Noncooperative Model Predictive Game With Markov Jump Graph

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***Abstract*—In this paper, the distributed stochastic model predictive control (MPC) is proposed for the noncooperative game problem of the discrete-time multi-player systems (MPSs) with the undirected Markov jump graph. To reflect the reality, the state and input constraints have been considered along with the external disturbances. An iterative algorithm is designed such that model predictive noncooperative game could converge to the so-called ε-Nash equilibrium in a distributed manner. Sufficient conditions are established to guarantee the convergence of the proposed algorithm. In addition, a set of easy-to-check conditions are provided to ensure the mean-square uniform bounded stability of the underlying MPSs. Finally, a numerical example on a group of spacecrafts is studied to verify the effectiveness of the proposed method.**

***Index Terms*—Markov jump graph, model predictive control (MPC), multi-player systems (MPSs), noncooperative game, ε-Nash equilibrium.**

I. Introduction

OVER past decades, the noncooperative game has been attracting extensive research attention in a variety of communities such as battlefield [1], air combat [2], and the security issue of the cyber-physical systems [3]. In engineering practice, the noncooperative game has been also widely used in large-scale systems where the central coordinator is absent [4]–[6]. Furthermore, in [7], the pioneering work has been studied with the social dilemma experiments to reveal how networks promote when there exists the noncooperative game in social contacts. Nash equilibrium (NE) is normally adopted to quantify the outcome of the noncooperative game [8]–[10]. On NE, no players could benefit by changing their actions if other players keep their strategies unchanged. It is worth noting that, in practical large-scale systems, each subsystem (normally regarded as player) is normally subjected to various constraints, and would operate in a distributed manner where global state and parameter information are not prerequisite [4], [11]–[14]. To handle such a problem, the model predictive control (MPC) has been introduced into the noncooperative game in some pioneering papers [15]–[19]. By designing iterative algorithms of the MPC, each player can obtain the NE by only adopting the local state information from its neighbors rather than the global state and parameter information. Nevertheless, as far as we are concerned, the sufficient conditions have not been established in existing literature to guarantee the convergence of such iterative algorithms. This formulates the main motivation of writing this paper.

As mentioned above, the MPC method is effective in handling the dynamical constraint [20]–[25]. Though the constraints in the noncooperative games can be dealt with by MPC, another problem arises. In the traditional game setup, it has been implicitly assumed that the global information of the underlying dynamical system should be possessed by all the controllers/players, which is impossible in most practical situations [26]. Furthermore, in the noncooperative game of control systems, there exist the inevitable bandwidth limitations on communication networks [27]. This inherent limitation may lead to data conflict, which is the root cause of phenomena caused by bad networks. The scheduling protocols are widely used to avoid data collisions. Typically, scheduling protocols include periodic protocol, quadratic protocol and stochastic protocol [28]. Among these protocols, the stochastic communication protocols (SCP) have caught the attention of researchers and been used in industrial systems [29]. In the context of SCP implementation, a protocol that uses Markov chains to represent "random switching" of communication topology among players has been widely used in vehicle platoon [30], multiple high-speed trains [31], vehicular ad hoc networks [32], complex network control systems [33] and multi-player systems (MPSs) [34]. Though the Markov jump graph has been studied for many MPSs [35]–[37], there is little literature on noncooperative games with constraints under Markov jump graphs, which formulates another motivation of writing our paper.

As such, it is by no means a trivial task to obtain the NE solution under the MPC scheme for MPSs over the Marko-

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vian jump switching graph. More specifically, the main technical challenges are identified as follows: 1) How to reflect the dynamical state and control constraints in the noncooperative game problem? 2) How to design an algorithm for MPSs such that each player could obtain the so-called NE solution over the Markovian jump graph? 3) How to ensure the convergence of the derived algorithm and overall controlled system?

The main contributions of this paper are given as follows: 1) the studied noncooperative game problem is reformulated in the framework of MPC under the state/control input constraints; 2) an iterative algorithm is put forward for the considered game problem such that the ε-NE can be attained in a distributed manner; and 3) the easy-to-check sufficient conditions are established to guarantee the convergence of the derived iterative algorithm and the stability of the underlying MPSs.

Notations: $\mathbb{Z}_{\geq 0}$ is the nonnegative integer set. $\mathbb{Z}[a,b]$ with $a,b \in \mathbb{Z}_{\geq 0}$ is the positive integer set $\{a, a+1, \ldots, b\}$. $\mathbb{Z}^+$ is the positive integer set. I represents the unit matrix with appropriate dimensions. $\|\cdot\|$ denotes the Euclidean norm. $\|\cdot\|_\infty$ denotes the infinite norm. $s \in \mathbb{Z}[1,n]$. $\mathbf{0}$ and $\mathbf{1}$ represent the matrices full of 0 and 1 with appropriate dimensions, respectively. $\overline{\lambda}(A)$ (respectively, $\underline{\lambda}(A)$) represents the maximum eigenvalue (respectively, minimum eigenvalue) of matrix A. For a constrained variable $A \in \mathbb{R}^n$, $\max(A)$ (or $\min(A)$) represents that each element in A takes the maximum value (or minimum value) in the constraint. diag$\{\cdot\}$ represents the diagonal matrix. For a matrix, * represents the symmetric part in the matrix. For variable $x \in \mathbb{R}^n$, x^*, $\tilde{x}$ and $\hat{x}$ represent the optimal value, candidate value and predictive value of x, respectively. $\Pr\{\cdot\}$ and $\mathbb{E}\{\cdot\}$ represent the probability and the expectation, respectively.

II. Problem Formulation and Preliminaries

A. Graph Topology

Consider the MPSs with N players. The stochastic switching topology $\mathcal{G}(r(k)) = \{\mathcal{V}(r(k)), \mathcal{E}(r(k)), \mathcal{A}(r(k))\}$ is considered, where $\mathcal{V}(r(k)) = \{v_1, v_2, \ldots, v_N\}$, $\mathcal{E}(r(k)) = \mathcal{V}(r(k)) \times \mathcal{V}(r(k))$ and $\mathcal{A}(r(k)) = [a_{ij}(r(k))]$ $(i,j \in \mathcal{V}(r(k)))$ represent the set of players, the set of edges and the weighted adjacency matrix, respectively. The parameter $\{r(k), k \in \mathbb{Z}_{\geq 0}\}$ is a discrete-time homogeneous Markov chain taking values in a finite set $M = \{1, 2, \ldots, q\}$ and its transition rate matrix $\Pi = \pi_{mn}$ is governed by

$$\pi_{mn} = \Pr\{r(k+1) = n | r(k) = m\} \geq 0,\ \forall m, n \in M$$

for $k \in \mathbb{Z}_{\geq 0}$ and $\Sigma_{n=1}^{q} \pi_{mn} = 1$, $\forall m \in M$. For an undirected graph, $a_{ij}(r(k)) = a_{ji}(r(k)) > 0$ indicates that the i-th and the j-th players can exchange information equally. Additionally, we have $a_{ii}(r(k)) = 0$ for all $i \in \mathcal{V}(r(k))$. $\mathcal{N}_i(r(k)) = \{j : (v_j, v_i) \in \mathcal{E}(r(k))\}$ represents a collection of all players capable of communicating with the i-th player.

B. System Model

Consider the MPSs with N players labeled by $\{1, 2, \ldots, N\}$. The underlying dynamics of the i-th player is described by

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + D_i \omega_i(k) \tag{1}$$

where $x_i(k) \in \mathbb{R}^{n_x}$, $u_i(k) \in \mathbb{R}^{n_u}$ and $\omega_i(k) \in \mathbb{R}^{n_x}$ represent the state variable, control input and disturbance of player i, respectively. A_i, B_i and D_i are matrices with appropriate dimensions. $k \in \mathbb{Z}_{\geq 0}$ represents the time instant. For player i, the state and control constraints are considered as

$$x_i(k) \in \mathcal{X}_i = \{x_i : b_x^{iT} x_i(k) \leq h_x^i\}$$

$$u_i(k) \in \mathcal{U}_i = \{u_i : b_u^{iT} u_i(k) \leq h_u^i\}$$

where b_x^i, b_u^i, h_x^i and h_u^i are all matrices with appropriate dimensions known a priori; The disturbance term is assumed to satisfy

$$\omega_i(k) \in \mathcal{W}_i = \{\omega_i(k) : b_\omega^{iT} \omega_i(k) \leq h_\omega^i\}$$

where b_ω^i is a known appropriate matrix and h_ω^i is a known constant vector.

C. Control Input

For player i $(\forall i \in \mathcal{V}(r(k)))$, the control input to be designed takes the form as

$$u_i(k) = K_i x_i(k) + c_i(k) \tag{2}$$

where K_i is a feedback gain to be designed such that $\Phi_i = A_i + B_i K_i$ is strictly stable; $c_i(k)$ is the correction control signal generated by solving the Prob 1 which would be presented subsequently.

D. Noncooperative Game

Denote the state constraint of the i-th player as $\mathcal{X}_i$. Note that $x_i(k)$ is affected by $x_j(k)$, $j \in \mathcal{N}_i(r(k))$ which is subjected to $\mathcal{X}_j$. Define $\Omega_{-i}^i = \prod_{j \in \mathcal{N}_i(r(k))} \mathcal{X}_j$ where $\prod$ is the Cartesian product. Denote $x_{-i}(k)$ as the set of the states of players in $\mathcal{V}(r(k))$ except the i-th player. Denote $x_{-i}^i(k)$ as the set of the states of players which are the neighbors of the i-th player. The game thus defined on $\mathcal{G}(r(k))$ is denoted by $\mathcal{F}(\mathcal{X}_i, J_i, \mathcal{G}(r(k)))$. For any given $x_{-i}^i(k) \in \Omega_{-i}^i$ and $\omega_i(k) \in \mathcal{W}_i$, player i aims to solve the following optimization problem where each player minimizes the individual cost function $J_i(k) = J_i(\vec{x}_i(k), \vec{u}_i(k), \vec{x}_{-i}^i(k), r(k))$ in a distributed and noncooperative manner. Here

$$\begin{aligned} J_i(k) = & \sum_{s=0}^{T_s-1} \sum_{j \in \mathcal{N}_i(r(k))} a_{ij}(r(k)) \|\hat{x}_i(k+s|k) - \hat{x}_j(k+s|k)\|_{Q_{ij}}^2 \\ & + \sum_{s=0}^{T_s-1} \left\{ \|\hat{x}_i(k+s|k)\|_{Q_i}^2 + \|\hat{u}_i(k+s|k)\|_{R_i}^2 \right\} \\ & + \|\hat{x}_i(k+T_s|k)\|_{P_i}^2 \end{aligned} \tag{3}$$

and

$$\vec{x}_i(k) = [\hat{x}_i^T(k|k)\ \hat{x}_i^T(k+1|k)\ \cdots\ \hat{x}_i^T(k+T_s|k)]^T$$

$$\vec{u}_i(k) = [\hat{u}_i^T(k|k)\ \hat{u}_i^T(k+1|k)\ \cdots\ \hat{u}_i^T(k+T_s-1|k)]^T$$

$$\vec{x}_j(k) = [\hat{x}_j^T(k|k)\ \hat{x}_j^T(k+1|k)\ \cdots\ \hat{x}_j^T(k+T_s-1|k)]^T$$

where $\hat{x}_i(k+s|k)$ represents the predictive state; $\hat{u}_i(k+s|k)$ represents the predictive input signal; $\hat{x}_j(k+s|k)$ denotes the predictive state obtained from the player j for $j \in \mathcal{N}_i(r(k))$; $Q_i > 0$, $Q_{ij} > 0$ and $R_i > 0$ are weighting matrices with appro-

priate dimensions; $P_i > 0$ is the terminal penalty matrix to be designed; and $T_s \in \mathbb{Z}^+$ is the predictive horizon. In addition, $\hat{u}_i(k+s|k) = K_i\hat{x}_i(k+s|k) + \hat{c}_i(k+s|k)$ for $s \in \mathbb{Z}[0, T_s - 1]$ and $\vec{c}_i(k) = [\hat{c}_i^T(k|k)\ \hat{c}_i^T(k+1|k)\ \cdots\ \hat{c}_i^T(k+T_s-1|k)]^T$.

The following definitions are presented which would be used in the sequel.

Definition 1 [38]: System (1) is said to be mean-square uniformly bounded if, for any $\Xi_{1,i} > 0$ ($i \in \{1, 2, \ldots, N\}$), there exists a constant $\Xi_{2,i} = \Xi_{2,i}(\Xi_{1,i})$ such that $\mathbb{E}\{\|x_i(k)\|\} < \Xi_{2,i}$ for $\mathbb{E}\{\|x_i(0)\|\} \le \Xi_{1,i}$ and $k \in \mathbb{Z}_{\ge 0}$ ($i \in \mathcal{V}(r(k))$), where $\Xi_{2,i}(\cdot)$ is the non-negative continuous functionals, mapping bounded sets in the Banach space of continuous function C into bounded sets in $[0, +\infty)$.

Definition 2 [39]: $(\vec{x}_i^{\mathrm{N}*}(k), \vec{u}_i^{\mathrm{N}*}(k), \vec{x}_{-i}^{i\mathrm{N}*}(k))$ is said to be the ε-NE of the game $F(\mathcal{X}_i, J_i, \mathcal{G}(r(k)))$ if, for every given $\vec{x}_{-i}^{i\mathrm{N}*}(k)$, there exists a non-negative constant ε_i such that

$$\begin{aligned} &J_i(\vec{x}_i^{\mathrm{N}*}(k), \vec{u}_i^{\mathrm{N}*}(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k)) \\ &\quad \le J_i(\vec{x}_i(k), \vec{u}_i(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k)) + \varepsilon_i,\ \forall i \in \mathcal{V}(r(k)) \end{aligned} \tag{4}$$

holds with $\forall \hat{x}_i(k+s|k) \in \mathcal{X}_i$ for $s \in \mathbb{Z}[0, T_s]$ and $\forall \hat{u}_i(k+s|k) \in \mathcal{U}_i$ for $s \in \mathbb{Z}[0, T_s - 1]$, where $J_i(\vec{x}_i(k), \vec{u}_i(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k))$ is the admissible cost of the i-th player; $\vec{x}_i^{\mathrm{N}*}(k)$ is the optimal strategy of player i, and $\vec{x}_{-i}^{i\mathrm{N}*}(k)$ are the optimal strategies of the neighbor of player i.

The following optimization problem which aims to find ε-NE in presence of the state and control input constraints, disturbances and Markovian jump graph is given as:

Prob 1:

$$(\vec{x}_i^{\mathrm{N}*}(k), \vec{u}_i^{\mathrm{N}*}(k)) = \arg\min \mathbb{E}\{J_i(\vec{x}_i(k), \vec{u}_i(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k))\} \tag{5}$$

s.t.

$$\hat{x}_i(k+s+1|k) = A_i\hat{x}_i(k+s|k) + B_i\hat{u}_i(k+s|k) \tag{6}$$

$$\hat{x}_i(k+s|k) \in \hat{\mathcal{X}}_i(k+s|k) \tag{7}$$

$$\hat{u}_i(k+s|k) \in \mathcal{U}_i \tag{8}$$

for $s \in \mathbb{Z}[0, T_s - 1]$ where

$$\begin{aligned} \hat{\mathcal{X}}_i(k+s|k) &= \Big\{\hat{x}_i(k+s|k) : b_x^{iT}\hat{x}_i(k+s|k) \\ &\qquad \le h_x^i - \tilde{\xi}_i(k+s|k)\Big\} \\ \tilde{\xi}_i(k+s|k) &= \max b_x^{iT} e_i(k+s|k),\ \omega_i(k+s|k) \in \mathcal{W}_i \\ e_i(k+s+1|k) &= \Phi_i e_i(k+s|k) + D_i\omega_i(k+s|k) \\ e_i(k+s|k) &= x_i(k+s|k) - \hat{x}_i(k+s|k) \end{aligned}$$

where $\omega_i(k+s|k)$ only indicates the possible future situation of the disturbance, not the prediction of the disturbance. Furthermore, we have $e_i(k|k) = \mathbf{0}$ due to $\hat{x}_i(k|k) = x_i(k)$.

Towards this end, the design objectives of this paper are given as: 1) design an iterative algorithm for MPSs (1) under (2) over the Markovian jump graph $\mathcal{G}(r(k))$ such that the noncooperative game can be solved in a distributed manner; 2) establish sufficient conditions to guarantee that the noncooperative game $F(\mathcal{X}_i, J_i, \mathcal{G}(r(k)))$ at each time step converges to the ε-NE in (4); and 3) establish sufficient conditions to guarantee that the MPSs (1) is mean-square uniformly bounded under designed input signal (2).

Remark 1: The motivation of organizing this paper are: 1) few studies have established the theoretical conditions for the convergence of noncooperative games with constraints, so we present a sufficient condition to ensure that the noncooperative game can obtain the ε-NE solution under constraints and disturbances; 2) few studies have obtained the ε-NE of noncooperative games only using the neighbors' state information in a totally distributed way, so we propose a totally distributed method to deal with the noncooperative game by only using the neighbors' state information; 3) few studies have considered the inevitable phenomenon in MPSs such as communication delay, link failures, packet dropouts and node failures [33], so we uses Markov jump graph to model these phenomenons and establish the closed-loop stability of the MPSs in the noncooperative game.

III. Main Results

A. Iterative Algorithm

Considering the dynamic constraints (6)–(8), it becomes impossible to obtain the NE solution of game $F(\mathcal{X}_i, J_i, \mathcal{G}(r(k)))$ by solving the traditional fixed point equation [40], not to mention the case where all the players in game $F(\mathcal{X}_i, J_i, \mathcal{G}(r(k)))$ are governed by the Markovian jump graph. Therefore, in this part, we turn to solve Prob 1 by designing a novel iterative algorithm that is fully distributed.

In this part, an iterative algorithm is proposed to solve Prob 1 such that each player can achieve the ε-NE at each time instant k. For illustration, we define Prob 2 to clearly show the iterative algorithm that is designed for the purpose of obtaining the solution of Prob 1.

Denote $t \in \mathbb{Z}[0, \bar{t}]$ as the iterative times with given constant $\bar{t} \in \mathbb{Z}^+$. Let $\vec{x}_i^*(t,k)$, $\vec{u}_i^*(t,k)$, $\vec{x}_{-i}^{i}(t,k)$ and $J_i^*(t,k)$ be the optimal values and the optimal costs at the t-th iterative of Prob 2 at time step k.

Prob 2:

$$\begin{aligned} &J_i(\vec{x}_i^*(t,k), \vec{u}_i^*(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k)) \\ &\quad = \min \mathbb{E}\Big\{J_i(\vec{x}_i(t,k), \vec{u}_i(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k))\Big\} \end{aligned} \tag{9}$$

s.t.

$$\hat{x}_i(k+s+1|t,k) = A_i\hat{x}_i(k+s|t,k) + B_i\hat{u}_i(k+s|t,k) \tag{10}$$

$$\hat{x}_i(k+s|t,k) \in \hat{\mathcal{X}}_i(k+s|t,k) \tag{11}$$

$$\begin{aligned} &\hat{u}_i(k+s|t,k) \in \mathcal{U}_i \\ &s \in \mathbb{Z}[0, T_s - 1] \end{aligned} \tag{12}$$

where

$$\begin{aligned} &J_i(\vec{x}_i(t,k), \vec{u}_i(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k)) \\ &\quad = \sum_{s=0}^{T_s-1}\Big\{\|\hat{x}_i(k+s|t,k)\|_{Q_i}^2 + \|\hat{u}_i(k+s|t,k)\|_{R_i}^2\Big\} \\ &\qquad + \|\hat{x}_i(k+T_s|t,k)\|_{P_i}^2 + \sum_{s=0}^{T_s-1}\sum_{j\in\mathcal{N}_i(r(k))} a_{ij}(r(k)) \\ &\qquad \times \|\hat{x}_i(k+s|t,k) - \hat{x}_j(k+s|t-1,k)\|_{Q_{ij}}^2 \end{aligned} \tag{13}$$

with

$$\vec{x}_i(t,k) = [\hat{x}_i^T(k|t,k)\ \hat{x}_i^T(k+1|t,k)\ \cdots\ \hat{x}_i^T(k+T_s|t,k)]^T$$

$$\vec{u}_i(t,k) = [\hat{u}_i^T(k|t,k)\ \hat{u}_i^T(k+1|t,k)\ \cdots\ \hat{u}_i^T(k+T_s-1|t,k)]^T$$

$$\vec{c}_i(t,k) = [\hat{c}_i^T(k|t,k)\ \hat{c}_i^T(k+1|t,k)\ \cdots\ \hat{c}_i^T(k+T_s-1|t,k)]^T$$

$$\vec{x}_j(t-1,k) = [\hat{x}_j^T(k|t-1,k)\ \cdots\ \hat{x}_j^T(k+T_s-1|t-1,k)]^T$$

$$\hat{\mathcal{X}}_i(k+s|t,k) = \left\{\hat{x}_i(k+s|t,k) : b_x^{iT}\hat{x}_i(k+s|t,k) \le h_x^i - \tilde{\xi}_i(k+s|t,k)\right\}$$

$$\tilde{\xi}_i(k+s|t,k) = \max b_x^{iT} e_i(k+s|t,k)$$

$$e_i(k+s+1|t,k) = \Phi_i e_i(k+s|t,k) + D_i\omega_i(k+s|t,k)$$

$$e_i(k+s|t,k) = x_i(k+s|t,k) - \hat{x}_i(k+s|t,k)$$

$$\omega_i(k+s|t,k) \in \mathcal{W}_i,\ e_i(k|t,k) = \mathbf{0}$$

for $i \in \mathcal{V}(r(k))$, $j \in \mathcal{N}_i(r(k))$ and $s \in \mathbb{Z}[0, T_s-1]$.

Note that $\hat{x}_i(k+s|t,k)$, $\hat{u}_i(k+s|t,k)$, $\hat{c}_i(k+s|t,k)$, $\hat{\mathcal{X}}_i(k+s|t,k)$, $\tilde{\xi}_i(k+s|t,k)$, $e_i(k+s|t,k)$ and $\omega_i(k+s|t,k)$ represent the t-th iterative of $\hat{x}_i(k+s|k)$, $\hat{u}_i(k+s|k)$, $\hat{c}_i(k+s|k)$, $\hat{\mathcal{X}}_i(k+s|k)$, $\tilde{\xi}_i(k+s|k)$, $e_i(k+s|k)$ and $\omega_i(k+s|k)$ of Prob 2 for $t \in \mathbb{Z}[0,\bar{t}]$, respectively. $\vec{x}_j^*(t-1,k)$ represents the $(t-1)$-th iterative of $\vec{x}_j^*(k)$ of Prob 2 for $t \in \mathbb{Z}[1,\bar{t}]$. In addition, $\vec{x}_j^*(0,k) = \vec{x}_j^{N*}(t,k-1)$ for $k \in \mathbb{Z}^+$, and $\vec{x}_j^*(t-1,0) = \vec{x}_j^*(0)$ for $k=0$. Considering $\hat{x}_i(k|t,k) = x_i(k)$, we have $e_i(k|t,k) = \mathbf{0}$ for $t \in \mathbb{Z}[0,\bar{t}]$.

It should be noted that Prob 2 is iteratively solved in the iterative algorithm, and the solution of Prob 2 is equivalent to that of Prob 1 when the termination condition of the iterative algorithm is satisfied. For simplicity, we denote

$$J_i^*(t,k,r(k)) = J_i(\vec{x}_i^*(t,k,r(k)), \vec{u}_i^*(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k)).$$

In the following, we present the iteration algorithm for the noncooperative game of the MPSs to obtain the MPC input signals. Notice that the convergence of Algorithm 1 is to be proven in Theorem 1.

Algorithm 1 The iterative algorithm for the noncooperative game

Initialization:

1. At $k=0$, set $\vec{x}_{-i}^i(0,0) = \mathbf{0}$ for player i. Set $\bar{t} \in \mathbb{Z}^+$ and the iterative accuracy $\bar{J}_i > 0$ for all $i \in \mathcal{V}(r(k))$. Player i solves Prob 2 to obtain $\vec{x}_i^*(0,0)$. Set $k = k+1$.

Iteration: At $k \in \mathbb{Z}^+$, the controller of player i executes the following steps:

2. Set the iterative step $t=0$.
3. Player i transmits $\vec{x}_i^*(t,k-1)$ to its neighbors and obtains $\vec{x}_{-i}^{i*}(t,k-1)$.
4. Set $t=t+1$. Player i uses $x_i(k)$ and $\vec{x}_{-i}^{i*}(t-1,k-1)$ to solve Prob 2 such that the player i obtains $\vec{x}_i^*(t,k)$ and $J_i^*(t,k)$.
5. If $t=\bar{t}$ or $|J_i^*(t,k) - J_i^*(t-1,k)| \le \bar{J}_i$, player i sends $\hat{u}_i^*(t,k)$ to the actuator. Go to 7. Otherwise, player i transmits $\vec{x}_i^*(t,k)$ to its neighbors, and obtains $\vec{x}_{-i}^{i*}(t,k)$ from its neighbors.
6. Set $t=t+1$. Player i uses $x_i(k)$ and $\vec{x}_{-i*}^{i}(t-1,k)$ to solve Prob 2 such that the player i obtains $\vec{x}_i^*(t,k)$ and $J_i^*(t,k)$. Go to 5.
7. Set $k=k+1$. Go to 2.

The diagram of the proposed iterative algorithm is shown in Fig. 1. In Fig. 1, the optimization process of player i at instant k is shown in detail. $-i \in \mathcal{N}_i(m)$ represents the neighbors of player i. At each iterative step t, player i and its neighbors check the termination conditions. If the termination conditions are not satisfied, player i exchanges its information with its neighbors, and use the obtained information to solve the optimization problem. If the termination conditions are satisfied, then the algorithm is terminated.

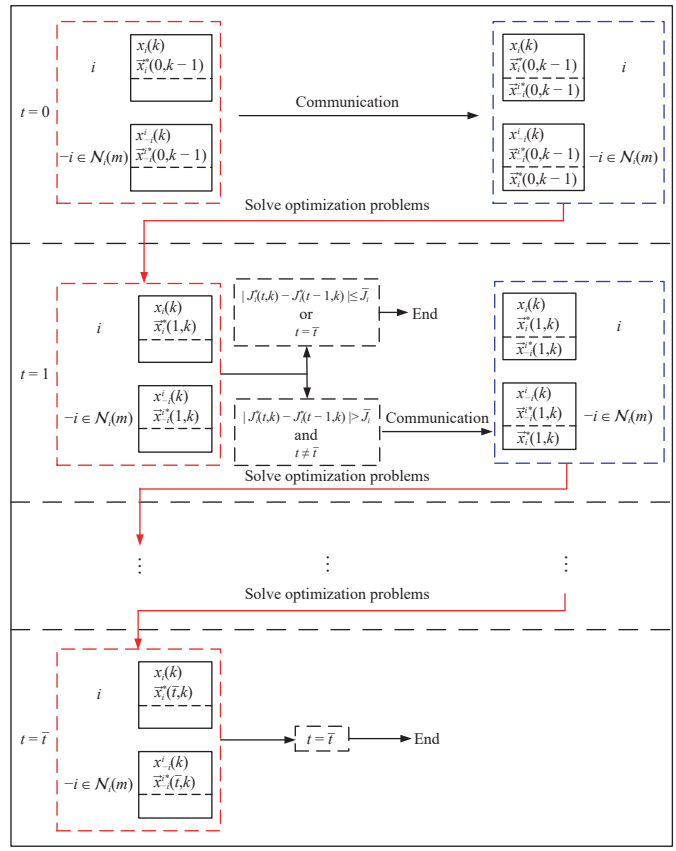

Fig. 1. The diagram of the iterative algorithm.

Remark 2: From Algorithm 1, it is easy to find that the player i only uses the state information obtained from its neighbors without requiring any knowledge of the system parameters of the neighboring players, e.g., A_j, B_j, Q_j, R_j ($j \in \mathcal{N}_i(r(k))$) and so on. The advantage of this is that it is more in line with the actual situation of online optimization and has less conservativeness.

Remark 3: The polynomial complexity of Algorithm 1 under the maximum number of iteration $\bar{t}$ is given as

$$O\left(\mathbb{E}\left\{\vec{c}_i^*(\bar{t},k)\right\}\right) = O(\bar{t}n_{i,\max})$$

where

$$n_{i,1,\max} = \max\{2n_u^3T_s^3q|\bar{\mathcal{N}}_i(m)|,\ 2n_un_xT_s^3q|\bar{\mathcal{N}}_i(m)|\}$$

$$n_{i,2,\max} = 2T_s^2n_u^2n_x,\ n_{i,3,\max} = 2n_u^3T_s^4$$

$$n_{i,\max} = \max\{n_{i,1,\max},\ n_{i,2,\max},\ n_{i,3,\max}\}$$

with $|\bar{\mathcal{N}}_i(m)|$ being the number of player i's neighbors.

In the following, Lemma 1 is proposed to prepare for the convergence proof of Algorithm 1.

Lemma 1: Prob 2 is equivalent to the following optimiza-

tion problem:

$$\begin{aligned}
&J_i(\vec{x}_i^*(t,k),\vec{u}_i^*(t,k),\vec{x}_{-i}^{i*}(t-1,k)(t-1,k),r(k))\\
&\quad=\min\mathbb{E}\{J_i(z_i(k|t,k),\vec{x}_{-i}^{i*}(t-1,k)(t-1,k),r(k))\}\\
&\quad=\min\mathbb{E}\Big\{\|z_i(k|t,k)\|^2_{\mathbb{E}\{\hat{Q}_i(m)\}}+\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m)\\
&\qquad\times\vec{x}_j^{*T}(t-1,k)\vec{Q}_{ij}\vec{x}_j^*(t-1,k)-2\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m)\\
&\qquad\times z_i^T(k|t,k)C_{i1}^T\vec{Q}_{ij}\vec{x}_j^*(t-1,k)\Big\},\ m\in M
\end{aligned}\tag{14}$$

s.t. $g_i(t,k,s)\le\mathbf{0}$ where

$$\vec{x}_i(t,k)=C_{xx}^i\hat{x}_i(k|t,k)+C_{xc}^i\vec{c}_i(t,k)$$

$$g_i(t,k,s)=B_{xu}^i\Psi_i^s z_i(k|t,k)-H_i+\xi_i(t,s)$$

$$\xi_i(t,s)=\max_{\omega_i(k+f|t,k)\in\mathcal{W}_i}B_\omega^i\sum_{f=0}^{s-1}\Phi_i^f D_i\omega_i(k+f|t,k)$$

$$\text{s.t. }\ \omega_i(k+s|k)\in\mathcal{W}_i,\ s\in\mathbb{Z}[1,T_s-1]$$

$$\xi_i(t,0)=\mathbf{0},\ z_i(k|t,k)=\left[\hat{x}_i^T(k|t,k)\ \ \vec{c}_i^T(t,k)\right]^T$$

$$\vec{Q}_{ij}=\mathrm{diag}\{Q_{ij},\dots,Q_{ij},\mathbf{0}\},\ \vec{R}_i=\mathrm{diag}\{R_i,\dots,R_i\}$$

$$C_{i1}=\left[C_{xx}^i\ \ C_{xc}^i\right],\ C_{i2}=\left[C_{ux}^iC_{xx}^i\ \ C_{ux}^iC_{xc}^i+I\right]$$

$$\vec{Q}_i=\mathrm{diag}\{Q_i,\dots,Q_i,P_i\}$$

$$\Psi_i=\begin{bmatrix}\Phi_i & B_iE_i\\ \mathbf{0}M_i & \end{bmatrix},\ B_{xu}^i=\begin{bmatrix}b_x^{iT}[I\ \mathbf{0}]\\ b_u^{iT}[K_i\ E_i]\end{bmatrix}$$

$$E_i=\begin{bmatrix}I & \mathbf{0} & \cdots & \mathbf{0}\end{bmatrix},\ M_i=\begin{bmatrix}\mathbf{0} & I\\ \mathbf{0} & \mathbf{0}\end{bmatrix}$$

$$H_i=\begin{bmatrix}h_x^{iT} & h_u^{iT}\end{bmatrix}^T,\ B_\omega^i=\begin{bmatrix}b_x^i & K_i^Tb_u^i\end{bmatrix}^T$$

$$C_{ux}^i=\begin{bmatrix}K_i & \cdots & \mathbf{0} & \mathbf{0}\\ \vdots & \ddots & \vdots & \vdots\\ \mathbf{0} & \cdots & K_i & \mathbf{0}\end{bmatrix},\ C_{xx}^i=\begin{bmatrix}I\\ \Phi_i\\ \vdots\\ \Phi_i^{T_s}\end{bmatrix}$$

$$C_{xc}^i=\begin{bmatrix}\mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ B_i & \cdots & \mathbf{0} & \mathbf{0}\\ \vdots & \ddots & \vdots & \vdots\\ \Phi_i^{T_s-2}B_i & \cdots & B_i & \mathbf{0}\\ \Phi_i^{T_s-1}B_i & \cdots & \Phi_iB_i & B_i\end{bmatrix}$$

$$\begin{aligned}\mathbb{E}\{\hat{Q}_i(m)\}=&\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m)C_{i1}^T\vec{Q}_{ij}C_{i1}\\&+C_{i2}^T\vec{R}_iC_{i2}+C_{i1}^T\vec{Q}_iC_{i1}.\end{aligned}\tag{15}$$

Proof: Substituting (10) into (13), (13) becomes

$$\begin{aligned}
&J_i(\vec{x}_i(t,k),\vec{u}_i(t,k),\vec{x}_{-i}^{i*}(t-1,k),r(k))\\
&\quad=\|C_{ux}^i(C_{xx}^i\hat{x}_i(k|t,k)+C_{xc}^i\vec{c}_i(t,k))+\vec{c}_i(t,k)\|^2_{\vec{R}_i}\\
&\qquad+\|C_{xx}^i\hat{x}_i(k|t,k)+C_{xc}^i\vec{c}_i(t,k)\|^2_{\vec{Q}_i}+\sum_{j\in\mathcal{N}_i(r(k))}a_{ij}(r(k))\\
&\qquad\times\|C_{xx}^i\hat{x}_i(k|t,k)+C_{xc}^i\vec{c}_i(t,k)-\vec{x}_j^*(t-1,k)\|^2_{\vec{Q}_{ij}}
\end{aligned}\tag{16}$$

with $\vec{x}_i(t,k)=C_{xx}^i\hat{x}_i(k|t,k)+C_{xc}^i\vec{c}_i(t,k)$. Then, (16) can be rewritten as (14) with $z_i(k|t,k)=[\hat{x}_i^T(k|t,k)\ \ \vec{c}_i^T(t,k)]^T$. Consider

$$\begin{aligned}
x_i(k+s|t,k)&=\hat{x}_i(k+s|t,k)+e_i(k+s|t,k)\\
\hat{x}_i(k+s+1|t,k)&=\Phi_i\hat{x}_i(k+s|t,k)+B_i\hat{c}_i(k+s|t,k)\\
e_i(k+s+1|t,k)&=\Phi_ie_i(k+s|t,k)+D_i\omega_i(k+s|t,k)
\end{aligned}$$

with $e_i(k|t,k)=\mathbf{0}$ and $\hat{x}_i(k|t,k)=x_i(k)$. It is easy to obtain that

$$\begin{aligned}
x_i(k+s|t,k)&=[I\ \ \mathbf{0}]z_i(k+s|t,k)+e_i(k+s|t,k)\\
u_i(k+s|t,k)&=[K_i\ \ E_i]z_i(k+s|t,k)+K_ie_i(k+s|t,k)\\
z_i(k+s+1|t,k)&=\Psi_iz_i(k+s|t,k).
\end{aligned}$$

Then, (11) and (12) are extended to

$$g_i(t,k,s)=B_{xu}^i\Psi_i^sz_i(k|t,k)-H_i+\xi_i(t,s),\ s\in\mathbb{Z}[0,T_s-1].\qquad\blacksquare$$

Remark 4: In this paper, the noncooperative game for the MPSs is considered. According to the proposed iterative algorithm in Algorithm 1, the iteration would be terminated when no players in $\mathcal{V}(r(k))$ can benefit from changing its action while other players in $\mathcal{V}(r(k))$ keep their strategies unchanged, which is exactly the definition of NE. Furthermore, we have proven that the noncooperative game can finally converge to the ε-NE. In the framework of MPC, the state and input constraints are taken into consideration for the noncooperative game. By using the proposed iterative algorithm, we do not have to obtain NE analytically, which is actually impossible to acquire if the input/state constraints and external disturbances are considered.

B. Convergence of the Iterative Algorithm

The first theorem provides the sufficient condition to guarantee that Algorithm 1 is convergent and the mean-square uniformly bounded stability is obtained for the noncooperative game at each time instant k as number of iteration $t\to+\infty$.

Theorem 1: If there exists a positive definite symmetry matrix $P_X(m)$ ($\forall m\in M$), such that

$$2\Phi_X^T(m)P_X(m)\Phi_X(m)-P_X(m)<0\tag{17}$$

holds with

$$\Phi_X(m)=\begin{bmatrix}\mathbf{0} & \tilde{A}_{12}(m) & \cdots & \tilde{A}_{1N}(m)\\ \tilde{A}_{21}(m) & \mathbf{0} & \cdots & \tilde{A}_{2N}(m)\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{A}_{N1}(m) & \tilde{A}_{N2}(m) & \cdots & \mathbf{0}\end{bmatrix}$$

$$\tilde{A}_{ij}(m)=\sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m)C_{xc}^i\left(\mathbb{E}\{\hat{Q}_i^{22}(m)\}\right)^{-1}C_{xc}^{iT}\vec{Q}_{ij}$$

$$\mathbb{E}\{\hat{Q}_i(m)\}=\begin{bmatrix}\mathbb{E}\{\hat{Q}_i^{11}(m)\} & \mathbb{E}\{\hat{Q}_i^{12}(m)\}\\ \mathbb{E}\{\hat{Q}_i^{21}(m)\} & \mathbb{E}\{\hat{Q}_i^{22}(m)\}\end{bmatrix},\ m\in M\tag{18}$$

where $\mathbb{E}\{\hat{Q}_i(m)\}$, $\vec{Q}_{ij}$ and C_{i1} are given in Lemma 1, then Algorithm 1 can guarantee that the noncooperative game of the MPSs converge to the ε-NE. Furthermore, $(\vec{x}_i^{\mathrm{N}*}(k), \vec{u}_i^{\mathrm{N}*}(k), \vec{x}_{-i}^{i\mathrm{N}*}(k))$ $(i \in \mathcal{V}(m))$ is the ε-NE of the game $F(\mathcal{X}_i, J_i, \mathcal{G}(r(k)))$ with $\varepsilon_i = L_i(D_x^i + D_u^i)$, where

$$D_x^i = \max_{x_i^1, x_i^2 \in \mathcal{X}_i} \|x_i^1 - x_i^2\|_\infty$$

$$D_u^i = \max_{u_i^1, u_i^2 \in \mathcal{U}_i} \|u_i^1 - u_i^2\|_\infty$$

for $i \in \mathcal{V}(m)$.

Proof: Rewrite $\mathbb{E}\{J_i(\vec{x}_i(t,k), \vec{u}_i(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k))\}$ as

$$\begin{aligned}
&\mathbb{E}\Big\{J_i(\vec{x}_i(t,k), \vec{u}_i(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k))\Big\} \\
&= \mathbb{E}\left\{\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) \vec{x}_j^{*T}(t-1,k) \vec{Q}_{ij} \vec{x}_j^{*}(t-1,k)\right\} \\
&\quad - 2\mathbb{E}\left\{\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) \hat{x}_i^T(k|t,k) C_{xx}^{iT} \vec{Q}_{ij} \vec{x}_j^{*}(t-1,k)\right\} \\
&\quad - 2\mathbb{E}\left\{\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) \vec{c}_i^T(t,k) C_{xc}^{iT} \vec{Q}_{ij} \vec{x}_j^{*}(t-1,k)\right\} \\
&\quad + \mathbb{E}\Big\{\hat{x}_i^T(k|t,k)\mathbb{E}\{\hat{Q}_i^{11}(m)\}\hat{x}_i(k|t,k)\Big\} \\
&\quad + 2\mathbb{E}\Big\{\vec{c}_i^T(t,k)\mathbb{E}\{\hat{Q}_i^{21}(m)\}\hat{x}_i(k|t,k)\Big\} \\
&\quad + \mathbb{E}\Big\{\vec{c}_i^T(t,k)\mathbb{E}\{\hat{Q}_i^{22}(m)\}\vec{c}_i(t,k)\Big\}
\end{aligned} \tag{19}$$

with

$$\mathbb{E}\{\hat{Q}_i(m)\} = \begin{bmatrix} \mathbb{E}\{\hat{Q}_i^{11}(m)\} & \mathbb{E}\{\hat{Q}_i^{12}(m)\} \\ \mathbb{E}\{\hat{Q}_i^{21}(m)\} & \mathbb{E}\{\hat{Q}_i^{22}(m)\} \end{bmatrix}$$

where $\mathbb{E}\{\hat{Q}_i^{12T}(m)\} = \mathbb{E}\{\hat{Q}_i^{21}(m)\}$. Similarly, we can rewrite $g_i(t,k,s)$ as

$$g_i(t,k,s) = B_{xu}^i \Psi_i^s \left[\hat{x}_i^T(k|t,k) \;\; \vec{c}_i^T(t,k)\right]^T - H_i + \xi_i(t,s).$$

Denote $F_i(s) = B_{xu}^i \Psi_i^s$ and $F_i(s) = [F_{i1}(s) \; F_{i2}(s)]$. Then, we have

$$g_i(t,k,s) = F_{i1}(s)\hat{x}_i(k|t,k) + F_{i2}(s)\vec{c}_i(t,k) - H_i + \xi_i(t,s). \tag{20}$$

According to the Karush-Kuhn-Tucher (KKT) condition [41], (14) suggests us to find a $\vec{c}_i^*(t,k)$ satisfying the following conditions:

$$\vec{c}_i^*(t,k) = \arg\min \mathbb{E}\{L_i(\vec{c}_i(t,k))\}$$

$$\begin{aligned}
L_i(\vec{c}_i(t,k)) &= J_i(\vec{x}_i(t,k), \vec{u}_i(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k)) \\
&\quad + \sum_{s=0}^{T_s-1} \delta_i^T(k+s|t,k) g_i(t,k,s)
\end{aligned}$$

$$g_i(t,k,s) \le \mathbf{0},\; \delta_i(k+s|t,k) \ge \mathbf{0} \tag{21}$$

$$\delta_i^T(k+s|t,k) g_i(t,k,s) = 0 \tag{22}$$

$$\mathbf{0} = \frac{\partial \mathbb{E}\Big\{\bar{L}_i(\vec{c}_i(t,k))\Big\}}{\partial \vec{c}_i(t,k)} \tag{23}$$

for $s \in \mathbb{Z}[0, T_s - 1]$, where $\delta_i(k+s|t,k)$ is the decision variable.

From (19) and (23), we have

$$\begin{aligned}
&\mathbb{E}\{\hat{Q}_i^{22}(m)\}\mathbb{E}\Big\{\vec{c}_i^*(t,k)\Big\} \\
&= -\mathbb{E}\left\{\mathbb{E}\{\hat{Q}_i^{12}(m)\}\hat{x}_i(k|t,k) + \frac{1}{2}\sum_{s=0}^{T_s-1} F_{i2}^T(s)\delta_i(k+s|t,k)\right\} \\
&\quad + \mathbb{E}\left\{\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) C_{xc}^{iT} \vec{Q}_{ij} \vec{x}_j^*(t-1,k)\right\}.
\end{aligned} \tag{24}$$

Additionally, (21) and (22) are equivalent to the following two cases:

Case I: $\delta_i(k+s|t,k) = \mathbf{0}$ and $g_i(t,k,s) < \mathbf{0}$, $s \in \mathbb{Z}[0, T_s - 1]$.

Case II: $\delta_i(k+s|t,k) \ge \mathbf{0}$ and $g_i(t,k,s) = \mathbf{0}$, $s \in \mathbb{Z}[0, T_s - 1]$.

For Case I, (24) can be rewritten as

$$\begin{aligned}
&\mathbb{E}\{\hat{Q}_i^{22}(m)\}\mathbb{E}\Big\{\vec{c}_i^*(t,k)\Big\} \\
&= \mathbb{E}\left\{\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) C_{xc}^{iT} \vec{Q}_{ij} \vec{x}_j^*(t-1,k)\right\} \\
&\quad - \mathbb{E}\Big\{\mathbb{E}\{\hat{Q}_i^{21}(m)\}\hat{x}_i(k|t,k)\Big\}.
\end{aligned} \tag{25}$$

For Case II, according to (20), we have

$$\mathbb{E}\Big\{F_{i2}(s)\vec{c}_i^*(t,k)\Big\} = \mathbb{E}\{H_i - \xi_i(t,s) - F_{i1}(s)\hat{x}_i(k|t,k)\} \tag{26}$$

for $s \in \mathbb{Z}[0, T_s - 1]$. Combining (24), (25) and (26), we have

$$\begin{aligned}
&\mathcal{M}_i(\hat{x}_i(k|t,k), \vec{c}_i^*(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k) = m) \\
&= \mathbb{E}\left\{\frac{1}{2}\sum_{s=0}^{T_s-1} F_{i2}^T(s)\delta_i(k+s|t,k)\right\} \\
&= \mathbb{E}\left\{\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) C_{xc}^{iT} \vec{Q}_{ij} \vec{x}_j^*(t-1,k)\right. \\
&\quad \left. - \mathbb{E}\{\hat{Q}_i^{22}(m)\}\vec{c}_i^*(t,k)\right\} - \mathbb{E}\Big\{\mathbb{E}\{\hat{Q}_i^{21}(m)\}\hat{x}_i(k|t,k)\Big\}.
\end{aligned}$$

The norm bound of $\mathbb{E}\{\frac{1}{2}\sum_{s=0}^{T_s-1} F_{i2}^T(s)\delta_i(k+s|t,k)\}$ can be obtained by

$$\mathcal{M}_i^*(t,k) = \max\Big\|\mathcal{M}_i(\hat{x}_i(k|t,k), \vec{c}_i^*(t,k), \vec{x}_{-i}^{i*}(t-1,k), r(k) = m)\Big\|$$

s.t.

$$\mathbb{E}\Big\{F_{i2}(s)\vec{c}_i^*(t,k)\Big\} = \mathbb{E}\Big\{H_i - \xi_i(t,s) - F_{i1}(s)\hat{x}_i^*(k|t,k)\Big\}$$

$$b_x^{jT}\hat{x}_j^*(k+s|t-1,k) \le h_x^j - \tilde{\xi}_j(k+s|t-1,k)$$

$$b_x^{iT}\hat{x}_i(k|t,k) \le h_x^i - \tilde{\xi}_i(k|t,k)$$

$$j \in \mathcal{N}_i(m),\; s \in \mathbb{Z}[0, T_s - 1].$$

Therefore, it is easy to obtain

$$\begin{aligned}
&\mathbb{E}\{\hat{Q}_i^{22}(m)\}\mathbb{E}\Big\{\vec{c}_i^*(t,k)\Big\} \\
&= \mathbb{E}\left\{\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) C_{xc}^{iT} \vec{Q}_{ij} \vec{x}_j^*(t-1,k)\right\} \\
&\quad - \mathbb{E}\Big\{\mathbb{E}\{\hat{Q}_i^{21}(m)\}\hat{x}_i(k|t,k) - \omega_x^i(t,k)\Big\}
\end{aligned}$$

where

$$\omega_x^i(t,k) \in \left\{ \mathbb{E}\{\frac{1}{2}\sum_{s=0}^{T_s-1} F_{i2}^T(s)\delta_i(k+s|t,k)\},\ \mathbf{0} \right\}.$$

Then, we have

$$\begin{aligned}\mathbb{E}\left\{\vec{c}_i^*(t,k)\right\} = \mathbb{E}\Bigg\{ &\sum_{j\in\mathcal{N}_i(m)}\sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m) \\ &\times\left(\mathbb{E}\{\hat{Q}_i^{22}(m)\}\right)^{-1} C_{xc}^{iT}\vec{Q}_{ij}\vec{x}_j^*(t-1,k)\Bigg\} \\ &-\mathbb{E}\left\{\left(\mathbb{E}\{\hat{Q}_i^{22}(m)\}\right)^{-1}\mathbb{E}\{\hat{Q}_i^{21}(m)\}\hat{x}_i(k|t,k)\right\} \\ &-\mathbb{E}\left\{\left(\mathbb{E}\{\hat{Q}_i^{22}(m)\}\right)^{-1}\omega_x^i(t,k)\right\}. \end{aligned} \tag{27}$$

From (15) and (27), it follows that:

$$\begin{aligned}\mathbb{E}\left\{\vec{x}_i^*(t,k)\right\} = \mathbb{E}\Bigg\{ &\sum_{j\in\mathcal{N}_i(m)}\tilde{A}_{ij}(m)\vec{x}_j^*(t-1,k) \\ &+\Theta_i\hat{x}_i(k|t,k)+\Gamma_i\omega_x^i(t,k)\Bigg\} \end{aligned} \tag{28}$$

with

$$\tilde{A}_{ij}(m) = \sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m)C_{xc}^i\left(\mathbb{E}\{\hat{Q}_i^{22}(m)\}\right)^{-1}C_{xc}^{iT}\vec{Q}_{ij}$$

$$\Theta_i(m) = C_{xx}^i - C_{xc}^i\left(\mathbb{E}\{\hat{Q}_i^{22}(m)\}\right)^{-1}\mathbb{E}\{\hat{Q}_i^{21}(m)\}$$

$$\Gamma_i(m) = -C_{xc}^i\left(\mathbb{E}\{\hat{Q}_i^{22}(m)\}\right)^{-1}.$$

Denote $X(t,k) = [\vec{x}_1^T(t,k)\ \vec{x}_2^T(t,k)\ \cdots\ \vec{x}_N^T(t,k)]^T$. Then, we have

$$\begin{aligned}&\mathbb{E}\{X(t+1,k)\} \\ &\quad = \mathbb{E}\left\{\Phi_X(m)X(t,k)+\tilde{\Theta}(m)X(0,k)+\Upsilon(m)W(t+1,k)\right\}\end{aligned}$$

with

$$\Upsilon(m) = \mathrm{diag}\{\Gamma_1(m),\Gamma_2(m),\ldots,\Gamma_N(m)\}$$

$$\tilde{\Theta}(m) = \mathrm{diag}\{\Theta_1(m),\Theta_2(m),\ldots,\Theta_N(m)\}$$

$$W(t,k) = [\omega_x^{1T}(t,k)\ \omega_x^{2T}(t,k)\ \cdots\ \omega_x^{NT}(t,k)]^T$$

and $\Phi_X(m)$ is given in (18). Denote

$$\tilde{W}(t,k) = \tilde{\Theta}(m)X(0,k)+\Upsilon(m)W(t+1,k).$$

Choose a Lyapunov candidate

$$V_X(t,k) = X^T(t,k)P_X(m)X(t,k)$$

with a positive definite matrix $P_X(m)$. Letting $\Delta V_X = \mathbb{E}\{V_X(t+1,k)-V_X(t,k)\}$ and (17) hold, we have

$$\begin{aligned}&\mathbb{E}\{V_X(t+1,k)-V_X(t,k)\} \\ &\quad\le \mathbb{E}\left\{X^T(t,k)\left(2\Phi_X^T(m)P_X(m)\Phi_X(m)-P_X(m)\right)X(t,k)\right\} \\ &\qquad+\mathbb{E}\left\{2\tilde{W}^T(t,k)P_X(m)\tilde{W}(t,k)\right\} \\ &\quad\le -\mathbb{E}\left\{\frac{\alpha(m)}{\overline{\lambda}(P_X(m))}\right\}\mathbb{E}\{V_X(t,k)\}+\mathbb{E}\left\{\varrho(m)\|\tilde{W}(t,k)\|_\infty^2\right\}\end{aligned}$$

where

$$\varrho(m) = 2\overline{\lambda}(P_X(m)),\ 0<\alpha(m)\le\min\{\varsigma(m),\overline{\lambda}(P_X(m))\}$$

$$\varsigma(m) = \underline{\lambda}(P_X(m)-2\Phi_X^T(m)P_X(m)\Phi_X(m)).$$

Then, we have

$$\begin{aligned}\mathbb{E}\{V_X(t+1,k)\} \le &\left(1-\mathbb{E}\left\{\frac{\alpha(m)}{\overline{\lambda}(P_X(m))}\right\}\right)\mathbb{E}\{V_X(t,k)\} \\ &+\mathbb{E}\left\{\varrho(m)\|\tilde{W}(t,k)\|_\infty^2\right\}\end{aligned}$$

which yields

$$\begin{aligned}\mathbb{E}\{V_X(t,k)\} \le &\left(1-\mathbb{E}\left\{\frac{\alpha(m)}{\overline{\lambda}(P_X(m))}\right\}\right)^t\mathbb{E}\{V_X(0,k)\} \\ &+\mathbb{E}\left\{\frac{\overline{\lambda}(P_X(m))\varrho(m)}{\alpha(m)}\|\tilde{W}(t,k)\|_\infty^2\right\}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}\mathbb{E}\left\{\|X(t,k)\|^2\right\} \le &\mathbb{E}\left\{\frac{\overline{\lambda}(P_X(m))}{\underline{\lambda}(P_X(m))}\left(1-\frac{\alpha(m)}{\overline{\lambda}(P_X(m))}\right)^t\right\}\|X(0,k)\|^2 \\ &+\mathbb{E}\left\{\frac{\varrho(m)\overline{\lambda}(P_X(m))}{\alpha(m)\underline{\lambda}(P_X(m))}\|\tilde{W}(t,k)\|_\infty^2\right\}.\end{aligned}$$

When $t\to+\infty$, it can be obtained that

$$\mathbb{E}\left\{\|X(+\infty,k)\|^2\right\} \le \mathbb{E}\left\{\frac{\varrho(m)\overline{\lambda}(P_X(m))}{\alpha(m)\underline{\lambda}(P_X(m))}\|\tilde{W}(t,k)\|_\infty^2\right\}.$$

Therefore, the mean-square uniformed bounded stability is proved for the noncooperative game and the convergence of Algorithm 1 is proved.

In the following, the proof of obtaining the ε-NE of the game $F(\mathrm{Con}_{\eta_i},J_i,\mathcal{G})$ by Algorithm 1 is given.

When Algorithm 1 meets the termination conditions, i.e., $t=\bar{t}$ or $|J_i^*(t,k,r(k))-J_i^*(t-1,k,r(k))|\le\bar{J}_i$ for $t\in\mathbb{Z}[0,\bar{t}]$, we have $\vec{x}_i^{\mathrm{N}*}(k)=\vec{x}_i^*(t,k)$, $\vec{u}_i^{\mathrm{N}*}(k)=\vec{u}_i^*(t,k)$ and $\vec{x}_j^{\mathrm{N}*}(k)=\vec{x}_j^*(t,k)$ for $i\in\mathcal{V}(r(k))$ and $j\in\mathcal{N}_i(r(k))$.

Note that (13) is a quadratic function and its value is limited on a compact subset of $\mathbb{R}^{n_x}\times\mathbb{R}^{n_x}$, it is concluded that (13) is Lipschitz continuous [4] and there exists $L_i>0$ such that

$$\begin{aligned}&|J_i(\vec{x}_i^{\mathrm{N}*}(k),\vec{u}_i^{\mathrm{N}*}(k),\vec{x}_{-i}^{i\mathrm{N}*}(k),r(k)) \\ &\qquad - J_i(\vec{x}_i(t,k),\vec{u}_i(t,k),\vec{x}_{-i}^{i\mathrm{N}*}(k),r(k))| \\ &\qquad\le L_i\|\vec{x}_i^{\mathrm{N}*}(k)-\vec{x}_i(t,k)\|_\infty+L_i\|\vec{u}_i^{\mathrm{N}*}(k)-\vec{u}_i(t,k)\|_\infty\end{aligned} \tag{29}$$

where $(\vec{x}_i(t,k),\vec{u}_i(t,k),\vec{x}_{-i}^{i\mathrm{N}*}(k))$ is the solution of Prob 2 when the termination condition of Algorithm 1 is not satisfied; and $J_i(\vec{x}_i^{\mathrm{N}*}(k),\vec{u}_i^{\mathrm{N}*}(k),\vec{x}_{-i}^{i\mathrm{N}*}(k),r(k))$ is the ε-NE for the noncooperative game $F(\mathcal{X}_i,J_i,\mathcal{G}(r(k)))$.

For $i\in\mathcal{V}$, defining $D_x^i=\max_{x^1,x^2\in\mathcal{X}_i}\|x^1-x^2\|_\infty$ and $D_u^i=\max_{u_i^1,u_i^2\in\mathcal{U}_i}\|u_i^1-u_i^2\|_\infty$, we have

$$\begin{aligned}&J_i(\vec{x}_i^{\mathrm{N}*}(k),\vec{u}_i^{\mathrm{N}*}(k),\vec{x}_{-i}^{i\mathrm{N}*}(k),r(k)) \\ &\qquad\le J_i(\vec{x}_i(t,k),\vec{u}_i(t,k),\vec{x}_{-i}^{i\mathrm{N}*}(k),r(k))+\varepsilon_i\end{aligned}$$

where $\varepsilon_i=L_i(D_x^i+D_u^i)$.

Note that $(\vec{x}_i(t,k),\vec{u}_i(t,k))$ is also the admissible solution of

Prob 1 when the termination condition of Algorithm 1 is not satisfied. We denote $\vec{x}_i(k) = \vec{x}_i(t,k)$ and $\vec{u}_i(k) = \vec{u}_i(t,k)$ which gives rise to

$$J_i(\vec{x}_i^{\mathrm{N}*}(k), \vec{u}_i^{\mathrm{N}*}(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k)) \leq J_i(\vec{x}_i(k), \vec{u}_i(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k)) + \varepsilon_i,\ \forall i \in \mathcal{V}(r(k)).$$

As such, the ε-NE of the game $F(\mathcal{X}_i, J_i, \mathcal{G}(r(k)))$ can be obtained by Algorithm 1. ■

Remark 5: In [16], [42] and [43], the iterative algorithms have been proposed for the noncooperative game under the MPC scheme. However, the convergence conditions of the iterative algorithms have not been provided explicitly. In this paper, we establish the convergence condition of the proposed iterative algorithm for the noncooperative game subjected to state/input constraints and disturbances under the MPC scheme and Markov jump graph. Furthermore, we consider the Markov jump graph of the noncooperative game, which is also different from the results in [16], [42] and [43]. Note that condition (17) can help to select the design of the weight parameters Q_i, R_i and Q_{ij} in cost function (3), which is meaningful in engineering practice.

Remark 6: In Theorem 1, we have obtained $\varepsilon_i = L_i(D_x^i + D_u^i)$. It is easy to obtain

$$D_x^i = \max_{x_i^1, x_i^2 \in \mathcal{X}_i} \|x_i^1 - x_i^2\|_\infty,\ D_u^i = \max_{u_i^1, u_i^2 \in \mathcal{U}_i} \|u_i^1 - u_i^2\|_\infty$$

$$L_i = \max\left\{\left\|\frac{\partial \mathbb{E}\{J_i(t,k)\}}{\partial \hat{x}_i}\right\|_\infty, \left\|\frac{\partial \mathbb{E}\{J_i(t,k)\}}{\partial \hat{u}_i}\right\|_\infty\right\}$$

$$\hat{x}_i \in \mathcal{X}_i,\ \hat{u}_i \in \mathcal{U}_i.$$

By calculating L_i, we can find that L_i is related with Q_i, Q_{ij}, R_i, P_i, $\hat{x}_i$ and $\hat{u}_i$ for $i = 1,2,3,4$ and $j \in \mathcal{N}_i$. Furthermore, D_x^i and D_u^i are related with $\hat{x}_i$ and $\hat{u}_i$. It is obvious that $\hat{x}_i$ and $\hat{u}_i$ are constrained by $\mathcal{X}_i = \{x_i : b_x^{iT} x_i(k) \leq h_x^i\}$ and $\mathcal{U}_i = \{u_i : b_u^{iT} u_i(k) \leq h_u^i\}$, respectively. Therefore, the accuracy ε_i of the ε-NE is related with Q_i, R_i, Q_{ij}, P_i, b_x^i, b_u^i, h_x^i and h_u^i in theory. Besides, the larger the weighting matrices, the greater the value of ε_i. Moreover, the pairs (b_x^i, h_x^i) and (b_u^i, h_u^i) are used to describe the range of the constraints $\mathcal{X}_i$ and $\mathcal{U}_i$, respectively. The larger the ranges of $\mathcal{X}_i$ and $\mathcal{U}_i$, the greater the value of ε_i.

Remark 7: When the iterative algorithm reaches the termination condition, it means that the termination condition is satisfied at some iteration step $\tilde{t}$ ($\tilde{t} \in \mathbb{Z}[2, \bar{t}]$). That is, player i uses $x_i(k)$ and $\vec{x}_j(\tilde{t}-1,k)$ as the inputs of Prob 2 to obtain the ε-NE of the noncooperative game. Similarly, this means that player i uses $x_i(k)$ and $\vec{x}_j(k) = \vec{x}_j(\tilde{t}-1,k)$ as the inputs of Prob 1 to obtain the ε-NE of the noncooperative game. Therefore, the solution of Prob 2 is equivalent to the solution of Prob 1 when the termination condition of the iterative algorithm is satisfied.

Remark 8: It is known that the optimization problem in the noncooperative game can be fragile. Under the influence of disturbance, each player can not achieve the optimal cost, and can only make the cost close to the optimal cost. ε-NE is used to describe the degree to which the real cost deviates from the optimal cost, and theoretically gives the cost in the worst case. Actually, ε-NE has been widely used in traffic [44], wireless systems [45], electricity [46] and commodity markets [47].

Remark 9: In this paper, the optimization problem of each player is a convex optimization problem with the strict convex cost function and convex closed constraints. Therefore, there must exist an optimal solution of the optimization problem for each player. Furthermore, we have proved the recursively feasibility of the optimization problem such that there must exist a feasible solution of the optimization problem for each player at each time step k. Considering the iterative algorithm proposed in the paper, each player solves its optimization problem several times during each time instant k until the termination condition is satisfied. It should be noted that the termination condition is designed by considering the definition of ε-NE in Definition 2. Therefore, combining with the existence of the optimal solution in each iteration step t at each time instant k and the termination condition designed by the definition of ε-NE, it can be proved that the ε-NE solution must exist of the noncooperative game.

Corollary 1: If the states of each player do not reach the constraint boundary, we have

$$\mathbb{E}\left\{\vec{x}_i^*(t,k)\right\} = \mathbb{E}\left\{\sum_{j \in \mathcal{N}_i(m)} \tilde{A}_{ij}(m)\vec{x}_j^*(t-1,k) + \Theta_i \hat{x}_i(k|t,k)\right\}$$

such that

$$\mathbb{E}\{X(t+1,k)\} = \mathbb{E}\{\Phi_X(m)X(t,k) + \tilde{\Theta}X(0,k)\}$$

which indicates

$$\mathbb{E}\{X(t+n,k)\} = \mathbb{E}\left\{\Phi_X^n X(t,k) + \left(\Phi_X^{n-1} + \cdots + I\right)\tilde{\Theta}X(0,k)\right\},\ n \in \mathbb{Z}^+.$$

It is obvious that $X(+\infty,k)$ is the NE as $n \to +\infty$. Therefore, in this case, the NE can be obtained by Algorithm 1.

Proof: The proof is similar to that in Theorem 1, and it is omitted here. ■

Remark 10: In Theorem 1, the sufficient conditions of obtaining the ε-NE of the MPSs are established. Comparing Theorem 1 and Corollary 1, it can be found that the existence of input/state constraints and disturbances makes the MPSs unable to obtain the pure NE. It is difficult to prove that the traditional concept NE solution must exist within the convex closed constraints. Therefore, in this paper, the ε-NE is considered to describe the results of noncooperative game.

C. Feasibility and Mean-Square Uniform Bounded Stability

In this part, we present the recursive feasibility of Prob 1 and the mean-square uniform bounded stability of the closed-loop system. Firstly, the following lemma is given to demonstrate the feasibility of Prob 1.

Lemma 2: If the Prob 1 has a feasible solution at time instant k, then it also has a feasible solution at time instant $k+1$.

Proof: From Lemma 1, we arrive at the conclusion that Prob 1 is equivalent to the optimization problem (14). Therefore, if feasibility of (14) holds, then the feasibility of (5) also holds. Next, we aim to prove the feasibility of (14). Choose a candidate input sequence for player i at step $k+1$ as $\vec{\tilde{c}}_i(k+1) =$

$M_i\vec{c}_i^*(k)$ such that we have

$$\tilde{z}_i(k+1) = \Psi_i z_i^*(k) + \bar{D}_i\omega_i(k),\ \bar{D}_i = \begin{bmatrix} D_i^T & \mathbf{0}^T \end{bmatrix}^T$$

with the optimal $z_i^*(k) = [x_i^{T*}(k)\ \vec{c}_i^{T*}(k)]^T$ and the feasible $\tilde{z}_i(k+1)$. By using Theorem 3.1 in [48], we can easily obtain that, if $z_i^*(k)$ satisfies

$$B_{xu}^i \Psi_i^s z_i^*(k) \le H_i - \xi_i(s)$$

for $s \in \mathbb{Z}[0, T_s - 1]$, then $\tilde{z}_i(k+1)$ satisfies

$$B_{xu}^i \Psi_i^s \tilde{z}_i(k+1) \le H_i - \xi_i(s)$$

for $s \in \mathbb{Z}[0, T_s - 1]$. Therefore, the feasibility of Prob 1 can be guaranteed by that of (14). ■

In the following theorem, sufficient conditions are established to guarantee the mean-square uniform boundedness of system (1) by using the proposed control input.

Theorem 2: If there exist $\Xi_i > 0$ and Υ_i such that the linear matrix inequality (LMI)

$$\begin{bmatrix} -\Xi_i & \Xi_i A_i^T + \Upsilon_i B_i^T & \Upsilon_i & \Xi_i \\ * & -\Xi_i & \mathbf{0} & \mathbf{0} \\ * & * & -R_i^{-1} & \mathbf{0} \\ * & * & * & -Q_i^{-1} \end{bmatrix} < 0 \tag{30}$$

holds, then system (1) is mean-square uniformly bounded and $K_i = \Upsilon_i^T \Xi_i^{-1}$. Furthermore, the state of system (1) converges to the set $\mathbb{S}_i$ with $\mathbb{S}_i = \{x_i(k) :\ \mathbb{E}\{\|x_i(k)\|_{Q_i}^2\} \le \bar{\sigma}_i\}$ where

$$\begin{aligned}\bar{\sigma}_i = {} & \bar{\lambda}(P_i)\bar{\omega}_i^2(T_s) + \frac{2\bar{\lambda}(P_i)\bar{\omega}_i(T_s)}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}}\|h_x^i - \bar{\xi}_i(T_s)\| \\ & + \sum_{s=1}^{T_s-1}\left\{\bar{\lambda}(Q_i)(\bar{\omega}_i(s))^2 + \frac{2\bar{\lambda}(Q_i)\bar{\omega}_i(s)}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}}\|h_x^i - \bar{\xi}_i(s)\|\right\} \\ & + \sum_{s=1}^{T_s-1}\sum_{j\in N_i(m)}\sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m)\left(\tilde{h}_x^i(s) + \tilde{h}_x^j(s)\right)^2\end{aligned}$$

$$\bar{\xi}_i(s) = \max\sum_{f=0}^{s-1} b_x^{iT}\Phi_i^f D_i\omega_i(f),\ \omega_i(f) \in \mathcal{W}_i,\ \bar{\xi}_i(0) = \mathbf{0}$$

$$\bar{\omega}_i(s) = \max\|\Phi_i^s D_i\omega_i(k)\|,\ \omega_i(k) \in \mathcal{W}_i,\ s \in \mathbb{Z}[0, T_s - 1]$$

$$\tilde{h}_x^i(s) = \frac{\sqrt{\bar{\lambda}(Q_{ij})}}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}}\|h_x^i - \bar{\xi}_i(s)\|$$

$$\tilde{h}_x^j(s) = \frac{\sqrt{\bar{\lambda}(Q_{ij})}}{\sqrt{\underline{\lambda}(b_x^j b_x^{jT})}}\|h_x^j - \bar{\xi}_j(s)\|. \tag{31}$$

Proof: Denote

$$\begin{aligned}& V_i^*(k, r(k) = m) \\ & \quad = J_i(\vec{x}_i^*(k), \vec{u}_i^*(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k) = m) \\ & \tilde{V}_i(k+1, r(k+1)|r(k) = m) \\ & \quad = J_i(\vec{\tilde{x}}_i(k+1), \vec{\tilde{u}}_i(k+1), \vec{\tilde{x}}_{-i}^{\mathrm{I}}(k+1), r(k+1)|r(k) = m)\end{aligned}$$

where $\vec{\tilde{x}}_i(k+1)$, $\vec{\tilde{u}}_i(k+1)$ and $\vec{\tilde{x}}_{-i}^{\mathrm{I}}(k+1)$ are the feasible sequence at $k+1$; $J_i(\vec{\tilde{x}}_i(k+1), \vec{\tilde{u}}_i(k+1), \vec{\tilde{x}}_{-i}^{\mathrm{I}}(k+1), r(k+1)|r(k) = m)$ is the feasible cost at $k+1$; and $J_i(\vec{x}_i^*(k), \vec{u}_i^*(k), \vec{x}_{-i}^{i\mathrm{N}*}(k), r(k) = m)$ is the optimal cost at k.

Denote

$$\mathbb{E}\{\Delta V_i(k)\} = \mathbb{E}\left\{\tilde{V}_i(k+1, r(k+1)|r(k) = m) - V_i^*(k, r(k) = m)\right\}.$$

Split $\mathbb{E}\{\Delta V_i(k)\} = \mathbb{E}\{\Delta_1\} + \mathbb{E}\{\Delta_2\} + \mathbb{E}\{\Delta_3\} + \mathbb{E}\{\Delta_4\}$ by

$$\begin{aligned}\mathbb{E}\{\Delta_1\} = {} & \mathbb{E}\left\{\sum_{s=1}^{T_s-1}\left\{\|\tilde{x}_i(k+s|k+1)\|_{Q_i}^2 - \|\hat{x}_i^*(k+s|k)\|_{Q_i}^2\right\}\right\} \\ & + \mathbb{E}\left\{\sum_{s=1}^{T_s-1}\left\{\|\tilde{u}_i(k+s|k+1)\|_{R_i}^2 - \|\hat{u}_i^*(k+s|k)\|_{R_i}^2\right\}\right\} \\ \mathbb{E}\{\Delta_2\} = {} & \mathbb{E}\left\{\|\tilde{x}_i(k+T_s|k+1)\|_{Q_i}^2 + \|\tilde{u}_i(k+T_s|k+1)\|_{R_i}^2\right\} \\ & + \mathbb{E}\left\{\|\tilde{x}_i(k+1+T_s|k+1)\|_{P_i}^2 - \|\hat{x}_i^*(k+T_s|k)\|_{P_i}^2\right\} \\ \mathbb{E}\{\Delta_3\} = {} & -\mathbb{E}\{\|\hat{x}_i^*(k|k)\|_{Q_i}^2 - \|\hat{u}_i^*(k|k)\|_{Q_i}^2\} \\ \mathbb{E}\{\Delta_4\} = {} & \mathbb{E}\left\{\sum_{s=1}^{T_s-1}\left\{\sum_{j\in N_i(m)}\sum_{\mu=1}^{q}\pi_{m\mu}a_{ij}(m)\right.\right. \\ & \times\|\tilde{x}_i(k+s|k+1) - \tilde{x}_j(k+s|k+1)\|_{Q_{ij}}^2 \\ & \left.\left. - \sum_{j\in N_i(m)} a_{ij}(m)\|\hat{x}_i^*(k+s|k) - \hat{x}_j^{\mathrm{N}*}(k+s|k)\|_{Q_{ij}}^2\right\}\right\}.\end{aligned}$$

For $\mathbb{E}\{\Delta_1\}$, we have $\tilde{u}_i(k+s|k+1) = \hat{u}_i^*(k+s|k)$ for $s \in \mathbb{Z}[1, T_s - 1]$ such that

$$\begin{aligned}\Delta_1 = {} & \mathbb{E}\left\{\sum_{s=1}^{T_s-1}\left\{\|\tilde{x}_i(k+s|k+1)\|_{Q_i}^2 - \|\hat{x}_i^{\mathrm{N}*}(k+s|k)\|_{Q_i}^2\right\}\right\} \\ = {} & \mathbb{E}\left\{\sum_{s=1}^{T_s-1}\left(\|\tilde{x}_i(k+s|k+1)\|_{Q_i} - \|\hat{x}_i^{\mathrm{N}*}(k+s|k)\|_{Q_i}\right)\right. \\ & \left.\times\left(\|\tilde{x}_i(k+s|k+1)\|_{Q_i} + \|\hat{x}_i^{\mathrm{N}*}(k+s|k)\|_{Q_i}\right)\right\} \\ \le {} & \mathbb{E}\left\{\sum_{s=1}^{T_s-1}\|\Phi_i^s D_i\omega_i(k)\|_{Q_i}\right. \\ & \left.\times\left(\|\Phi_i^s D_i\omega_i(k)\|_{Q_i} + 2\|\hat{x}_i^*(k+s|k)\|_{Q_i}\right)\right\} \\ \le {} & \mathbb{E}\left\{\sum_{s=1}^{T_s-1}\|\Phi_i^s D_i\omega_i(k)\|_{Q_i}\times\left(\|\Phi_i^s D_i\omega_i(k)\|_{Q_i}\right.\right. \\ & \left.\left. + \frac{2\sqrt{\bar{\lambda}(Q_i)}}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}}\|h_x^i - \tilde{\xi}_i(k+s|k)\|\right)\right\}\end{aligned}$$

where

$$\tilde{\xi}_i(k+s|k) = \max\sum_{f=0}^{s-1} b_x^{iT}\Phi_i^f D_i\omega_i(k+f|k)$$

$$\omega_i(k+f|k) \in \mathcal{W}_i,\ s \in \mathbb{Z}[1, T_s - 1],\ \tilde{\xi}_i(k|k) = \mathbf{0}.$$

Denote

$$\bar{\xi}_i(s) = \max \sum_{f=0}^{s-1} b_x^{iT} \Phi_i^f D_i \omega_i(f),$$

$$\omega_i(f) \in \mathcal{W}_i,\ s \in \mathbb{Z}[1, T_s - 1],\ \bar{\xi}_i(0) = \mathbf{0}.$$

Denote $\bar{\omega}_i(s) = \max \|\Phi_i^s D_i \omega_i(k)\|,\ \omega_i(k) \in \mathcal{W}_i,\ s \in \mathbb{Z}[0, T_s - 1]$. Then, we have

$$\Delta_1 \le \sum_{s=1}^{T_s-1} \left\{ \bar{\lambda}(Q_i)(\bar{\omega}_i(s))^2 + \frac{2\bar{\lambda}(Q_i)\bar{\omega}_i(s)}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}} \|h_x^i - \bar{\xi}_i(s)\| \right\}.$$

For Δ_2, we have

$$\begin{aligned}
\Delta_2 &= \mathbb{E}\Big\{\|\tilde{x}_i(k+T_s|k+1)\|_{Q_i}^2 + \|\tilde{u}_i(k+T_s|k+1)\|_{R_i}^2 \\
&\quad + \|\tilde{x}_i(k+1+T_s|k+1)\|_{P_i}^2\Big\} - \mathbb{E}\Big\{\|\hat{x}_i^{\mathrm{N}*}(k+T_s|k)\|_{P_i}^2\Big\} \\
&\quad + \mathbb{E}\Big\{\|\tilde{x}_i(k+T_s|k+1)\|_{P_i}^2 - \|\tilde{x}_i(k+T_s|k+1)\|_{P_i}^2\Big\} \\
&\le \mathbb{E}\Big\{\|\tilde{x}_i(k+T_s|k+1) - \hat{x}_i^{\mathrm{N}*}(k+T_s|k)\|_{P_i} \\
&\quad \times \Big(\|\tilde{x}_i(k+T_s|k+1)\|_{P_i} + \|\hat{x}_i^{\mathrm{N}*}(k+T_s|k)\|_{P_i}\Big)\Big\} \\
&\quad - \mathbb{E}\Big\{\|\tilde{x}_i^T(k+T_s|k+1)\|_{\bar{Q}_i}^2\Big\} \\
&\le \mathbb{E}\Big\{\Big(\|\Phi_i^{T_s} D_i \omega_i(T_s)\|_{P_i} + 2\|\hat{x}_i^{\mathrm{N}*}(k+T_s|k)\|_{P_i}\Big) \\
&\quad \times \|\Phi_i^{T_s} D_i \omega_i(T_s)\|_{P_i} - \|\Phi_i^{T_s} D_i \omega_i(T_s)\|_{\bar{Q}_i}^2\Big\} \\
&\le \mathbb{E}\left\{\left(\|\Phi_i^{T_s} D_i \omega_i(T_s)\|_{P_i} + \frac{2\sqrt{\bar{\lambda}(P_i)}}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}} \|h_x^i - \bar{\xi}_i(T_s)\|\right)\right. \\
&\quad \left. \times \|\Phi_i^{T_s} D_i \omega_i(T_s)\|_{P_i}\right\} - \mathbb{E}\Big\{\|\Phi_i^{T_s} D_i \omega_i(T_s)\|_{\bar{Q}_i}^2\Big\} \\
&\le \bar{\lambda}(P_i)\bar{\omega}_i^2(T_s) + \frac{2\bar{\lambda}(P_i)\bar{\omega}_i(T_s)}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}} \|h_x^i - \bar{\xi}_i(T_s)\|
\end{aligned}$$

where $-\bar{Q}_i = \Phi_i^T P_i \Phi_i + Q_i + K_i^T R_i K_i - P_i$.

For Δ_3, we have

$$\Delta_3 \le -\mathbb{E}\Big\{\|\hat{x}_i^{\mathrm{N}*}(k|k)\|_{Q_i}^2\Big\} = -\mathbb{E}\Big\{\|x_i(k)\|_{Q_i}^2\Big\}.$$

For Δ_4, we have

$$\begin{aligned}
\Delta_4 &\le \mathbb{E}\Bigg\{\sum_{s=1}^{T_s-1} \sum_{j \in \mathcal{N}_i(m)} \sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) \\
&\quad \times \|\tilde{x}_i(k+s|k+1) - \tilde{x}_j(k+s|k+1)\|_{Q_{ij}}^2\Bigg\} \\
&\le \mathbb{E}\Bigg\{\sum_{s=1}^{T_s-1} \sum_{j \in \mathcal{N}_i(m)} \sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) \\
&\quad \times \Big(\|\tilde{x}_i(k+s|k+1)\|_{Q_{ij}} + \|\tilde{x}_j(k+s|k+1)\|_{Q_{ij}}\Big)^2\Bigg\} \\
&\le \sum_{s=1}^{T_s-1} \sum_{j \in \mathcal{N}_i(m)} \sum_{\mu=1}^{q} \pi_{m\mu} a_{ij}(m) \Big(\tilde{h}_x^i(s) + \tilde{h}_x^j(s)\Big)^2
\end{aligned}$$

where

$$\tilde{h}_x^i(s) = \frac{\sqrt{\bar{\lambda}(Q_{ij})}}{\sqrt{\underline{\lambda}(b_x^i b_x^{iT})}} \|h_x^i - \bar{\xi}_i(s)\|$$

$$\tilde{h}_x^j(s) = \frac{\sqrt{\bar{\lambda}(Q_{ij})}}{\sqrt{\underline{\lambda}(b_x^j b_x^{jT})}} \|h_x^j - \bar{\xi}_j(s)\|.$$

Therefore, we have

$$\mathbb{E}\{\Delta V_i(k)\} \le -\mathbb{E}\{\|\hat{x}_i^*(k|k)\|_{Q_i}^2\} + \bar{\sigma}_i$$

with $\bar{\sigma}_i$ given in (31), which suggests that the states would converge to the set $\mathbb{S}_i$, and hence system (1) is mean-square uniformly bounded. ■

Remark 11: In this paper, the distributed MPC method is studied for the noncooperative MPSs. Although the noncooperative game based on MPC has been studied in [15], [16], the proposed method in this paper has the distinguishing advantages: 1) the proposed distributed stochastic MPC scheme is capable of handling the state/input constraints which are pervasive in engineering practice; 2) the convergence of the proposed iterative algorithm to obtain the ε-NE is proven; 3) the feasibility and stability of the MPSs are proven under the Markov jump graph.

Remark 12: In Theorem 2, we obtain (30) to guarantee the stability of the closed-loop system, which is a sufficient condition. On the one hand, the stability of the closed-loop system can be guaranteed if (30) holds. Actually, the main factor affecting the conservatism of (30) is the term $Q_i + K_i^T R_i K_i$. The smaller the weight matrices Q_i and R_i, the easier it is to solve (30). Furthermore, (30) can still be applied to most control systems when the weight matrices Q_i and R_i are selected properly, such as aircraft systems [49], spacecraft systems [50], circuit systems [51] and so on.

IV. Numerical Example

In this part, the numerical simulations on space on-orbit assembly are carried out. The goal is to formulate one virtue spacecraft with 4 noncooperative spacecrafts. Consider a network whose topology is represented by an undirected Markov jump graph with the set $\mathcal{V}(r(k)) = \{1,2,3,4\}$ and the adjacency matrix is given as $\mathcal{A}(r(k)) = [a_{ij}(r(k))]$. Furthermore, the finite set M is given as $M = \{1,2,\ldots,5\}$ and its transition rate matrix is given as

$$\Pi = \begin{bmatrix} 0.2 & 0.1 & 0.5 & 0.1 & 0.3 \\ 0.3 & 0.4 & 0.1 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.2 & 0.2 \\ 0.2 & 0.3 & 0.2 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.1 & 0.3 & 0.2 \end{bmatrix}.$$

The corresponding weighted adjacency matrices $A(r(k))$ are given as

$$\mathcal{A}(r(k)=1) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathcal{A}(r(k)=2)=\begin{bmatrix}0&1&0&0\\1&0&0&1\\0&0&0&1\\0&1&1&0\end{bmatrix}$$

$$\mathcal{A}(r(k)=3)=\begin{bmatrix}0&1&0&1\\1&0&0&1\\0&0&0&1\\1&1&1&0\end{bmatrix}$$

$$\mathcal{A}(r(k)=4)=\begin{bmatrix}0&1&0&1\\1&0&1&0\\0&1&0&1\\1&0&1&0\end{bmatrix}$$

$$\mathcal{A}(r(k)=5)=\begin{bmatrix}0&0&1&1\\0&0&0&0\\1&0&0&1\\1&0&1&0\end{bmatrix}.$$

Therefore, the relative motion equations of 4 spacecrafts and their preset points can be established. Therefore, the relative motion equations of four spacecrafts and their preset points can be established. Consider the C-W equation in [50] as

$$\ddot{X}_i(t)-2\tilde{\omega}_i\dot{Y}_i(t)=u_{i,X}(t)+\omega_{i,X}(t)$$
$$\ddot{Y}_i(t)+2\tilde{\omega}_i\dot{X}_i(t)-3\tilde{\omega}_i^2Y_i(t)=u_{i,Y}(t)+\omega_{i,Y}(t)$$
$$\ddot{Z}_i(t)+\tilde{\omega}_i^2z_i(t)=u_{i,Z}(t)+\omega_{i,Z}(t)$$

for $i=1,2,3,4$. Therefore, we have

$$\dot{x}_i(t)=\tilde{A}_ix_i(t)+\tilde{B}_iu_i(t)+\tilde{D}_i\omega_i(t)$$

with

$$\tilde{A}_i=\begin{bmatrix}0&1&0&0&0&0\\0&0&0&2\tilde{\omega}_i&0&0\\0&0&0&1&0&0\\0&0&-2\tilde{\omega}_i&3\tilde{\omega}_i^2&0&0\\0&0&0&0&0&1\\0&0&0&0&-\tilde{\omega}_i^2&0\end{bmatrix}$$

$$\tilde{B}_i=\tilde{D}_i=\begin{bmatrix}0&0&0\\1&0&0\\0&0&0\\0&1&0\\0&0&0\\0&0&1\end{bmatrix}$$

$$x_i=[X_i\ \dot{X}_i\ Y_i\ \dot{Y}_i\ z_i\ \dot{Z}_i]^T,\ \omega_i(t)=[\omega_{i,X}\ \omega_{i,Y}\ \omega_{i,Z}]^T$$
$$u_i=[u_{i,X}\ u_{i,Y}\ u_{i,Z}]^T,\ \tilde{A}_1=\tilde{A}_2=\tilde{A}_3=\tilde{A}_4.$$

Let

$$X_1=x_{i,1},\ \dot{X}_i=x_{i,2},\ Y_i=x_{i,3}$$
$$\dot{Y}_i=x_{i,4},\ z_i=x_{i,5},\ \dot{Z}_i=x_{i,6}.$$

Choose $\tilde{\omega}_i=7.2722\times10^{-5}$ [50]. Selecting the sampling period $T=0.01$ s, we obtain the discrete-time state-space representation of the C-W functions as

$$\boldsymbol{x}_i(k+1)=A_i\boldsymbol{x}_i(k)+B_iu_i(k)+D_i\omega_i(k)$$

with $A_i=T\tilde{A}_i+I$, $B_i=T\tilde{B}_i$ and $D_i=T\tilde{D}_i$. The constraints are given as

$$-h_x^i\le x_i(k)\le h_x^i,\ -h_u^i\le u_i(k)\le h_u^i,\ -h_\omega^i\le\omega_i(k)\le h_\omega^i$$
$$h_u^i=100\times[10\ \ 5\ \ 10\ \ 5\ \ 10\ \ 5]^T,\ h_\omega^i=[10\ \ 5\ \ 10\ \ 5\ \ 10\ \ 5]^T$$
$$h_x^i=100\times[10\ \ 10\ \ 5\ \ 5\ \ 10\ \ 10\ \ 5\ \ 5\ \ 10\ \ 10\ \ 5\ \ 5]^T.$$

Select $Q_i=I$, $R_i=I$ and $Q_{ij}=I$ ($j\in\mathcal{N}_i(r(k))$) such that (17) holds. According to (30), we have

$$K_i^T=\begin{bmatrix}-1.5094&-0.00006&0\\-2.3856&-0.0001&0\\0.00002&-1.5092&0\\-0.0001&-2.3855&0\\0&0&-1.5094\\0&0&-2.3856\end{bmatrix},\quad i=1,2,3,4$$

$$\Xi_i=\begin{bmatrix}\tilde{\Xi}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&\tilde{\Xi}&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\tilde{\Xi}\end{bmatrix},\ \tilde{\Xi}=\begin{bmatrix}0.0059&-0.0038\\-0.0038&0.0065\end{bmatrix}$$

$$\Upsilon_i=\begin{bmatrix}0.00008&0&0\\-0.0099&0&0\\0&0.00008&0\\0&-0.0099&0\\0&0&0.00008\\0&0&-0.0099\end{bmatrix},\quad i=1,2,3,4.$$

Select the initial states as

$$x_1(0)=[600\ \ 0\ \ 200\ \ 0\ \ 500\ \ 0]^T$$
$$x_2(0)=-[700\ \ 0\ \ 200\ \ 0\ \ 600\ \ 0]^T$$
$$x_3(0)=[300\ \ 0\ \ 50\ \ 0\ \ 10\ \ 0]^T$$
$$x_4(0)=-[200\ \ 0\ \ 200\ \ 0\ \ 100\ \ 0]^T.$$

Furthermore, we choose the termination conditions of Algorithm 1 as $\bar{t}=50$ and $\bar{J}_i=0.0001$. By selecting $P_X(m)=I$, (17) holds.

In order to ensure the accuracy of the results, 100 Monte Carlo simulations are carried out and the average results of these 100 Monte Carlo simulations are shown. Furthermore, $x_{i,s}$ ($s\in\{1,2,3,4,5,6\}$) is used to denote the s-th component of x_i. The corresponding results of system i ($i=1,2,3,4$) are shown in Figs. 2–9. Specifically, Figs. 2–5 plot the states of the four spacecraft systems and Figs. 6–9 plot $J_i^*(t,2)-J_i^*(t-1,2)$. Furthermore, Table I gives the cost error of the four systems in the iteration at $k=2$. From Figs. 6–9 and Table I, it is easy to find that the spacecraft obtains the optimal cost iteratively at each time instant of the noncooperative game $F(\mathcal{X}_i,J_i,\mathcal{G}(r(k)))$. As a result, from Figs. 2–9 and Table I, the effectiveness of the proposed method is verified.

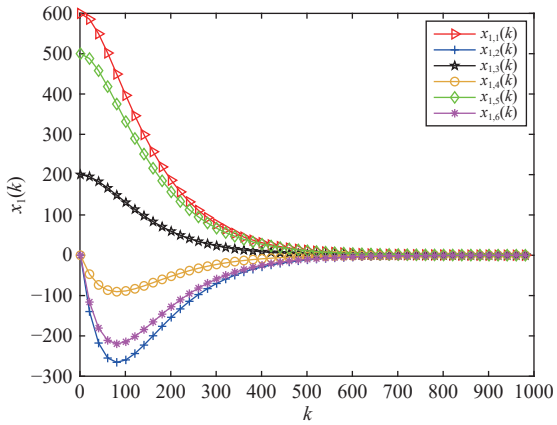

Fig. 2. The states of System 1.

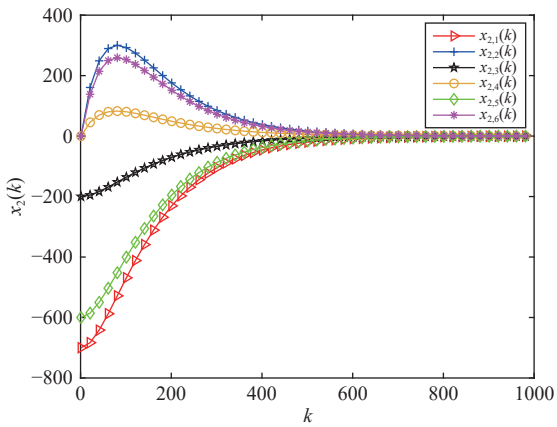

Fig. 3. The states of System 2.

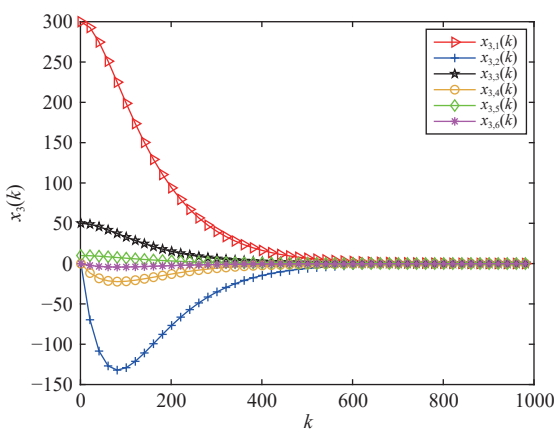

Fig. 4. The states of System 3.

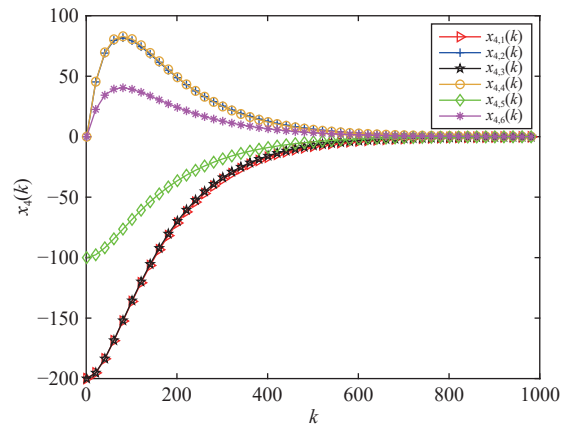

Fig. 5. The states of System 4.

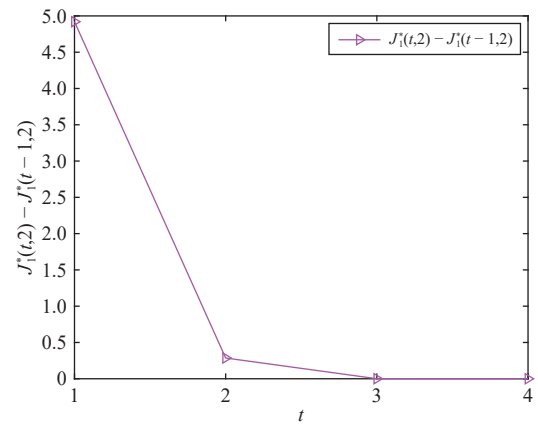

Fig. 6. The iteration error $J_1^*(t,2)-J_1^*(t-1,2)$ of System 1.

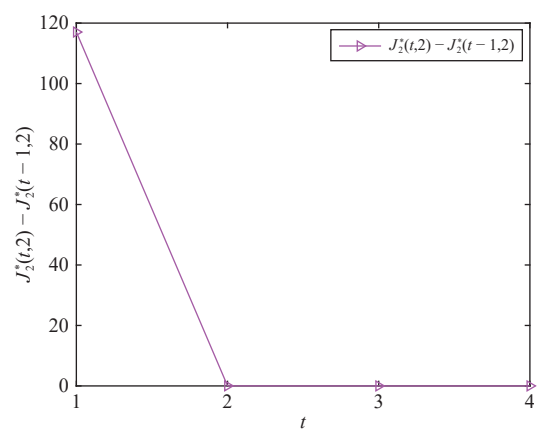

Fig. 7. The iteration error $J_2^*(t,2)-J_2^*(t-1,2)$ of System 2.

V. Conclusion

This paper has studied the noncooperative game problem of the MPSs, where a distributed stochastic MPC scheme has been designed to cope with the state/input constraints and the disturbances. In presence of the considered complexities, the noncooperative game problem has been solved by using an iterative algorithm and the ε-NE has been obtained in a distributed manner. Furthermore, sufficient conditions have been provided for the proposed iterative algorithm such that each player could converge to the ε-NE. In particular, sufficient conditions have been established to guarantee the mean-square uniform boundedness for the MPSs by using the proposed MPC inputs. Finally, a numerical example has been studied to verify the effectiveness of the proposed method.

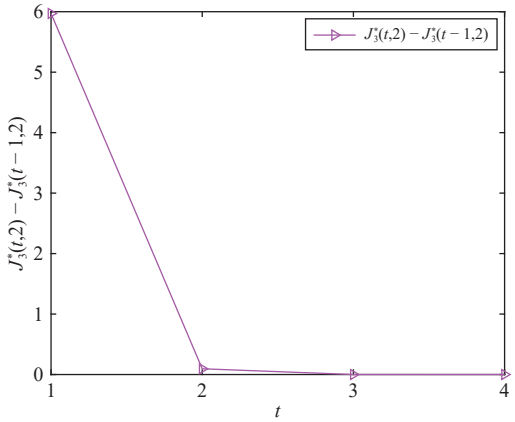

Fig. 8. The iteration error $J_3^*(t,2) - J_3^*(t-1,2)$ of System 3.

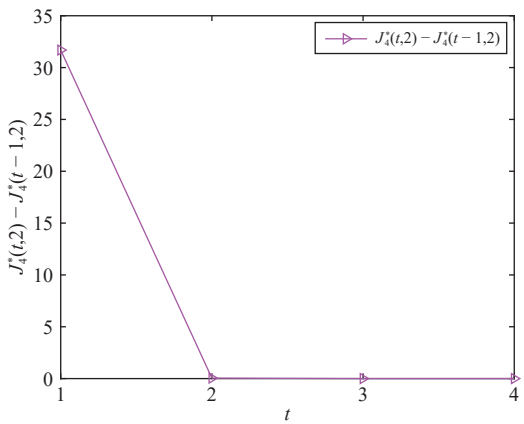

Fig. 9. The iteration error $J_4^*(t,2) - J_4^*(t-1,2)$ of System 4.

TABLE I
THE ITERATION ERROR OF SYSTEMS AT $k = 2$

System	Iteration			
	$t = 1$	$t = 2$	$t = 3$	$t = 4$
$i = 1$	4.9205	0.2860	0.0006	0.000
$i = 2$	117.0392	0.0264	0.0009	0.000
$i = 3$	5.9714	0.0952	0.0002	0.000
$i = 4$	31.6920	0.0454	0.0002	0.000

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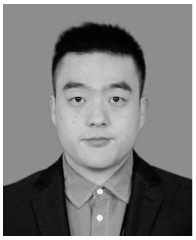

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