Letter

Intermittent Control for Fixed-Time Synchronization of Coupled Networks

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Dear Editor,

This letter deals with fixed-time synchronization (Fd-TS) of complex networks (CNs) under aperiodically intermittent control (AIC) for the first time. The average control rate and a new Lyapunov function are proposed to overcome the difficulty of dealing with fixed-time stability/synchronization of CNs for AIC. Based on the Lyapunov and graph-theoretical methods, a Fd-TS criterion of CNs is given. Moreover, the method of this letter is also applicable to the study of finite-time synchronization of CNs for AIC. Finally, the theoretical results are applied to study the Fd-TS of oscillator systems, and simulation results are given to verify the effectiveness of the results.

Recently, the dynamics of CNs have attracted extensive attention due to their wide applications in real-world networks. As one of the most important collective behaviors of CNs, synchronization has received considerable interest in many fields [1]. It should be noted that the most existing results about the synchronization of CNs studied asymptotic synchronization and exponential synchronization [2], and they are often classified into infinite-time synchronization. In many practical problems, achieving synchronization within a finite time is more desirable and useful. Therefore, finite-time synchronization (Fe-TS) has been investigated by many researchers. In contrast with infinite-time synchronization, Fe-TS has been reported to possess faster convergence and better performance against uncertainties and disturbances.

Nevertheless, a significant limitation of Fe-TS is that the settling time depends on the initial values. In many practical systems, the initial values may be difficult to obtain in advance. Fortunately, this problem was overcome by Polyakov [3] through introducing the concept of fixed-time stability and presenting fundamental results on fixed-time stability. Inspired by Polyakov's novel fixed-time stability theory, there are some follow-up works about fixed-time stability for various CNs [4]–[6]. Compared to Fe-TS, the settling time of Fd-TS is determined by the designed controller parameters, which do not rely on the initial values and can be estimated in advance. Furthermore, many practical systems such as microgrid systems and spacecraft dynamics usually desire to achieve fixed-time convergence. Consequently, it is meaningful and necessary to further explore the Fd-TS of CNs both in theory and methods.

In general, it is difficult to realize self-synchronization for CNs due to the complexity of node dynamics and topologies. Therefore, many kinds of control techniques have been employed to achieve the synchronization of CNs in [7]. Different from the continuous control schemes, the discontinuous control methods such as intermittent control (IC) [2], [8], and impulsive control have been extensively studied because they can reduce control cost as well as the number of information exchanges. It is known that IC can be divided into periodically IC (PIC), and AIC [2], and AIC takes PIC as a special case; thus, it is more general to consider AIC. For AIC, scholars mainly considered asymptotic synchronization [2], exponential synchronization [9], and Fe-TS. However, there are few results focusing on the Fd-TS for CNs by AIC. The existing theory to study asymptotic synchronization, exponential synchronization, and Fe-TS cannot be directly extended to Fd-TS. Moreover, the Fd-TS theoretical framework based on AIC is not established. Therefore, it is urgently necessary to develop a new theory and methods to investigate the Fd-FS of CNs via AIC, which motivates this work. The main contributions of this letter are as follows.

1) Unlike the existing literature dealing with finite-time stability/synchronization for IC, in this letter, we establish a theoretical framework of fixed-time stability/synchronization for AIC for the first time. 2) Compared with the existing literature [4], an auxiliary function is introduced to consider the Fd-TS of CNs under IC, which allows the control function in the rest intervals of IC to be zero. In the existing literature [4], the control function in the rest intervals of IC is not zero, which may be regarded as a switching control rather than an IC in a general sense. Thus, the control strategy proposed in this letter is more general. 3) In [2], [9], some scholars mainly considered the asymptotic synchronization or exponential synchronization for AIC. Moreover, the existing literature have certain restrictions on the control intervals, such as $\sup_i\{\zeta_{i+1}-\mu_i\} \le con1$, $\sup_i\{\zeta_{i+1}-\zeta_i\} \le con2$, where $con1$ and $con2$ are positive constants. For parameters ζ_i and μ_i, see Fig. 1. This letter uses the average control rate ϑ in (5), not the infimum of the control rate $\eta_i = (\mu_i-\zeta_i)/(\zeta_{i+1}-\zeta_i)$, which is easy to satisfy the condition of the theorem because of $\vartheta \ge \liminf_{i\to\infty}\{\eta_i\}$. In addition, the idea of using the average control rate in this letter also can apply to the study of finite-time stability/synchronization for AIC, which is more general.

Fig. 1. Schematic diagram of AIC. Blue and yellow areas represent control and rest intervals, respectively.

Mathematical model: The CNs model is considered as follows:

$$\dot{x}_k(t) = f_k(x_k(t),t) + \sum_{h=1}^{n} b_{kh}\alpha_{kh}(x_k(t),x_h(t)),\ \ k\in\mathcal{N} \tag{1}$$

where $x_k(t) = (x_{k1}(t),x_{k2}(t),\ldots,x_{km}(t))^T \in \mathbb{R}^m$ and $\mathcal{N} = \{1,2,\ldots,n\}$; coupled function $\alpha_{kh}: \mathbb{R}^m\times\mathbb{R}^m \to \mathbb{R}^m$; $f_k: \mathbb{R}^m\times\mathbb{R}^+ \to \mathbb{R}^m$ and $b_{kh} \ge 0$ is coupling weight and $b_{kk} = 0$ for all $k\in\mathcal{N}$.

System (1) is considered as a master system, and we consider the following system as slave system:

$$\dot{z}_k(t) = f_k(z_k(t),t) + \sum_{h=1}^{n} b_{kh}\alpha_{kh}(z_k(t),z_h(t)) + u_k(t),\ \ k\in\mathcal{N} \tag{2}$$

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in which $z_k(t) \in \mathbb{R}^m$ and $u_k(t)$ is an AIC strategy.

Let $y_k(t) = z_k(t) - x_k(t)$ be error vector. Based on systems (1) and (2), we can obtain

$$\dot{y}_k(t) = \tilde{f}_k(x_k(t), z_k(t), t) + \sum_{h=1}^{n} b_{kh}\tilde{\alpha}_{kh}(x_k(t), z_k(t), x_h(t), z_h(t)) + u_k(t),\ \ k \in \mathcal{N} \tag{3}$$

in which $\tilde{f}_k(x_k, z_k, t) = f_k(z_k, t) - f_k(x_k, t)$ and $\tilde{\alpha}_{kh}(x_k, z_k, x_h, z_h) = \alpha_{kh}(z_k, z_h) - \alpha_{kh}(x_k, x_h)$. The AIC $u_k(t)$ is considered as follows:

$$u_k(t) = \begin{cases} -\varrho_1 y_k(t) - \varrho_2 \mathrm{sign}(y_k(t))[y_k(t)]^p \\ -\varrho_3 \mathrm{sign}(y_k(t))[y_k(t)]^q,\ \ t \in [\zeta_i, \mu_i),\ \ i \in \mathbb{N} \\ 0,\quad t \in [\mu_i, \zeta_{i+1}) \end{cases} \tag{4}$$

where $\mathrm{sign}(y_k) = \mathrm{diag}(\mathrm{sign}(y_{k1}), \mathrm{sign}(y_{k2}), \ldots, \mathrm{sign}(y_{km}))$ and $\mathbb{N} = \{0, 1, 2, \ldots\}$; $[\zeta_i, \mu_i)$ and $[\mu_i, \zeta_{i+1})$ stand for the ith control interval and rest interval, respectively; $\zeta_0 = 0$, $[y_k]^p = (|y_{k1}|^p, |y_{k2}|^p, \ldots, |y_{km}|^p)^T$; $\varrho_s > 0$ $(s = 1, 2, 3)$, $q > 1$, and $0 < p < 1$.

Assumption 1: Functions f_k and α_{kh} satisfy the Lipschitz conditions with Lipschitz constants $\beta_k > 0$ and $\varphi_{kh} > 0$, respectively.

Definition 1 [8]: For AIC strategy (4), there are $\vartheta \in (0, 1)$ and $T_\vartheta \geq 0$ such that

$$T_{con}(t_1, t_2) \geq \vartheta(t_1 - t_2) - T_\vartheta,\ \ \forall t_1 > t_2 \geq t_0 \tag{5}$$

in which $T_{con}(t_1, t_2)$ stands for the total control interval length on $[t_2, t_1)$, ϑ represents the average control rate, and T_ϑ is called the elasticity number.

Definition 2: The master system (1) and slave system (2) are said to achieve Fe-TS, if there is a settling time $T(y(0)) > 0$ such that $\lim_{t \to T(y(0))} y(t) = 0$ and $y(t) \equiv 0$ for $t \geq T(y(0))$, where $y(t) = (y_1^T(t), y_2^T(t), \ldots, y_n^T(t))^T$. The time $T(y(0))$ is called the settling time of synchronization, which is dependent of $y(0)$. Especially, if there is a fixed time $T^* > 0$ which is independent of $y(0)$, then systems (1) and (2) achieve the Fd-TS.

Analysis of Fd-TS: This section gives two lemmas to study the Fd-TS under AIC (4). Then, a Fd-TS criterion of CNs is given.

Lemma 1: Assume that continuous function $\Psi(t) \geq 0$ satisfies

$$\begin{cases} \dot{\Psi}(t) \leq -a_1 \Psi^q(t) - a_2 \Psi^p(t),\ \ t \in [\zeta_i, \mu_i) \\ \dot{\Psi}(t) \leq 0,\qquad\qquad\qquad\qquad t \in [\mu_i, \zeta_{i+1}) \end{cases} \tag{6}$$

in which $a_1 > 0$, $a_2 > 0$, $q > 1$, and $0 < p < 1$. Then, $\Psi(t) = 0$ when $t = T^*$, where settling time T^* satisfies

$$T^* \leq \frac{1 + a_1(q-1)T_\vartheta}{a_1(q-1)\vartheta} + \frac{1 + a_2(1-p)T_\vartheta}{a_2(1-p)\vartheta}$$

where T_ϑ and ϑ are defined in Definition 1.

Proof: See Section II in the Supplementary material. ■

Lemma 2: Assume that continuous function $\Psi(t) \geq 0$ satisfies

$$\begin{cases} \dot{\Psi}(t) \leq -a_1 \Psi^q(t) - a_2 \Psi^p(t) - a_3 \Psi(t),\ \ t \in [\zeta_i, \mu_i) \\ \dot{\Psi}(t) \leq a_4 \Psi(t),\qquad\qquad\qquad\qquad t \in [\mu_i, \zeta_{i+1}) \end{cases} \tag{7}$$

in which $a_i > 0$ $(i = 1, 2, 3, 4)$, $q > 1$, and $0 < p < 1$. If there exists $\breve{\varepsilon} > 0$ satisfying the following inequalities:

$$-a_3 + \breve{\varepsilon}(1 - \vartheta) < 0 \text{ and } a_4 - \breve{\varepsilon}\vartheta < 0$$

then $\Psi(t) = 0$ when $t \geq T^*$, where settling time T^* satisfies

$$T^* \leq \frac{1 + \hat{a}_1(q-1)T_\vartheta}{\hat{a}_1(q-1)\vartheta} + \frac{1 + a_2(1-p)T_\vartheta}{a_2(1-p)\vartheta}$$

where $\hat{a}_1 = a_1 \exp\{(1-q)\breve{\varepsilon}T_\vartheta\}$, T_ϑ and ϑ are defined in Definition 1.

Proof: See Section III in the Supplementary material. ■

Now, a main result is state as follows.

Theorem 1: If the digraph $(\mathcal{G}, A)$ with $A = (b_{kh}\varphi_{kh})_{n \times n}$ is strongly connected, and there exists $\breve{\varepsilon} > 0$ satisfying the following inequalities:

$$-a_3 + \breve{\varepsilon}(1 - \vartheta) < 0 \text{ and } a_4 - \breve{\varepsilon}\vartheta < 0 \tag{8}$$

in which $a_3 = \min\{a_{3k}\}$, $a_4 = \max\{a_{4k}\}$, $a_{3k} = 2(\varrho_1 - \beta_k) - 4\sum_{h=1}^{n} b_{kh}\varphi_{kh} > 0$, and $a_{4k} = 2\beta_k + 4\sum_{h=1}^{n} b_{kh}\varphi_{kh}$, then systems (1) and (2) achieve the Fd-TS, and settling time T^* satisfies

$$T^* \leq \frac{1 + \hat{a}_1(q-1)T_\vartheta}{\hat{a}_1(q-1)\vartheta} + \frac{1 + a_2(1-p)T_\vartheta}{a_2(1-p)\vartheta} \tag{9}$$

with $\hat{a}_1 = a_1 \exp\{(1-q)\breve{\varepsilon}T_\vartheta\}$, $a_2 = 2\sigma^1_{\min}\varrho_2$, $a_1 = 2\sigma^2_{\min}\varrho_3(mn)^{\frac{1-q}{2}}$, $\sigma^1_{\min} = \min_{k \in \mathcal{N}}\{c_k^{\frac{1-p}{2}}\}$, $\sigma^2_{\min} = \min_{k \in \mathcal{N}}\{c_k^{\frac{1-q}{2}}\}$, and $c_k > 0$ can be found in [10].

Proof: See Section IV in the Supplementary material. ■

Remark 1: Theorem 1 requires two inequalities in (8) to be true. We can get that if the average control rate ϑ is greater, the conditions are easier to meet when other parameters are fixed, which shows that the design of IC has an essential impact on the Fd-TS of CNs.

Remark 2: In the proof of Lemma 2, we use an auxiliary function $U(t) = \exp\{\Theta(t)\}\Psi(t)$, where function $\Theta(t)$ is defined in Section III-1 of the Supplementary material. Moreover, when $t \in [\mu_i, \zeta_{i+1})$, we can get that

$$\dot{\Theta}(t) \leq -\breve{\varepsilon}\vartheta \tag{10}$$

which enables $\dot{U}(t) \leq -(\breve{\varepsilon}\vartheta - a_4)U(t)$ to be established in the rest intervals $[\mu_i, \zeta_{i+1})$. Similar ideas were discussed in semilinear systems [8]. This letter uses the technique to overcome the difficulty of studying Fd-TS of CNs under AIC.

Remark 3: In Lemma 2, we give a synchronization criterion to achieve the Fd-TS of the systems under AIC for the first time. In the existing results [2], [9], some scholars mainly considered the asymptotic synchronization or exponential synchronization for IC. Moreover, the existing results have certain restrictions on the control and rest intervals, such as $\sup_i\{\zeta_{i+1} - \mu_i\} = con1$, $\sup_i\{\zeta_{i+1} - \zeta_i\} = con2$, where $con1$ and $con2$ are positive constants. In this letter, we use the average control rate instead of the infimum of the control rate $\eta_i = (\mu_i - \zeta_i)/(\zeta_{i+1} - \zeta_i)$, which is easier to satisfy the conditions of the theorem because of $\vartheta \geq \inf_{i \in \mathbb{N}}\{\eta_i\}$. In addition, the idea of using an average control rate in this letter also applies to the study of Fe-TS, which is more general. However, as far as we know, no author has considered the general case for Fe-TS under AIC by using the technique of this letter.

Remark 4: We give an important differential inequality (7) in Lemma 2, which can deal with Fd-TS of CNs for AIC. Moreover, the fixed time T^* depends on the average control rate ϑ and elasticity number T_ϑ. In addition, we find that the larger parameters a_1 and a_2 in (7), the smaller the fixed time T^*. And when the average control rate ϑ is greater, the settling time T^* will be smaller.

Remark 5: Recently, some scholars have considered the Fd-TS for CNs under AIC [4]. For the IC, this letter allows the control function in the rest intervals of the IC to be zero. In the existing results [4], the control function in the rest intervals of IC is not zero, which may be regarded as a switching control. In addition, the average control rate of AIC is considered, which provides less conservative results.

Application and numerical simulations: See Section IV in the Supplementary material.

Conclusions: We considered the Fd-TS of CNs under AIC. The average control rate and a new Lyapunov function were proposed to overcome the difficulty of dealing with fixed-time stability/synchronization of CNs for AIC. Meanwhile, a Fd-TS criterion of CNs was given. Finally, we applied the theoretical results to study the Fd-TS of oscillator systems, and simulation results were given to verify the effectiveness of the results. Considering the influence of the delay factor, the Fd-TS of delayed CNs under AIC will be studied in the future.

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Supplementary material: The supplementary material of this letter can be found in links https://maifile.cn/est/d3296793221015/pdf.

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