Letter

Distributed Dimensionality Reduction Filtering for CPSs Under DoS Attacks

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Dear Editor,

This letter concerns about the distributed dimensionally reduction filtering for a class of cyber physical systems with denial of service (DoS) attacks. Considering each sensor exchanging measurements with neighbors subjected to finite network energy, the dimensionally reduction method is applied. In this case, under random DoS attack, a finite horizon Kalman filtering is designed. Additionally, the optimal transmitting schedules of sensors are proposed to minimize the trace of estimation error covariance, which are solved by applying the mixed strategy. Finally, a simulation example is given to show the effectiveness of proposed distributed filtering.

A wireless sensor network is composed of a number of spatially distributed intelligent sensors, which has been widely used in practice. Since the distributed filtering has the robustness of filter failures and lower computation costs, it has achieved particular attention and wealth of researches have been proposed in [1], [2]. It is noticed that any communication network can only carry finite amount of information at each instant. However, for a large-scale cyber-physical system (CPS), the dimensions of measurement may be high, which cannot be transmitted completely, simultaneously. In order to deal with the problem, Chen *et al.* [3] proposed the dimensionally reduction strategy (DRS) for distributed fusion filtering, in which, the network bandwidth limitation between the sink node and the fusion center was considered. Moreover, the authors in [4] pointed out that for a high-dimensional signal, the DRS may be efficient in traffic reduction as compared with the quantization method. It is noticed that the distributed filtering with high-dimensional measurements are rarely investigated, which is one of motivations of this letter.

In addition to the limited bandwidth influencing on the performance of CPSs, the network security is an another important issue to be addressed. In recent years, a number of researches have been studied the impact of specific malicious attacks such as DoS attacks [5], [6] and false data injection (FDI) attacks [7]. As pointed out in [8] that the DoS attacks are more likely to happen in control systems. With assumption the behavior of DoS attacker described by the Bernoulli random process, Zhang *et al.* [6] gave a method to achieve the optimal jamming schedule from the perspective of attackers and a stability sufficient condition under the optimal attack schedule. From the perspectives of attacker and defender, the game theory was applied in [9] to find the Nash equilibrium between two players. The aforementioned references are investigated under an assumption that the Kalman filtering had entered the steady state. It is worthy noticing that before the Kalman filtering entering steady state, the malicious attacker would make a great destructive. For this reason, the distributed filtering for CPS under DoS attack is investigated in this letter.

Motivated by the above observation, this letter studies the distributed dimensionally reduction filtering for CPSs under DoS attacks. In comparison with those relevant existing results, the contributions of this letter can be summarized as follows. Firstly, a scenario more similar to this work were considered in [10], [11], where the network transmission burden were reduced through event-triggered mechanisms. However, the event-triggered mechanism cannot select and transmit the partial measurements, which is not suitable for a large-scale CPSs with high-dimensional measurements. Secondly, different with [3], [4], the DRS is firstly introduced to the problem of distributed filtering, where the measurements are selected and transmitted. Third, by using the mixed strategy, the optimal DRS is proposed to minimize the trace of estimation error covariance.

Problem formulation: The dynamic of a class of linear discrete-time CPSs is given as

$$x(k+1) = Ax(k) + Bw(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, and $w(k) \in \mathbb{R}^\omega$ is a zero-mean white noise with covariance $Q_w > 0$, A and B are system matrices with appropriate dimensions.

A sensor network is utilized to estimate the states of CPSs, which can be described by a directed graph $\mathcal{G} = (\mathcal{V}, \xi, \mathcal{A})$, which consists of a set of sensors $\mathcal{V} = \{1, 2, \ldots, N\}$ and a set of edges $\xi \in \mathcal{V} \times \mathcal{V}$. The adjacent matrix is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$. If $a_{ij} = 1$, sensor j is neighboring of sensor i. Each sensor measures the outputs of system through

$$y_i(k) = C_i x(k) + v_i(k) \tag{2}$$

where $y_i(k) \in \mathbb{R}^m$ is the measurements of sensor i, and $v_i(k) \in \mathbb{R}^m$ is the channel noise with zero-mean and covariance $Q_{v_i} > 0$, and matrix C_i is given with appropriate dimensions. The system disturbance $\omega(k)$ is uncorrelated with $v_i(k)$, which means that $E\{\omega(k)v_i^T(k)\} = 0$. Assume that the pair matrices (A, B) is controllable and the pair matrices $(A, C_i)_{i\in\mathcal{V}}$ is observable.

It is noted that in this letter, each sensor measures the system outputs and then transmits its measurement to neighbors. In fact, the dimensions of measurement $y_i(k)$ may be high in large-scale CPSs. However, the network channel bandwidth is limited, which only carries a finite amount of information at each instant. In this case, it is impossible to transmit the data packet $y_i(k)$ completely. To reduce the network transmission burden, the DRS [3] is applied in this letter. Denote the variable h as the number of dimensions of $y_i(k)$ can be selected and transmitted at each instant, where $h < m$. Therefore, for each sensor, there exist $\Delta = C_m^h = h!/(h!(m-h)!)$ possible cases to be allowed to send components of $y_i(k)$. Note that at each instant, only one case of the groups of Δ can be selected, and the selected signal is denoted as $\tilde{y}_i(k) \in \mathbb{R}^h$. Define $h_{i,l}^j(k) = 1$ or $h_{i,l}^j(k) = 0$ to represent the lth dimension of $y_i(k)$ is whether or not selected and transmitted to sensor j. Therefore, $h_{i,l}^j(k)$ must satisfy

$$\sum_{l=1}^{m} h_{i,l}^j(k) \le h, \quad i, j \in \mathcal{N}. \tag{3}$$

Additionally, the network channel is fragile for the malicious attacker. In this letter, the DoS attacks are considered which satisfy the Bernoulli stochastic distribution. In order to describe the stochastic process, the notation $\lambda_{ij}(k)$ is defined, whose expectation is $\bar{\lambda}_{ij}$. If $\lambda_{ij}(k) = 1$, the attacker does not send the malicious information and the measurement $\tilde{y}_i(k)$ can be transmitted to its neighbor j, successfully. Otherwise, the measurement $\tilde{y}_i(k)$ is discarded by DoS attacker.

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Moreover, a zero order hold (ZOH) mechanism, equipped in each sensor, holds the latest receiving information. Thus, at instant k, ZOH i receives information from sensor j as $z_{i,j}(k)=\lambda_{ji}(k)h_j(k)y_j(k)+(I-\lambda_{ji}(k)h_j(k))z_{i,j}(k-1)$, where $h_j(k)=\text{diag}\{h^i_{j,1}(k), h^i_{j,2}(k), \dots, h^i_{j,m}(k)\}$. Sensor i collects its neighboring information denoting as $Z_i(k)=\sum_{j\in\mathcal{N}_i} z_{i,j}(k)$. Define $\Pi_{ij}=\text{col}\{\underbrace{0,\dots,0}_{j-1}, I_j, \underbrace{0,..0}_{n_i-j}\}$ and $Y_i(k)=\text{col}\{y_j(k)\}_{j\in\mathcal{N}_i}$. Then, one can get that

$$Z_i(k)=\sum_{j\in\mathcal{N}_i} H_{ji}(k)\Pi_{ij}Y_i(k)+\sum_{j\in\mathcal{N}_i}(I-H_{ji}(k))\Pi_{ij}Z_i(k-1) \tag{4}$$

where $Y_i(k)=\bar{G}_i x(k)+\tilde{v}_i(k)$, $\bar{G}_i=\text{row}\{C_j\}_{j\in\mathcal{N}_i}$, $\bar{v}_i(k)=\text{col}\{v_j(k)\}_{j\in\mathcal{N}_i}$, and $H_{ji}(k)=\lambda_{ji}(k)h_j(k)$. Furthermore, denote $\eta_i(k)=[x^T(k)x\ Z_i^T(k-1)]^T$, one can obtain the following augment system:

$$\eta_i(k+1)=\mathcal{A}_i(k)\eta_i(k)+\mathcal{B}_i(k)\tilde{v}_i(k),\ \ Z_i(k)=C_i(k)\eta_i(k)+\hat{v}_i(k) \tag{5}$$

where $\mathcal{A}_i(k)=\begin{bmatrix} A & 0\\ \sum_{j\in\mathcal{N}_i} H_{ji}(k)\bar{G}_i & \sum_{j\in\mathcal{N}_i}(I-H_{ji}(k))\Pi_{ij}\Pi^\top_{ij}\end{bmatrix}$, $\mathcal{B}_i(k)=\begin{bmatrix} B & 0\\ 0 & \sum_{j\in\mathcal{N}_i} H_{ji}(k)\Pi_{ij}\Pi^T_{ij}\end{bmatrix}$, $\hat{v}_i(k)=\begin{bmatrix} w(k)\\ \bar{v}_i(k)\end{bmatrix}$, $C_i(k)=[\sum_{j\in\mathcal{N}_i} H_{ji}(k)\bar{G}_i\times \sum_{j\in\mathcal{N}_i}(I-H_{ji}(k))\Pi_{ij}\Pi^T_{ij}]$, $\mathcal{D}_i(k)=\begin{bmatrix}0 & \sum_{j\in\mathcal{N}_i} H_{ji}(k)\Pi_{ij}\Pi^T_{ij}\end{bmatrix}$.

The purpose of this letter is to propose a distributed filtering for CPS (1) under consideration with the network limited energy and DoS attacks. There exist two problems to be solved in this letter described as follows. 1) Assume that $h_j(t)$, $j\in\mathcal{N}_i$, is known in priori, and then the aim is to propose the distributed finite-horizon filtering for system (5). 2) Design the transmission schedule $h_j(k)$ to minimize the trace of estimation error covariance subject to (3).

Distributed finite-horizon filtering design: Define $\hat{\eta}_i(k|k-1)$ and $\hat{\eta}_i(k|k)$ as the one-step predicted estimation and estimation states, respectively. Then, the distributed filtering of augment system (5) is derived based on the finite-horizon Kalman filtering [12].

Theorem 1: For a CPS under the DRS and DoS attacks, the distributed finite horizon Kalman filtering of the augment system (5) can be obtained as

$$\begin{aligned}
\hat{\eta}_i(k|k)&=\hat{\eta}_i(k|k-1)+K_i(k)\tilde{Z}_i(k)\\
\hat{\eta}_i(k+1|k)&=\hat{\eta}_i(k+1|k-1)+F_i(k)\tilde{Z}_i(k)\\
P_i(k+1|k)&=\bar{\mathcal{A}}_i(k)P_i(k|k-1)\bar{\mathcal{A}}^\top_i(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\\
&\quad\times\hat{\mathcal{A}}_i(k)E\{\eta_i(k)\eta_i^T(k)\}\hat{\mathcal{A}}_i^T(k)\Pi^T_{ij}h_j(k)+\bar{\mathcal{B}}_i(k)R_v\bar{\mathcal{B}}_i^T(k)\\
&\quad+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{\mathcal{B}}_i^\top(k)R_v\hat{\mathcal{B}}_i^T(k)\Pi^T_{ij}h_j(k)\\
P_i(k|k)&=P_i(k|k-1)+P_i(k|k-1)\bar{C}_i^T(k)K_i^T(k)+K_i(k)\bar{C}_i(k)\\
&\quad\times P_i(k|k-1)+K_i(k)\Omega_i(k)K_i^T(k)
\end{aligned} \tag{6}$$

where $\tilde{Z}_i(k)=Z_i(k)-\bar{C}_i\hat{\eta}_i(k|k-1)$, $\Omega_i(k)=\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\times\hat{C}_i(k)E\{\eta_i(k)\eta_i^T(k)\}\hat{C}_i^T(k)\Pi_{ij}h_j(k)+\bar{C}_i(k)P_i(k|k-1)\bar{C}_i^T(k)+\bar{\mathcal{D}}_i(k)Q_v\times\bar{\mathcal{D}}_i^T(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{\mathcal{D}}_i(k)Q_v\hat{\mathcal{D}}_i^T(k)\Pi_{ij}h_j(k)$, $\Theta_i(k)=\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{C}_iE\{\eta_i(k)\eta_i^T(k)\}\Pi^T_{ij}h_j^T(k)\hat{C}_i+\bar{C}_i(k)P_i(k|k-1)\bar{C}_i^T(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\hat{\mathcal{D}}_iQ_v\hat{\mathcal{D}}_i\Pi^T_{ij}h_j^T(k)+\sum_{j,l\in\mathcal{N}_i,j\neq l}\bar{\lambda}_{ji}\bar{\lambda}_{il}h_j(k)\Pi_{ij}\times\hat{\mathcal{D}}_i(k)Q_v\hat{\mathcal{D}}_i^T(k)\Pi^T_{ij}h_l^T(k)$, $F_i(k)=\Big[\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{\mathcal{A}}_i(k)E\times\{\eta_i(k)\eta_i^T(k)\}\hat{C}_i^T(k)\Pi_{ij}h_j(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\Pi_{ij}\hat{\mathcal{B}}_iQ_v\hat{\mathcal{D}}_i^T(k)\Pi_{ij}h_j(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{\mathcal{D}}_i(k)Q_v\hat{\mathcal{D}}_i^T(k)\Pi_{ij}h_j(k)\Big]\Theta_i^{-1}(k)$, $K_i(k)=P_i(k|k-1)\bar{C}_i^T(k)\Theta_i^{-1}(k)$, $\hat{\mathcal{A}}_i=\begin{bmatrix}0 & 0\\ \bar{G}_i & \Pi^\top_{ij}\end{bmatrix}$, $\hat{\mathcal{B}}_i=\begin{bmatrix}0 & 0\\ 0 & \Pi^\top_{ij}\end{bmatrix}$ $\hat{C}_i=[\bar{G}_i\ \ \Pi^T_{ij}]$ $\hat{\mathcal{D}}_i=[0\ \Pi^T_{ij}]$$\bar{\mathcal{A}}_i(k)=\begin{bmatrix} A & 0\\ \sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\Pi_{ij}\bar{G}_i & \sum_{j\in\mathcal{N}_i}(I-\bar{\lambda}_{ji}h_j(k))\Pi_{ij}\Pi^T_{ij}\end{bmatrix}$, $\bar{\mathcal{B}}_i(k)=\begin{bmatrix} B & 0\\ 0 & \sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\Pi_{ij}\Pi^T_{ij}\end{bmatrix}$, $\bar{\mathcal{D}}_i(k)=[0\ \ \sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\Pi_{ij}\Pi^T_{ij}]$, $\bar{C}_i(k)=\Big[\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\Pi_{ij}\bar{G}_i\quad \sum_{j\in\mathcal{N}_i}(I-\bar{\lambda}_{ji}h_j(k))\Pi_{ij}\Pi^T_{ij}\Big]$.

Proof: Taking projection of both side of (4) to the space $L(Z_i(1), Z_i(2), Z_i(3),\dots,Z_i(k-1))$, one obtains $\hat{Z}_i(k|k-1)=\bar{C}_i(k)\hat{\eta}_i(k|k-1)+\text{proj}\{v_i(k)|Z_i(1),Z_i(2),\dots,Z_i(k-1)\}$. Since $\hat{v}_i(k)\perp L(\hat{v}_i(k-1),\hat{v}_i(k-2),\dots,\hat{v}_i(1),\eta_i(1))_{\hat{v}_i(k-1)}$, $\text{proj}\{v_i(k)|Z_i(1),Z_i(2),\dots,Z_i(k-1)\}=0$, where $L(\cdot)_{v_i(k)}$ represents that the linear space $L(\cdot)$ is dependent on the stochastic parameters in the set $v_i(k)$. Therefore, one has $\hat{Z}_i(k|k-1)=\bar{C}_i(k)\hat{\eta}_i(k|k-1)$. Defining $\tilde{Z}_i(k)=Z_i(k)-\hat{Z}_i(k|k-1)$, the following equation can be obtained $\tilde{Z}_i(k)=(C_i(k)-\bar{C}_i(k))\eta_i(k)+\bar{C}_i(k)e_i(k|k-1)+\hat{v}_i(k)$, where $e_i(k|k-1)=\eta_i(k)-\hat{\eta}_i(k|k-1)$. Since $\hat{v}_i(k)\perp\eta_i(k|k-1)$, $\hat{v}_i(k)\perp e_i(k|k-1)$, and $E\{C_i(k)-\bar{C}_i(k)\}=0$, defining $\Theta_i(k)=E\{\tilde{Z}_i(k)\times\tilde{Z}_i^T(k)\}$, one yields $\Theta_i(k)=\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{C}_i(k)E\{\eta_i(k)\times\eta_i^T(k)\}\hat{C}_i^\top(k)\Pi^T_{ij}(k)h_j(k)+\tilde{C}_i(k)P_i(k|k-1)\tilde{C}_i^{T}(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\Pi_{ij}\times\hat{\mathcal{D}}_i(k)Q_v\hat{\mathcal{D}}_i^T(k)\Pi^T_{ij}h_j(k)+\sum_{l,j\in\mathcal{N}_i,j\neq l}\bar{\lambda}_{ji}\bar{\lambda}_{li}h_j(k)\Pi_{ij}\hat{\mathcal{D}}_iQ_v\hat{\mathcal{D}}_i^T(k)\Pi_{il}h_l(k)$. The authors in [12] gave a method to obtain the one-step predicted estimation $\hat{\eta}_i(k+1|k)$ as $\hat{\eta}_i(k+1|k)=\hat{\eta}_i(k+1|k-1)+F_i(k)\tilde{Z}_i(k)$, $F_i(k)=E\{\eta_i(k+1)\tilde{Z}_i^T(k)\}E\{\tilde{Z}_i(k)\tilde{Z}_i^T(k))\}^{-1}$. Therefore, it is important to derive $F_i(k)$. One can obtain that $F_i(k)=\Big[\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\times\hat{\mathcal{A}}_i(k)E\{\eta_i(k)\eta_i^T(k)\}\times\hat{C}_i^T(k)\Pi_{ij}h_j(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}h_j(k)\Pi_{ij}\hat{\mathcal{B}}_iQ_v\hat{\mathcal{D}}_i^T(k)\times\Pi^T_{ij}h_j(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{\mathcal{D}}_i(k)Q_v\hat{\mathcal{D}}_i^T(k)\Pi^T_{ij}h_j(k)\Big]\Theta_i^{-1}(k)$. Since $\tilde{v}_i(k|k-1)\perp L(Z_i(1),Z_i(2),\dots,Z_i(k-1))$, one takes both side of the augment equation (4) in the space $L(Z_i(1),Z_i(2),\dots,Z_i(k-1))$, having $\hat{\eta}_i(k+1|k-1)=\bar{\mathcal{A}}_i(k)\hat{\eta}_i(k|k-1)$. From [12], the state estimation $\hat{\eta}_i(k|k)$ can be determined as $\hat{\eta}_i(k|k)=\hat{\eta}_i(k|k-1)+K_i(k)\tilde{Z}_i(k)$, $K_i(k)=E\{\eta_i(k)\tilde{Z}_i^T(k)\}E\{\tilde{Z}_i(k)\tilde{Z}_i^T(k))\}^{-1}$. Since $\hat{\eta}_i(k|k-1)\perp e_i(k|k-1)$, one can get that $K_i(k)=P_i(k|k-1)\bar{C}_i^T(k)\Theta_i^{-1}(k)$. Following, the procedures for the covariance matrices $P_i(k+1|k)$ and $P_i(k|k)$ are proposed. One can derive that $e_i(k+1|k)=\bar{\mathcal{A}}_i(k)e_i(k|k-1)+\hat{\mathcal{A}}_i(k)\eta_i(k)-F_i(k)\tilde{Z}_i(k)+\mathcal{B}_i(k)\tilde{v}_i(k)$. Therefore, the covariance matrix $P_i(k+1|k)=E\{e_i(k+1|k)e_i^T(k+1|k)\}$ can be proposed as $P_i(k+1|k)=\bar{\mathcal{A}}_i(k)P_i(k|k-1)\bar{\mathcal{A}}_i^\top(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\times\hat{\mathcal{A}}_i(k)E\{\eta_i(k)\eta_i^T(k)\}\hat{\mathcal{A}}_i^T(k)\times\Pi^T_{ij}h_j(k)+\bar{\mathcal{B}}_i(k)R_v\bar{\mathcal{B}}_i^T(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}\hat{\mathcal{B}}_i^T(k)R_v\hat{\mathcal{B}}_i^T(k)\times\Pi^T_{ij}h_j(k)$. Furthermore, one can get that $P_i(k|k)=P_i(k|k-1)+P_i(k|k-1)\bar{C}_i^T(k)K_i^T(k)+K_i(k)\bar{C}_i(k)P_i(k|k-1)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})h_j(k)\Pi_{ij}K_i(k)\times\hat{C}_i(k)E\{\eta_i(k)\eta_i^T(k)\}\hat{C}_i^T(k)\Pi_{ij}h_j(k)K_i^T(k)+K_i(k)\bar{C}_i(k)P_i(k|k-1)\bar{C}_i^T(k)\times K_i^T(k)+K_i(k)\bar{\mathcal{D}}_i(k)Q_v\bar{\mathcal{D}}_i^T(k)K_i^T(k)+\sum_{j\in\mathcal{N}_i}\bar{\lambda}_{ji}(1-\bar{\lambda}_{ji})K_i(k)h_j(k)\Pi_{ij}\times\hat{\mathcal{D}}_i(k)Q_v\hat{\mathcal{D}}_i^T(k)\Pi_{ij}h_j(k)K_i^T(k)$. ■

From (6), one can get that the estimation error covariance $P_i(k|k)$ is related to the transmission data packet $h_j(k)$, $j\in\mathcal{N}_i$, which implies that different $h_j(k)$ induces different estimation error covariance. In order to optimize the performance of distributed filtering, the optimal $h_j(k)$, $j\in\mathcal{N}_i$, is designed to minimize the trace of estimation error covariance $P_i(k|k)$.

Design of dimensionality reduction strategy: The design of DRS is to minimize the trace of estimation error covariance $P_i(k|k)$. The optimization problem can be described by

$$\min_{h_j(k)}\text{trace}(P_i(k|k))\ \text{s.t.}\ \sum_{l=1}^{m}h^i_{j,l}(k)\le h, h^i_{j,l}(k)=\{0,1\},\ j\in\mathcal{N}_i. \tag{7}$$

Lemma 1: Strategies $\{h_j(k)|j\in\mathcal{N}_i\}$ constitute games over sensor i with utility $P_i(k|k)$, and there exist at last one Nash equilibrium.

Proof: $P_i(k|k)$ is the cumulative utility determined by DRS, which indicates that the transmission schedules among neighbors have independent influences on estimation error. Then, $\{h_j(k)|j\in\mathcal{N}_i\}$ constitute games with interface $P_i(k|k)$. Since $h_j(k)$ subjects to discrete action space, the uniqueness of Nash equilibrium is classified from cases in pure strategies and mixed strategies. For the game in pure strategies, Nash equilibriums certainly exist under finite action spaces. However, order interchangeability leads to the possibility of multiple equilibriums. For that in mixed strategies, strategies in probability space subject to convex set, the uniqueness of Nash equilibrium is guaranteed with convexity of $P_i(k|k)$. ■

It is noticed that there exist Δ possible cases to be selected for each sensor. Therefore, one can derive $\bar{\Delta}_i$ possible cases for the estimation error covariance $P_i(k|k)$, where $\bar{\Delta}_i = \Delta^{\bar{n}_i}$ and $\bar{n}_i$ is the number of the neighbors of sensor i. In this letter, the mixed strategies are defined as $H_{i,\text{mixed}}(\gamma_i^1, \gamma_i^2, \ldots, \gamma_i^{\bar{\Delta}_i}) = \{H_{i,\text{pure}}^r(k)$ with possibility γ_i^r, where $H_{i,\text{pure}}^r(k)$ is the pure strategy under the r-th transmission schedule, $r = 1, 2, \ldots, \bar{\Delta}_i$, $\sum_{r=1}^{\bar{\Delta}_i} \gamma_i^r = 1$, and $\gamma_i^r \in [0, 1]$. It is noticed that the different combination of γ_i^r constitute different mixed strategies. Therefore, the number of mixed strategies is infinite.

Due to the number of pure strategies is finite, the value of trace$(P_i(k|k))$ is easily determined under each pure strategy. By defining $P_i^r(k|k)$ as the value of $P_i(k|k)$ under the r-th pure strategy, the optimization problem (7) can be converted as

$$\min_{\Gamma_i} \sum_{r=1}^{\bar{\Delta}_i} \gamma_i^r \text{trace}(P_i^l(k|k)) \quad \text{s.t.} \quad \sum_{r=1}^{\bar{\Delta}_i} \gamma_i^r = 1. \tag{8}$$

where $\Gamma_i = \{\gamma_i^1, \gamma_i^2, \ldots, \gamma_i^{\bar{\Delta}_i}$. The optimal problem can be calculated by using the Lagrange multipliers method.

Simulation: This section is to verify the effectiveness of the derived distributed filtering. The satellite system is applied, as presented in [11]. A network of 5 sensors is applied, where any two sensors can communicate. The measurement matrix of each sensor is $C_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Assume that only one dimension of measurement can be selected and transmitted.

Note that there exist 2^4 pure strategies for each sensor. By applying the mixed strategy, the probability of each cases at each instant can be obtained, and then the distributed filtering under DRSs is derived. The system states and estimated states are shown in Fig. 1. As can be seen from Fig. 1, the estimated sates can catch up with the system states, which means that the proposed reduction dimensioned distributed filtering under the DoS attack can achieve good effectiveness.

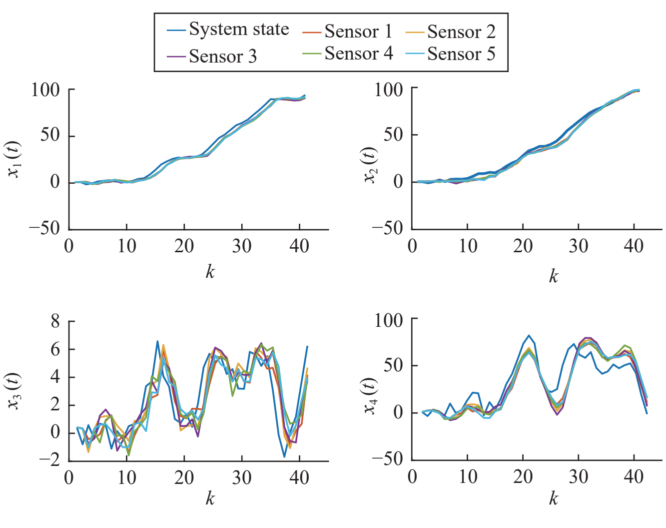

Fig. 1. The comparisons between estimation state and system states.

Conclusion: In this letter, the dimensionally reduction distributed finite-horizon filtering for CPSs under DoS attacks is investigated. Considering the CPSs with high-dimensional measurements, the DRS is applied, which is to select and transmit partial measurement. In this scenario, the distributed finite-horizon Kalman filtering is proposed under DoS attacks. Moreover, the mixed strategy is used to derive the optimal transmission schedule for CPSs to minimize the trace of estimation error covariance. Finally, the effectiveness of the proposed distributed filtering is presented.

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