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Neural event-triggered optimal filtering co-design of Markovian jump systems with hidden mode detections

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Abstract

The modified H_∞ filtering problem for a class of Markovian jump systems with unknown nonlinear dynamics is investigated in the work by developing the neural event-triggered filter co-design method. Moreover, the true system modes are assumed to be inaccessible such that the estimated jumping modes are utilized for the mode-dependent filters. In particular, a novel event-triggered mechanism is introduced to improve filtering communication efficiency, where the unknown nonlinearity approximation is conducted by a neural network. By virtue of employing Lyapunov–Krasovskii method, sufficient filtering conditions are constructed to ensure the optimal H_∞ performance under the mean-square framework, based on which desired mode-dependent filter gains, event-triggering, and neural network parameters are co-designed with an aid of matrix techniques. Illustrative simulations with two practical examples are finally carried out to validate the usefulness and advantages of our developed approach.

Keywords

Markovian jump system, neural event-triggered scheme, optimal filtering, unknown nonlinearity, hidden mode detections

Introduction

During the last decades, Markovian jump systems have attracted increasing research attention owing to their theoretical significance and practical value (Shi and Li, 2015; Teel et al., 2014; Wang et al., 2020b; Zhang et al., 2016). Indeed, Markovian jump systems have the notable modeling ability in describing complex dynamical systems with subsystems under perturbed jumping parameters or changing environments, which have a wide background in numerous real-world applications. For instance, typical Markovian jump systems can be applied to model the robotic systems (Jiang et al., 2020), power systems (Ugrinovskii* and Pota, 2005), biological systems (Shen et al., 2019), and other engineering areas (Xu et al., 2022; Zhu et al., 2022). Especially, mode-dependent control schemes have been verified to be effective with the help of system jumping modes. Nevertheless, it is worth mentioning that it is always difficult or impossible to acquire the true system mode information in time, which would lead to the mode mismatch between original systems and devised controllers or filters. Under this context, it is necessary to further investigate the mode-dependent analysis and synthesis issues with mode mismatch phenomena (Wang et al., 2021; Wu et al., 2017; Zhang et al., 2019a). As one of the nonsynchronous mode-dependent control methods, much effort has been paid to the so-called asynchronous control for Markovian or semi-Markovian systems. More precisely, the estimated system modes are utilized with the form of conditional probability in the controllers instead of true system modes, such that more applicability can be achieved accordingly. To name a few, a novel asynchronous passive control method is proposed for Markovian jump systems based on a hidden Markovian model, where mode information is assumed to be unaccessible (Wu et al., 2016). Moreover, the asynchronous finite-time filtering problem of Markovian jump systems is addressed by estimated system modes (Ren et al., 2019). Furthermore, the asynchronous state estimator is developed for Markovian jump neural networks with randomly occurring sensor saturations (Men et al., 2018). These remarkable methods can well deal with the nonsynchronous features and improve the control performance.

Another hot research topic lies in the fact of network-induced constraints with limited communication resources for networked control systems. Recently, to cope with the general time-triggered schemes, many remarkable methods have been developed for novel event-triggered strategies (Ge et al., 2021; Peng and Li, 2018; Zou et al., 2020). By setting certain event functions and thresholds for the signal transmissions, more communication efficiency can be increased to some extent. As a result, various event-triggered control strategies have been reported and many distinguishing achievements can be found

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in the literature. In particular, the optimal control problem can be well solved by the event-triggered mechanism with more efficiency (Wang et al., 2022; Zhao et al., 2022). For the Markovian jump systems with multiple system modes, some researchers have focused on mode-dependent event-triggered mechanisms for less conservatism results (Dai et al., 2019; Dong and Gong, 2020). However, it should be pointed out that it is always difficult to determine the mode-dependent event-triggering strategies under nonsynchronous mode jumping (Cao et al., 2020; Ji et al., 2020; Liu, 2016). Therefore, it is important to develop asynchronous approaches during the control procedures. It can be found that considering the asynchronous features for Markovian jump systems can further increase the design applicability with more practical implementations. On the other hand, recently, event-triggered schemes with neural networks have been reported with promising control performance. Unfortunately, to the authors' best knowledge, few efforts have been devoted to the mode-dependent event-triggered filtering topics of Markovian jump systems with unknown nonlinear dynamics, especially for the cases with asynchronous features or neural network learning methods, which motivates us for this research.

Inspired by the above results, this work aims at solving the optimal H_∞ filtering for Markovian jump systems by proposing a novel mode-dependent neural event-triggered strategy along with asynchronous controllers. In comparison with most reported literature, our main contributions could be listed by the following parts:

(1) A novel event-triggered scheme is established for the Markovian jump systems with unknown nonlinear dynamics, where the neural network learning strategy is taken into account for practical implementations. Generally speaking, our proposed filter design method can provide a new way to combine the neural network learning and the event-triggered strategy, such that the unknown system dynamics can be released.
(2) The asynchronous co-design method for mode-dependent filters and event-triggering parameters is proposed to ensure the modified H_∞ performance. For comparative research, in the context of asynchronous mode-dependent design for the Markovian jump system, our developed modified H_∞ performance can deal with the external disturbance and neural network approximation error in a unified framework.
(3) To show the merits of our developed filtering design, illustrative examples with practical system models are also provided with validation simulation results. It is noteworthy that the established sufficient conditions can be solved by convex optimization with the formulated online neural network learning law, which can relax the requirement of precise nonlinear dynamics for the filter design procedure.

The rest parts of our work are organized as follows. In section "Problem statement and preliminaries," the nonlinear Markovian jump system model is introduced and the formulated filtering problem is presented. In section "Main results," by applying the Lyapunov–Krasovskii function, analysis and synthesis of theoretical derivations are given with mathematical proofs. In section "Simulation examples," the simulations are performed with three examples. In section "Conclusion," the overall paper is concluded at the end.

Problem statement and preliminaries

Consider the following Markovian jump system with unknown nonlinear dynamics

$$\begin{cases} \dot{x}(t) = A(\sigma(t))x(t) + f(x(t)) + B(\sigma(t))w(t) \\ y(t) = C(\sigma(t))x(t) + D(\sigma(t))w(t) \\ z(t) = L(\sigma(t))x(t) \\ x(0) = x_0, \sigma(0) = \sigma_0 \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the system state, $f(x(t))$ represents the unknown nonlinear function, $w(t) \in \mathbb{R}^m$ stands for the external disturbance belonging to $L_2[0, \infty)$, $y(t) \in \mathbb{R}^l$ corresponds to the output measurement, $z(t) \in \mathbb{R}^s$ is the signal to be estimated, and all $A(\sigma(t))$, $B(\sigma(t))$, $C(\sigma(t))$, $D(\sigma(t))$, and $L(\sigma(t))$ are constant matrices for a fixed $\sigma(t)$. In addition, $\sigma(t)$ is a continuous-time discrete-state Markov process on $(\mathbb{O}, \mathbb{F}, \mathbb{P})$, which takes values in $\mathcal{S} = \{1, \ldots, N\}$ and its transition probability matrix is denoted by $\Pi = (\pi_{ij})$, $\forall i, j \in \mathcal{S}$ with

$$\Pr(\sigma(t+h) = j | \sigma(t) = i) = \begin{cases} \pi_{ij} h + o(h), & i \neq j, \\ 1 + \pi_{ii} h + o(h), & i = j, \end{cases}$$

and

$$\pi_{ii} = -\sum_{j=1, i \neq j}^{N} \pi_{ij}, \forall i, j \in \mathcal{S}$$

For simplicity, denote $\sigma(t) = i$ as i index, and it follows that

$$\begin{cases} \dot{x}(t) = A(i)x(t) + f(x(t)) + B(i)w(t) \\ y(t) = C(i)x(t) + D(i)w(t) \\ z(t) = L(i)x(t) \\ x(0) = x_0, \sigma(0) = \sigma_0 \end{cases}$$

Consequently, a mode-dependent Luenberger-type filter is designed with consideration of event-triggered communication as follows

$$\begin{cases} \dot{\hat{x}}(t) = A(i)\hat{x}(t) + \hat{f}(\hat{x}(t)) + K(i)(y(t_k h) - C(i)\hat{x}(t_k h)) \\ \hat{z}(t) = L(i)\hat{x}(t) \end{cases} \tag{2}$$

where $\hat{f}(\hat{x}(t))$ represents the approximate unknown nonlinearity and $K(i)$ denotes the mode-dependent filter gains to be designed. It should be pointed out that the Luenberger-type observers have considerable advantages in system structure and implementation (Tarantino et al., 2000; Zhang et al., 2019b). The following event-triggered strategy is utilized under the networked framework. In detail, based on the sampling period $h > 0$, an event generator with ZOH is adopted to update the released signals with $t_k h$, $k = 0, 1, 2, \ldots$. By applying the ZOH with desired sampling period h to keep the sampled latest signal, Zeno's behavior can also be avoided for the triggering transmissions.

By defining the filter error as $x_e = x(t) - \hat{x}(t)$ and $z_e = z(t) - \hat{z}(t)$, it can be obtained that

$$\begin{cases} \dot{x}_e(t) = A(i)x_e(t) + f(x(t)) - \hat{f}(\hat{x}(t)) \\ \qquad -K(i)(C(i)x_e(t_k h) + D(i)w(t_k h)) + B(i)w(t) \\ z_e(t) = L(i)x_e(t) \end{cases} \tag{3}$$

Subsequently, the corresponding triggering function is designed by

$$\epsilon_k^T(t)W_1(i)\epsilon_k(t) \geq \varepsilon(i)y_e^T(t_k h)W_2(i)y_e(t_k h) \tag{4}$$

where

$$\epsilon_k(t) = y_e(t_k h + jh) - y_e(t_k h)$$
$$y_e(t_k h) = C(i)x_e(t_k h) + D(i)w(t_k h)$$

$0 \leq \varepsilon(\sigma(t)) < 1$ and $W_1(i) > 0$, $W_2(i) > 0$ are mode-dependent scaling matrices.

Remark 1. *It is worthwhile to point out that the event-triggered strategies always focus on improving the communication efficiency with less updated information. For Markovian jump systems, the mode-dependent triggering function can utilize the mode information for more design feasibility. In addition, the proposed event-triggered strategy can be further extended to the cases with network constraints, that is, network-induced delays and so on (Li et al., 2018; Mazenc et al., 2022).*

By employing the virtual delay approach within triggering intervals $[t_k h, t_{k+1} h)$ (Fridman 2010), it can be derived that $C(i)x_e(t_k h) + D(i)w(t_k h) = C(i)x_e(t - d(t)) + D(i)w(t - d(t)) + \tau_k(t)$, which can lead to that

$$\tau_k^T(t)W_1(i)\tau_k(t) \leq \varepsilon(i)(y_e(t - d(t)) + \tau_k(t))^T \times W_2(i)(y_e(t - d(t)) + \tau_k(t))$$

where $0 \leq d(t) = t - t_k h < h$ means the virtual delay.

As a result, the developed filter can be further modified by

$$\begin{cases} \dot{\hat{x}}(t) = A(i)\hat{x}(t) + magenta\hat{f}(\hat{x}(t)) + K(i)(y(t - d(t)) \\ \qquad + \tau_k(t) - C(i)\hat{x}(t - d(t))), \\ \hat{z}(t) = L(i)\hat{x}(t) \end{cases} \tag{5}$$

Since $f(x(t))$ is unknown for the filter system, a neural network is applied for unknown approximate reconstruction (Noriega and Wang, 1998; Yang et al., 2016). Then, it follows that

$$f(x(t)) = \mathcal{W}^* \varphi(x(t)) + \delta_1(t) \tag{6}$$

where $\delta_1(t)$ denotes the approximation error, $\varphi(x(t))$ implies the nonlinear neuron vector, and

$$\mathcal{W}^* = \arg \min_{\mathcal{W}(t) \in \Omega_\mathcal{W}} \left\{ \sup_{x \in \Omega_x} \| \delta(t) \| \right\}$$

represents the optimized neuron weights in the neural network. Under this context, $\hat{f}(\hat{x}(t))$ can be obtained to approximate $f(x(t))$ as follows

$$\hat{f}(\hat{x}(t)) = \mathcal{W}(t)\varphi(\hat{x}(t)) \tag{7}$$

where $\mathcal{W}(t)$ represents the weight matrix of the neural network. Hence, it follows that

$$\begin{cases} \dot{x}_e(t) = A(i)x_e(t) - K(i)C(i)x_e(t - d(t)) \\ \qquad + \tilde{\mathcal{W}}(t)\varphi(\hat{x}(t)) + \delta(t) - K(i)\tau_k(t) \\ \qquad + B(i)w(t) - K(i)D(i)w(t - d(t)), \\ z_e(t) = L(i)x_e(t) \end{cases} \tag{8}$$

where $\delta(t) = \delta_1(t) + \delta_2(t)$, $\delta_2(t) = \mathcal{W}^*(\varphi(x(t)) - \varphi(\hat{x}(t)))$ and $\tilde{\mathcal{W}}(t) = \mathcal{W}^*(t) - \mathcal{W}(t)$.

Remark 2. *As the neural networks have an adaptive adjustment mechanism to approximate the nonlinearities with desired accuracy and generalization ability, they can be adopted for superior unknown dynamics with input-output data.*

Furthermore, notice the fact that the true modes of system (1) are always difficult to acquire. To deal with the hidden modes, the observed modes are utilized by the mode-dependent filter. As such, the nonsynchronous filter gain $K(\rho)$ with observed modes is applied instead of $K(i)$, where $\varsigma(t) = \rho \in \mathcal{T} = \{1, \ldots, M\}$ is another stochastic process with the following conditional probability

$$\Pr\{\varsigma(t) = \rho | \sigma(t) = i\} = \lambda_{i\rho} \tag{9}$$

$$\sum_{\rho=1}^{\mathcal{M}} \lambda_{i\rho} = 1 \tag{10}$$

Eventually, the resulting filtering error system can be rewritten as follows

$$\begin{cases} \dot{x}_e(t) = A(i)x_e(t) - K(\rho)C(i)x_e(t - d(t)) \\ \qquad + \tilde{\mathcal{W}}(t)\varphi(\hat{x}(t)) + \delta(t) - K(\rho)\tau_k(t) \\ \qquad + B(i)w(t) - K(\rho)D(i)w(t - d(t)) \\ z_e(t) = L(i)x_e(t) \end{cases} \tag{11}$$

Remark 3. *In practice, the nonsynchronous phenomenon between the observer and original systems can be found due to information dropouts and delays. As a result, it is reasonable to consider the hidden mode detection strategy to deal with an unpredictable mode mismatch.*

Moreover, the optimal modified H_∞ performance is introduced for disturbance attenuation.

Definition 1. *The filtering error system (11) is said to satisfy the optimal modified H_∞ performance in mean-square sense if there exists a constant $\gamma > 0$ such that*

$$\begin{aligned} &\int_0^\infty \mathbb{E}\{z_e^T(t)z_e(t)\}dt \\ &< \gamma^2 \int_0^\infty \mathbb{E}\{w^T(t)w(t) + w^T(t - d(t))w(t - d(t)) \\ &\quad + \delta^T(t)\delta(t)\}dt \end{aligned} \tag{12}$$

under zero initial conditions.

Remark 4. *It is noted that for some practical systems, the corresponding dynamics with uncertainties should be considerably taken into account for design robustness (Tutsoy, 2015, 2016; Tutsoy et al., 2018). Under this context, the above-modified H_∞ performance can be further extended to deal with the uncertainties, such that the modeling errors can be effectively decreased in the meantime.*

To this end, the objective of this paper is to determine the asynchronous filter gain $K(\rho)$ such that the modified H_∞ performance can be achieved accordingly.

Main results

In this section, our main theoretical results will be established based on matrix convex optimization techniques.

Theorem 1. *System (11) can satisfy the modified H_∞ performance with mode-dependent filter gain $K(\rho)$, $\rho \in \mathcal{T}$ and parameter $\gamma > 0$ according to Definition 1, if there exist mode-dependent matrix $P(i) > 0$, $i \in \mathcal{S}$ and matrices $Q > 0$, $R > 0$, $S > 0$, $\Lambda > 0$, such that the following linear matrix inequality holds, where*

$$\Theta(i,\rho) = \begin{bmatrix} \Theta_1(i,\rho) & \Theta_2(i,\rho) \\ * & \Theta_3(i,\rho) \end{bmatrix} < 0$$

with

$$\Theta_1(i,\rho) = \begin{bmatrix} \Theta_{11}(i,\rho) & \Theta_{12}(i,\rho) \\ * & \Theta_{13}(i,\rho) \end{bmatrix},$$

$$\Theta_{11}(i,\rho) = \begin{bmatrix} \Theta_{111}(i,\rho) & P(i) + A(i)^T S \\ * & -2S + h^2 R \end{bmatrix},$$

$$\Theta_{111}(i,\rho) = Q - R + \sum_{j=1}^{N} \pi_{ij} P(j) + L^T(i)L(i),$$

$$\Theta_{12}(i,\rho) = \begin{bmatrix} R \\ -\sum_{\rho=1}^{M} \lambda_{i\rho} S K(\rho) C(i) \end{bmatrix},$$

$$\Theta_{13}(i,\rho) = -2R + \varepsilon(i) C^T(i) W_2(i) C(i),$$

$$\Theta_2(i,\rho) = [\Theta_{21}(i,\rho), \Theta_{22}(i,\rho)],$$

$$\Theta_{21}(i,\rho) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & S & -\sum_{\rho=1}^{M} \lambda_{i\rho} Y(\rho) \\ R & 0 & \varepsilon(i) C^T(i) W_2(i) \end{bmatrix}$$

$$\bar{\Theta}_{22}(i,\rho) = \begin{bmatrix} 0 & 0 \\ SB(i) & -\sum_{\rho=1}^{M} \lambda_{i\rho} S K(\rho) D(i) \\ 0 & \varepsilon(i) C^T(i) W_2(i) D(i) \end{bmatrix},$$

$$\Theta_3(i,\rho) = \begin{bmatrix} \Theta_{31}(i,\rho) & \Theta_{32}(i,\rho) \\ * & \Theta_{33}(i,\rho) \end{bmatrix},$$

$$\Theta_{31}(i,\rho) = \begin{bmatrix} -Q-R & 0 & 0 \\ * & -\gamma^2 I & 0 \\ * & * & -W_1(i) + \varepsilon(i) W_2(i) \end{bmatrix},$$

$$\Theta_{32}(i,\rho) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \varepsilon(i) W_2(i) D(i) \end{bmatrix},$$

$$\Theta_{33}(i,\rho) = \begin{bmatrix} -\gamma^2 I & 0 \\ * & -\gamma^2 I + \varepsilon(i) D^T(i) W_2(i) D(i) \end{bmatrix}$$

and the online neural network learning law is designed as follows

$$\dot{\mathcal{W}}(t) = \Lambda S \dot{x}_e(t) \varphi^T(\hat{x}(t))$$

Proof. For certain mode i, choose the mode-dependent Lyapunov–Krasovskii function as follows

$$V(i,t) = V_1(i,t) + V_2(i,t) + V_3(i,t) + V_4(i,t) \quad (13)$$

where

$$V_1(i,t) = x_e^T(t) P(i) x_e(t)$$
$$V_2(i,t) = \int_{t-h}^{t} x_e^T(s) Q x_e(s) ds$$
$$V_3(i,t) = \bar{h} \int_{-\bar{h}}^{0} \int_{t+s}^{t} \dot{x}_e^T(\eta) R \dot{x}_e(\eta) d\eta ds$$
$$V_4(i,t) = tr\{\tilde{\mathcal{W}}^T(t) \Lambda^{-1} \tilde{\mathcal{W}}(t)\}$$

In addition, the weak infinitesimal operator $\mathcal{L}$ for mode-dependent $V(i,t)$ is defined by

$$\mathcal{L}V(i,t) = \lim_{\Delta \to 0} \frac{1}{\Delta} \{\mathbb{E}\{V(\sigma(t+\Delta), t+\Delta) | \sigma(t) = i\} - V(i,t)\}$$

Subsequently, taking the above weak infinitesimal operator along with evolution of $V(i,t)$, it yields that

$$\mathcal{L}V_1(i,t) = 2x_e^T(t) P(i) \dot{x}_e(t) + \sum_{j=1}^{N} \pi_{ij} x_e^T(t) P(j) x_e(t)$$
$$\mathcal{L}V_2(i,t) = x_e^T(t) Q x_e(t) - x_e^T(t-h) Q x_e(t-h)$$
$$\mathcal{L}V_3(i,t) = h^2 \dot{x}_e^T(t) R \dot{x}_e(t) - h \int_{t-h}^{t} \dot{x}_e^T(s) R \dot{x}_e(t) ds$$
$$\mathcal{L}V_4(i,t) = 2tr\{\dot{\tilde{\mathcal{W}}}^T(t) \Lambda^{-1} \tilde{\mathcal{W}}(t)\}$$

Furthermore, based on the resulting filtering error system dynamics, one has that

$$2\mathbb{E}\{\dot{x}_e^T(t)S(A(i)x_e(t) - K(\rho)C(i)x_e(t-d(t)) + \tilde{\mathcal{W}}(t)\varphi(\hat{x}(t)) + \delta(t) - K(\rho)\tau_k(t) + B(i)w(t) - K(\rho)D(i)w(t-d(t)) - \dot{x}_e(t))\} = 0$$

Based on the online neural network learning law, one has that

$$\mathcal{L}V_4(i,t) = 2tr\{\dot{\tilde{\mathcal{W}}}^T(t)\Lambda^{-1}\tilde{\mathcal{W}}(t)\} = 2tr\{-\varphi(\hat{x}(t))\dot{x}_e^T(t)S\tilde{\mathcal{W}}(t)\}$$

which implies that

$$\mathbb{E}\{\dot{x}_e^T(t)S\tilde{\mathcal{W}}(t)\varphi(\hat{x}(t)) - tr\{\varphi(\hat{x}(t))\dot{x}_e^T(t)S\tilde{\mathcal{W}}(t)\}\} = 0$$

By applying Jensen's inequality Kim (2016), it can be obtained that

$$-h\int_{t-h}^{t}\dot{x}_e^T(s)R\dot{x}_e(s)ds \leqslant \begin{bmatrix} x_e(t) \\ x_e(t-d(t)) \\ x_e(t-h) \end{bmatrix}^T \begin{bmatrix} -R & R & 0 \\ * & -2R & R \\ * & * & -R \end{bmatrix} \begin{bmatrix} x_e(t) \\ x_e(t-d(t)) \\ x_e(t-h) \end{bmatrix}$$

In addition, it can be obtained by the event-triggering condition that

$$\varepsilon(i)(y_e(t-d(t)) + \tau_k(t))^T W_2(i)(y_e(t-d(t)) + \tau_k(t)) - \tau_k^T(t)W_1(i)\tau_k(t) > 0$$

such that the triggering matrices can be introduced in the convex optimization conditions.

By virtue of the trace property of matrices in the Lyapunov–Krasovskii function $V(i,t)$ and the neural network learning the law of $\dot{\mathcal{W}}(t) = \Lambda S\dot{x}_e(t)\varphi^T(\hat{x}(t))$ in Theorem 1, it can be deduced that

$$\mathcal{L}V(i,t) + z_e^T(t)z_e(t) - \gamma^2 w^T(t)w(t) - \gamma^2 w^T(t-d(t))w(t-d(t)) - \gamma^2\delta^T(t)\delta(t) < \eta^T(t)\Theta(i,\rho)\eta(t)$$

where $\eta(t) = [x_e^T(t), \dot{x}_e^T(t), x_e^T(t-d(t)), x_e^T(t-h), \delta^T(t), \tau_k^T(t), w^T(t), w^T(t-d(t))]^T$ with $\Theta(i,\rho)$ defined in Theorem 1.

Hence, it can be verified that if $\Theta(i,\rho) < 0$ holds, then $z_e^T(t)z_e(t) - \gamma^2 w^T(t)w(t) - \gamma^2 w^T(t-d(t))w(t-d(t)) - \gamma^2\delta^T(t)\delta(t) < 0$ can be satisfied by integrating $\Theta(i,\rho) < 0$ between 0 and ∞ under the zero initial condition. As a result, the modified H_∞ performance can be ensured according to Definition 1 and the neural network parameters and the event-triggering parameters can be designed by solving the convex optimization problem, which completes the proof.

The modified H_∞ performance is introduced to the mode-dependent filtering problem based on a neural event-triggered co-design approach, such that neural network approximation errors and external disturbances can be attenuated with a satisfied level. Based on the established optimal conditions in Theorem 1, the following theorem is presented to synthesize the mode-dependent filter gains.

Theorem 2. *System (11) can satisfy the modified H_∞ performance and parameter $\gamma > 0$ according to Definition 1, if there exist mode-dependent matrices $P(i) > 0$, ..., $Y(\rho)$, $\rho \in \mathcal{T}$ and matrices $Q > 0$, $R > 0$, $S > 0$, $\Lambda > 0$, such that the following linear matrix inequality holds, where*

$$\bar{\Theta}(i,\rho) = \begin{bmatrix} \bar{\Theta}_1(i,\rho) & \bar{\Theta}_2(i,\rho) \\ * & \bar{\Theta}_3(i,\rho) \end{bmatrix} < 0$$

with

$$\bar{\Theta}_1(i,\rho) = \begin{bmatrix} \bar{\Theta}_{11}(i,\rho) & \bar{\Theta}_{12}(i,\rho) \\ * & \bar{\Theta}_{13}(i,\rho) \end{bmatrix}$$

$$\bar{\Theta}_{11}(i,\rho) = \begin{bmatrix} \bar{\Theta}_{111}(i,\rho) & P(i) + A(i)^T S \\ * & -2S + h^2 R \end{bmatrix}$$

$$\bar{\Theta}_{111}(i,\rho) = Q - R + \sum_{j=1}^{N}\pi_{ij}P(j) + L^T(i)L(i)$$

$$\bar{\Theta}_{12}(i,\rho) = \begin{bmatrix} R \\ -\sum_{\rho=1}^{M}\lambda_{i\rho}Y(\rho)C(i) \end{bmatrix}$$

$$\bar{\Theta}_{13}(i,\rho) = -2R + \varepsilon(i)C^T(i)W_2(i)C(i)$$

$$\bar{\Theta}_2(i,\rho) = [\bar{\Theta}_{21}(i,\rho), \bar{\Theta}_{22}(i,\rho)]$$

$$\bar{\Theta}_{21}(i,\rho) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & S & -\sum_{\rho=1}^{M}\lambda_{i\rho}Y(\rho) \\ R & 0 & \varepsilon(i)C^T(i)W_2(i) \end{bmatrix}$$

$$\bar{\Theta}_{22}(i,\rho) = \begin{bmatrix} 0 & 0 \\ SB(i) & -\sum_{\rho=1}^{M}\lambda_{i\rho}Y(\rho)D(i) \\ 0 & \varepsilon(i)C^T(i)W_2(i)D(i) \end{bmatrix}$$

$$\bar{\Theta}_3(i,\rho) = \begin{bmatrix} \bar{\Theta}_{31}(i,\rho) & \bar{\Theta}_{32}(i,\rho) \\ * & \bar{\Theta}_{33}(i,\rho) \end{bmatrix}$$

Algorithm 1. The optimal filter design algorithm.

Require:
The parameters of state-space system model;
The parameters of constructed neural network;
The parameters of asynchronous conditional probability;
Ensure:
1: Solving the linear matrix inequality conditions in Theorem 2;
2: Solving the convex optimization conditions in Remark 5;
3: **return** mode-dependent filter gains $K(\rho)$, event-triggering matrices $W_1(i)$, $W_2(i)$, H_∞ performance γ;

$$\bar{\Theta}_{31}(i,\rho) = \begin{bmatrix} -Q-R & 0 & 0 \\ * & -\gamma^2 I & 0 \\ * & * & -W_1(i)+\varepsilon(i)W_2(i) \end{bmatrix}$$

$$\bar{\Theta}_{32}(i,\rho) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \varepsilon(i)W_2(i)D(i) \end{bmatrix}$$

$$\bar{\Theta}_{33}(i,\rho) = \begin{bmatrix} -\gamma^2 I & 0 \\ * & -\gamma^2 I+\varepsilon(i)D^T(i)W_2(i)D(i) \end{bmatrix}$$

and the online neural network learning law is designed as follows

$$\dot{\mathcal{W}}(t) = \Lambda S\dot{x}_e(t)\varphi^T(\hat{x}(t))$$

With above feasible solutions, the mode-dependent filter gain $K(\rho)$, $\rho \in \mathcal{T}$ can be obtained by

$$K(\rho) = S^{-1}Y(\rho).$$

Proof. By letting $SK(\rho) = Y(\rho)$, the proof can be directly followed by Theorem 1.

Remark 5. *The derived criterion in Theorem 2 is represented in the form of strict linear matrix inequality, such that the mode-dependent filter parameters can be solved conveniently with mathematical software. Furthermore, by applying recent advances in Lyapunov–Krasovskii function construction methods, i.e. time-dependent Lyapunov functional Lee and Park (2017), the Wirtinger-based integral inequality Seuret and Gouaisbaut (2013), the derived conservatism can be further deduced accordingly. The optimized value of γ can be solved based on the above convex optimization condition as*

$$\min \gamma, \quad s.t. \bar{\Theta}(i,\rho) < 0, i \in \mathcal{S}, \rho \in \mathcal{T}$$

such that the minimal value of γ in the modified H_∞ performance can be obtained. Furthermore, the overall optimal filter design can be formulated as follows: [H]

Simulation examples

In what follows, the simulation results are provided to demonstrate the advances of the developed co-design filtering approach.

Example 1. *Consider the Markovian jump system (1), where the system parameters are given by*

$$A(1) = \begin{bmatrix} -1.7 & 0 \\ 0 & -1.4 \end{bmatrix}, A(2) = \begin{bmatrix} -1.5 & 0 \\ 0 & -2.1 \end{bmatrix}$$
$$B(1) = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, B(2) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$
$$C(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix}, C(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$D(1) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, D(2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}$$
$$L(1) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix}, L(2) = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.4 \end{bmatrix}$$

and the unknown nonlinear function $f(x(t))$ is set by

$$f(x(t)) = 0.2\begin{bmatrix} \tan x_1(t) \\ \tan x_2(t) \end{bmatrix}$$

Furthermore, the transition probability and conditional probability matrices are supposed to be

$$\Pi = \begin{bmatrix} 0.6 & -0.6 \\ -0.7 & 0.7 \end{bmatrix}$$

and

$$\Lambda = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{bmatrix}$$

In the simulation, the parameters of neural network are designed by

$$\Lambda = 2I$$
$$\varphi(x(t)) = \begin{bmatrix} \frac{1}{1+e^{-x_1(t)}} \\ \frac{1}{1+e^{-x_2(t)}} \end{bmatrix}$$
$$W(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and the external disturbances are assumed to be $w(t) = [0.1\sin(t), 0.1\sin(t)]^T$. As a result, by choosing the modified H_∞ performance as $\gamma = 1$ and sampling period as $h = 0.2$ s, the mode-dependent filter gains and the event-triggering parameters are solved by Theorem 2 as

$$K(1) = \begin{bmatrix} -0.4409 & 0 \\ 0 & 1.0754 \end{bmatrix}, K(2) = \begin{bmatrix} 0.5580 & 0 \\ 0 & -0.7315 \end{bmatrix}$$
$$W_1(1) = \begin{bmatrix} 1.0209 & 0 \\ 0 & 1.0196 \end{bmatrix}, W_1(2) = \begin{bmatrix} 1.0462 & 0 \\ 0 & 1.0443 \end{bmatrix}$$
$$W_2(1) = \begin{bmatrix} 0.7833 & 0 \\ 0 & 0.7212 \end{bmatrix}, W_2(2) = \begin{bmatrix} 0.6963 & 0 \\ 0 & 0.6921 \end{bmatrix}$$

With these parameters, the simulation results are shown in Figures 1–4. It can be seen in Figure 2 that the filtering error dynamics can converge to zeros despite external disturbance, while the true system modes and filtering modes are shown in Figure 1. Figure 3 depicts the event-triggering instants, such that one can find that our developed filtering strategy can considerably decrease the information communication with desired filtering performance. Figure 4 illustrates the neural network learning error dynamics for the filter, where the unknown nonlinear function is well approximated. Moreover, a comparison result with a neural network and without unknown nonlinear dynamics approximation in the filter design is given in Figure 5, where one can find that the adoption of neural network can effectively improve the filter applicability.

Example 2. *In what follows, a practical example of the tunnel diode circuit is borrowed to verify the applicability of our proposed filtering method, where the dynamics are described by*

$$i_d(t) = 0.01V_d(t) + \rho(i)V_d^3(t)$$

where $\rho(i)$ denotes the characteristic parameter with two modes $i = 1, 2$Wang et al. (2020a). Then, by setting the circuit parameters $C = 100\,\mathrm{mF}$, $L = 1H$, $R = 10\Omega$, $\rho(1) = 0.004$, $\rho(2) = 0.005$, the state space dynamics can be further represented as follows

$$\begin{cases} \dot{x}(t) = A(i)x(t) + f(i, x(t)) + B(i)w(t) \\ y(t) = C(i)x(t) + D(i)w(t) \\ z(t) = L(i)x(t) \\ x(0) = x_0, \sigma(0) = \sigma_0 \end{cases}$$

with

$$A(1) = \begin{bmatrix} -\frac{0.01}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}, A(2) = \begin{bmatrix} -\frac{0.01}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix},$$

$$B(1) = \begin{bmatrix} 0 \\ \frac{0.1}{L} \end{bmatrix}, B(2) = \begin{bmatrix} 0 \\ \frac{0.1}{L} \end{bmatrix},$$

$$C(1) = \begin{bmatrix} 1 & 0 \end{bmatrix}, C(2) = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$D(1) = 0.1, D(2) = 0.1,$$

$$L(1) = I, L(2) = I,$$

$$f(\sigma(t), x(t)) = \begin{bmatrix} -\frac{\rho(i)}{C}x_1^2(t) & 0 \\ 0 & 0 \end{bmatrix} x(t)$$

Moreover, the transition probability matrix is described by

$$\Pi = \begin{bmatrix} 0.5 & -0.5 \\ -0.6 & 0.6 \end{bmatrix}$$

and the conditional probability matrix is given as

$$\Lambda = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.5 \end{bmatrix}$$

Other main simulation parameters can be chosen as same as that in Example 1, then by solving Theorem 2, the following filter gains and event-triggering matrices can be calculated by

$$K(1) = \begin{bmatrix} 2.9816 \\ -1.8245 \end{bmatrix}, K(2) = \begin{bmatrix} 3.7840 \\ -2.0436 \end{bmatrix}$$

$$W_1(1) = 108.3703, W_1(2) = 100.1931$$

$$W_2(1) = 0.9314, W_2(2) = 0.6289$$

With initial conditions $[-0.1, 0.2]^T$, the filtering error dynamics, the event-triggering instants and the neural learning approximation error responses are depicted in Figures 6–9, which can lead to the conclusion that our developed filtering method achieves desired performance.

Example 3. *In addition, another practical example of VTOL (vertical take-off and landing) helicopter is also employed with nonlinear dynamics (Yan et al., 2019). Then, the state space model can be described as follows*

$$\begin{cases} \dot{x}(t) = A(i)x(t) + f(i, x(t)) + B(i)w(t) \\ y(t) = C(i)x(t) + D(i)w(t) \\ z(t) = L(i)x(t) \\ x(0) = x_0, \sigma(0) = \sigma_0 \end{cases}$$

where $x(t) = [x_1(t), x_2(t), x_4(t), x_4(t)]^T$ and $x_1(t), x_2(t), x_4(t), x_4(t)$ represent the horizontal velocity, vertical velocity, pitch rate, and pitch angle of VTOL, respectively. The two-mode jumping is conducted by air speed under 135, and 170 knot, with

$$A(1) = \begin{bmatrix} -1.4 & 1.4 & 0.2 & -0.1 \\ -0.5 & -1.41 & 0 & -2.0 \\ 0.1 & 0.37 & -1.71 & 1.42 \\ 0 & -0.2 & -0.1 & -0.3 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} -1.4 & 1.4 & 0.2 & -0.1 \\ -0.5 & -1.41 & 0 & -2.0 \\ 0.1 & 0.51 & -1.71 & 2.52 \\ 0 & -0.2 & -0.1 & -0.3 \end{bmatrix}$$

$$B(1) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, B(2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$C(1) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, C(2) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$D(1) = \begin{bmatrix} 0 & 0 & 0.1 & 0 \\ 0 & 0.1 & 0 & 0 \end{bmatrix}, D(2) = \begin{bmatrix} 0 & 0 & 0.1 & 0 \\ 0 & 0.1 & 0 & 0 \end{bmatrix}$$

$$L(1) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, L(2) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$f(i, x(t)) = 0.1 \sin(|x(t)|)$$

Moreover, the transition probability matrix is described by

$$\Pi = \begin{bmatrix} 0.7 & -0.7 \\ -0.9 & 0.9 \end{bmatrix}$$

and the conditional probability matrix is given as

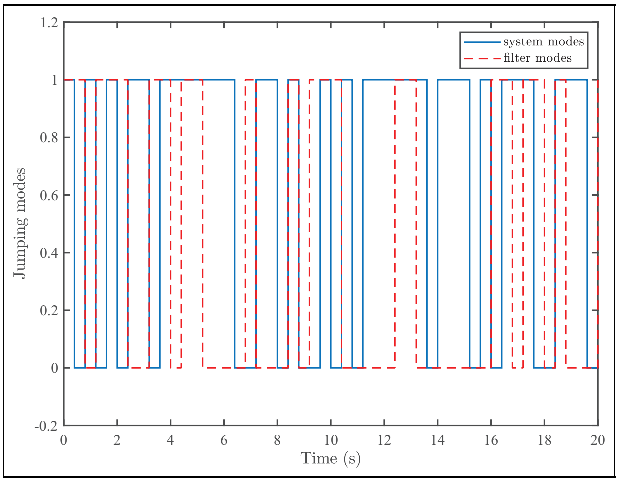

Figure 1. The jumping modes of Markovian jump system and its filter.

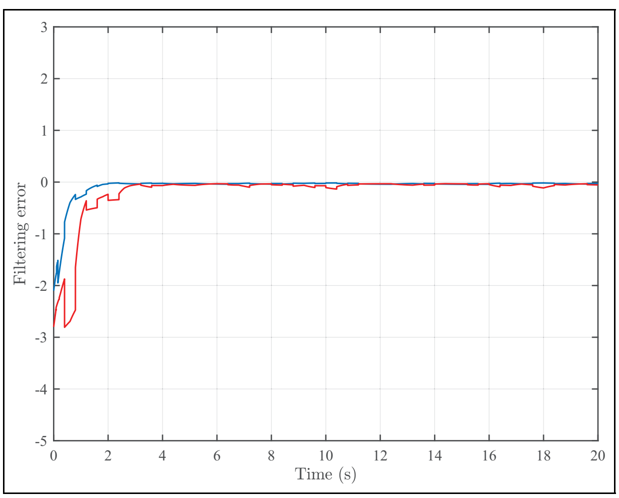

Figure 2. The filtering error dynamics.

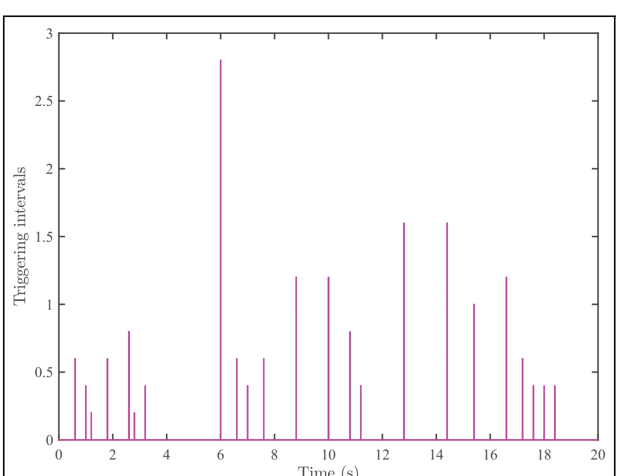

Figure 3. The event-triggering instants and intervals.

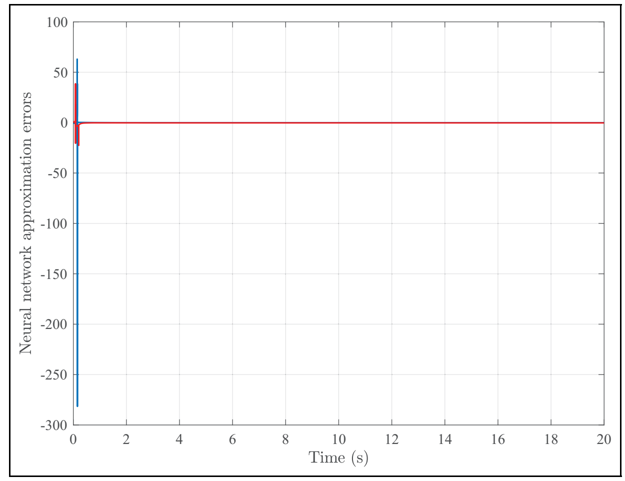

Figure 4. The neural network approximation error dynamics.

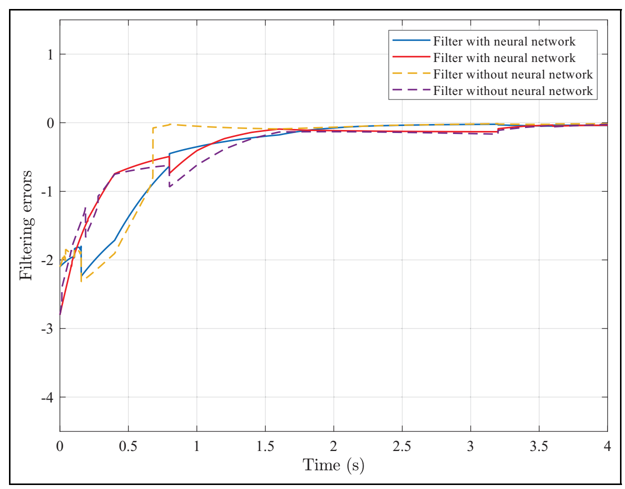

Figure 5. Comparison result of neural network approximation.

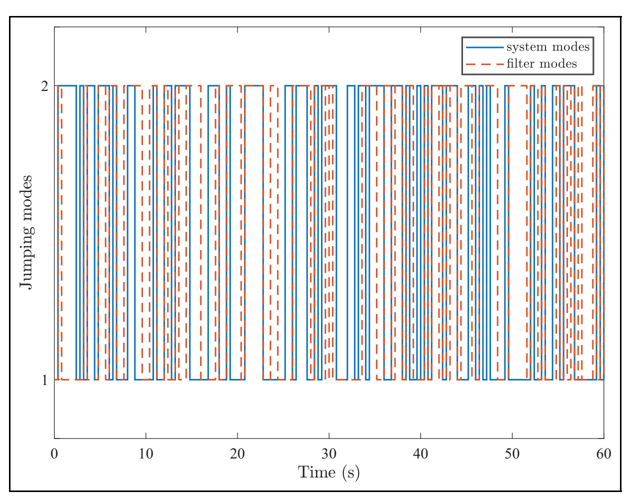

Figure 6. The jumping modes of Markovian jump system and its filter.

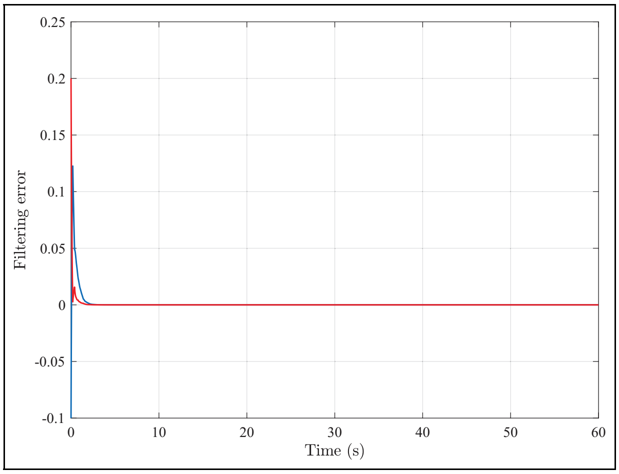

Figure 7. The filtering error dynamics.

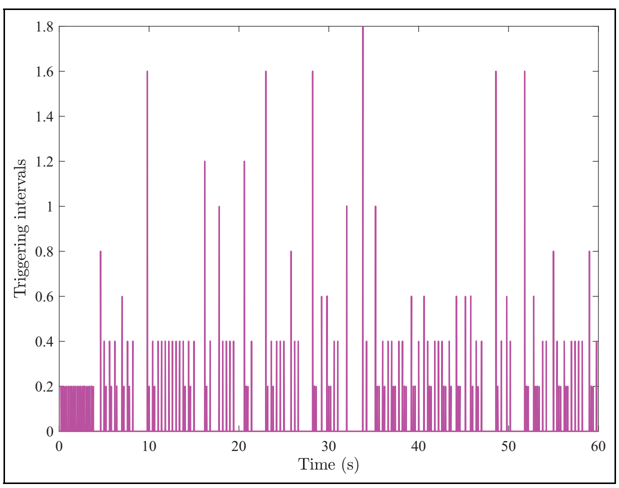

Figure 8. The event-triggering instants and intervals.

$$\Lambda = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

Similarly, with other same parameters and zero initial conditions, the corresponding filter gains and event-triggering matrices can be calculated by

$$K(1) = \begin{bmatrix} 1.2486 & 1.1917 \\ -0.0646 & 0.6464 \\ 0.9756 & 0.1259 \\ -0.1589 & 0.6250 \end{bmatrix}, K(2) = \begin{bmatrix} 0.1240 & 1.5719 \\ -1.3793 & 0.8936 \\ 2.0158 & 0.0261 \\ 0.3450 & 0.4080 \end{bmatrix}$$

$$W_1(1) = \begin{bmatrix} 666.7043 & 10.2832 \\ 10.2832 & 670.9950 \end{bmatrix}, W_1(2) = \begin{bmatrix} 668.9408 & 10.4773 \\ 10.4773 & 670.8186 \end{bmatrix}$$

$$W_2(1) = \begin{bmatrix} 0.8640 & -0.5557 \\ -0.5557 & 1.5731 \end{bmatrix}, W_2(2) = \begin{bmatrix} 0.5795 & -0.3710 \\ -0.3710 & 1.0532 \end{bmatrix}$$

Based on these setting parameters and main similar settings in Example 1, the simulation results are depicted in Figures 10–

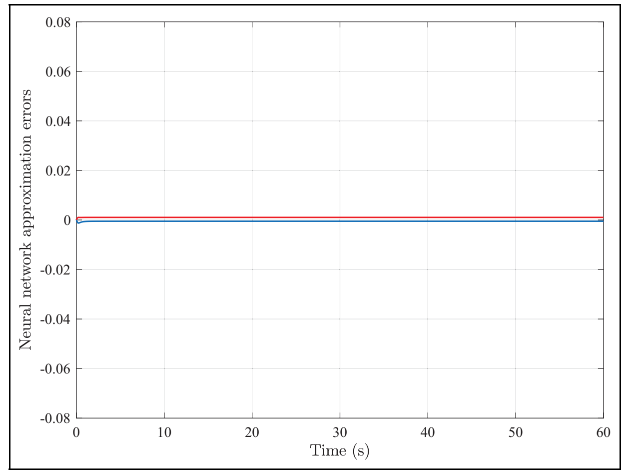

Figure 9. The neural network approximation error dynamics.

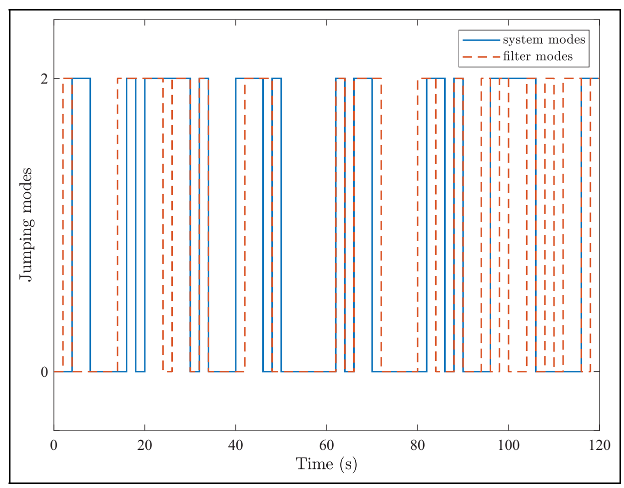

Figure 10. The jumping modes of Markovian jump system and its filter.

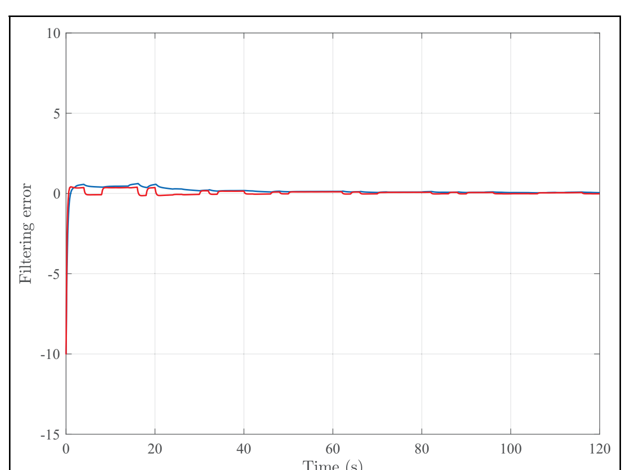

Figure 11. The filtering error dynamics.

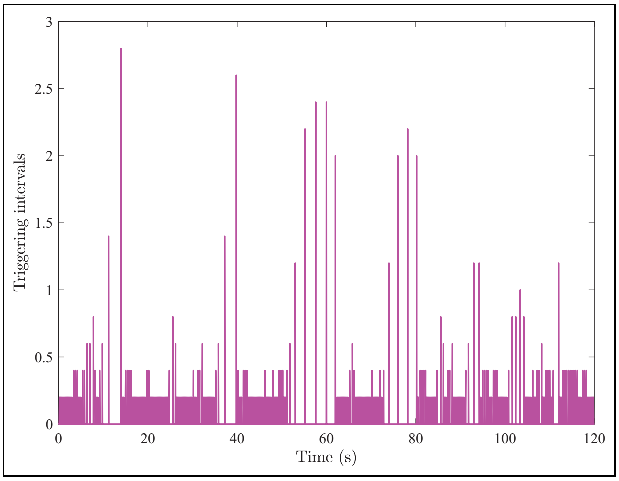

Figure 12. The event-triggering instants and intervals.

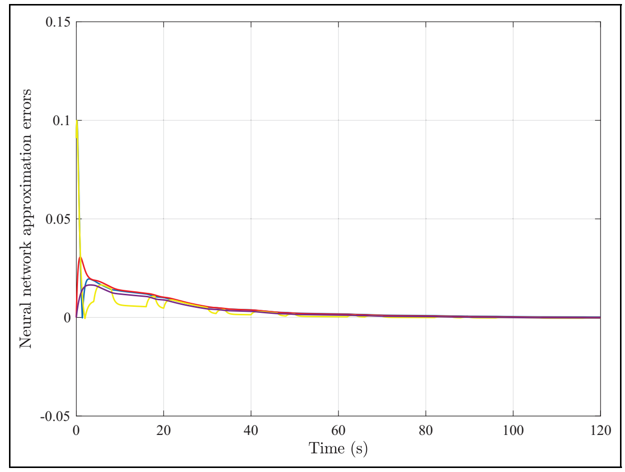

Figure 13. The neural network approximation error dynamics.

13. It can be seen from these resulting states that the true states of the VTOL are well filtered with an effective neural learning process and desired H_∞ performance index, while the communication efficiency is considerably improved accordingly. Therefore, all these simulation validation results can support our theoretical results.

Conclusion

This paper is concerned with the optimal filter co-design issue of Markovian jump systems with unknown nonlinearity and unaccessible mode information. Afterward, the event-triggered scheme is adopted such that less updated control information can be utilized. More precisely, the signal transmission is conducted by event-triggered scheme based on the ZOH. Based on the hidden mode information estimation model, the nonsynchronous mode-dependent filters are further developed with online neural network learning rules, where conditional probability modes are employed instead of true modes. Sufficient analysis and synthesis criteria are derived by convex techniques, such that the modified H_∞ performance can be achieved. Eventually, the effectiveness of the theoretical results is shown via three simulation examples. In our future research, some interesting results are extending current nonsynchronous models to more complicated conditions, that is, how to deal with the partly known conditional probabilities for mode information mismatch in a more general form.

Declaration of conflicting interests

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Appendix

Notation

$\mathbb{R}^N$	N dimensional Euclidean space matrices
$\mathbb{A} > 0$	Positive symmetric definite matrix $\mathbb{A}$
$(\mathbb{O}, \mathbb{F}, \mathbb{P})$	Complete probability space
$\mathbb{E}\cdot$	Mathematics expectation
$sym\mathbb{A}$	$\mathbb{A} + \mathbb{A}^T$
$L_2[0, \infty)$	Space of square-integrable functions on $[0, \infty)$
$tr\{\mathbb{A}\}$	The trace of matrix $\mathbb{A}$
$*$	Symmetry term in matrix