Letter

Moving Target Landing of a Quadrotor Using Robust Optimal Guaranteed Cost Control

Kewei Xia, Seong-Min Lee, *Member, IEEE*, Wonmo Chung, Yao Zou, *Member, IEEE*, and Hungsun Son, *Member, IEEE*

Dear Editor,

This letter proposes a robust control strategy for the autonomous landing of a quadrotor on a moving target. Specifically, a force command that consists of a cascade dynamics estimator and an optimal guaranteed cost control law is exploited for the position-loop tracking. Then, an orientation constraint torque command is employed for the attitude-loop tracking such that the quadrotor refrains from flipping during the landing operation. Stability analysis indicates that the overall closed-loop system is asymptotically stable. Finally, flight experiments validate and access the theoretical results.

Recent years have witnessed the rapid development of unmanned aerial vehicles (UAVs), especially in military and civil fields, such as forest reconnaissance, urban surveillance, and transportation [1]–[3]. To improve the tracking performance of the UAV, numerous control algorithms have been reported. A multiple-observer based controller was exploited to overcome the wind disturbance and payload [4]. A robust control algorithm was investigated to counteract the rotor fault and disturbance [5]. In particular missions like refueling and maritime rescue, the autonomous landing of the UAV is significant and inevitable [6]. However, the unavailable target's acceleration could prevent the direct application of these algorithms [4] and [5].

To land a quadrotor on a moving platform, an adaptive control strategy was designed [7]. A formation controller was proposed for the UAV tracking and landing on a ground vehicle [8]. As for the shipboard landing of the UAV, a robust adaptive control approach with mission planning was provided [9]. The control barrier function was introduced into an adaptive neural network control algorithm to solve the unmanned helicopter ship landing with visibility constraint [10]. It is worth noting that the orientation of the UAV should be maintained within a safe range during the landing operation that refrains from flipping over. Unfortunately, all the previous mentioned control strategies are developed without this consideration [4], [5] and [7]–[10]. Besides, the control strategies in [7], [9] and [10] have not been validated by flight experiments, a complicated implementation with multiple parameter tuning could be faced due to the developed control forms [7], [9] and [10]. Therefore, how to design a robust control strategy with simple implementation and parameter tuning is still significant yet challenging, which is the motivation of this study.

Motivated by the above observation, this letter focuses on the control development with the experiment validation for a quadrotor landing on a moving target. The main contributions are summarized as follows: 1) In contrast to [7], [9] and [10], the bounded force command consisting of a nominal PD control and a robust dynamics estimator not only provides a simple implementation and parameter tuning but also ensures the system robustness. 2) Compared with [4], [5] and [7]–[10], the torque command is designed with orientation constraint which is formulated based on the physical meaning. This could prevent the quadrotor from flipping over during the landing operation which further enhances the reliability of the control system.

Problem formulation: In terms of the Newton-Euler formulation, the dynamics of the quadrotor is described as

$$\begin{cases} \dot{p} = v \\ \dot{v} = -g\hat{e}_3 + \dfrac{T}{m}R\hat{e}_3 + a_d \end{cases} \tag{1}$$

$$\begin{cases} \dot{Q} = \dfrac{1}{2}Q \odot \omega^+ \\ J\dot{\omega} = -\omega^\times J\omega + \tau + \tau_d \end{cases} \tag{2}$$

where $p \in \mathbb{R}^3$ and $v \in \mathbb{R}^3$ represent the position and velocity of the quadrotor in the inertial frame $\mathcal{F}_i$, g represents the local gravitational acceleration, m represents the mass, $T \in \mathbb{R}$ represents the applied thrust in the body frame $\mathcal{F}_b$ of the quadrotor along $\hat{e}_3 \triangleq [0,0,1]^T$, $Q \in \mathbb{Q} \triangleq \{[\sigma, q^T]^T \in \mathbb{R}\times\mathbb{R}^3 | \sigma^2 + q^T q = 1\}$ and $\omega \in \mathbb{R}^3$ represent the unit quaternion and angular velocity, $\odot$ represents the quaternion product operator, $\omega^+ = [0, \omega^T]^T$, $R \in \mathrm{SO}(3)$ represents the rotation matrix from frame $\mathcal{F}_b$ to frame $\mathcal{F}_i$, $J \in \mathbb{R}^{3\times 3}$ represents the inertial matrix, $\tau \in \mathbb{R}^3$ represents the applied torque, a_d and τ_d represent the uncertain acceleration and torque.

Suppose that the target's position p_0, velocity $\dot{p}_0$ and acceleration $\ddot{p}_0$ are all bounded. Define $\tilde{p} = p - p_0$ and $\tilde{v} = v - \dot{p}_0$ as the relative position and relative velocity. In view of (1), the relative position dynamics can be given by $\dot{\tilde{p}} = \tilde{v}$ and $\dot{\tilde{v}} = -g\hat{e}_3 + u + d_a + T(R - R_c)\hat{e}_3/m$, where $u = TR_c\hat{e}_3/m \triangleq [u_x, u_y, u_z]^T$, $d_a \triangleq [d_{a1}, d_{a2}, d_{a3}]^T = a_d - \ddot{p}_0$, and R_c is the command rotation matrix. It follows that $T = m\|u\|$. According to [5], the perturbation term $T(R - R_c)\hat{e}_3/m \to 0$ if $T(t) \in \mathcal{L}_\infty$, $\forall t \geq 0$ and $R \to R_c$, as $t \to \infty$. According the hierarchical framework, the attitude command $Q_c = [\sigma_c, q_c^T]^T \in \mathbb{Q}$ is extracted as $\sigma_c = \sqrt{(1 + u_z/\|u\|)/2}$ and $q_c = [-u_y, u_x, 0]^T/(2\|u\|\sigma_c)$. The angular velocity command is given by $\omega_c = 4\mathcal{P}_c^T \dot{Q}_c$, where $\mathcal{P}_c = \mathcal{P}(Q_c) = [-q_c^T; \sigma_c I_3 + q_c^\times]/2$.

Control objective: In this letter, the control objective is to design a thrust command T and a torque command τ such that the quadrotor can track the moving target while finally landing on the desired zone of it. More specifically, given a reference descending velocity $v_d = [0, 0, v_{zd}]^T$, if the control commands T and τ are developed such that $\lim_{t\to\infty} \tilde{p}(t) = 0$, $\lim_{t\to\infty} \tilde{v}(t) = v_d$, $\lim_{t\to\infty} Q(t) = Q_c(t)$, and $\lim_{t\to\infty} \omega(t) = \omega_c(t)$, then the landing objective is achieved.

Main results: In this section, the main design procedures of the force and torque commands are provided.

Force command development: Define $\bar{p} = \tilde{p} - p_d$ and $\bar{v} = \tilde{v} - v_d$ as the tracking errors, where $p_d = [0, 0, \int_0^t v_{zd}(s)ds]^T$. The nominal error position dynamics can be given by $\dot{\bar{p}} = \bar{v}$ and $\dot{\bar{v}} = u_0 + d_a$, where $u_0 = -g\hat{e}_3 - \dot{v}_d + u$. Define $x = [\bar{p}^T, \bar{v}^T]^T$. Its time derivative satisfies $\dot{x} = Ax + B(u_0 + d_a)$, where $A = \begin{bmatrix} 0_3 & I_3 \\ 0_3 & 0_3 \end{bmatrix}$ and $B = \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}$. The force controller is taken in the form $u_0 = u_n + u_s$; where u_n is the nominal part that provides a stabilization controller of the sliding mode dynamics, and u_s is the robust part that compensates for the effect of uncertainty. Introduce an integral sliding surface $\mathfrak{s} = \Pi(x(t) - x(t_0) - \int_{t_0}^t (Ax + Bu_n)ds)$, where $\Pi \in \mathbb{R}^{3\times 6}$ is a constant matrix ensuring the positive definiteness of ΠB. Design the sliding mode control law $u_s = \begin{cases} -\hat{d}_a - \frac{\bar{d}_a B^T \Pi^T \mathfrak{s}}{\|B^T \Pi^T \mathfrak{s}\|}, & \mathfrak{s} \neq 0, \\ -\hat{d}_a, & \mathfrak{s} = 0, \end{cases}$ where $\hat{d}_a = \mathfrak{F}_{\bar{d},\epsilon}(-(\hat{v}_1^v - \hat{v}_2^v)/(\eta_1 - \eta_2) + v_1^w)$ is the estimate of d_a generated by the cascade dynamics estima-

Corresponding author: Hungsun Son.

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K. Xia is with the Advanced Research Institute of Multidisciplinary Sciences, Beijing Institute of Technology, Beijing 100081, China (e-mail: kwxia@bit.edu.cn).

S.-M. Lee, W. Chung, and H. Son are with the Department of Mechanical Engineering, Ulsan National Institute of Science and Technology, Ulsan 44919, South Korea (e-mail: leesm128@unist.ac.kr; momone@unist.ac.kr; hson@unist.ac.kr).

Y. Zou is with the Institute of Artificial Intelligence, University of Science and Technology Beijing, Beijing 100083, China (e-mail: zouyao@ustb.edu.cn).

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tor [11], $\mathfrak{F}(\cdot)$ is the smooth saturation function [12], η_1 and η_2 are positive constants satisfying $\eta_1 > \eta_2$, $\hat{\upsilon}_1^v$ and $\hat{\upsilon}_2^v$ are the outputs of filters $\eta_1 \dot{\hat{\upsilon}}_1^v + \hat{\upsilon}_1^v = \hat{v}$ and $\eta_2 \dot{\hat{\upsilon}}_2^v + \hat{\upsilon}_2^v = \hat{v}$, $\hat{v} = \upsilon_1^v - \eta_1(\upsilon_1^v - \upsilon_2^v)/(\eta_1 - \eta_2)$, υ_1^v and $\upsilon_2^v(0)$ are the outputs of filters $\eta_1 \dot{\upsilon}_1^v + \upsilon_1^v = \bar{v}$ and $\eta_2 \dot{\upsilon}_2^v + \upsilon_2^v = \bar{v}$, $\upsilon_1^w(0)$ is the output of filter $\eta_1 \dot{\upsilon}_1^w + \upsilon_1^w = -u_0$, $\bar{d}_a$ is a positive constant satisfying $\bar{d}_a > \|\tilde{d}_a\|$, and $\tilde{d}_a = \hat{d}_a - d_a$. Then, we have the following theorem.

Theorem 1: The sliding surface $\mathfrak{s}(t) = 0$, $\forall t \geq 0$ is achieved by the proposed robust control law u_s.

Proof: Choose a Lyapunov function $\mathrm{L}_s = \mathfrak{s}^T \mathfrak{s}/2$. It is trivial to show that $\dot{\mathrm{L}}_s \leq 0$ for any $\mathfrak{s}$. Note that $\mathfrak{s}(x(t_0), t_0) = 0$ in terms of $\mathfrak{s}$. This implies that the sliding mode is achieved for $t \geq t_0$. ■

Next, let $\dot{\mathfrak{s}} = 0$. By finding the equivalent control $u_s^e = -d_a$ on the sliding surface, the equivalent sliding dynamics is given by $\dot{x} = Ax + Bu_n$. Define a cost function

$$\mathrm{L}_c = \int_0^\infty (x(t)^T \mathcal{R}_x x(t) + u_n(t)^T \mathcal{R}_u u_n(t)) dt \tag{3}$$

where $\mathcal{R}_x \in \mathbb{R}^{6\times 6}$ and $\mathcal{R}_u \in \mathbb{R}^{3\times 3}$ are positive definite diagonal matrices. The nominal control law is designed in following form:

$$u_n = Kx \tag{4}$$

where $K \in \mathbb{R}^{3\times 6}$ is a constant matrix. Furthermore, to ensure the nonsingularity property, the following condition should be guaranteed $-\bar{u}_n < u_{ni} < \bar{u}_n$, $i = 1,2,3$, where $\bar{u}_n$ is a positive constant. Let $Z = \mathrm{diag}(z_1, z_2, z_3)$ be a constant matrix satisfying $0 < Z < \bar{U}_n$ with $\bar{U}_n = \bar{u}_n^2 I_3$. Then, we have the following theorem.

Theorem 2: If there exist a constant $\gamma > 0$, a symmetric matrix $P > 0$ and matrices K, Z such that $\Upsilon + \mathcal{R}_x + K^T \mathcal{R}_u K < 0$, $x^T(0)Px(0) \leq \gamma$, $\gamma K P^{-1} K^T \leq Z$ and $Z < \bar{U}_n$ hold simultaneously, where $\Upsilon = PA + A^T P + K^T B^T P + PBK$, then the proposed control law (4) with magnitude constraint ensures that 1) The closed-loop error position dynamics is asymptotically stable; and 2) The cost function (3) satisfies $\mathrm{L}_c < \gamma$ with a guaranteed cost value γ.

Proof: Choose a Lyapunov function candidate $\mathrm{L}_x = x^T Px$. The derivative of L_x along the closed-loop trajectory can be derived as $\dot{\mathrm{L}}_x < -x^T(\mathcal{R}_x + K^T \mathcal{R}_u K)x$. This implies that $\dot{\mathrm{L}}_x < -\underline{\lambda}(\mathcal{R}_x + K^T \mathcal{R}_u K) \times \mathrm{L}_x / \bar{\lambda}(P)$. It can be concluded from the Lyapunov stability theory that the closed-loop dynamics is asymptotically stable, i.e., $\lim_{t\to\infty} x(t) = 0$. In addition, note that $\mathrm{L}_x(t) < \mathrm{L}_x(0) = x^T(0)Px(0) \leq \gamma$. It further follows from (4) that $\|u_{ni}\|^2 = \|K_i x\|^2 = \|K_i P^{-\frac{1}{2}} P^{\frac{1}{2}} x\|^2 \leq K_i P^{-1} \times K_i^T x^T Px \leq \gamma K_i P^{-1} K_i^T \leq z_i < \bar{u}_n^2$, where K_i is the i-th row of K. Therefore, the control constraint is ensured. Next, in view of (4), we have $\mathrm{L}_x(\infty) - \mathrm{L}_x(0) < -\int_0^\infty (x(t)^T \mathcal{R}_x x(t) + u_n(t)^T \mathcal{R}_u u_n(t)) dt$. Based on (3), it then follows that $\mathrm{L}_c = \int_0^\infty (x(t)^T \mathcal{R}_x x(t) + u_n(t)^T \mathcal{R}_u u_n(t)) dt < \mathrm{L}_x(0) \leq \gamma$. Therefore, the guaranteed cost is achieved, i.e., $\mathrm{L}_c < \gamma$. ■

To facilitate the feasibility of the conditions in Theorem 2, we next provide the transformed conditions which are formulated by linear matrix inequalities (LMIs) directly [13].

Proposition 1: If there exist a constant $\gamma > 0$, matrices $0 < Y \in \mathbb{R}^{6\times 6}$, $N \in \mathbb{R}^{3\times 6}$, Z such that the following conditions hold:

$$\begin{bmatrix} \Upsilon^* & Y & N^T \\ Y & -\gamma \mathcal{R}_x^{-1} & 0_{6\times 3} \\ N & 0_{3\times 6} & -\gamma \mathcal{R}_u^{-1} \end{bmatrix} < 0 \tag{5}$$

$$\begin{bmatrix} 1 & x^T(0) \\ x(0) & Y \end{bmatrix} \geq 0 \tag{6}$$

$$\begin{bmatrix} Z & N \\ N^T & Y \end{bmatrix} \geq 0 \tag{7}$$

$$Z - \bar{U}_n < 0 \tag{8}$$

where $\Upsilon^* = AY + Y^T A^T + BN + N^T B^T$, then, by choosing $K = NY^{-1}$, the optimal guaranteed cost control is achieved in the sense of Theorem 2.

Remark 1: In fact, the optimal guaranteed cost issue can be transformed as the following optimization problem:

$$\min_{\substack{\gamma > 0, Y > 0, N, Z > 0 \\ \text{s.t. } (5)-(8)}} \gamma. \tag{9}$$

This implies that a set of solutions (γ, Y, N, Z) guarantees a local optimal guaranteed cost control $u_n = NY^{-1}x$ which ensures the minimization of the upper bound of the performance function. A sufficient condition can be provided by solving the LMIs (5)−(8).

Torque command development: To refrain the quadrotor from flipping over during the landing operation, an orientation constraint torque command is proposed. First, define a unit boresight vector as z_B which points towards the opposite direction of axis b_z in frame $\mathcal{F}_b$. Given a normalized vector x_I in frame $\mathcal{F}_i$, a half-cone angle should be guaranteed strictly smaller than $\theta \in (0, \pi/2)$ [14]. Let z_I be the vector z_B expressed in frame $\mathcal{F}_i$. Then, it can be given by $z_I = R^T z_B = (\sigma^2 - q^T q) z_B + 2q^T z_B q + 2\sigma q^\times z_B$. The attitude within the mandatory zone is equivalent to following condition $x_I \times z_I > \cos(\theta) > 0$. By substituting z_I, it follows that $x_I^T(\sigma^2 - q^T q) z_B + 2q^T \times z_B x_I^T q + 2\sigma q^T (z_B^\times x_I) > \cos(\theta)$. It can be further rewritten into the following compact form $\mathbb{Q}_m = \left\{ Q \in \mathbb{Q} | Q^T MQ > \cos(\theta) \right\}$ where $M = \begin{bmatrix} x_I^T z_B & (x_I^\times z_B)^T \\ x_I^\times z_B & x_I z_B^T + z_B x_I^T - (x_I^T z_B) I_3 \end{bmatrix}$. Introduce a novel transformation function.

$$\xi = \xi(Q) = \frac{\mathrm{k}Q}{\cos(\theta) - Q^T MQ} \tag{10}$$

where k is a positive constant. It is trivial to show that $Q(t) \in \mathbb{Q}_m$, $\forall t \geq 0$ can be guaranteed by $\xi(t) \in \mathcal{L}_\infty$, $\forall t \geq 0$ given $Q(0) \in \mathbb{Q}_m$. It can be proven that its gradient ∇_ξ is a positive definite matrix on $Q \in \mathbb{Q}_m$. This implies that $\xi(Q)$ is a one-one mapping on $Q \in \mathbb{Q}_m$. Define $\xi_c = \xi(Q_c) = \mathrm{k}Q_c / (\cos(\theta) - Q_c^T MQ_c)$. According to the one-one property of $\xi(\cdot)$, $\xi \to \xi_c$ indicates $Q \to Q_c$ without exceeding the mandatory orientation zone. Define a new manifold $\zeta = \omega + \Xi^{-1}(\kappa_1 \tilde{\xi}^\wedge - \dot{\xi}_c^\wedge)$, where $\kappa_1 > 0$, $\Xi \in \mathbb{R}^{3\times 3}$ is composed of the $2-4$ rows of $\nabla_\xi \mathcal{P}(Q)$, the superscript $\wedge$ is the inverse operation of the superscript $+$, $\tilde{\xi} = \xi - \xi_c$, and $\dot{\xi}_c = \nabla_{\xi c} \mathcal{P}_c \omega_c$. Then, we have the error attitude dynamics $\dot{\tilde{\xi}}^\wedge = \Xi\zeta - \kappa_1 \tilde{\xi}^\wedge$ and $J\dot{\zeta} = \chi + \tau + d_\tau$, where $\chi = \kappa_1 J \Xi^{-1} \dot{\tilde{\xi}}^\wedge - J\Xi^{-1} \dot{\Xi} \Xi^{-1} (\kappa_1 \tilde{\xi}^\wedge - \dot{\xi}_c^\wedge)$ and $d_\tau = -\omega^\times J\omega + \tau_d - J\Xi^{-1} \ddot{\xi}_c^\wedge$.

Similarly, the orientation constraint toque command $\tau = \tau_n + \tau_s$ is developed under the integral sliding mode framework. Define a sliding surface $\mathfrak{z} = J\zeta(t) - J\zeta(t_0) - \int_{t_0}^t (\chi + \tau_n) ds$. Then, design the sliding mode control law as $\tau_s = \begin{cases} -\hat{d}_\tau - \frac{\bar{d}_\tau \mathfrak{z}}{\|\mathfrak{z}\|}, & \mathfrak{z} \neq 0, \\ -\hat{d}_\tau, & \mathfrak{z} = 0, \end{cases}$ where $\hat{d}_\tau$ is the estimate of d_τ generated by the cascade dynamics estimator, $\tilde{d}_\tau = \hat{d}_\tau - d_\tau$, and $\bar{d}_\tau$ is a positive constant satisfying $\bar{d}_\tau > \|\tilde{d}_\tau\|$. Then, we have the following theorem.

Theorem 3: The sliding surface $\mathfrak{z}(t) = 0$, $\forall t \geq 0$ is achieved by the proposed control law τ_s.

Proof: The proof is similar to the procedure of Theorem 1. ■

Next, the equivalent attitude sliding dynamics $J\dot{\zeta} = \chi + \tau_n$ is found when $\dot{\mathfrak{z}} = 0$. Design the following nominal torque command:

$$\tau_n = -\Xi^T \tilde{\xi}^\wedge - \kappa_2 \zeta - \chi \tag{11}$$

where κ_2 is a positive constant. Then, we have the following theorem.

Theorem 4: The control law (11) ensures that the closed-loop equivalent attitude sliding surface dynamics is asymptotically stable, i.e., $\forall Q(0) \in \mathbb{Q}_m$, $Q(t) \to Q_c(t)$ and $\omega(t) \to \omega_c(t)$, as $t \to \infty$.

Proof: Choose a Lyapunov function candidate $\mathrm{L}_a = \tilde{\xi}^{\wedge T} \tilde{\xi}^\wedge / 2 + \zeta^T J\zeta/2$. It's time derivative along the closed-loop trajectory satisfies $\dot{\mathrm{L}}_a \leq -2\min(\kappa_1, \kappa_2) \mathrm{L}_a / \max(1, \bar{\lambda}(J)) < 0$, $\forall \mathrm{L}_a \neq 0$. This indicates that the closed-loop nominal attitude dynamics is asymptotically stable, i.e., $\lim_{t\to\infty} \tilde{\xi}^\wedge(t) = 0$ and $\lim_{t\to\infty} \zeta(t) = 0$. This implies that $\xi^\wedge(t) \to \xi_c^\wedge(t) \Rightarrow Q(t) \to Q_c(t)$, as $t \to \infty$. Recalling the definition of ζ gives that $\omega(t) \to \Xi^{-1} \dot{\xi}_c^\wedge \to \Xi_c^{-1} \dot{\xi}_c^\wedge$, where Ξ_c is the 2-4 rows of $\nabla_{\xi c} \mathcal{P}_c$. It follows from $\dot{\xi}_c^\wedge = \Xi_c \omega_c$ that $\omega(t) \to \omega_c(t)$, as $t \to \infty$. ■

Experiment results: The flight experiments by implementing the proposed strategy and PID control are conducted on the quadrotor and ground vehicle system. The experiment setup consists of an F330 DJI quadrotor, a manned electrical vehicle platform, the motion capture system, and the control center.

The inertia parameters of the quadrotor are $m = 1.041$ kg and $J = \mathrm{diag}(0.0073, 0.0073, 0.0147)$ kgm^2. To avoid the possible control chattering, the sliding mode controllers are modified as $u_s = -\hat{d}_a - \bar{d}_a B^T \Pi^T \mathfrak{s}/(\|B^T \Pi^T \mathfrak{s}\| + 0.05)$ and $\tau_s = -\hat{d}_\tau - \bar{d}_\tau \mathfrak{z}/(\|\mathfrak{z}\| + 0.05)$. The control parameters are chosen as follows: $\eta_1 = 20$, $\eta_2 = 10$, $\bar{d} = 3$, $\epsilon = 0.1$, $\bar{d}_a = 0.001$, $\mathrm{k} = 1$, $\theta_\pi/2$, $\beta_1 = 0.4$, $\beta_2 = 0.2$, $\kappa_1 = 3$, $\kappa_2 = 5$ and $\bar{d}_\tau = 1$. In terms of the LMIs, by choosing $\mathcal{R}_x = \mathrm{diag}(1, 1, 1, 0.2, 0.2, 0.2)$, $\mathcal{R}_u = I_3$ and $\bar{u}_n = 4$, we have $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$, where $K_1 = \begin{bmatrix} -0.1978 & 0 & -0.0005 \\ 0 & -0.1974 & 0 \\ 0.0001 & 0 & -0.3831 \end{bmatrix}$ and $K_2 = \begin{bmatrix} -0.0696 & 0 & 0 \\ 0 & -0.0695 & 0 \\ 0 & 0 & -0.1390 \end{bmatrix}$. To further highlight the robustness of the proposed strategy, an additional weight (0.158 kg) is fixed on the quadrotor. The 4 m/s persistent wind with respect to the arena center is generated by a large electrical fan. Experiment results are presented in Figs. 1 and 2.

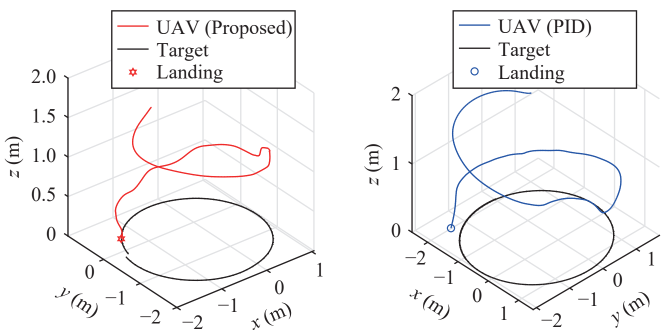

Fig. 1. Trajectories of the quadrotor and the target.

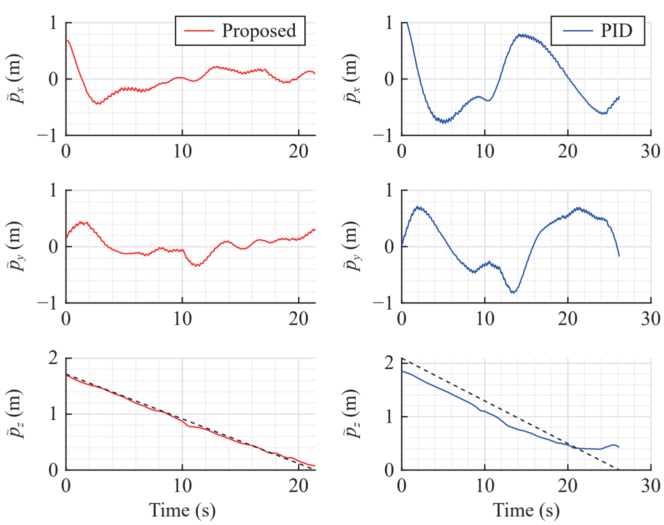

Fig. 2. Relative position.

Fig. 1 draws the trajectories of the quadrotor and the target. Fig. 2 collects the relative positions. It can be seen that the quadrotor is tracking the target while gradually descending to it during the experiment. When the landing condition $\tilde{p}_z \le 0.05$ m is satisfied, the quadrotor is terminated and finally lands on the desired zone of the target. It can observed that the proposed strategy guarantees a smaller transient and higher control accuracy than the PID control. Therefore, the effectiveness of the proposed control strategy has been verified by real-time experiments.

Conclusion: In this letter, the control development for the autonomous landing of a quadrotor on a moving target has been investigated. Based on the integral sliding mode framework, a robust hierarchical strategy consisting of a bounded PD force command and an orientation constraint torque command is proposed, where the cascade dynamics estimator is exploited to compensate for the system uncertainty. Flight experiments with comparison results demonstrate the advantages and highlights of the proposed strategy.

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