Data-driven adaptive-critic optimal output regulation towards water level control of boiler-turbine systems☆

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ABSTRACT

This paper considers a new adaptive-critic optimal output regulation scheme for continuous-time general linear systems with unknown dynamics and unmeasurable disturbance. The objective of researching the optimal output regulation problem is pursuing the stability of the closed-loop system while enabling the output to optimally track the reference signal with disturbance rejection. The problem is solved by employing an approximate optimal regulator that consist of optimal feedback control obtained by a data-driven learning algorithm and optimal feedforward control achieved by solving regulator equation. The missing system matrices of the plant are exactly figured out by employing state/input data. Moreover, by employing the minimal polynomial of the exosystem matrix, the parameter of interference in the output equation is available. Then, a predefined cost function is presented based on the regulator equation that aims to design the optimal control law. The stability analysis demonstrate that the presented control scheme can stabilize the closed-loop system, meanwhile the output asymptotically tracks the reference signal. Finally, the effectiveness of the developed optimal control scheme is validated through an application towards the drum water level control of boiler-turbine systems.

1. Introduction

Owing to the rapid development of power generation technology, the new energy industry has became the spotlight in resent years. However in 2019, China's thermal power generation still accounted for more than 70 percent of the whole electricity (Chen, Yan, Guo, & Liu, 2020), which means the coal-fired power plant is still an indispensable part of the electricity supply. As the core of power generation units, co-ordinated control of the boiler-turbine system (BTS) exert an enormous function on the safe and economical operation of the power plant (Gao, Zeng, Ping, Zhang, & Liu, 2020), hence it becomes a hotspot of the research in related field.

Adaptive-critic designs (ACDs) (Dhar, Verma, & Behera, 2017; Liu, Zhang, & Tu, 2020; Wei, Zhu, Li, & Liu, 2021b; Yang & He, 2018; Zhang, Zhang, Xiao, & Jiang, 2019), a method of reinforcement learning theory (Guo, Yan, & Cui, 2019; Sutton, Barto, et al., 1998; Wei, Li, & Liu, 2020a; Zhang, Zhang, Cai, & Su, 2019; Zhou, Subagdja, Tan,

& Ong, 2021), is an effective approach to address model-free adaptive optimization problem (Chen, Hu et al., 2020). Since having almost the same structure and property in addressing optimal control problems, adaptive dynamic programming (ADP) is regarded as the synonym for ACDs (Liu, Wei, Wang, Yang, & Li, 2017). Several originally innovative, model-free learning algorithms are proposed for optimization problem of systems with unknown dynamics (Massenio, Naso, Lewis, & Davoudi, 2020; Modares & Lewis, 2014; Wei et al., 2020b; Xue, Luo, Liu, & Li, 2020). Djordjevic et al. (2022) consider control problem of the hydraulic servo actuators (HSA) with unknown dynamics. Through combining ADP technology with output feedback, an online learning data-driven controller based on measured input and output data is designed. In the work of Modares and Lewis (2014), a novel ADP algorithm is applied to solutions of optimal tracking problems with partially unknown system dynamics. An ADP based event-triggered

control scheme is proposed by Xue et al. (2020) to address the optimization problem of nonlinear systems with unknown parameter and input constraints. Zhan, Wu, Jiang, and Jiang (2015) study the optimal control problem of network systems under channel noise. In the work of Wei et al. (2020b), authors employ ADP method to achieve the optimal controller for ice-storage air conditioning system.

The output regulation is a significant theory in the field of cybernetics that applies to address output tracking problem combined with interference suppression while maintaining closed-loop stability (Huang, 2004). Meanwhile, some researchers pursue optimal performance of the output regulator through minimizing a reasonable designed cost function. This kind of problems are known as optimal output regulation problem. Quit a few related researches are studied , see Liu and Huang (2019), Saberi, Stoorvogel, Sannuti, and Shi (2003). A learning-based method is developed by utilizing input/partial-state data for the linear system with unknown dynamics and unmeasurable disturbance (Gao & Jiang, 2016). Jiang et al. (2022) investigate the learning-based adaptive optimal output regulation problem of disturbed linear continuous-time systems with convergence rate limitation. Without accurately modeling the system or a stabilizing feedback control gain, a new online iteration algorithm is presented, which can learn both the optimal feedback and feedforward control gain by measurable data. However, the controllers presented by Gao and Jiang (2016) and Jiang et al. (2022) are only suitable for the system excluded control input in output equation. This strong pre-condition limits its application in practice. Wu, Liang, and Hu (2021) improve the method proposed by Gao and Jiang (2016) to solve the optimization problem of general linear system with unknown dynamics.

In BTS, the boiler and turbine are considered as an integrated whole to address the discrepancies of their dynamic characteristics. Thus, the major objective of BTS is to balance the supply and demand between the power plant and grid. To track the load command, some progressive control schemes are proposed (Ławryńczuk, 2017; Pan, Shen, Wu, Nguang, & Chen, 2020; Zhu, Wu, & Shen, 2019). In the work of Ławryńczuk (2017), an efficient nonlinear predictive control law is presented through linearizing the local state-space online. An effective state-feedback controller for boiler-turbine nonlinear systems is developed by Zhu et al. (2019). Pan et al. (2020) propose an internal-model robust adaptive control approach to handle nonlinearity and long-time delay of BTS. Meanwhile, model predict control (MPC) algorithms are introduced to enhance energy-saving capacity of power generation in some researches. In the work of Zhang, Decardi-Nelson, Liu, Shen, and Liu (2020), a zone economic model predictive control scheme is presented to optimize the coal economy while satisfying the power generation demand from the grid. Klaučo and Kvasnica (2017) design a MPC-based optimal controller to improve safety and economical performance of a boiler-turbine system. Nevertheless, most existing control methods of BTS focus on command tracking or economy raising, which can be abstracted as state regulation/tracking problems. The drum water level, which only appears in the output equation of BTS, is rarely investigated as a crucial index for the safe operation of the system (Moradi, Saffar-Avval, & Bakhtiari-Nejad, 2012; Wang, Wu, & Ma, 2020). Moreover, most researches ignored the presence of disturbance that is unavoidable in the process of power generation.

In practical applications, input saturation, and fault detection are important problems to be considered (Wei, Lu, Zhou, Cheng, & Wang, 2021a; Zhang, Mu, Gao, & Wang, 2021). Zhuang, Tao, Chen, Stojanovic, and Paszke (2022) propose a novel iterative learning control (ILC) approach based on successive projection scheme for repetitive systems. By a simulation model, the presented algorithm is verified at occasions with and without input constraints. A new consensus control method of multi-agent systems with and without input saturation is proposed in Lu, Wu, Zhan, Han, and Yan (2021). The asynchronous fault detection (FD) observer design is studied for Markov jump systems (MJSs) by Cheng et al. (2021). A multi-objective scheme to the FD problem is designed through combining the H_∞ attenuation index and H_- increscent index.

According to its output tracking and optimization property, the application of optimal output regulation towards BTS can make power requirement economically suffice and stabilize drum water level with existence of disturbance simultaneously. However, the operation mechanism of BTS is sophisticated and tough to model, even if the precise model is established, the control protocol is difficult to design because of its complexity. In general, the model will be transformed into an affine nonlinear form, then linearize it at equilibrium point. But the accuracy can hardly be guaranteed in this process. Therefore, it is meaningful to design a control scheme for system with unknown dynamics and unmeasurable disturbance.

Motivated by aforementioned references, especially Gao and Jiang (2016), in this paper, we present a novel optimal output regulation control scheme for systems with unknown dynamics and unmeasurable disturbance to track the reference signal while optimizing the predefined cost function. By employing ACDs algorithm and output regulation method, a new adaptive optimal output tracker with interference suppression is designed. Meanwhile, we give a rigorous convergence analysis which can demonstrate the stability of presented controller. Finally, simulation results show that the output of the closed-loop system can asymptotically track the signal representing the desired water level with unmeasurable disturbance.

Comparing with the previous work, we underline the major contributions of this manuscript from the following aspects. First, we present an ACDs-based adaptive optimal controller for BTS. Different from Klaučo and Kvasnica (2017), Ławryńczuk (2017), linearizing the affine nonlinear model before designing the control scheme, the controller proposed in this paper is computed by measured state/input data and no longer require the state equation of BTS. Djordjevic et al. (2022) also utilize the data-driven method to design an online learning controller for HSA without knowledge of dynamics. However, the considered model is a linear discrete system, which is different from BTS. Second, by conclusions drawn from Gao and Jiang (2016), Jiang et al. (2022) and Wu et al. (2021), we achieve the approach to solve the optimal output regulation problem for model-free system with unmeasurable disturbance. Third, to the best of our knowledge, this manuscript is the first trial integrating output regulation and ACDs for control of drum water level in BTS. By integrating output regulation and ACDs, our proposed method can pursue the stability of the closed-loop system while enabling the output to optimally track the reference signal with disturbance rejection and no longer require the state equation of BTS to design the controller.

Notations. In this paper, $\mathbb{Z}_+$ represents the set of non-negative integers. $\mathbb{C}^-$ means the open left-half complex plane. For matrix $A \in \mathbb{R}^{n \times n}$, $\sigma(A)$ is complex spectrum of A. $\mathrm{col}(\cdot)$ denotes a concatenation. $\otimes$ denotes the Kronecker product operator. For matrix B, $\mathrm{vec}(B) = [b_1^T, b_2^T, \ldots, b_m^T]$, where $b_i \in \mathbb{R}^r$ is the columns of $B \in \mathbb{R}^{r \times m}$, $\ker(B)$ represents the kernel of matrix B. For symmetric matrix $C \in \mathbb{R}^{n \times n}$, $\mathrm{vecs}(C) = [c_{11}, 2c_{12}, \ldots, 2c_{1n}, c_{22}, 2c_{23}, \ldots, 2c_{i-1,i}, c_{ii}]^T \in \mathbb{R}^{\frac{i(i+1)}{2}}$, in which c_{nm} represents the nth row and mth column of matrix C. For column vector $x \in \mathbb{R}^i$, $\mathrm{vecv}(x) = [x_1^2, x_1 x_2, \ldots, x_1 x_n, x_2^2, x_2 x_3, \ldots, x_{i-1} x_i, x_i^2]^T \in \mathbb{R}^{\frac{i(i+1)}{2}}$, where x_n represents the nth element of the vector.

2. Problem formulation

In general, BTS can be considered as a three-input and three-output system (Åström & Bell, 1987; Wu, Shen, Li, & Lee, 2013) with the state vector $x(t) \in \mathbb{R}^3$, the output vector $y(t) \in \mathbb{R}^3$ and the input vector $u(t) \in \mathbb{R}^3$, where the states $x = [x_1, x_2, x_3]^T$ represent drum steam pressure, power of steam turbine and fluid density in drum respectively. While the inputs $u = [u_1, u_2, u_3]^T$ consist of flow rate of fuel, flow rate of steam and feedwater to drum. The output $y = [y_1, y_2, y_3]^T$ is drum steam pressure, power of steam turbine and water level of the drum. The schematic of BTS is demonstrated in Fig. 1.
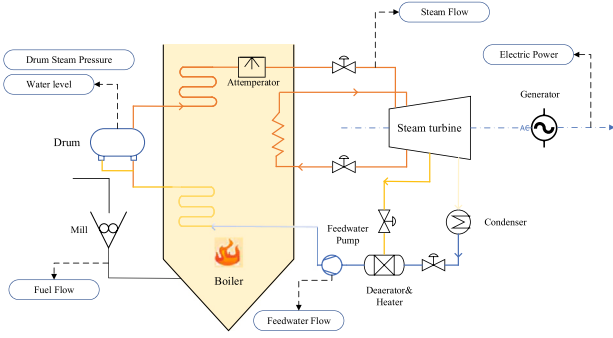
Fig. 1. The schematic diagram of a boiler-turbine power system.

2.1. Boiler-turbine system and drum water level control problem

BTS integrates the boiler and turbine into a coherent entirety to guarantee the global performance of the power plant. Since the conversion of water into saturated water vapor in the drum involves the calculation of physical parameters, which are discontinuous within the temperature range, it is difficult to establish an accurate mathematical model. In order to make the state track the load command from grid or manual setting and stable at the desired equilibrium point, most methods of current related research linearize the simplified model near the equilibrium point and then design the controller by the model predictive control (MPC) combined with fuzzy membership function (Hou, Gong, Huang, & Zhang, 2019; Kong & Yuan, 2019). It is notable that these methods require a high precision of the model, and the accuracy has great influence on the control effect. To solve this problem, the system model can be regarded as a linear model with unknown dynamics, meanwhile employ ADP algorithm to calculate an approximate optimal control law by state/input data.

Remark 1. Though the actual model of drum water level should be nonlinear model, in order to enable the designed controller to timely respond to the requirements of state adjustment, most current researches are linearizing the simplified model near the equilibrium point and then designing the controller by some optimal control methods (Hou et al., 2019; Kong & Yuan, 2019). The effectiveness of these methods are proved in several papers through practical application. Different from the methods that linearize the affine nonlinear model before designing the control scheme, the controller proposed in this manuscript no longer require the state equation of BTS and avoid the influence on the control effect caused by the inaccuracy of the equilibrium point.

The drum water level indirectly reflects the balance between steam load and water supply. Meanwhile, keeping the level at a proper value (generally called zero benchmark) is one of the prerequisites to ensure the stability of the power units. Overall, the main objective of drum water level control for BTS is satisfying power demand while stabilizing drum water level at zero benchmark with unmeasurable disturbance. Therefore, control effect of the drum water level has significant impact on safe operation. The process of conversion between water and steam in the drum is shown in Fig. 2.

According to the property of BTS and output regulation analyzed above, we can convert the drum water level control problem into the general linear output regulation problem.

2.2. Optimal output regulation problem

As assuming above, we first study a continuous-time linear system written as

$$
\begin{aligned}
\dot{x} &= \bar{A}x + \bar{B}u + \bar{E}\theta, \\
\dot{\theta} &= \hat{\Omega}\theta, \\
e &= \bar{C}x + \bar{D}u + \bar{F}\theta,
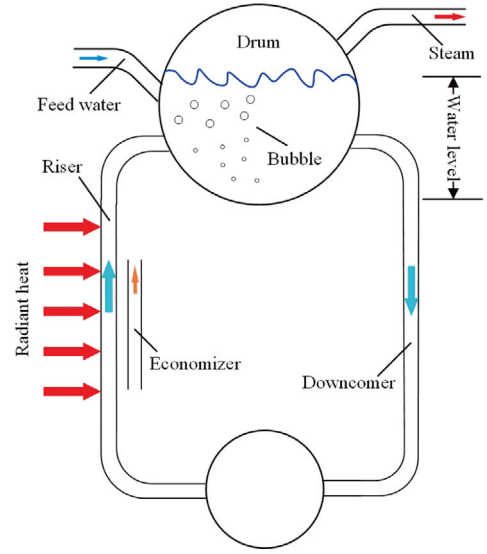\end{aligned}
\tag{1}
$$

Fig. 2. The schematic of water-steam system in a coal-fired power plant.

in which $x \in \mathbb{R}^n$ is the measurable state vector, $u \in \mathbb{R}^m$ is control input, $e \in \mathbb{R}^r$ is the tracking error. $\bar{A} \in \mathbb{R}^{n \times n}$, $\bar{B} \in \mathbb{R}^{n \times m}$, $\bar{E} \in \mathbb{R}^{n \times q}$, $\bar{C} \in \mathbb{R}^{r \times n}$, $\bar{D} \in \mathbb{R}^{r \times m}$, $\bar{F} \in \mathbb{R}^{r \times q}$ and $\hat{\Omega} \in \mathbb{R}^{q \times q}$ are constant matrices. To reflect the unknown dynamics of the system, $\bar{A}$, $\bar{B}$ and $\bar{E}$ are assumed unknown throughout this paper. $\theta \in \mathbb{R}^q$ is the state of exosystem, $d = \bar{E}\theta$ denotes the exogenous disturbances and $y_d = -\bar{F}\theta$ represents the reference signal. Some necessary assumptions are given as follows.

Assumption 1. The signal of exosystem θ is unmeasurable.

Assumption 2. $(\bar{A}, \bar{B})$ is stabilizable.

Assumption 3. $\text{rank} \begin{pmatrix} \lambda I_n - \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{pmatrix} = n + r$, in which $\lambda \in \sigma(\hat{\Omega})$.

Assumption 4. The minimal polynomial of $\hat{\Omega}$ can be obtained, which is

$$
\alpha_n(\hat{s}) = \prod_{i=1}^{M} (\hat{s} - \hat{\lambda}_i)^{a_i} \prod_{j=1}^{N} (\hat{s}^2 - 2\hat{\mu}_j \hat{s} + \hat{\mu}_j^2 + \hat{\omega}_j^2)^{b_j}
$$

in which the degree of the polynomial $\alpha_n(\hat{s})$ is q_n, $q_n \leq q$.

According to Assumption 4, we are able to obtain $w \in \mathbb{R}^{q_n}$ and $\Omega \in \mathbb{R}^{q_n \times q_n}$ satisfying

$$
\begin{aligned}
\dot{w} &= \Omega w, \\
\theta &= Gw,
\end{aligned}
\tag{2}
$$

where $G \in \mathbb{R}^{q \times q_n}$ is an unknown constant matrix.

Combining (2) into (1), the system can be transformed into the following form

$$
\begin{aligned}
\dot{x} &= \bar{A}x + \bar{B}u + Ew, \\
\dot{w} &= \Omega w, \\
e &= \bar{C}x + \bar{D}u + Fw,
\end{aligned}
\tag{3}
$$

where $E = \bar{E}G$ and $F = \bar{F}G$.

The optimal control law is designed for system (3) in the form as follow

$$
u = -K_x x + K_v w
\tag{4}
$$

where the closed-loop system has exponential stability and the tracking error $e(t)$ asymptotically converges to 0. $K_x \in \mathbb{R}^{m \times n}$, $K_v \in \mathbb{R}^{m \times q_n}$ can be called feedback gain matrix and feedforward gain matrix, respectively.

Theorem 1 (*Huang, 2004*). *Under Assumptions 3 and 4, find a K_x that satisfies $\sigma(\bar{A} + \bar{B}K_x) \subset \mathbb{C}^-$. Then, the output regulation problem can be addressed by control scheme (4), if there exist $X \in \mathbb{R}^{n \times q_n}$, $U \in \mathbb{R}^{m \times q_n}$ that satisfy the following matrix equation:*

$$X\Omega = \bar{A}X + \bar{B}U + E,$$
$$0 = \bar{C}X + \bar{D}U + F, \tag{5}$$

with the feedforward gain K_v given by

$$K_v = U + K_x X. \tag{6}$$

Remark 2. Matrices X and U can be regarded as the result of solving regulator Eqs. (5) in this manuscript. Moreover, (X, U) and (K_v, K_x) are related to each other by Eq. (6). Then, we have the following conclusion. Under Assumptions 2–3 in this manuscript, let the feedback gain K_x be such that $(A + BK_x)$ is exponentially stable. The linear output regulation is solvable by a feedback control of the form $u = -K_x x + K_v \omega$, if and only if there exist two matrices X and U that satisfy the linear matrix Eq. (5), with the feedforward gain K_v being given by $K_v = U + K_x X$.

For any initial state x_0 and w_0, controller (4) that address the regulator Eq. (5) can make $\lim_{t \to \infty}(u(t) - Uw(t)) = 0$, meanwhile $\lim_{t \to \infty}(x(t) - Xw(t)) = 0$.

Let $\hat{x} = x - X^* w$, $\hat{u} = u - U^* w$, by (3) and (4), we can form the error system as follows

$$\dot{\hat{x}} = \bar{A}\hat{x} + \bar{B}\hat{u} \tag{7}$$
$$e = \bar{C}\hat{x} + \bar{D}\hat{u}. \tag{8}$$

The control objective of BTS in this paper is stabilizing drum water level while minimizing the prescribed performance function which represents energy cost in the process of operation. To achieve this goal, we need to optimize (X^*, U^*) of Problem 1. Meanwhile, solving the optimization Problem 2 to obtain the optimal control law (Krener, 1992). These two problems are constructed as follows.

Problem 1.

$$\operatorname*{minimize}_{(X,U)} \quad \operatorname{vec}^T \begin{pmatrix} X \\ U \end{pmatrix} \operatorname{vec} \begin{pmatrix} X \\ U \end{pmatrix}$$
subject to   (5).

Problem 2.

$$\operatorname*{minimize}_{(\hat{x},\hat{u})} \quad J(\hat{x}, \hat{u}) = \int_0^\infty (\hat{x}^T Q \hat{x} + \hat{u}^T R \hat{u}) dt$$
subject to   (7) and (8)

where $Q \in \mathbb{R}^{n \times n}$ is a positive semi-definite matrix, $R \in \mathbb{R}^{m \times m}$ is a positive matrix and $(\bar{A}, \sqrt{Q})$ is observable.

By analysis above, control objective of the closed-loop system in this note can be completed if we present a control scheme $u = -K_x^* x + K_v^* w$ where:

(1) K_x^* can be obtained by optimizing Problem 2
(2) $K_v^* = U^* + K_x^* X^*$, in which (X^*, U^*) is optimization solution of Problem 1.

3. Data-driven optimal output regulator design

In this section, a new adaptive optimal output regulator with interference suppression is designed. First, by employing an ACDs-based data-driven algorithm, the absent system matrices $\bar{A}, \bar{B}$ and E are obtain. Then, we utilize the classical regulator equation to achieve the objective control law. Finally, the stability of the designed output regulation system is analyzed theoretically.

3.1. Solution of LQR problem

As Problem 2 is a normal LQR problem, the optimal feedback gain K_x^* can be computed through the method of Lewis, Vrabie, and Vamvoudakis (2012)

$$K_x^* = R^{-1} B^T P, \tag{9}$$

where P is the positive definite symmetric matrix and the unique solution of the equation written as follows

$$\bar{A}^T P + P\bar{A} - P\bar{B}R^{-1}\bar{B}^T P + Q = 0. \tag{10}$$

As (10) is nonlinear in terms of P, it is hard to compute P from (10). To approximate it, several algorithms are designed. One of them is recalled in the following.

Lemma 1 (*Kleinman, 1968*). *Let $K_{x0} \in \mathbb{R}^{m \times n}$ be an arbitrary feedback gain that can stable the system. Positive definite matrix P_j can be solved by the following Lyapunov equation*

$$(\bar{A} - \bar{B}K_{xj})^T P_j + P_j(\bar{A} - \bar{B}K_{xj}) + Q + K_{xj}^T R K_{xj} = 0, \tag{11}$$

for each $j = 1, 2, \ldots$

$$K_{xj} = R^{-1}\bar{B}^T P_{j-1}. \tag{12}$$

Therefore, the properties below hold:
   (1) $\sigma(\bar{A} - \bar{B}K_{xj}) \subset \mathbb{C}^-$;
   (2) $P^* \le P_{j+1} \le P_j$;
   (3) $\lim_{j \to \infty} K_{xj} = K_x^*$, $\lim_{j \to \infty} P_j = P^*$.

3.2. ACDs-based solution of optimal feedback gain

Under the condition that the coefficient of matrices $\bar{A}, \bar{B}$ and E are absent, we propose an ACDs-based data-driven algorithm that can approximately compute the matrix P^* and the optimal feedback gain K_x^* in (9).

Let $\bar{A}_j = \bar{A} - \bar{B}K_{xj}$. Then we can transform system (3) into

$$\dot{x} = \bar{A}_j x + \bar{B}(K_{xj} x + u) + Ew. \tag{13}$$

Let $Q_j = Q + K_{xj}^T R K_{xj}$. According to (10) and (13), on $[t, t + \Delta t]$, we have

$$x^T(t + \Delta t)P_j x(t + \Delta t) - x^T(t)P_j x(t)$$
$$= \int_t^{t+\Delta t} [x^T(\bar{A}_j^T P_j + P_j \bar{A}_j)x + 2(u + K_{xj}x)^T \bar{B}P_j x$$
$$+ 2w^T E^T P_j x]d\tau$$
$$= \int_t^{t+\Delta t} [-x^T Q x + 2(u + K_{xj}x)^T R K_{x(j+1)}x$$
$$+ 2w^T E^T P_j x]d\tau. \tag{14}$$

By introducing Kronecker product, we convert Eq. (14) as the following form

$$x^T Q_j x = (x^T \otimes x^T)\operatorname{vec}(Q_j)$$
$$(u + K_{xj})^T R K_{x(j+1)} x = [(x^T \otimes x^T)(I_n \otimes (K_{xj}R)) +$$
$$(x^T \otimes u^T)(I_n \otimes R)]\operatorname{vec}(K_{x(j+1)})$$
$$w^T E^T P_j x = (x^T \otimes w^T)\operatorname{vec}(E^T P_j). \tag{15}$$

Furthermore, for positive integer l, we define

$$\delta_{xx}(t_l) = [\text{vecv}(x(t_1)) - \text{vecv}(x(t_0)), \text{vecv}(x(t_2)) - \\ \text{vecv}(x(t_1)), \dots, \text{vecv}x(t_l) - \text{vecv}(x(t_{l-1}))]^T,$$

$$\Gamma_{xx}(t_l) = \left[\int_{t_0}^{t_1} x \otimes x d\tau, \dots, \int_{t_{l-1}}^{t_l} x \otimes x d\tau\right]^T,$$

$$\Gamma_{\bar{x}}(t_l) = \left[\int_{t_0}^{t_1} \text{vecv}(x) d\tau, \dots, \int_{t_{l-1}}^{t_l} \text{vecv}(x) d\tau\right]^T, \quad (16)$$

$$\Gamma_{xu}(t_l) = \left[\int_{t_0}^{t_1} x \otimes u d\tau, \dots, \int_{t_{l-1}}^{t_l} x \otimes u d\tau\right]^T,$$

$$\Gamma_{xw}(t_l) = \left[\int_{t_0}^{t_1} x \otimes w d\tau, \dots, \int_{t_{l-1}}^{t_l} x \otimes w d\tau\right]^T,$$

in which $t_0 < t_1 < \cdots < t_l$ are positive integers. By (15) and (16), we transform (14) as

$$\Psi_j \begin{pmatrix} \text{vecs}(P_j) \\ \text{vec}(K_{x(j+1)}) \\ \text{vec}(E^T P_j) \end{pmatrix} = \Phi_j, \quad (17)$$

where $\Psi_j = [\delta_{xx}(t_l), -2\Gamma_{xx}(t_l)(I_n \otimes (K_{xj}{}^T R)) - 2\Gamma_{xu}(t_l)(I_n \otimes R), -2\Gamma_{xw}(t_l)]$ and $\Phi_j = -\Gamma_{xx}(t_l)\text{vec}(Q_j)$.

Lemma 2.

$$\text{rank}(\Gamma_{xx}(t_l)) = \text{rank}(\Gamma_{\bar{x}}(t_l)) \le \frac{n(n+1)}{2}.$$

Proof. For $k = 0, 1, \dots, n-2$ and $j = k+2, \dots, n$, the $(kn+j)$th row of Γ_{xx}^T is $(\int_{t_0}^{t_1} x_j x_{k+1} d\tau \cdots \int_{t_{s-1}}^{t_s} x_j x_{k+1} d\tau)$, which is same as the $((j-1)n+k+1)$th row of Γ_{xx}^T. Obviously, there are $\frac{n(n-1)}{2}$ pairs in total. It is noted that $\Gamma_{\bar{x}}$ is obtained by omitting the $((j-1)n+k+1)$th row. Therefore, $\text{rank}(\Gamma_{xx}) = \text{rank}(\Gamma_{\bar{x}}) \le n^2 - \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$. The proof is completed.

Lemma 3. *If there exists a $l^* \in \mathbb{Z}_+$, for all $l > l^*$, any sequence $t_0 < t_1 < \cdots < t_l$*

$$\text{rank}\left([\Gamma_{xx}(t_l), \Gamma_{xu}(t_l), \Gamma_{xw}(t_l)]\right) = \frac{n(n+1)}{2} + (m + q_n)n. \quad (18)$$

Matrix Ψ_j $j \in \mathbb{Z}_+$ is full column rank.

Proof. The detail of this proof is similar to Lemma 3 in Gao and Jiang (2016), so we omit it here.

According to Lemma 3, if the related assumptions hold, Eq. (17) can be solved as follows

$$\begin{pmatrix} \text{vecs}(P_j) \\ \text{vec}(K_{x(j+1)}) \\ \text{vec}(E^T P_j) \end{pmatrix} = (\Psi_j^T \Psi_j)^{-1} \Psi_j^T \Phi_j. \quad (19)$$

Both P_j and K_{k+1} can be computed by (19). From (12), we obtain that

$$\bar{B} = P_j^{-1} K_{x(j+1)}^T R. \quad (20)$$

It is noted that after iteratively figuring out the matrix P_j, matrix E is obtained simultaneously.

Theorem 2. *If there exists a $l^* \in \mathbb{Z}_+$ such that for all $l > l^*$ any sequence $t_0 < t_1 < \cdots < t_l$ that satisfies Eq. (18), by a initial admissible feedback matrix $K_{x0} \in \mathbb{R}^{m \times n}$, the sequence $(P_j, K_{x(j+1)})(j = 0, 1, \dots)$ obtained from Algorithm 1 converge to (P^*, K_x^*).*

Proof. In Lemma 3, the matrix Ψ_j is full column rank. Therefore, P_j and $K_{x(j+1)}$ can be obtained from (19). If Assumption 3 holds, P_j and $K_{x(j+1)}$ satisfy (11) and (12), respectively. Thus, the convergence of P_j and $K_{x(j+1)}$ is proved by Lemma 1.

Algorithm 1 ACDs-based data-driven algorithm for the optimal feedback gain

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1: Utilize $u = -K_{x0}x + \xi$ on $[t_0, t_l]$ with bounded exploration noise ξ and $\sigma(\bar{A} - \bar{B}K_{x0}) \subset \mathbb{C}^-$ to create initial admissible control;
2: Compute $\delta_{xx}(t_l), \Gamma_{xx}(t_l), \Gamma_{xu}(t_l), \Gamma_{xw}(t_l)$ until the rank condition (18) is guaranteed;
3: Solve E, P_j and $K_{x(j+1)}$ from (19);
4: Let $j \leftarrow j + 1$, repeat computing P_j until $\|P_{j+1} - P_j\| \le \varepsilon$ with $j \ge 1$ and a small positive constant ε;
5: Calculate $\bar{B}$ from (20).

3.3. Solving system matrix $\bar{A}$

To solve the optimal regulator (4), the unknown matrices $\bar{A}, \bar{B}$ and E should be achieved in advance. Algorithm 1 is applied to obtain matrices $\bar{B}$ and E. Next, another method is presented to solve the matrix $\bar{A}$.

Before moving on, we introduce some customized matrices for concision. Let $S_1 = I_n$, $S_2 = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$, $S_3 = \begin{pmatrix} 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$,

$\dots, S_{\frac{n^2-n+2}{2}} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}$. $S_1, S_2, \dots, S_{\frac{n^2-n+2}{2}} \in \mathbb{R}^{n \times n}$. As the derivation in (14), following from (3), we have

$$x^T(t + \Delta t)S_i x(t + \Delta t) - x^T(t)S_i x(t) = \\ \int_t^{t + \Delta t} \left[x^T(\bar{A}^T S_i + S_i \bar{A})x + 2(u^T \bar{B} + \omega^T E^T)S_i x\right] d\tau, \quad (21)$$

in which $i = 1, 2, \dots, \frac{n^2-n+2}{2}$.

Similar to Eq. (15), we transform (21) into Kronecker product representation

$$\begin{aligned} x^T(\bar{A}^T S_i + S_i \bar{A})x &= 2(x^T \otimes x^T)\text{vec}(\bar{A}^T S_i) \\ &= (\text{vec}(x))^T \text{vecs}(\bar{A}^T S_i + S_i \bar{A}), \\ u^T \bar{B} S_i x &= (x^T \otimes u^T)\text{vec}(\bar{B}S_i), \\ w^T E^T S_i x &= (x^T \otimes w^T)\text{vec}(E^T S_i), \end{aligned} \quad (22)$$

Then (22) can be rewritten as the following form

$$\hat{\Phi}\text{vecs}(\bar{A}^T S_i + S_i \bar{A}) = \hat{\Psi} \begin{pmatrix} \text{vecs}(P_j) \\ \text{vec}(K_{x(j+1)}) \\ \text{vec}(E^T P_j) \end{pmatrix}, \quad (23)$$

where $\hat{\Psi} = [\delta_{xx}(t_{l1}), -2\Gamma_{xu}(t_{l1}), -2\Gamma_{xw}(t_{l1})]$ and $\hat{\Phi} = \Gamma_{\bar{x}}(t_{l1})$.

Lemma 4. *If there exists a $l_1^* \in \mathbb{Z}_+$ such that for all $l > l_1^*$, for any sequence $t_0 < t_1 < \cdots < t_{l1}$*

$$\text{rank}(\hat{\Phi}) = \frac{n(n+1)}{2}. \quad (24)$$

Therefore,

$$\text{vecs}(\bar{A}^T S_i + S_i \bar{A}) = (\hat{\Phi}^T \hat{\Phi})^{-1} \hat{\Phi}^T \hat{\Psi} \begin{pmatrix} \text{vecs}(S_i) \\ \text{vec}(\bar{B}S_i) \\ \text{vec}(E^T S_i) \end{pmatrix}. \quad (25)$$

Proof. Since the matrix $\hat{\Phi}$ has full column rank, $\hat{\Phi}^T \hat{\Phi}$ is nonsingular, we can multiply the left side of (23) with $(\hat{\Phi}^T \hat{\Phi})^{-1} \hat{\Phi}^T$ and obtain (25).

As $\bar{B}, E$ are obtained by Algorithm 1, $\hat{\Phi}, \hat{\Psi}$ and $\begin{pmatrix} \text{vecs}(S_i) \\ \text{vec}(\bar{B}S_i) \\ \text{vec}(E^T S_i) \end{pmatrix}$ can

all be computed. Let $\bar{A} = (a_{kj})_{n \times n}$ and $S_i = (s_{kj}^{(i)})_{n \times n}$. obviously, $\text{vecs}(\bar{A}^T S_i + S_i \bar{A}) = s_i$, in which $s_i = \text{col}(2\sum_{k=1}^{n} a_{k1} s_{k1}^{(i)}, 2(\sum_{k=1}^{n} a_{k1} s_{k2}^{(i)} + a_{k2} s_{k1}^{(i)}), \ldots, 2(\sum_{k=1}^{n} a_{kj} s_{km}^{(i)} + a_{km} s_{kj}^{(i)}), \ldots, 2\sum_{k=1}^{n} a_{kn} s_{kn}^{(i)}) \in \mathbb{R}^{\frac{n(n+1)}{2}}$. Substituting $S_1 = I_n$ into (25), a_{12} and a_{21} are available. Similarly, substituting $S_{\frac{n^2-n+2}{2}}$ into (25), $a_{n-1,n}$ and $a_{n,n-1}$ can be figured out. Then, the system matrix $\bar{A}$ is reached.

3.4. Optimal feedforward gain

Since unknown matrices $\bar{A}, \bar{B}, E$ and the approximate optimal feedback gain matrix K_x^* is obtained by the analysis above, we are able to figure out the optimal feedforward gain matrix K_v^*. To obtain it, we first solve Problem 1, which is subjected to regulator Eq. (5).

The regulator Eq. (5) can be put into the following form (Huang, 2004):

$$\begin{bmatrix} I_n & 0_{n \times m} \\ 0_{r \times n} & 0_{r \times m} \end{bmatrix} \begin{bmatrix} X \\ U \end{bmatrix} \Omega$$
$$- \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix} \begin{bmatrix} X \\ U \end{bmatrix} = \begin{bmatrix} E \\ F \end{bmatrix}. \tag{26}$$

Using the properties of Kronecker product, we can transform (26) into a standard linear algebraic equation of the form

$$Tx = b, \tag{27}$$

where

$$T = \Omega^T \otimes \begin{bmatrix} I_n & 0_{n \times m} \\ 0_{r \times n} & 0_{r \times m} \end{bmatrix} - I_{q_n} \otimes \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix},$$

$$x = \text{vec}\left(\begin{bmatrix} X \\ U \end{bmatrix}\right), b = \text{vec}\left(\begin{bmatrix} E \\ F \end{bmatrix}\right).$$

The ACDs-based algorithm for optimal output regulation with unknown system matrices is presented as follows.

Algorithm 2 ACDs-based data-driven algorithm for general linear optimal output regulation problem.

1: Utilize $u = -K_{x0}x + \xi$ on $[t_0, t_l]$ with bounded exploration noise ξ and $\sigma(\bar{A} - \bar{B}K_{x0}) \subset \mathbb{C}^-$ to create initial admissible control;
2: Solve the optimal feedback gain K_x^* and matrices $\bar{B}, E$ by algorithm 1;
3: Calculate the matrix $\bar{A}$ by (25);
4: Solve Problem 1 by (26) and (27) to find (X^*, U^*);
5: Compute K_v by (6), the approximate optimal output regulator is $u = -K_x^* x + K_v^* w$.

Remark 3. The Algorithm 1 and 2 presented in this paper both contain three quantities that should be specified to begin the algorithm. They consist of one arbitrary feedback gain K_{x0} that can stable the system, a bounded exploration noise ξ and the data sampling time period $[t_0, t_l]$. Two of them can affect the outcome as assigning different initial value. If the amplitude of the exploration noise is too large, the state will be far from the equilibrium point and the system will be unstable. Therefore, the amplitude of the noise components is limited from -0.5 to 0.5, -0.3 to 0.3 and -2 to 2, respectively in simulation process. If the time period is too short, the amount of data is not enough to solve the unknown matrices $\bar{A}, \bar{B}$ and E. Hence, the time period should be set long enough.

3.5. Stability analysis

In this subsection, the stability of system (3) with the optimal output regulator achieved by using ACDs-based data driven algorithm 2 is analyzed theoretically.

Theorem 3. *Under Assumptions 1–4 and conditions in Lemmas 3–4, the optimal output regulation problem of system (3) can be obtained by the following control scheme*

$$u = -K_x^* x + K_v^* w$$

where K_x^* and K_v^* are computed by Algorithm 1 and Algorithm 2, respectively. Then we have:

(1) The closed-loop system is stabilized by the presented control scheme;
(2) The tracking error $e(t)$ converges to 0 asymptotically.

Proof. (1) Suppose Assumption 3, $\sigma(\bar{A} - \bar{B}K_x^*) \subset \mathbb{C}^-$ can be directly proved by Lemma 1 and Theorem 2.

(2) Since Assumptions 1–3 and conditions in Lemmas 3–4 hold, the approximate optimal feedback control gain K_x^* can be achieved. The system (3) in closed-loop with the approximate optimal control law ensure that

$$\dot{\hat{x}} = (\bar{A} - \bar{B}K_x^*)\hat{x},$$
$$e = (\bar{C} - \bar{D}K_x^*)\hat{x}.$$

From (1), we get $\lim_{t \to \infty} \bar{x}(t) = 0$, and therefore $\lim_{t \to \infty} e(t) = 0$. The proof is completed.

4. Numerical analysis

In the section above, we achieve the approximate optimal output regulation control scheme of system (3) theoretically by ACDs-based data driven algorithm 2. To demonstrate the effectiveness of the presented scheme, we apply it towards the boiler-turbine system in this section.

We collect operation data from a 160MW boiler-turbine coordinate system. The output equation of this system can be written as (Åström & Bell, 1987)

$$y_1 = x_1,$$
$$y_2 = x_2, \tag{28}$$
$$y_3 = 0.05(0.13073x_3 + 100\alpha_s + q_e/9 - 67.975),$$

where $q_e = ((0.854u_2 - 0.147)x_1 + 45.59u_1 - 2.51u_3 - 2.096)$ and $\alpha_s = (1 - 0.00154x_3)(0.8x_1 - 25.6)/x_3(1.039 - 0.0012304x_1)$. Considering the equilibrium point under load 60MW with the state $x_d = [102, 60, 438.93]^T$ and control $u_0 = [0.3102, 0.671, 0.3971]$, we can linearize the output equation by Taylor expansion at the set equilibrium point. Then we obtain the linear output equation

$$y = \bar{C}x + \bar{D}u,$$

where

$$\bar{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.0065 & 0 & 0.0054 \end{bmatrix},$$

$$\bar{D} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.2786 & 0.5323 & -0.0153 \end{bmatrix}.$$

We assume that state of BTS fluctuates around the equilibrium point x_d, so the exosystem can be written as

$$\dot{w} = \begin{bmatrix} 0 & -\pi & 0 & 0 \\ \pi & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5\pi \\ 0 & 0 & 1.5\pi & 0 \end{bmatrix} w.$$

Remark 4. It is noted that $d = \bar{E}\theta$ denotes the exogenous disturbances, where $\dot{w} = \Omega w, \theta = Gw$ and $G, \bar{E}$ are unknown constant matrices. The dynamic model of ω given in the simulation part is used to describe the exosystem. Therefore, we just know the eigenvalue of the system matrix of disturbance and the unknown constant matrices $G, \bar{E}$ can reflect the unpredictability of the disturbance.

(a) Optimal control law u_1

(b) Optimal control law u_2

(c) Optimal control law u_3

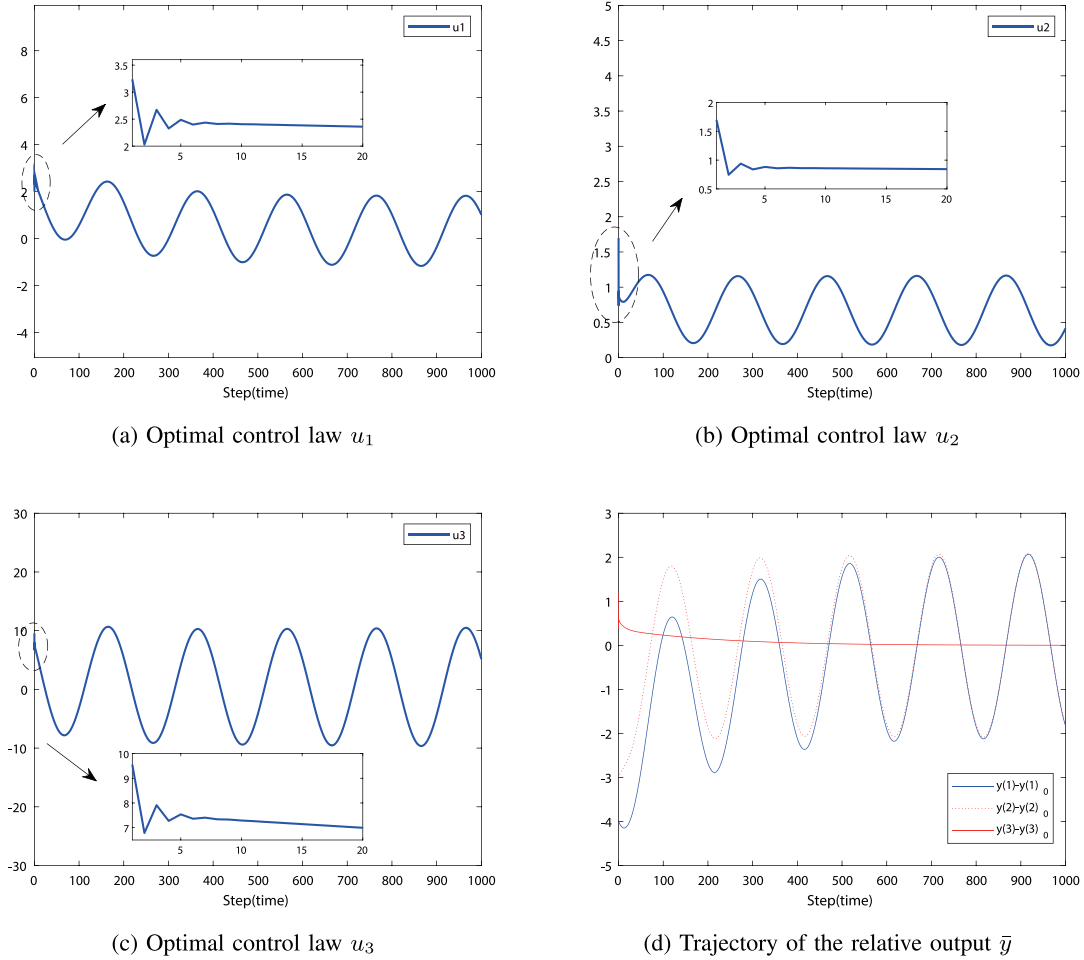(d) Trajectory of the relative output $\bar{y}$

Fig. 3. The ACDs-based optimal output regulation control law and relative output of the BTS.

The initial state is $w(0) = [1, 0, 1, 0]^T$. To stabilize drum water level at the zero benchmark under the existence of state fluctuation, we can generate the reference signal as

$$y_d = Fw = \begin{bmatrix} \sqrt{3} & 1 & 0 & 0 \\ \sqrt{3} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} w.$$

Remark 5. The prescribed performance function used in this paper is written as $J(\hat{x}, \hat{u}) = \int_0^\infty (\hat{x}^T Q \hat{x} + \hat{u}^T R \hat{u}) dt$, where $\hat{x} = x - X^* w$, $\hat{u} = u - U^* w$. Finding the optimal control scheme for BTS that minimizes this performance function can make power requirement economically suffice and stabilize drum water level with existence of disturbance simultaneously. The requirement of quickly or slowly responding to demand for certain state component can be achieved by changing the related element on the diagonal of the weight matrices Q. Similarly, changing the related element on the diagonal of the weight matrices R can make the corresponding input factor have a strong or weak willing to save the relevant energy. For the sake of generality, we chose matrices Q and R as identity matrix with appropriate dimensions.

The matrices $\bar{A}$, $\bar{B}$, and E are unknown. Meanwhile Q and R are chosen as identity matrix. The noise ξ consist of sinusoidal waves with diverse frequencies. The feedback gain matrix K_{xj} updates after each iteration and the optimal one K_x^* is

$$K_x^* = \begin{bmatrix} 0.5758 & 0.0166 & 0.0332 \\ -0.0131 & 0.5722 & -0.0527 \\ -0.0347 & 0.0498 & 0.5740 \end{bmatrix},$$

from the initial admissible one

$$K_{x0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The solution of (X, U) is unique and optimal. Then K_v^* can be computed and the result is

$$K_v^* = \begin{bmatrix} -0.2133 & -1.7815 & 0 & 0 \\ -1.4552 & -0.2826 & 0 & 0 \\ 7.1495 & -7.0930 & 0 & 0 \end{bmatrix}.$$

Fig. 3 displays the optimal control law generated by ACDs-based learning Algorithm 2 and the relative output of the system (compared with the equilibrium point). Fig. 4 shows the state trajectory of BTS with designed optimal controller. Since the state of BTS is interfered by sinusoidal disturbance, the control scheme need regular fluctuation to keep the drum water level stabilizing at the zero benchmark. Fig. 5 demonstrates that the ACDs-based approximate optimal controller render the output of BTS asymptotically track the reference signal. Therefore, the drum water level can be stabilized at the benchmark point with unknown disturbance.

To more clearly illustrate the presented algorithm in this paper, we add an extended example to further prove the effectiveness of the presented scheme. In this case, we assume that the transfer of equilibrium point will occur during the operation, which happens commonly in practice.

As shown in Fig. 6, the transfer occurs at 400th step, where the tracking error suddenly increases because the system equation switches. After that the designed algorithm recalculates the system
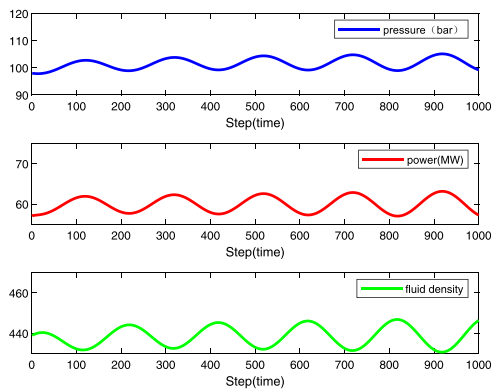
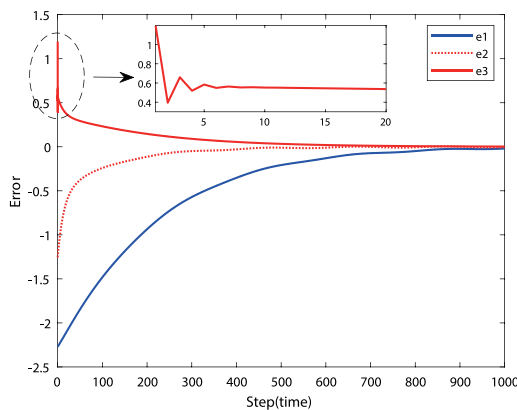**Fig. 4.** State trajectory of BTS with optimal controller.

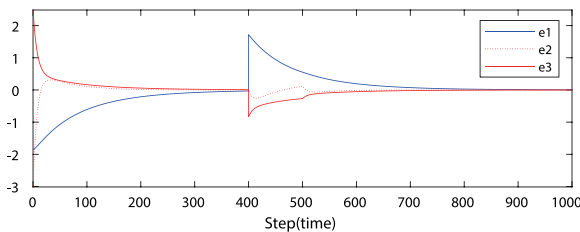**Fig. 5.** Output tracking errors of BTS with optimal controller.

Fig. 6. Output tracking errors of BTS with equilibrium point transfer.

matrix through the data obtained at the new equilibrium point and the corresponding optimal output regulator is obtained. When the newly calculated input is applied to the system, the output tracking error gradually converges to 0. About 500 steps later, the output tracking error converges to 0. Through the extended experiment, it is proved that the algorithm is also effective even when the equilibrium point transfer occurs during the operation.

5. Conclusion

In this note, a data-driven optimal output regulation algorithm is presented towards water level control of BTS with unknown dynamics and disturbance. To obtain the optimal feedback control gain, we utilize an ACDs-based learning method to update the feedback gain during each iteration. Then, by solving optimization Problem 1 under the regulator Eq. (5), the feedforward control gain is also achieved. Moreover, convergence analysis is presented to validate stability of the optimal control law. Finally, results of the application to water level control of boiler-turbine system are shown to prove effectiveness of

the designed method. The main limitation of the proposed method is that the unmeasurable disturbance of the system should be linear and the eigenvalues of it need be known in advance, which renders the application of this method is limited. The implication for practice is highlighted in the way that optimal output regulation towards BTS can make power requirement economically suffice and stabilize drum water level with existence of disturbance simultaneously. Future work will focus on the solution of optimal output regulation for nonlinear system with completely unknown disturbances.

CRediT authorship contribution statement

Qinglai Wei: Conceptualization, Methodology. **Xin Wang:** Data curation, Writing – original draft, Software. **Yu Liu:** Software, Validation. **Gang Xiong:** Supervision, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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