

Consensus Control of Multi-Agent Systems With Two-Way Switching Directed Topology[★]

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Abstract: In this paper, a novel control protocol method is developed to solve distributed consensus control problems for multi-agent systems with switching directed topologies. Because of the external disturbance, the information cannot transmit completely so that the proposition of two-way topology is necessary. Besides, the external disturbance is considered naturally and we give a control protocol and parameters of noise intensity function in this paper. Furthermore, an example is given to support the proposed theorem. Finally, the simulation results prove the correctness of the theory.

Keywords: Distributed consensus control; multi-agent systems; two-way topology; fixed and switching directed topologies;

1. INTRODUCTION

The consensus problem of multi-agent systems has received compelling interest and has been studied extensively over the past years due to the rapid developments of computer science and its wide application in many areas. Consensus problem is the fundamental problem in multi-agent systems. The essence of it is to construct proper controller so that all agents can attain a consensus decision value by using the information of each agent and its neighbours (Tu et al., 2017; Li et al., 2019; Zheng et al., 2017). Recently, distributed consensus control for multi-agent systems has attracted more and more attention compared with the traditional centralized coordination control approaches.

In the consensus problem of multi-agent systems, the major objective is to make all agents converge to a common state (Shariati and Zhao, 2018; Shen et al., 2019). The consensus control of multi-agent systems has been researched a lot in recent years and has obtained many good results (Li et al., 2020; Lan et al., 2019). For instance, in (Moreau, 2005), it presents the necessary and/or sufficient conditions for the convergence of the individual agents' states to a common value. In (Xin and Li, 2016), it is shown that the observer-based protocol solves the consensus problem when the sampling period is sufficiently small and the average dwell time of the switching signal is sufficiently large. Many researches have received, however, the consensus analysis for directed topology is more

challenging than the case of undirected topology (Olfati-Saber and Murray, 2004). Group consensus tracking issue of continuous-time second-order multi-agent systems under directed fixed topology was studied in (Huang et al., 2018). In addition, with the widespread application of machine learning, various learning methods are introduced to settle consensus problems. For instance, (Wei et al., 2015) solves the optimization problem of a class of stochastic nonlinear systems by reinforcement learning algorithm. Moreover, for the case of multi-agent systems with multiple leaders, the distributed exponential finite-time containment of multi-agent systems was addressed in (Liu et al., 2015).

Some preliminary results of multi-agent systems with directed topology have reported in (Zeng et al., 2017; Liu et al., 2016). According to the absence or presence of leaders in a multi-agent system, consensus control can be divided into leaderless consensus and leader-following consensus. In (Jadbabaie et al., 2003), it introduced the leaderless consensus problem of multi-agent systems under directed topology. While in this paper, we introduce the directed topology with a leader, which broaden the applications by guaranteeing all the individual dynamics converge to a desired trajectory.

Since the switching topology of multi-agent systems is more practical than the fixed topology in the real environment, we analyse the property of the multi-agent with switching topology in this paper. Inspired by the preliminary literature (Lin et al., 2016), consensus control of multi-agent systems with two-way switching directed topologies is introduced in this paper.

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The configuration of this paper is given as follows. Graphs and preliminaries of heterogeneous multi-agent systems are presented in section 2. The multi-agent systems problem with directed topology is given in section 3. Consensus control with switching directed topology of multi-agent systems is presented in section 4. Section 5 provides the simulation studies. Conclusion is given in section 6.

2. GRAPHS AND PRELIMINARIES OF HETEROGENEOUS MULTI-AGENT SYSTEMS

2.1 Graphs

Let G_r be a directed graph with a set of $V = \{v_1, v_2, \dots, v_N\}$, a set of directed edges $\varepsilon \subseteq V \times V$, the adjacency element a_{ij} of weighted adjacency matrix $\Delta = [a_{ij}]$ is non-negative value. The edges of G_r are denoted by e_{ij} . A edge e_{ij} in graph G_r is denoted by the ordered pair of nodes (v_j, v_i) , where v_j and v_i are called the parent and child nodes, respectively. The adjacency element corresponding to the edge of the graph is positive, i.e., $a_{ij} > 0$ if and only if $e_{ij} \in \varepsilon$. Moreover, we assume that $a_{ii} = 0$ for all the i . If there is information flowing from vertex i to vertex j , $a_{ij} > 0$; otherwise $a_{ij} = 0$. Laplacian matrix $L = \mathbb{D} - \Delta$, $\mathbb{D} = [\mathbb{D}_{ij}]$ is a diagonal matrix, $\mathbb{D}_{ii} = \sum_{j=1}^N a_{ij}$. Therefore, the Laplacian matrix $L = [l_{ij}]$ can be written as

$$l_{ij} = \begin{cases} \sum_{k=1, k \neq i}^n a_{ik}, & i = j \\ -a_{ij}, & i \neq j \end{cases}$$

For the multi-agent system with a leader and N followers, we define that 0 represents the leader and $1, 2, \dots, N$ represent N followers, respectively. We define a diagonal matrix $D = \text{diag}[d_i]$, $i = 1, 2, \dots, N$. $d_i > 0$ if it is connected between the leader and the i th follower. We can define that $d_i > 0$ if there is information flowing between the leader and the i th follower; otherwise, $d_i = 0$. The G_r is defined on the vertices $0, 1, \dots, N$.

Besides, for the whole paper, $R^{N \times N}$ is the set of $n \times n$ real matrix. I_N is the identity matrix of dimension N . In this paper, S is positive symmetric matrix. $A \otimes B$ is the Kronecker product of matrix A and B .

There are some matrices defined for simplification, i.e., $H_{\tau(t)} = L_{\tau(t)} + D_{\tau(t)} \in R^{N \times N}$, $R^1 = \text{diag}(R_1, R_2, \dots, R_N)$, $R^2 = [R_{ji}] \in R^{N \times N}$ $i, j = 1, 2, \dots, N$.

3. THE MULTI-AGENT SYSTEMS PROBLEM WITH DIRECTED TOPOLOGY

In this paper, multi-agent system with directed topology is considered, which contains an active leader indexed by 0 and N agents indexed by $1, 2, \dots, N$ in graph G_r . The dynamics of the i th agent is given by

$$\dot{x}_i(t) = Ax_i(t) + C(t)\varphi(x_i(t), t) + Bu_i(t) \quad (1)$$

where $x_i \in R^n$ is the state of the i th agent, $i = 1, 2, \dots, N$ and $u_i \in R^m$ is the control input of the i th agent which can only use local information of its neighbours and itself. A and B are constant real matrices. $C(t) \in R^{n \times n}$ is the function of time. The $\varphi(x_i(t), t) =$

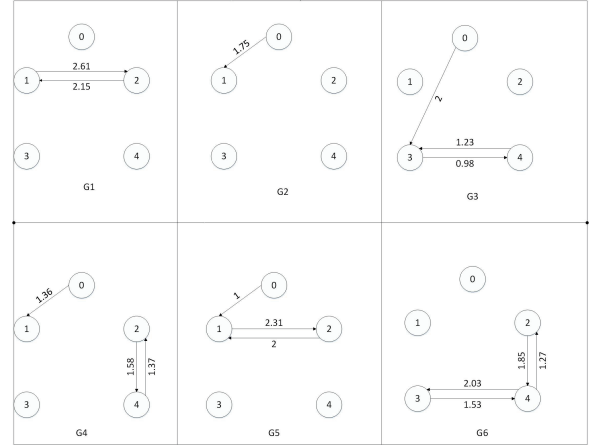


Fig. 1. Switching directed topology of multi-agent systems

$(\varphi_0(x_i(t), t), \varphi_1(x_i(t), t), \dots, \varphi_n(x_i(t), t))^T, i = 1, 2, \dots, N$ is the noise intensity function vector. The dynamics of the leader denoted by 0 is represented as

$$\dot{x}_0(t) = Ax_0(t) + C(t)\varphi(x_0(t), t) \quad (2)$$

where x_0 is the state of the leader. Because the dynamics of the leader is independent of the others, i.e., $u_0(t) \equiv 0$. There is no control input for the leader. $\varphi(x_0(t), t) : R^n$ is the noise function vector of the leader.

4. CONSENSUS CONTROL WITH SWITCHING TOPOLOGY OF MULTI-AGENT SYSTEMS

In this section, we design a control protocol with switching directed topology of multi-agent system which can be seen in figure 1. Before moving on, let $C(t)$ be the scalar zero mean with Gaussian white noise matrix and there are some Assumptions applied to the subsection.

Assumption 1. There is a nonnegative constant ρ , which satisfies the Lipschitz condition

$$\|\varphi(y, t) - \varphi(z, t)\| \leq \rho \|y - z\| \quad (3)$$

the condition holds for all $y, z \in R^n, t > 0$.

Assumption 2. The vertex 0 associated with the leader is global reachable vertex.

Assumption 3. The internal dynamics matrix A has eigenvalues with negative real part.

We give the general assumptions to analyse the switching directed topology of the multi-agent system.

- (1) Consider a switching signal $\tau : [t_0, \infty) \rightarrow \Psi$, it's piecewise constant. Ψ is a finite set of all possible interconnection topologies of the multi-agent system, which initial time is t_0 . All the possible graphs are defined on the vertices $0, 1, \dots, N$ by $\{G_p : p \in \Psi\}$ and $\{G_a : p \in \Psi\}$ is used to denote subgraphs that defined on vertices $1, 2, \dots, N$.
- (2) The time-interval $[t_0, \infty)$ is composed of an infinite sequence of bounded, non-overlapping, contiguous time-intervals $[t_j, t_{j+1})$ for $j = 0, 1, \dots$ with $t_0 = 0$. What's more, there is a sequence of non-overlapping subintervals for each interval $[t_k, t_{k+1})$, the subintervals are $[t_k^0, t_k^1), [t_k^1, t_k^2), \dots, [t_k^{m_k-1}, t_k^{m_k})$, $t_k = t_k^0$, $t_{k+1} = t_k^{m_k}$, satisfying $t_k^{j+1} - t_k^j \geq \mu$, $0 \leq j \leq m_k - 1$ for some integers $m_k \geq 1$ and constant $\mu \geq 0$.

Therefore, the interconnection topology is fixed during each of such subintervals. We define each subinterval $[t_k^j, t_k^{j+1})$ by G_k^j and the graph defined by $G_{\tau(t)}$ is fixed.

Assumption 4. The graphs are jointly connected across each interval $[t_k, t_{k+1})$, $k = 0, 1, \dots$

The consensus control with two-way directed topology of multi-agent system can be achieved, if the following equation is satisfied for initial condition $x_i(0)$, $i = 1, 2, \dots, N$.

$$\lim_{t \rightarrow \infty} E\{|x_i(t) - x_0(t)|^2\} = 0, i = 0, 1, \dots, N$$

where $E[\cdot]$ is the mathematical expectation of a given random variable.

Now, we give the control protocol of the multi-agent with the system (1) and (2). The protocol is designed as

$$u_i = K \left[\sum_{j=1}^N \alpha_{ij} a_{ij} (x_i - x_j) + \alpha_i d_i (x_i - x_0) \right], \quad (4)$$

$$\dot{\alpha}_{ij} = \eta_{ij} a_{ij} (x_i - x_j)^T \Gamma (x_i - x_j),$$

$$\dot{\alpha}_i = \eta_i d_i (x_i - x_0)^T \Gamma (x_i - x_0) \quad i = 1, 2, \dots, N,$$

where $K \in R^{m \times n}$ is the feedback gain matrix. a_{ij} is the (i, j) entry of the adjacency matrix Δ and d_i is the i th diagonal entry of the leader adjacency matrix D . α_{ij} is the time-varying coupling weight between agent i and agent j and α_i is the time-varying coupling weight between agent i and agent 0. Because of $a_{ij} \neq a_{ji}$, the values of $\dot{\alpha}_{ij}$ and $\dot{\alpha}_{ji}$ are different, i.e., $\dot{\alpha}_{ij} \neq \dot{\alpha}_{ji}$. Besides, η_{ij} and η_i are both positive constants.

From the system (1), we can rewritten the system as

$$dx_i(t) = [Ax_i(t) + Bu_i(t)]dt + \varphi(x_i(t), t)dW(t) \quad (5)$$

where $dW(t) = C(t)dt$ and $C(t)$ is Gauss white noise generated by Brown motion $W(t) \in R^{n \times n}$. According to the knowledge we learned, it's easy to get that $E\{W(t)\} = 0$, $E\{[W(t)]^2\} = dt$. The system of the leader is rewritten as

$$dx_0(t) = Ax_0(t)dt + \varphi(x_0(t), t)dW(t). \quad (6)$$

Let $e_i(t) = x_i(t) - x_0(t)$, integrating equations (5) and (6), the state error is as follows

$$de_i(t) = Ae_i(t)dt + Bu_i(t)dt + \tilde{\varphi}(e_i(t), t)dW(t), \quad (7)$$

where $\tilde{\varphi}(e_i(t), t) = \varphi(x_i(t), t) - \varphi(x_0(t), t)$, substituting (4) into (7), we can get

$$de_i = Ae_i dt + BK \left[\sum_{j=1}^N \alpha_{ij} a_{ij} (e_i - e_j) + \alpha_i d_i e_i \right] dt + \tilde{\varphi}(e_i, t) dW(t) \quad i = 1, 2, \dots, N. \quad (8)$$

The positive symmetric matrix $S > 0$ is the solution to the inequality as follows

$$S < \xi I, \quad SA + A^T S - 2SBB^T S + \xi \gamma < 0, \quad (9)$$

where $\xi > 0$ is the tuning parameter and $\gamma = \rho I_4$. The feedback gain matrix can be considered as

$$K = -B^T S. \quad (10)$$

For the symmetry and neat, the constant gain matrix in (4) is represented as

$$\Gamma = SBB^T S. \quad (11)$$

Theorem 5. Consider the multi-agent system given by (1) and (2), under the Assumptions 2-4, the problem of consensus control is solved by the controller (4). Besides, the coupling weighting α_{ij} and α_i converge to some finite constants.

Proof. Given the Lyapunov function as follows

$$V(t) = \sum_{i=1}^N e_i^T(t) S e_i(t) + \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{(\alpha_{ij} - b)^2}{\eta_{ij}} + \sum_{i=1}^N \frac{(\alpha_i - b)^2}{\eta_i}, \quad (12)$$

where b is a positive constant, using (8), the derivative of $V(t)$ is calculated as follows

$$\begin{aligned} dV(t) = & \sum_{i=1}^N 2e_i^T S de_i + \sum_{i=1}^N \tilde{\varphi}^T(e_i, t) S \tilde{\varphi}(e_i, t) dt \\ & + 2 \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{(\alpha_{ij} - b)}{\eta_{ij}} d\alpha_{ij}(t) \\ & + \sum_{i=1}^N \frac{2(\alpha_i - b)}{\eta_i} d\alpha_i(t) \end{aligned} \quad (13)$$

using (4), we get

$$\begin{aligned} & \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{(\alpha_{ij} - b)}{\eta_{ij}} d\alpha_{ij}(t) \\ = & \sum_{i=1}^N \sum_{j=1, j \neq i}^N (\alpha_{ij} - b) a_{ij} e_i^T \times SBB^T S(e_i - e_j) dt \\ & + \sum_{i=1}^N \sum_{j=1, j \neq i}^N (\alpha_{ji} - b) a_{ji} e_i^T \times SBB^T S(e_i - e_j) dt \end{aligned} \quad (14)$$

Substituting (14) into (13), the derivative of $V(t)$ is derived as follows

$$\begin{aligned} dV(t) = & 2 \sum_{i=1}^N e_i^T S A e_i dt + \sum_{i=1}^N \tilde{\varphi}^T(e_i, t) S \tilde{\varphi}(e_i, t) dt \\ & + 2 \sum_{i=1}^N e_i^T S \tilde{\varphi}(e_i, t) dW(t) \\ & - 2b \sum_{i=1}^N \sum_{j=1}^N a_{ij} e_i^T SBB^T S(e_i - e_j) dt \\ & - 2b \sum_{i=1}^N d_i e_i^T SBB^T S e_i dt \\ & + \sum_{i=1}^N \sum_{j=1}^N 2(\alpha_{ji} - b) a_{ji} e_i^T SBB^T S(e_i - e_j) dt. \end{aligned} \quad (15)$$

According to the Assumption 1, we get

$$\sum_{i=1}^N \tilde{\varphi}^T(e_i, t) S \tilde{\varphi}(e_i, t) dt \leq \xi \sum_{i=1}^N e_i^T \gamma e_i dt, \quad (16)$$

where ξ is the maximum eigenvalue of the matrix S , $\gamma = \rho I_4$ and we obtain $E\{W(t)\} = 0$. Hence, the (15) can be rewritten as

$$\begin{aligned}
 dE\{V(t)\} \leq & E\left\{\sum_{i=1}^N e_i^\top (2SA + \xi\gamma) e_i dt \right. \\
 & - 2b \sum_{i=1}^N \sum_{j=1}^N H_{ij} e_i^\top SBB^\top S e_j dt \\
 & + 2 \sum_{i=1}^N M_i e_i^\top SBB^\top S e_i dt \\
 & \left. - 2 \sum_{i=1}^N \sum_{j=1}^N M_{ji} e_i^\top SBB^\top S e_j dt\right\}, \quad (17)
 \end{aligned}$$

where H_{ij} denotes the (i, j) th entry of the matrix $H_{\tau(t)} = L_{\tau(t)} + D_{\tau(t)}$. Let $R_{ji} = (\alpha_{ji} - b)a_{ji}$ and $R_i = \sum_{j=1}^N R_{ji}$. Denoting $\varepsilon = (e_1^\top, e_2^\top, \dots, e_N^\top)$, $R^1 = \text{diag}(R_1, R_2, \dots, R_N)$ and $R^2 = [R_{ji}]$, $i, j = 1, 2, \dots, N$. We can get

$$\begin{aligned}
 dE\{V(t)\} \leq & E\{\varepsilon^\top [I_N \otimes (SA + A^\top S + \xi\gamma) \\
 & - 2(bH - M^1 + M^2) \otimes SBB^\top S] \varepsilon dt\}. \quad (18)
 \end{aligned}$$

Let $bH - R^1 + R^2 = Q^\top \Lambda Q$, where Q is an orthogonal matrix and we assume that Q exists. Denoting $\delta = (Q \otimes I_n)\varepsilon$, the (18) becomes

$$\begin{aligned}
 dE\{V(t)\} \leq & E\{\delta^\top [I_N \otimes (A^\top S + SA + \xi\gamma) \\
 & - 2b\Lambda \otimes SBB^\top S] \delta dt\} \\
 \leq & E\left\{\sum_{i \in l(\tau(t))}^N \delta_i^\top (A^\top S + SA \right. \\
 & \left. - 2b\lambda_i SBB^\top S + \xi\gamma) \delta_i dt\right\} \\
 \leq & -E\left\{\sum_{i \in l(\tau(t))}^N \delta_i^\top \pi \delta_i dt\right\}, \quad (19)
 \end{aligned}$$

where λ_i are the eigenvalues of matrix $bH - R^1 + R^2$ and $-\pi$ is the maximum eigenvalue of the matrix $SA + A^\top S - 2SBB^\top S + \xi\gamma$. What's more, $b\lambda_i > 1$, $i = 1, 2, \dots, N$ hold, because the coupling weights α_{ij} and α_i are nondecreasing, the constant b can be sufficiently large to make $b\lambda_i > 1$, $\forall i \in l(\tau(t))$. Therefore, $dE\{V_2(t)\} \leq 0$, α_{ij} and α_i converge to some constants.

Then, we are going to prove that $e(t)$ converges to 0 in the mean square. we can get that for any $\sigma > 0$, there is a positive number M due to $E\{V_2(t)\}$ limits. Hence, for $\forall k \geq M$

$$E\{V_2(t_{k+1}) - V_2(t_k)\} < \sigma. \quad (20)$$

Consider $e(t)$, we can get

$$\begin{aligned}
 E\left\{\pi \left[\int_{t_k^0}^{t_k^1} \sum_{i \in l(\tau(t_k^0))} \delta_i^\top(t) \delta_i(t) dt + \dots \right. \right. \\
 \left. \left. + \int_{t_k^{m_k-1}}^{t_k^{m_k}} \sum_{i \in l(\tau(t_k^{m_k-1}))} \delta_i^\top(t) \delta_i(t) dt \right] \right\} \leq \sigma,
 \end{aligned}$$

for $\forall k \geq M$, we have

$$\begin{aligned}
 E\left\{\pi \left[\int_{t_k^0}^{t_k^0 + \mu} \sum_{i \in l(\tau(t_k^0))} \delta_i^\top(t) \delta_i(t) dt + \dots \right. \right. \\
 \left. \left. + \int_{t_k^{m_k-1}}^{t_k^{m_k-1} + \mu} \sum_{i \in l(\tau(t_k^{m_k-1}))} \delta_i^\top(t) \delta_i(t) dt \right] \right\} \leq \sigma,
 \end{aligned}$$

then

$$\lim_{t \rightarrow \infty} \int_t^{t+\mu} E\left\{\sum_{i \in l(\tau(t))} \delta_i^\top(s) \delta_i(s)\right\} ds = 0. \quad (21)$$

According to the Assumption 4, we get

$$\lim_{t \rightarrow \infty} \int_t^{t+\mu} E\left\{\sum_{i=1}^N r_i \delta_i^\top(s) \delta_i(s)\right\} ds = 0,$$

where r_1, \dots, r_N are positive number and we can get $E\{r_i \delta_i^\top(s) \delta_i(s)\} = 0$ when s tend to infinity. Hence, δ_i converges to the mean square, so does e_i , $i = 1, 2, \dots, N$.

5. SIMULATION STUDIES

In this part, we give an example to demonstrate the effectiveness of the approach proposed in this paper.

Example. First, given a multi-agent systems of 4 followers denoted as 1, 2, 3, 4 and a leader labelled as 0, the dynamics of the i th agent satisfies the (1) and (2)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.26 & 32.5 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 7.2 \\ 0 \\ 0 \end{bmatrix}.$$

It is easy to get that (A, B) is stabilisable and the eigenvalues of matrix A have negative real part. The intensity function $\varphi(x_i(t), t) = [1.3 \sin(x_{i1}(t)), 0, 1.5 \sin(x_{i3}(t)), 0]^\top$, which satisfies the Lipschitz condition. The possible interaction graphs are $\{G_1, G_2, G_3, G_4, G_5, G_6\}$ shown in Figure 1. The interaction topologies are switched in order of $G_1 \rightarrow G_2 \rightarrow G_3 \rightarrow G_4 \rightarrow G_5 \rightarrow G_6 \rightarrow G_1 \dots$, and each graph is kept for $\frac{1}{3}s$. The initial states are $x_0 = [1, 2, 3, 4]^\top$, $x_1 = [2.1, 2.5, 4, 3.5]^\top$, $x_2 = [1.8, 2.5, 5, 4.6]^\top$, $x_3 = [3, 3, 6, 5]^\top$, $x_4 = [1.2, 3, 4, 5.2]^\top$.

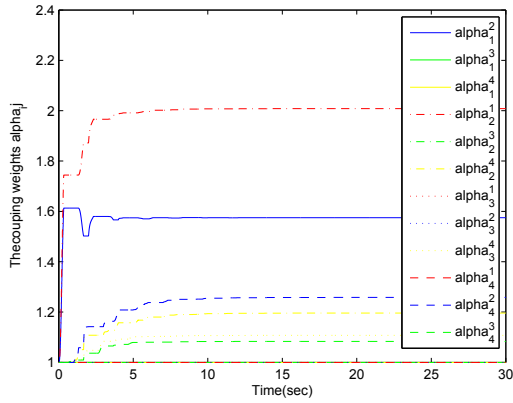
According to the condition of the paper, using Matlab we get

$$S = \begin{bmatrix} 1.3783 & -0.0163 & -0.2925 & -0.0512 \\ -0.0163 & 0.0249 & 0.0131 & -0.0115 \\ -0.2925 & 0.0131 & 0.4117 & 0.0855 \\ -0.0512 & -0.0115 & 0.0855 & 1.4729 \end{bmatrix},$$

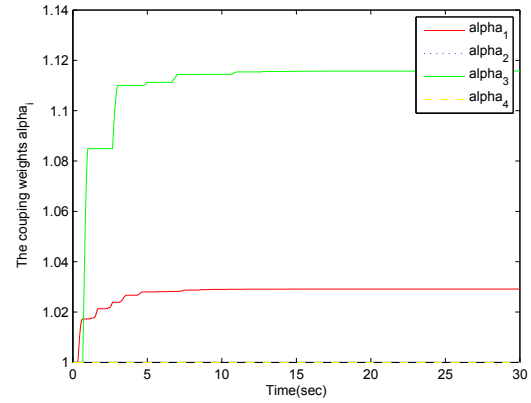
$$K = [0.1174, -0.1793, -0.0943, 0.0828],$$

$$\Gamma = \begin{bmatrix} 0.0138 & -0.0210 & -0.0111 & 0.0097 \\ -0.0210 & 0.0321 & 0.0169 & -0.0148 \\ -0.0111 & 0.0169 & 0.0089 & -0.0078 \\ 0.0097 & -0.0148 & -0.0078 & 0.0069 \end{bmatrix}.$$

From the Figure 2 and 3, we can know that the error state of followers e_i , $i = 1, 2, 3, 4$ reaches 0 and trajectories of the coupling weights α_{ij} , $i, j = 1, 2, 3, 4$ and α_i , $i = 1, 2, 3, 4$ reach some constant within limit time, which prove the correctness of the theorem.

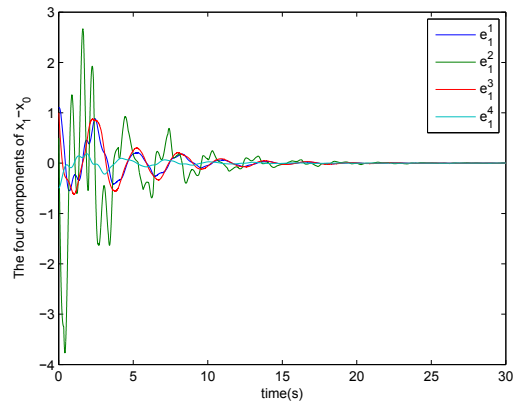


(a) The trajectories of α_{ij}

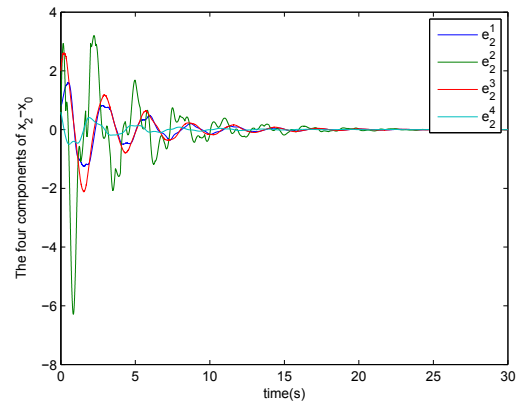


(b) The trajectories of α_i

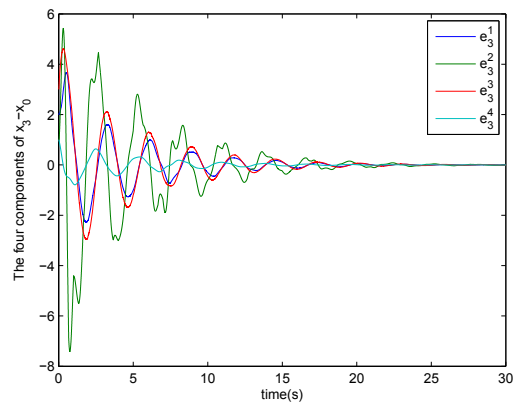
Fig. 2. The trajectories of coupling weights during the state of each agent be gradually consistent



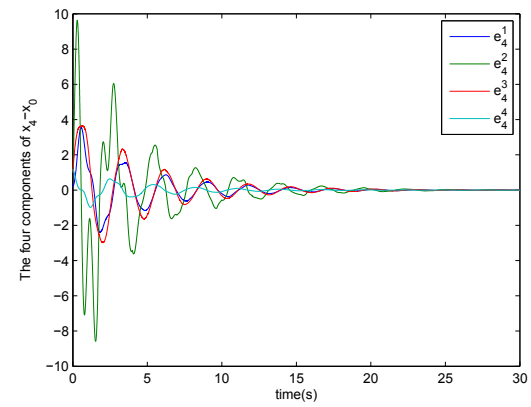
(a) The trajectories of e_1



(b) The trajectories of e_2



(c) The trajectories of e_3



(d) The trajectories of e_4

Fig. 3. The state error trajectories of the followers during the state of each agent be gradually consistent

6. CONCLUSION

In this paper, we discuss the consensus control of multi-agent systems with directed-switching topology. In addition, we design the control protocol to solve the problem and the consensus control is reached under the methods we propose. Finally, the simulations are given to prove that the controllers are efficient to settle consensus control problems of multi-agent systems with directed-switching topology.

Future work will attempt to further research multi-agent systems with optimization and time-varying topology.

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