and R. Colbaugh, "Web Analytics for Security Informatics," in *Intelligence and Security Informatics Conference (EISIC), 2011 European*, 2011, pp. 214–219.

- [3] D. M. Romero, W. Galuba, S. Asur, and B. A. Huberman, "Influence and passivity in social media," in *Proceedings of the 20th international conference companion on World wide web*, New York, NY, USA, 2011, pp. 113–114.
- [4] M. Gomez-Rodriguez, J. Leskovec, and A. Krause, "Inferring Networks of Diffusion and Influence," *ACM Trans Knowl Discov Data*, vol. 5, no. 4, pp. 21:1–21:37, Feb. 2012.
- [5] M. G. Rodriguez, J. Leskovec, D. Balduzzi, and B. Schölkopf, "Uncovering the structure and temporal dynamics of information propagation," *Netw. Sci.*, vol. 2, no. 01, pp. 26–65, 2014.
- [6] H. Kwak, C. Lee, H. Park, and S. Moon, "What is Twitter, a Social Network or a News Media?," in *Proceedings of the 19th International Conference on World Wide Web*, New York, NY, USA, 2010, pp. 591–600.
- [7] G. Sugihara and R. M. May, "Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series," *Nature*, vol. 344, no. 6268, pp. 734–741, Apr. 1990.
- [8] G. Sugihara, R. May, H. Ye, C. Hsieh, E. Deyle, M. Fogarty, and S. Munch, "Detecting Causality in Complex Ecosystems," *Science*, vol. 338, no. 6106, pp. 496–500, Oct. 2012.
- [9] C. Wang and B. A. Huberman, "How Random are Online Social Interactions?," *Sci. Rep.*, vol. 2, Sep. 2012.
- [10] M. Cha, H. Haddadi, F. Benevenuto, and P. K. Gummadi, "Measuring User Influence in Twitter: The Million Follower Fallacy," in *International Conference on Weblogs and Social Media*, 2010.
- [11] G. Jeh and J. Widom, "Scaling Personalized Web Search," in *Proceedings of the 12th International Conference on World Wide Web*, New York, NY, USA, 2003, pp. 271–279.
- [12] R. Ghosh and K. Lerman, "Parameterized centrality metric for network analysis," *Phys. Rev. E*, vol. 83, no. 6, p. 066118, Jun. 2011.
- [13] S. Wu, J. M. Hofman, W. A. Mason, and D. J. Watts, "Who says what to whom on twitter," in *Proceedings of the 20th international conference on World wide web*, New York, NY, USA, 2011, pp. 705–714.
- [14] D. Kempe, J. Kleinberg, and É. Tardos, "Maximizing the spread of influence through a social network," in *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining*, New York, NY, USA, 2003, pp. 137–146.
- [15] X. Zheng, Y. Zhong, D. Zeng, and F.-Y. Wang, "Social influence and spread dynamics in social networks," *Front. Comput. Sci.*, vol. 6, no. 5, pp. 611–620, Oct. 2012.
- [16] K. Hlaváčková-Schindler, M. Paluš, M. Vejmelka, and J. Bhattacharya, "Causality detection based on information-theoretic approaches in time series analysis," *Phys. Rep.*, vol. 441, no. 1, pp. 1–46, Mar. 2007.
- [17] C. W. J. Granger, "Investigating Causal Relations by Econometric Models and Cross-spectral Methods," *Econometrica*, vol. 37, no. 3, p. 424, Aug. 1969.
- [18] G. Ver Steeg and A. Galstyan, "Information transfer in social media," in *Proceedings of the 21st international conference on World Wide Web*, New York, NY, USA, 2012, pp. 509–518.
- [19] S. He, X. Zheng, D. Zeng, K. Cui, Z. Zhang, and C. Luo, "Identifying Peer Influence in Online Social Networks Using Transfer Entropy," in *Intelligence and Security Informatics*, G. A. Wang, X. Zheng, M. Chau, and H. Chen, Eds. Springer Berlin Heidelberg, 2013, pp. 47–61.
- [20] F. Takens, "Detecting strange attractors in turbulence," in *Dynamical Systems and Turbulence, Warwick 1980*, D. Rand and L.-S. Young, Eds. Springer Berlin Heidelberg, 1981, pp. 366–381.
- [21] S. González-Bailón, J. Borge-Holthoefer, A. Rivero, and Y. Moreno, "The Dynamics of Protest Recruitment through an Online Network," *Sci. Rep.*, vol. 1, Dec. 2011.
- [22] Jie Tang, "Social Influence Locality for Modeling Retweeting Behaviors," presented at the IJCAI'13, 2013.
- [23] K. Lerman, R. Ghosh, and T. Surachawala, "Social Contagion: An Empirical Study of Information Spread on Digg and Twitter Follower Graphs," in *Proceedings of 4th International Conference on Weblogs and Social Media*, 2010.