Letter

Adaptive Predefined-Time Optimal Tracking Control for Underactuated Autonomous Underwater Vehicles

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Dear Editor,

This letter concentrated on the adaptive predefined-time optimal tracking control for underactuated autonomous underwater vehicles (AUVs). According to adaptive dynamic programming (ADP) with actor-critic neural networks (NNs), by constructing a novel barrier-type cost function, an adaptive predefined-time optimal constraint control approach was developed. With the help of predefined-time stability theory via tanh(·) function, the developed control approach could ensure all state errors did not beyond the preset error boundary, and the controlled system was semi-global practical predefined-time stable. At the same time, the minimum of cost function could be guaranteed. Finally, simulation results were given to verify the effectiveness of the developed control approach.

Related work: With the continuous development of marine survey technology, underactuated AUVs, as a more efficient tool for ocean navigation and exploration, have paid considerable attentions for scholars. At the same time, some significance works have been obtained, see [1] and [2]. Reference [1] studies a leader-follower formation control issue for multi AUVs. Based on the designed kinematic and dynamic model of AUVs in [1], the authors in [2] propose a robust adaptive trajectory tracking control approach for AUVs. It should be noted that the above developed control methods do not consider the settling time and minimum energy consumption of AUVs. On the one hand, to deal with the infinite-time converge issue, fixed-time stable theory is developed in [3], the settling time in [3] do not depend on the initial values, which only depend on the design parameters. Thus, Li *et al.* [4] investigate the adaptive fuzzy fixed-time decentralized control issue for nonlinear systems. Huang *et al.* [5] study the robust adaptive fixed-time control issue for underactuated autonomous surface vessels. Note that the settling time in the above developed practical fixed-time control methods depends on the unknown weight vectors. Thus, [6] develops the predefined-time stable theory for nonlinear systems, the settling time is a design parameter, which does not depend on unknown weight vectors and other parameters. Inspired by [6], Xie and Chen [7] design a nonsingular adaptive predefined-time control law for rigid spacecrafts. On the other hand, scholars always hope to use smaller control energy on the premise of achieving satisfactory performance indicators, thus, Bellman [8] first present the dynamic programming theory. In addition, to solve the dimension disaster issue in dynamic programming (DP) theory, Werbos [9] developed the reinforcement learning (RL) algorithm by combining NNs for nonlinear systems. Mazouchi *et al.* [10] study the distributed adaptive optimal control issue for multi-agent systems, and Wen *et al.* [11] propose a simplified adaptive optimal control method for nonlinear systems. In addition, the above considered systems are also limited to affine ones, which cannot be applied to solve the nonlinear systems with unmatched conditions. Thus, Yang *et al.* [12] develop an ADP-based optimal tracking control strategy for multi-UAVs. Then, [13] studies the robust adaptive optimal full state constraint control issue for nonlinear systems. When considering the effective balance between quality control and energy control, Cao *et al.* [14] develop RL-based robust adaptive fixed-time optimal control method for robotic manipulators by constructing nonsingular sliding mode surface. However, by far, there are no available results on adaptive optimal predefined-time controllers for nonlinear systems with error constraints. Motivated by the above analysis, this letter first studies the predefined-time-based adaptive optimal tracking control problem for underactuated AUVs. Compared with the existing works, the main contributions of this letter can be highlighted as follows: 1) Based on the predefined-time stable theory and barrier cost function, a predefined-time adaptive trajectory tracking control method is first proposed for AUVs. It should be noted that the work [14] studies the adaptive optimal fixed-time control for nonlinear systems, which cannot preset the system convergence time, the settling time is subjected to the design parameters and unknown weight, the settling time in this letter is a main design parameter, which does not depend on the other design parameters and unknown weight. 2) With the help of hyperbolic tangent function, the singular issue can be effectively avoided. In addition, by using prescribed performance technique, the developed control approach can ensure all state errors can converge to a preset error boundary in predefined-time.

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Problem statement: Based on the body and earth fixed coordinates, inspired by [1] and [2], AUV usually can be modeled as follows:

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \boldsymbol{R}(\psi)\boldsymbol{\upsilon}\\ \boldsymbol{\tau}+\boldsymbol{d}(t) &= \boldsymbol{M}\dot{\boldsymbol{\upsilon}}+\boldsymbol{C}(\boldsymbol{\upsilon})\boldsymbol{\upsilon}+\boldsymbol{D}(\boldsymbol{\upsilon})\boldsymbol{\upsilon}\end{aligned} \tag{1}$$

where $\boldsymbol{\upsilon}=[u,v,r]^T$ represents the AUV's velocity vector with yaw rate r, sway velocity v and surge velocity u; $\boldsymbol{\eta}=[x,y,\psi]^T$ represents the AUV's position vector with yaw angle $\psi\in[0,2\pi)$ and position (x,y); $\boldsymbol{d}(t)=[d_1(t),d_2(t),d_3(t)]^T$ represents external disturbance vector; $\boldsymbol{\tau}=[\tau_u,0,\tau_r]^T$ represents the control input vector with yaw moment, sway force and surge force. Positive definite inertia matrix $\boldsymbol{M}=\mathrm{diag}\{m_{1,1},m_{2,2},m_{3,3}\}$. The damping matrix $\boldsymbol{D}(\boldsymbol{\upsilon})=\mathrm{diag}\{d_{1,1}(u), d_{2,2}(v),d_{3,3}(r)\}$, the matrix of Coriolis and centripetal terms $\boldsymbol{C}(\boldsymbol{\upsilon})$ and rotation matrix $\boldsymbol{R}(\psi)$ are described as

$$\boldsymbol{C}(\boldsymbol{\upsilon})=\begin{bmatrix}0&0&-m_{2,2}v\\0&0&m_{1,1}u\\m_{2,2}v&-m_{1,1}u&0\end{bmatrix},\boldsymbol{R}(\psi)=\begin{bmatrix}\cos(\psi)&-\sin(\psi)&0\\\sin(\psi)&\cos(\psi)&0\\0&0&1\end{bmatrix}$$

where $d_{1,1}(u)=-X_u-X_{u|u|}|u|$, $d_{2,2}(v)=-Y_v-Y_{v|v|}|v|$ and $d_{3,3}(r)=-N_r-N_{r|r|}|r|$ with hydrodynamic derivatives $X_u,X_{u|u|}$, $Y_v,Y_{v|v|}$, $N_r,N_{r|r|}$; $m_{1,1}=m-X_{\dot{u}}$, $m_{2,2}=m-Y_{\dot{v}}$, $m_{3,3}=I_z-N_{\dot{r}}$ with AUV's mass m, added masses $X_{\dot{u}}$, $Y_{\dot{v}}$ and $N_{\dot{r}}$, and moment of inertia in yaw I_z.

Assumption 1 [1]: The elements of the desired trajectory $\boldsymbol{\eta}_d=[x_d, y_d,\psi_d]^T$ is bounded, their first and second-order derivatives are also bounded.

Assumption 2 [2]: There exists unknown constant vector $d^*=[d_1^*, d_2^*,d_3^*]^T$, the external disturbance vector $\boldsymbol{d}(t)=[d_1(t),d_2(t),d_3(t)]^T$ satisfies $\|\boldsymbol{d}(t)\|\le\|d^*(t)\|$.

Control objective: This letter will design an adaptive predefined-time constraint optimal tracking control law for AUVs (1) such that

1) AUV can track the desired trajectory in predefined time;

2) All closed-loop signals are bounded in predefined time;

3) The cost function is minimal, and all state errors remain within a preset region.

Definition 1 [6], [7]: For nonlinear system $\dot{\chi}(t)=f(\chi,t)$ with $f(0)=0$, the equilibrium point $\chi(0)=\chi_0$ is said to be the practical predefined-time stable (PTS) if the state trajectory satisfies $\|\chi(\chi_0,t)\|\le\delta$ for $\forall t\ge \mathrm{T}_{\max}$ with predefined time $\mathrm{T}_{\max}$ and constant $\delta>0$.

Lemma 1 [6], [7]: For any constants $\beta\in(0,1)$, $\pi>0$, $\bar{\pi}>0$ and $D>0$, there exists a continuous function $V(\chi)$, we have

$$\dot{V}(\chi)\le-\frac{\bar{\pi}}{\beta\mathrm{T}_{\max}}V^{1+\frac{\beta}{2}}-\frac{\pi}{\beta\mathrm{T}_{\max}}V^{1-\frac{\beta}{2}}+D \tag{2}$$

thus, the nonlinear system $\dot{\chi}(t)=f(\chi,t)$ is globally predefined-time stable (PPTS).

Lemma 2 [4]: For $\chi_i\in\mathbb{R}$, there exist constants $p\in(0,1]$ and $q>1$, one has

$$\sum_{i=1}^m|\chi_i|^q\ge m^{1-q}(\sum_{i=1}^m|\chi_i|)^q,\quad \sum_{i=1}^m|\chi_i|^p\ge(\sum_{i=1}^m|\chi_i|)^p. \tag{3}$$

Main results: First, define the coordinate transformation as

$$\begin{cases} \boldsymbol{e} = \boldsymbol{\eta} - \boldsymbol{\eta}_\alpha \\ z_2 = \boldsymbol{\upsilon} - \hat{\boldsymbol{\alpha}} \end{cases} \tag{4}$$

where $\boldsymbol{\eta}_\alpha = [x_d, y_d, \psi_\alpha]^T$, $\boldsymbol{e} = [e_x, e_y, e_\psi]^T$, ψ_α is called to be approach angle, and defined as

$$\psi_\alpha = \operatorname{atan2}(e_y, e_x)\tanh\left(\frac{e_x^2 + e_y^2}{\delta}\right) + \psi_d\left(1 - \tanh\left(\frac{e_x^2 + e_y^2}{\delta}\right)\right) \tag{5}$$

where $\operatorname{atan2}(e_y, e_x)$ represents the angle value of (e_x, e_y). Obviously, $\psi_\alpha = \psi_d$ when $e_x = e_y = 0$. Define error variable as

$$\xi_{je}(t) = \frac{e_j(t)}{\kappa_{je}(t)}, \quad \xi_{z_2}(t) = \frac{z_2(t)}{\kappa_{z_2}(t)}, \quad j = x, y, \psi \tag{6}$$

where $\kappa_{je}(t) = (\kappa_{je0} - \kappa_{je\infty})\exp(-a_j t) + \kappa_{je\infty}$ and $\kappa_{z_2}(t) = (\kappa_{z_20} - \kappa_{z_2\infty}) \times \exp(-a_{z_2}t) + \kappa_{z_2\infty}$ are performance functions, κ_{je0}, $\kappa_{je\infty}$, κ_{z_20}, $\kappa_{z_2\infty}$, a_j and a_{z_2} are positive constants, then we have $|e_j(0)| < \kappa_{je0}$ and $|z_2(0)| < \kappa_{z_20}$.

Step 1: For AUVs, design the cost function as

$$\begin{aligned} \mathcal{V}_1(\boldsymbol{\xi_e}(0)) &= \min_{\boldsymbol{\alpha} \in \Omega_1(U_1)} \left\{ \int_t^{t_f} p_1(\boldsymbol{\xi_e}(s), \boldsymbol{\alpha}(\boldsymbol{\xi_e})) ds \right\} \\ &= \int_t^{t_f} p_1(\boldsymbol{\xi_e}(s), \boldsymbol{\alpha}^*(\boldsymbol{\xi_e})) ds \end{aligned} \tag{7}$$

where $\boldsymbol{\xi_e} = [\xi_{x,e}, \xi_{y,e}, \xi_{\psi,e}]^T$, $p_1(\boldsymbol{\xi_e}, \boldsymbol{\alpha}^*) = \delta_1 \frac{\boldsymbol{\xi}_e^T \boldsymbol{\xi}_e}{2(\boldsymbol{1} - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^2} + r_1(\boldsymbol{\alpha}^*)^2$, $\delta_1 > 0$ and $r_1 > 0$ are constants. $\Omega_1(U_1)$ is the admissible control of $\boldsymbol{\alpha}^*$. t_f is the terminal time. Define the HJB equation as

$$\begin{aligned} H(\boldsymbol{\xi_e}, \boldsymbol{\alpha}, \frac{\partial \mathcal{V}_1}{\partial \boldsymbol{\xi_e}}) = \delta_1 \frac{\boldsymbol{\xi}_e^T \boldsymbol{\xi}_e}{2(\boldsymbol{1} - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^2} + r_1(\boldsymbol{\alpha}^*)^2 \\ + \frac{\partial \mathcal{V}_1}{\partial \boldsymbol{\xi_e}} \frac{1}{\boldsymbol{\kappa_e}} [\boldsymbol{R}(\psi)\boldsymbol{\alpha}^* - \dot{\boldsymbol{\eta}}_\alpha - \boldsymbol{\xi_e}\dot{\boldsymbol{\kappa}}_{\boldsymbol{e}}]. \end{aligned} \tag{8}$$

By solving $\frac{\partial H(\cdot)}{\partial \alpha^*} = 0$, we can obtain the ideal virtual control law as

$$\boldsymbol{\alpha}^* = -\frac{1}{2r_1 \boldsymbol{\kappa_e}} \frac{\partial \mathcal{V}_1}{\partial \boldsymbol{\xi_e}} \boldsymbol{R}(\psi). \tag{9}$$

To achieve the optimal control objective, let construct the $\frac{\partial \mathcal{V}_1}{\partial \boldsymbol{\xi_e}}$ as

$$\begin{aligned} \frac{\partial \mathcal{V}_1}{\partial \boldsymbol{\xi_e}} = \boldsymbol{R}^{-2}(\psi)\boldsymbol{\kappa_e} \Bigg\{ \boldsymbol{\kappa_e} \Bigg[\frac{2r_1 2^\beta \pi}{\beta \mathrm{T}_{\max}} \frac{(\frac{1}{2})^{1+\frac{\beta}{2}} \boldsymbol{\xi}_e^{1+\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2+\beta} \boldsymbol{\varpi_e}} + \frac{2r_1 \pi}{\beta \mathrm{T}_{\max}} \\ \times \frac{(\frac{1}{2})^{1-\frac{\beta}{2}} \boldsymbol{\xi}_e^{1-\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2-\beta} \boldsymbol{\varpi_e}} \tanh\left(\frac{\pi}{\beta \mathrm{T}_{\max}} \frac{(\frac{1}{2})^{1-\frac{\beta}{2}} \boldsymbol{\xi}_e^{2-\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2-\beta} \varsigma_1} \right) \Bigg] \\ - 2r_1(\dot{\boldsymbol{\eta}}_\alpha + \boldsymbol{\xi_e}\dot{\boldsymbol{\kappa}}_{\boldsymbol{e}}) + \frac{2\tau_1}{\boldsymbol{\kappa_e}} \boldsymbol{\xi_e} \boldsymbol{\varpi_e} + \mathcal{V}_1^*(\boldsymbol{X}) \Bigg\} \end{aligned} \tag{10}$$

where $\tau_1 > 0$ and $\varsigma_1 > 0$ are constants. $\mathcal{V}_1^* = -\frac{2r_1 2^\beta \pi}{\beta T_{\max}} \left(\frac{1}{2}\right)^{1+\frac{\beta}{2}} \frac{\boldsymbol{\kappa_e} \boldsymbol{\xi}_e^{1+\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2+\beta} \boldsymbol{\varpi_e}} - \frac{2r_1 \pi}{\beta \mathrm{T}_{\max}} \left(\frac{1}{2}\right)^{1-\frac{\beta}{2}} \frac{\boldsymbol{\kappa_e} \boldsymbol{\xi}_e^{1-\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2-\beta} \boldsymbol{\varpi_e}} \times \tanh\left[\frac{\pi}{\beta \mathrm{T}_{\max}} \frac{\boldsymbol{\xi}_e^{2-\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2-\beta} \varsigma_1} \left(\frac{1}{2}\right)^{1-\frac{\beta}{2}} \right] + 2r_1(\dot{\boldsymbol{\eta}}_\alpha + \boldsymbol{\xi_e}\dot{\boldsymbol{\kappa}}_{\boldsymbol{e}}) - \frac{2\tau_1}{\boldsymbol{\kappa_e}} \boldsymbol{\xi_e} \boldsymbol{\varpi_e} + \frac{1}{\boldsymbol{\kappa_e}} \frac{\partial \mathcal{V}_1}{\partial \boldsymbol{\xi_e}} \boldsymbol{R}^2(\psi), \boldsymbol{X} = [\boldsymbol{\xi_e}, \boldsymbol{\kappa_e}]^T$, $\boldsymbol{\varpi_e} = \frac{1 + \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^2}$

Based on [13], RBF NN is utilized to identify $\mathcal{V}_1^*(\boldsymbol{X})$, one has

$$\mathcal{V}_1^*(\boldsymbol{X}) = \boldsymbol{W}_1^{*T} \boldsymbol{S}_1(\boldsymbol{X}) + \varepsilon_1(\boldsymbol{X}) \tag{11}$$

where $\varepsilon_1(\boldsymbol{X})$ is the identify error vector and satisfies $\|\varepsilon_1(\boldsymbol{X})\| \le \varepsilon_1^*$ with constant $\varepsilon_1^* > 0$. $\boldsymbol{S}_1(\boldsymbol{X}) = [S_{1,1}(\boldsymbol{X}), S_{1,2}(\boldsymbol{X}), S_{1,3}(\boldsymbol{X})]^T$ is the basis function vector, and $\boldsymbol{W}_1^* = \operatorname{diag}\{W_{1,1}^*, W_{1,2}^*, W_{1,3}^*\}$ is the unknown parameter matrix.

Since $\boldsymbol{W}_1^*$ is the unknown parameter matrix, ADP algorithm with actor-critic NNs is adopted to achieve the optimal control goal, critic NN $\hat{\boldsymbol{W}}_{Jc1}^T \boldsymbol{S}_1(\boldsymbol{X})$ is utilized to online learning optimal cost function, then, design the critic NN updating law $\dot{\hat{\boldsymbol{W}}}_{Jc1}$ as

$$\dot{\hat{\boldsymbol{W}}}_{Jc1} = -\hbar_{c1} \boldsymbol{S}_1(\boldsymbol{X}) \boldsymbol{S}_1^T(\boldsymbol{X}) \hat{\boldsymbol{W}}_{Jc1} \tag{12}$$

where $\hbar_{c1} > 0$ is the learning rate.

And actor NN $\hat{\boldsymbol{W}}_{Ja1}^T \boldsymbol{S}_1(\boldsymbol{X})$ is utilized to online turning the ideal virtual control $\boldsymbol{\alpha}^*$, thus, design the optimal virtual control law as

$$\begin{aligned} \hat{\boldsymbol{\alpha}} = \boldsymbol{R}^{-1}(\psi) \Bigg\{ \boldsymbol{\kappa_e} \Bigg[-\frac{2^\beta \pi}{\beta \mathrm{T}_{\max}} \left(\frac{1}{2}\right)^{1+\frac{\beta}{2}} \frac{\boldsymbol{\xi}_e^{1+\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2+\beta} \boldsymbol{\varpi_e}} - \frac{\pi}{\beta \mathrm{T}_{\max}} \\ \times \left(\frac{1}{2}\right)^{1-\frac{\beta}{2}} \frac{\boldsymbol{\xi}_e^{1-\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2-\beta} \boldsymbol{\varpi_e}} \tanh\left(\frac{\pi}{\beta \mathrm{T}_{\max}} \frac{\boldsymbol{\xi}_e^{2-\beta}}{(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^{2-\beta} \varsigma_1} \right. \\ \left. \times \left(\frac{1}{2}\right)^{1-\frac{\beta}{2}} \right) \Bigg] + (\dot{\boldsymbol{\eta}}_\alpha + \boldsymbol{\xi_e}\dot{\boldsymbol{\kappa}}_{\boldsymbol{e}}) - \frac{\tau_1 \boldsymbol{\xi_e}}{r_1 \boldsymbol{\kappa_e}} \boldsymbol{\varpi_e} - \frac{\hat{\boldsymbol{W}}_{Ja1}^T \boldsymbol{S}_1(\boldsymbol{X})}{2r_1} \Bigg\}. \end{aligned} \tag{13}$$

Design the actor NN updating law $\dot{\hat{\boldsymbol{W}}}_{Ja1}$ as

$$\begin{aligned} \dot{\hat{\boldsymbol{W}}}_{Ja1} = &-\hbar_{a1} \boldsymbol{S}_1(\boldsymbol{X}) \boldsymbol{S}_1^T(\boldsymbol{X}) (\hat{\boldsymbol{W}}_{Ja1} - \hat{\boldsymbol{W}}_{Jc1}) \\ &-\hbar_{c1} \boldsymbol{S}_1(\boldsymbol{X}) \boldsymbol{S}_1^T(\boldsymbol{X}) \hat{\boldsymbol{W}}_{Jc1} \end{aligned} \tag{14}$$

where $\hbar_{a1} > 0$ is the learning rate and satisfies $\hbar_{a1} > \hbar_{c1}$.

Define the Hamiltonian approximation error as

$$\mathcal{E}_1 = H\left(\boldsymbol{\xi_e}, \hat{\boldsymbol{\alpha}}, \frac{\partial \hat{\mathcal{V}}_1}{\partial \boldsymbol{\xi_e}}\right) - H\left(\boldsymbol{\xi_e}, \boldsymbol{\alpha}, \frac{\partial \mathcal{V}_1}{\partial \boldsymbol{\xi_e}}\right) = H\left(\boldsymbol{\xi_e}, \hat{\boldsymbol{\alpha}}, \frac{\partial \hat{\mathcal{V}}_1}{\partial \boldsymbol{\xi_e}}\right). \tag{15}$$

Similar to [12], define positive-definite function $\boldsymbol{P}(t) = Tr\{(\hat{\boldsymbol{W}}_{Ja1} - \hat{\boldsymbol{W}}_{Jc1})^T (\hat{\boldsymbol{W}}_{Ja1} - \hat{\boldsymbol{W}}_{Jc1})\}$, then, we have

$$\dot{\boldsymbol{P}} = -\frac{\hbar_{a1}}{2} Tr\left\{ \frac{\partial \boldsymbol{P}(t)}{\partial \hat{\boldsymbol{W}}_{Ja1}} \boldsymbol{S}_1(\boldsymbol{X}) \boldsymbol{S}_1^T(\boldsymbol{X}) \frac{\partial \boldsymbol{P}(t)}{\partial \hat{\boldsymbol{W}}_{Ja1}} \right\} \le 0 \tag{16}$$

Obviously, it can ensure $\frac{\partial H(\boldsymbol{\xi_e}, \hat{\boldsymbol{\alpha}}, \frac{\partial \hat{\mathcal{V}}_1}{\partial \boldsymbol{\xi_e}})}{\partial \hat{\boldsymbol{W}}_{Ja1}} = \boldsymbol{0}_{3\times3}$.

Step 2: Design the predefined-time optimal control law $\boldsymbol{\tau}$ as

$$\begin{aligned} \boldsymbol{\tau} = \boldsymbol{\kappa_z} \Bigg[-\frac{2^\beta \pi}{\beta \mathrm{T}_{\max}} \frac{(\frac{1}{2})^{1+\frac{\beta}{2}} \boldsymbol{\xi}_z^{1+\beta}}{(1 - \boldsymbol{\xi}_z^T \boldsymbol{\xi}_z)^{2+\beta} \boldsymbol{\varpi_z}} - \frac{(\frac{1}{2})^{1-\frac{\beta}{2}} \pi}{\beta \mathrm{T}_{\max}} \frac{\boldsymbol{\xi}_z^{1-\beta}}{(1 - \boldsymbol{\xi}_z^T \boldsymbol{\xi}_z)^{2-\beta} \boldsymbol{\varpi_z}} \\ \times \tanh\left(\frac{\pi}{\beta \mathrm{T}_{\max}} \frac{(\frac{1}{2})^{1-\frac{\beta}{2}} \boldsymbol{\xi}_z^{2-\beta}}{(1 - \boldsymbol{\xi}_z^T \boldsymbol{\xi}_z)^{2-\beta} \varsigma_1} \right) \Bigg] + \hat{\boldsymbol{W}}^T \boldsymbol{S} - \frac{\tau_2 \boldsymbol{\xi}_z}{r_2 \boldsymbol{\kappa_z}} \boldsymbol{\varpi_z} \\ - \frac{1}{2r_2} \hat{\boldsymbol{W}}_{Ja2}^T \boldsymbol{S}_2 - \omega_{1,0} \tanh\left(\frac{\boldsymbol{\varpi_z}}{\boldsymbol{\kappa_z} \varsigma_2} \boldsymbol{\xi}_z \omega_{1,0} \right) \end{aligned} \tag{17}$$

where $\boldsymbol{\xi}_z = [\xi_{u,z}, \xi_{v,z}, \xi_{r,z}]^T$, $\boldsymbol{\varpi_z} = \frac{1 + \boldsymbol{\xi}_z^T \boldsymbol{\xi}_z}{(1 - \boldsymbol{\xi}_z^T \boldsymbol{\xi}_z)^2}$. $\hat{\boldsymbol{W}}$ is the estimation of $\boldsymbol{W}^*$, $\boldsymbol{W}^*$ will be defined later. $\varsigma_2 > 0$ is the constant.

Design the updating laws of actor-critic NNs and adaptive law as

$$\dot{\hat{\boldsymbol{W}}}_{Jc2} = -\hbar_{c2} \boldsymbol{S}_2(\boldsymbol{Z}) \boldsymbol{S}_2^T(\boldsymbol{Z}) \hat{\boldsymbol{W}}_{Jc2} \tag{18}$$

$$\begin{aligned} \dot{\hat{\boldsymbol{W}}}_{Ja2} = &-\hbar_{a2} \boldsymbol{S}_2(\boldsymbol{Z}) \boldsymbol{S}_2^T(\boldsymbol{Z}) (\hat{\boldsymbol{W}}_{Ja2} - \hat{\boldsymbol{W}}_{Jc2}) \\ &-\hbar_{c2} \boldsymbol{S}_2(\boldsymbol{Z}) \boldsymbol{S}_2^T(\boldsymbol{Z}) \hat{\boldsymbol{W}}_{Jc2} \end{aligned} \tag{19}$$

$$\dot{\hat{\boldsymbol{W}}} = \boldsymbol{\Gamma}(\boldsymbol{S}(\boldsymbol{Z}) \tfrac{\boldsymbol{\varpi_z}}{\boldsymbol{\kappa_z}} \boldsymbol{\xi}_z - \boldsymbol{K}\hat{\boldsymbol{W}}) \tag{20}$$

where $\hbar_{c2} > 0$ and $\hbar_{a2} > 0$ are the learning rates and satisfy $\hbar_{a2} > \hbar_{c2}$. $\boldsymbol{K}$ and $\boldsymbol{\Gamma}$ are positive-definite matrices.

Theorem 1: For AUV system (1), Assumptions 1 and 2 hold, if we adopt predefined-time optimal controller (17), virtual control law (13), actor-critic NNs updating laws (12), (14), (18) and (19), adaptive law (20), the developed control method has the following properties:

1) AUV can track $\boldsymbol{\eta}_d = [x_d, y_d, \psi_d]^T$ in predefined time;

2) All state errors are bounded and the cost functions are minimal.

Proof (Step 1): Choose the Lyapunov function as

$$V_1 = \frac{\boldsymbol{\xi}_e^T \boldsymbol{\xi}_e}{2(1 - \boldsymbol{\xi}_e^T \boldsymbol{\xi}_e)^2} + \frac{1}{2} \tilde{\boldsymbol{W}}_{Ja1}^T \tilde{\boldsymbol{W}}_{Ja1} + \frac{1}{2} \tilde{\boldsymbol{W}}_{Jc1}^T \tilde{\boldsymbol{W}}_{Jc1} \tag{21}$$

where $\tilde{\boldsymbol{W}}_{Ja1} = \boldsymbol{W}_1^* - \hat{\boldsymbol{W}}_{Ja1}$ and $\tilde{\boldsymbol{W}}_{Jc1} = \boldsymbol{W}_1^* - \hat{\boldsymbol{W}}_{Jc1}$ are the estimation errors, $\hat{\boldsymbol{W}}_{Ja1}$ and $\hat{\boldsymbol{W}}_{Jc1}$ are estimations of $\boldsymbol{W}_1^*$. Based on the designed virtual control law $\hat{\boldsymbol{\alpha}}$ and learning laws, we have

$$\dot{V}_1 \le -\frac{\epsilon_1 \pi}{\beta \mathrm{T}_{\max}} V_1^{1-\frac{\beta}{2}} - \frac{\bar{\epsilon}_1 \bar{\pi}}{\beta \mathrm{T}_{\max}} V_1^{1+\frac{\beta}{2}} + D_1 \tag{22}$$

where $\bar{\epsilon}_1 = \min\{2^\beta, \frac{\hbar_{c1}\zeta_b}{2}\}$ and $\epsilon_1 = \min\{1, \frac{\hbar_{c1}\zeta_b}{2}\}$, $D_1 = \frac{\beta}{2}(\frac{2-\beta}{2})^{(2-\beta)/\beta} + \frac{\hbar_{c1}\zeta_b}{2}(\frac{\|\bar{\boldsymbol{W}}_{Ja1}\|^2}{2})^{1+\beta/2} + \frac{\beta}{2}(\frac{2-\beta}{2})^{(2-\beta)/\beta} + \frac{\hbar_{c1}\zeta_b}{2}(\frac{\|\bar{\boldsymbol{W}}_{Jc1}\|^2}{2})^{1+\frac{\beta}{2}} + 0.2785\varsigma_1 + \frac{(\hbar_{c1}+\hbar_{a1})\zeta_b}{2} \boldsymbol{W}_1^{*T} \boldsymbol{W}_1^* + \|\boldsymbol{R}(\psi)\| \bar{\kappa}_{z_2}$, $\|\boldsymbol{\kappa}_{z_2}\| \le \bar{\kappa}_{z_2}$ with constants $\bar{\kappa}_{z_2} > 0$, $\|\bar{\boldsymbol{W}}_{Ja1}\| > 0$ and $\|\bar{\boldsymbol{W}}_{Jc1}\| > 0$.

Proof (Step 2): Choose the Lyapunov function as

$$V_2=\frac{\xi_z^T M\xi_z}{2(1-\xi_z^T\xi_z)^2}+\frac{\tilde{W}^T\Gamma^{-1}\tilde{W}}{2}+\frac{\tilde{W}_{Ja2}^T\tilde{W}_{Ja2}}{2}+\frac{\tilde{W}_{Jc2}^T\tilde{W}_{Jc2}}{2} \tag{23}$$

where $\tilde{W}$, $\tilde{W}_{Ja2}$ and $\tilde{W}_{Jc2}$ are the estimation errors, $\hat{W}_{Ja2}$ and $\hat{W}_{Jc2}$ are estimations of W_2^*.

Let $F(Z)=-M^{-1}[C(\upsilon)\upsilon+D(\upsilon)\upsilon+M\dot{\alpha}+M\xi_z\dot{\kappa}_z]$ with $Z=[\upsilon,\dot{\alpha},\xi_z,\kappa_z]^T$. RBFNN $\hat{F}(Z|\hat{W})=\hat{W}^T S(Z)$ is adopted to identify $F(Z)$, and one has

$$F(Z)=W^{*T}S(Z)+\varepsilon(Z) \tag{24}$$

where $S(Z)=[S_1(Z),S_2(Z),S_3(Z)]^T$ is the basis function vector, $\varepsilon(Z)=[\varepsilon_1,\varepsilon_2,\varepsilon_3]^T$ is the identify error vector, $W^{*T}=\mathrm{diag}\{W_u^{*T},W_v^{*T},W_r^{*T}\}$ is the ideal weight matrix.

In addition, according to Assumption 2, define $\omega_{1,0}^*=\varepsilon(Z)+d(t)$, which satisfies $\|\omega_{1,0}^*\|\le\omega_{1,0}$ with constant $\omega_{1,0}>0$, one has

$$\frac{\varpi_z}{\kappa_z}\xi_z\omega_{1,0}^*-\frac{\varpi_z}{\kappa_z}\xi_z\omega_{1,0}\tanh\left(\frac{\varpi_z}{\kappa_z\varsigma_2}\xi_z\omega_{1,0}\right)\le 0.2785\varsigma_2. \tag{25}$$

Similar to Step 1, $\dot{V}_2$ is

$$\dot{V}_2\le-\frac{\epsilon_2\pi}{\beta\mathrm{T}_{\max}}V_2^{1-\frac{\beta}{2}}-\frac{\bar{\epsilon}_2\bar{\pi}}{\beta\mathrm{T}_{\max}}V_2^{1+\frac{\beta}{2}}+D_2 \tag{26}$$

where $D_2=0.2785\varsigma_2+\frac{(\hbar_{c2}+\hbar_{a2})\zeta_b}{2}W_2^{*T}W_2^*+\frac{1}{2}W^{*T}KW^*+\frac{\beta}{2}(\frac{2-\beta}{2})^{\frac{2-\beta}{\beta}}+[\frac{\delta^*}{2}K]^{1+\frac{\beta}{2}}+\frac{\beta}{2}(\frac{2-\beta}{2})^{\frac{2-\beta}{\beta}}+\frac{\beta}{2}(\frac{2-\beta}{2})^{\frac{2-\beta}{\beta}}+\frac{\hbar_{c2}\zeta_b}{2}(\frac{\|\tilde{W}_{Ja2}\|^2}{2})^{1+\frac{\beta}{2}}+\frac{\hbar_{c2}\zeta_b}{2}\times(\frac{\|\tilde{W}_{Jc2}\|^2}{2})^{1+\frac{\beta}{2}}$, $\bar{\epsilon}_2=\min\{2^{\beta}(\lambda_{\max}(M)),\frac{\hbar_{c2}\zeta_b}{2},(\lambda_{\min}(K)\lambda_{\max}(\Gamma^{-1}))^{1+\beta/2}\}$ and $\epsilon_2=\min\{1/(\lambda_{\max}(M)),\frac{\hbar_{c2}\zeta_b}{2},(\lambda_{\min}(K)\lambda_{\max}(\Gamma^{-1}))^{1-\beta/2}\}$.

Let $V=\sum_{i=1}^2 V_i$, from (22) and (26), we have

$$\dot{V}\le-\frac{\epsilon\pi}{\beta\mathrm{T}_{\max}}V^{1-\frac{\beta}{2}}-\frac{\bar{\epsilon}\bar{\pi}}{\beta\mathrm{T}_{\max}}V^{1+\frac{\beta}{2}}+D \tag{27}$$

where $\epsilon=\min\{\epsilon_1,\epsilon_2\}$, $\bar{\epsilon}=\min\{\bar{\epsilon}_1,\bar{\epsilon}_2\}$ and $D=D_1+D_2$.

From (27), define the following sufficiently small region as:

$$\Omega=\left\{V|V\le\min\{[\frac{2\beta D\mathrm{T}_{\max}}{\epsilon\pi}]^{2/(2-\beta)},[\frac{2\beta D\mathrm{T}_{\max}}{\bar{\epsilon}\bar{\pi}}]^{2/(2+\beta)}\}\right\}.$$

Obviously, it ensure that V can converge into the set Ω in predefined time $\bar{\mathrm{T}}\le\sqrt{2}\mathrm{T}_{\max}$. Thus, it also ensure the errors ξ_e, ξ_z, $\hat{W}$, $\hat{W}_{Jai}$, $\hat{W}_{Jci}$ and other signals are all bounded in predefined time $\sqrt{2}\mathrm{T}_{\max}$. ■

Simulation example: For AUVs (1), the dynamic parameters are given as: $m_{1,1}=200$ Kg, $m_{2,2}=260$ Kg, $m_{3,3}=100$ Kg, $d_{1,1}=(60+110|u|)$ Kg/s, $d_{2,2}=(120+200|v|)$ Kg/s, $d_{3,3}=(80+120|r|)$ Kg/s. Choose the desired trajectory vector $\eta_d=[x_d,y_d,\psi_d]^T$ as $x_d=0.5t$, $y_d=10\sin(0.004\pi t)$, $\psi_d=\mathrm{atan}(y_d/x_d)$. The environmental disturbance $d(t)=[5\sin(0.5\pi t)+6\cos(\pi t),\sin(0.5t)+2\sin(0.1t),0]$. The initial conditions are given as: $\eta(0)=[-2,2,2]^T$, $\upsilon(0)=[0,0,0]^T$, $\hat{W}(0)=\mathrm{diag}\{0.02,0.05,0.03\}$, $\hat{W}_{Ja1}(0)=\mathrm{diag}\{0.1,0.3,0.2\}$, $\hat{W}_{Ja2}(0)=\mathrm{diag}\{0.2,0.3,0.1\}$, $\hat{W}_{Jc1}(0)=\mathrm{diag}\{0.3,0.1,0.2\}$, $\hat{W}_{Jc2}(0)=\mathrm{diag}\{0.5,0.2,0.3\}$. And design parameters are given as: $\beta=0.85$, $\mathrm{T}_{\max}=5$, $\pi=0.5$, $\varsigma_1=0.2$, $\varsigma_2=0.6$, $\tau_1=5$, $r_1=9.5$, $\tau_2=7.5$, $r_2=5.5$, $\hbar_{a1}=0.8$, $\hbar_{c1}=0.6$, $\hbar_{a2}=0.6$, $\hbar_{c2}=0.5$, $\Gamma=\mathrm{diag}\{10,10,20\}$, $K=\mathrm{diag}\{20,20,20\}$.

The performance functions $\kappa_{xe}(t)=(6-0.2)e^{-t}+0.2$, $\kappa_{ye}(t)=(10-0.5)e^{-t}+0.5$, $\kappa_{\psi e}(t)=(8-0.6)e^{-0.8t}+0.6$, $\kappa_{u,z_2}(t)=(5-0.3)e^{-1.5t}+0.3$, $\kappa_{v,z_2}(t)=(10-0.5)e^{-t}+0.5$, $\kappa_{r,z_2}(t)=(8-0.6)e^{-0.8t}+0.6$.

The simulation results are displayed by Figs. 1 and 2.

Conclusion: We have studied the ADP-based adaptive predefined-time optimal tracking control issue for AUVs. By constructing a novel barrier cost function with prescribed-performance technique, an error-constraint-based predefined-time adaptive optimal control approach has been developed. With the help of predefined-time stable theory, it has been proved all state errors can converge into a preset error boundary, and all system signals are bounded in predefined time.

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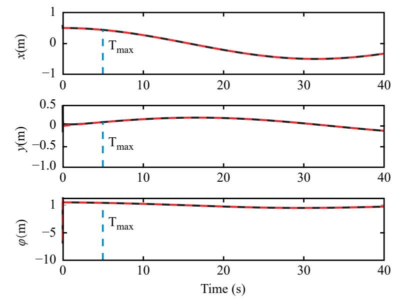

Fig. 1. Actual states and desired signals.

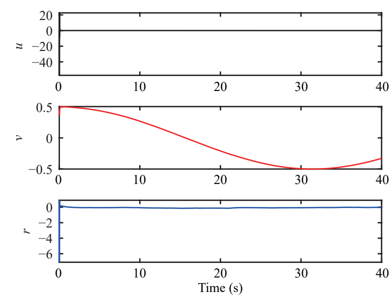

Fig. 2. AUV's velocities u, v, r.

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