Letter

Nonconvex Noise-Tolerant Neural Model for Repetitive Motion of Omnidirectional Mobile Manipulators

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Dear Editor,

Quadratic programming problems (QPs) receive a lot of attention in various fields of science computing and engineering applications, such as manipulator control [1]. Recursive neural network (RNN) is considered to be a powerful QPs solver due to its parallel processing capability and feasibility of hardware implementation [2]. In particular, a large number of RNN models, such as gradient neural network, are proposed as powerful alternatives for online solving QPs [3]. However, it is worth noting that most of the above neural networks are essentially designed for solving static QPs with time-invariant parameters. These neural algorithms cannot solve time-varying (TV) QPs because they cannot adapt to change in parameters, such as kinematic control of redundant arms [4]. Zeroing neural network (ZNN) is specially designed for real-time solution of time-varying problems. It uses the time derivative (TD) of time-varying parameters to solve the zero-finding problem [5]. The Taylor-type discrete-time ZNN (DTZNN) model is proposed in [6], which outperforms other models are inherently used to address the static QPs, such as Newton iterations. Although the DTZNN model makes full use of the TD information of the problem to be solved, it still does not explicitly consider the influence of noise. In the real-time solution of nonlinear system, there are system errors or external disturbances in hardware implementation, which can be regarded as noise [7]. Different RNN models are constructed by choosing different error functions (EFs) or utilizing different activation functions (AFs) in existing models, but the design process is roughly similar. However, the AF should be a monotone increasing odd function. Therefore, the ZNN-based model can be drawn by relaxing the convex constraint of the AF for TVQPs with equality and inequality constraints (EAICs) in the presence of noises.

Problem description: Considering joint angle, joint velocity, wheel angle and wheel-velocity limits of the RMC for the OMMs, which can be viewed as the following TVQPs with EAICs [5]:

$$\begin{aligned}&\min \frac{1}{2}\pi^T(t)M(t)\pi(t)+\Xi^T(t)\pi(t)\\ &\text{s.t.}\quad K(t)\pi(t)=\eta(t),\ \ W(t)\pi(t)\le \nu(t)\end{aligned} \tag{1}$$

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where Hessian matrix $M(t)\in\mathbb{R}^{n\times n}$ is positive-definite, and $\Xi(t)\in\mathbb{R}^n$, $K(t)\in\mathbb{R}^{m\times n}$ being of full row rank, $\eta(t)\in\mathbb{R}^m$ is all smoothly nonstationary. $\pi(t)\in\mathbb{R}^n$ is to be solved in real time; Symbol T indicates the transpose operator of a matrix or a vector; $W(t)=[I,I]^T\in\mathbb{R}^{p\times n}$; I represents identity matrix; $\nu(t)=[\ell^+,\ell^-]^T\in\mathbb{R}^p$ with ℓ^- and ℓ^+ denote the lower and upper limits of the joint, wheel, joint-velocity and wheel-velocity, respectively.

Owing to the disturbed FB function $\aleph(\cdot)^{\alpha}_{FB}$ and the Karush-Kuhn-Tucker condition, the TVQPs (1) can be rewritten as follows:

$$\begin{cases} M(t)\pi(t)+\Xi^T(t)\pi(t)+K^T(t)\zeta^*(t)+W^T(t)\varsigma^*(t)=0\\ K(t)\pi^*(t)-\eta(t)=0\\ \aleph^{\alpha}_{FB}((\nu(t)-W(t)\pi^*(t)),\ \varsigma^{*T}(t))=0.\end{cases} \tag{2}$$

Equation (2) can be reorganized as the following equation:

$$\Upsilon(\varpi(t),t)=Q(t)\varpi(t)-\mu(t) \tag{3}$$

where $\mu(t)=\begin{bmatrix} -\Xi(t),\eta(t),z(t)-\nu(t)\end{bmatrix}^T$ and

$$Q(t)=\begin{bmatrix} M(t) & K^T(t) & W^T(t)\\ K(t) & 0 & 0\\ W(t) & 0 & I\end{bmatrix},\quad \varpi(t)=\begin{bmatrix}\pi(t)\\ \zeta(t)\\ \varsigma(t)\end{bmatrix}$$

$z(t)=\sqrt{E(t)\circ E(t)+\varsigma(t)\circ\varsigma(t)}$, $E(t)=W(t)\pi(t)-\nu(t)$, where the symbol "$\circ$" represents the Hadamard product.

Design and theoretical analyses:

Nonconvex activation function noise-tolerant zeroing neural network (NAFNTZNN) model: The NAFNTZNN dynamic model can be designed as

$$\dot{\Upsilon}(t)=-\gamma \mathrm{O}_{\Omega}(\Upsilon(t))-\lambda\int_0^t \Upsilon(\delta)d\delta \tag{4}$$

where $\gamma>0$ and $\lambda>0$. The NAF $\mathrm{O}_{\Omega}(\cdot)$ is the projection from set Λ to set Ω. The NAF is defined as

$$\mathrm{O}_{\Omega}(a_i)=\begin{cases}\eta_1, & \text{if } \gamma\Gamma^p(a_i)>\eta_1\\ \gamma\Gamma^p(a_i), & \text{if } -\eta_2<\gamma\Gamma^p(a_i)<\eta_1\\ -\eta_2, & \text{if } \gamma\Gamma^p(a_i)<\eta_2\end{cases} \tag{5}$$

where

$$\Gamma^p(a_i)=\begin{cases}|a_i|, & \text{if } |a_i|>0\\ 0, & \text{if } |a_i|=0\\ -|a_i|, & \text{if } |a_i|<0\end{cases} \tag{6}$$

and $p\in(0\quad 1)$; $\eta_1=\eta_2>0$. Correlation parameters are $\eta_1=\eta_2=1$ and $p=0.2$, respectively. The NAFNTZNN model with external disturbance $\chi(t)$, which can be called as measurement noise (MN), can be obtained as follows:

$$\begin{aligned}Q(t)\dot{\varpi}(t)=&\dot{\mu}(t)-\dot{Q}(t)\varpi(t)-\gamma\mathrm{O}_{\Omega}(Q(t)\varpi(t)-\mu(t))\\ &-\lambda\int_0^t Q(\delta)\varpi(\delta)-\mu(\delta)d\delta+\chi(t).\end{aligned} \tag{7}$$

Convergent analyses of NAFNTZNN in the exist of MN:

Theorem 1: For the TVQPs with EAICs (1), the proposed NAFNTZNN (7), starting from a randomly initial state $\varpi(0)$ globally converges to a TV optimal solution $\varpi^*(t)$, where the first n elements composing the TV theoretical solutions $\pi^*(t)$ to TVQPs with EAICs (1).

Proof: The Lyapunov function (LF) can be constructed as

$$L(t)=\frac{1}{2}\Upsilon^T(t)\Upsilon(t)+\frac{1}{2}\lambda\Big(\int_0^t\Upsilon(\delta)d\delta\Big)^T\int_0^t\Upsilon(\delta)d\delta\ge 0.$$

The above equation can be organized into a compact form

$$L(t)=\frac{1}{2}\|\Upsilon(t)\|^2+\frac{1}{2}\lambda\left\|\int_0^t\Upsilon(\delta)d\delta\right\|^2\ge 0. \tag{8}$$

According to the LF selected above, when $\Upsilon(t)\ne 0$, $L(t)>0$ is obtained, and only when $\Upsilon(t)=0$, $L(t)=0$, it indicates that $L(t)$ is positive. Then, calculate its time derivative as

$$\dot{L}(t) = \Upsilon(t)\dot{\Upsilon}(t) + \lambda\Upsilon(t)\Big(\int_0^t \Upsilon(\delta)d\delta\Big)\Upsilon(t)(-\gamma \mathrm{O}_\Omega(\Upsilon(t))$$
$$-\lambda\int_0^t \Upsilon(\delta)d\delta) + \lambda\Upsilon(t)\int_0^t \Upsilon(\delta)d\delta \le -\frac{1}{2}\gamma(\mathrm{O}_\Omega(\Upsilon(t)))^2 \le 0$$

where $\lambda > 0$. On the based of Lyapunov stability theorem (LST), it deduces that (2) converges to zero globally, which clarifies the theoretical solution of NAFNTZNN model (7) converges to TVQPs with EAICs (1). ∎

Robustness analyses of NAFNTZNN in the exist of MN: The perturbation model of NAFNTZNN (7) is as follows:

$$Q(t)\dot{\varpi}(t) = -(\Delta Q(t) + \dot{Q}(t))\varpi(t) - \gamma \mathrm{O}_\Omega(Q(t)\varpi(t) - \mu(t))$$
$$-\lambda\int_0^t Q(\delta)\varpi(\delta) - \mu(\delta)\mathrm{d}\delta + \dot{\mu}(t) + \Delta\chi(t). \tag{9}$$

Theorem 2: Considering TVQPs with EAICs (1), and assuming that the model is perturbed by model disturbance $\Delta\chi(t)$ and differentiation error $\Delta Q(t)$. If $\|\Delta Q(t)\|_F \le b_1 \in \mathbb{R}$, $\|\Delta\chi(t)\|_2 \le b_2 \in \mathbb{R}$, $\|Q^{-1}(t)\|_F \le b_3 \in \mathbb{R}$, $\|\mu(t)\|_2 \le b_4 \in \mathbb{R}$, for all $t \in [0,+\infty]$ and $0 < b_1, b_2, b_3, b_4 < +\infty$, then the upper bound (UB) of the absolute value of TV error function (TVEF) $\Upsilon(\varpi(t),t) = Q(t)\varpi(t) - \mu(t)$ of the proposed NAFNTZNN (7) with an NAF and it will converge to 0 as $t \to +\infty$, that is, the first n elements composing of the TV theoretical solution $\pi^*(t)$ to (1) as $t \to +\infty$.

Proof: The derivative of (8) can be obtained as follows:

$$\dot{L}(t) = -\Upsilon^T(t)Z(t)\Upsilon(t) - \Upsilon^T(t)\Delta Q(t)Q^{-1}(t)\mu(t)$$
$$-\gamma\Upsilon^T(t)\mathrm{O}_\Omega(\Upsilon(t)) + \Upsilon^T(t)\Delta\chi(t) \tag{10}$$

where $Z(t) = \Delta Q(t)Q^{-1}(t)$. Due to the following inequality $|\lambda_{\max}(\cdot)| \le \|\cdot\|_F$, the first term on the right-hand side of (10) is rewritten as

$$\Upsilon^\top(t)\frac{Z^T(t) + Z(t)}{2}\Upsilon(t) \le \Upsilon^T(t)\Upsilon(t)\|Z(t)\|_F. \tag{11}$$

Due to the theory of matrix analysis, the following inequality $\|\Delta Q(t)Q^{-1}(t)\|_F \le \|\Delta Q(t)\|_F\|Q^{-1}(t)\|_F$. Thus, (11) is rewritten as follows:

$$\Upsilon^T(t)\frac{Z^T(t) + Z(t)}{2}\Upsilon(t) \le \Upsilon^T(t)\Upsilon(t)b_1b_3. \tag{12}$$

The last term on the right-hand side of (10) is based on the maximum value of the elements of the TV vector whose each element is less than or equal to the TV vector, and $\max_{1\le i\le m+n+p}|\Delta\chi_i(t)| \le \|\Delta\chi_i(t)\|$, set $\sum_{i=1}^{m+n+p}|\Upsilon_i(t)|b_2 = g$, then

$$\Upsilon^T(t)\Delta\chi(t) \le \sum_{i=1}^{m+n+p}|\Upsilon_i(t)|\max_{1\le i\le m+n+p}\|\Delta\chi_i(t)\| \le g \tag{13}$$

where b_2 is the UB of $\Delta\chi(t)$. The second term on the right-hand side of (10) has the following formula:

$$\Upsilon^T(t)Z(t)\mu(t)\|\mu(t)\| \le \sum_{i=1}^{m+n+p}|\Upsilon_i(t)|b_1b_2b_3. \tag{14}$$

Substituting (12)–(14) into (10), then

$$\dot{L}(t) \le \sum_{i=1}^{m+n+p}|\Upsilon_i(t)|b_2 - \sum_{i=1}^{m+n+p}|\Upsilon_i(t)|b_1b_2b_3 - \gamma\Upsilon^T(t)$$
$$\times \mathrm{O}_\Omega(\Upsilon(t)) - \Upsilon^T(t)\Upsilon(t)b_1b_3 \le \sum_{i=1}^{m+n+p}|\Upsilon_i(t)|\mathrm{O}(\Upsilon_i(t)) \tag{15}$$

where $\mathrm{O}(\Upsilon_i(t)) = -(|\Upsilon_i(t)|b_1b_3 + b_1b_2b_3 + \gamma\mathrm{O}_{\Omega i}(|\Upsilon_i(t)|) - b_2)$. Therefore, it can be discussed in the following two cases.

1) Given that $\mathrm{O}(\Upsilon_i(t)) \ge 0$, then $\dot{L}(t) \le 0$ can be obtained. Based on the LST, if $L(t) \ge 0$ and $\dot{L}(t) \le 0$, the TVEF $\varpi(t)$ formed by the vector converges to zero, which shows that the TV state variable $\varpi(t)$ converges to the optimal solution $\varpi^*(t)$. Furthermore, the model will reach a steady state, i.e., the LF will not decrease with $L(t) = 0$.

2) Assumed that $\mathrm{O}(\Upsilon_i(t)) < 0$, then $\dot{L}(t) \le 0$ has a positive UB. It can be divided into the following two cases, i.e., $\dot{L}(t) \le 0$ or $\dot{L}(t) > 0$. i) If $\dot{L}(t) \le 0$ and $L(t) \ge 0$, based on the LST, the TVEF with $\Upsilon(t)$ as a vector converges to zero, and the model reaches a steady state. ii) If $\dot{L}(t) > 0$, based on the definition of the NAF, following three cases are discussed.

Case 1: If $a_i < 0$ and $\Gamma^p(a_i) = -|a_i|^p$, then $\gamma\Gamma^p(a_i) = -\gamma|a_i|^p$, the NAF can be divided into the following two cases:

1) If $-\eta_1 < -\gamma|a_i|^p < \eta_1$, the NAF is defined as $\mathrm{O}_\Omega(a_i) = -\gamma|a_i|^p$, (15) is rewritten as

$$\dot{L}(t) \le -\sum_{i=1}^{m+n+p}|\Upsilon_i(t)|(|\Upsilon_i(t)|y - z|\Upsilon_i(t)|^p - r) \tag{16}$$

where $y = b_1b_3$, $z = \gamma^2$, $r = b_2 - b_1b_2b_3$.

i) If $|\Upsilon_i(t)| > 1$, then $|\Upsilon_i(t)| > |\Upsilon_i(t)|^p$, (16) can be rearranged as

$$\dot{L}(t) \le -\sum_{i=1}^{m+n+p}|\Upsilon_i(t)|(y - z)(|\Upsilon_i(t)| - \hbar) \tag{17}$$

where $\hbar = r/(y - z)$. In the case of $\hbar > 0$, considering the TD of LF $\dot{L}(t) > 0$, it deduces that the UB on the TD of the LF $\dot{L}(t)$ will decrease as $|\Upsilon_i(t)|$ of the TVEF increases. So, there exists a certain moment satisfies $\dot{L}(t) \le 0$, which shows the model is stable again, therefore, $|\Upsilon_i(t)|$ has an UB. When $\dot{L}(t) = 0$, the left-hand side of (16) is equal to zero. When $|\Upsilon_i(t)| = \hbar/2$, $|\Upsilon_i(t)|(|\Upsilon_i(t)| - \hbar) = 0$, which can be seen as a quadratic function of $|\Upsilon_i(t)|$, reaches negative minimum value. Besides, $|\Upsilon_i(t)|(|\Upsilon_i(t)| - \hbar) > 0$ always holds if and only if $|\Upsilon_i(t)| > 0$. In addition, the jth term $|\Upsilon_j(t)|$ serves as an UB on $|\Upsilon(t)|$, $i = 1, 2, \ldots, m+n+p$ if and only if the rest of $i = 1, 2, \ldots, m+n+p-1$ terms of $|\Upsilon_i(t)|(|\Upsilon_i(t)| - \hbar)$ obtain the minimum values. When $\dot{L}(t) = 0$, (17) can be rewritten as

$$|\Upsilon_j(t)|^2 - |\Upsilon_j(t)|\hbar + \sum_{i=1, i\ne j}^{m+n+p}|\Upsilon_i(t)|(|\Upsilon_i(t)| - \hbar) = 0. \tag{18}$$

Substitute $|\Upsilon_i(t)| = \hbar/2$ into (18), when $i = 1, 2, 3, \ldots, n+m+p$ and $i \ne j$, then (18) can be rewritten as

$$\sum_{i=1}^{m+n+p}|\Upsilon_i(t)|(|\Upsilon_i(t)| + \hbar) = |\Upsilon_j(t)|^2 - |\Upsilon_j(t)|\hbar - \upsilon_1 = 0$$

where $\upsilon_1 = \frac{m+n+p-1}{4}\hbar^2$, thereby, the UB $|\Upsilon_i(t)| = \frac{\hbar - \sqrt{\hbar^2 + 4\upsilon_1}}{2}$ and $|\Upsilon_{n+m+p}(t)|$ will driven to zero as $t \to \infty$.

ii) If $0 < |\Upsilon_i(t)| < 1$, then $|\Upsilon_i(t)| < |\Upsilon_i(t)|^p < 1$, (17) can be computed as

$$\dot{L}(t) \le -\sum_{i=1}^{m+n+p}|\Upsilon_i(t)|g(|\Upsilon_i(t)| - l) \tag{19}$$

where $g = b_1b_3$, $l = (b_2 + r^2 - b_1b_2b_3)/(b_1b_3)$. The analysis method is the same as the previous i), which is omitted here.

2) If $-\gamma|a_i|^p < \varphi_2$, the NAF is defined as $\mathrm{O}_\Omega(a_i) = -\eta_2$, therefore, (15) can be obtained as

$$\dot{L}(t) \le -\sum_{i=1}^{m+n+p}|\Upsilon_i(t)|(|\Upsilon_i(t)|\lambda_2 - \lambda_1) \tag{20}$$

where $\lambda_1 = b_2 - b_1b_2b_3 - \gamma\eta_2$ and $\lambda_2 = b_1b_3$. Assume that the condition $b_2 - b_1b_2b_3 - \gamma\eta_2 > 0$ holds. The following analysis is similar to Case 1, which is omitted here.

Case 2: If $a_i = 0$ and $\Gamma^p(a_i) = 0$, then $\gamma\Gamma^p(a_i) = 0$, the NAF is defined as $\mathrm{O}_\Omega(a_i) = 0$, therefore, (15) is rewritten as $\dot{L}(t) \le 0$, it means that the model is stable.

Case 3: If $a_i > 0$, the analysis method is the same as the previous Case 1, which is omitted here. ∎

Numerical validation: In this section, the effectiveness, superiority and physical realizability of the developed NAFNTZNN model (7) in solving TVQP problem with EAICs are verified by an RMC of the OMMs. Specifically, the initial values of variables are set as $\gamma = 10^3$, $\lambda = 4\times 10^3$, and the initial state is $x_0 = [1.4; 1.4; -1.4; -1.4; \pi/12; \pi/12; \pi/12; \pi/3]$ rad. Joint-angle limits $\pi^+ = [3.5; 3.5; 4.5; 4.5; 0.4; 1; 1; 1.5]$ rad with $\pi^+ = -\pi^-$; The joint-velocity upper limit $\dot{\pi}^+$ of

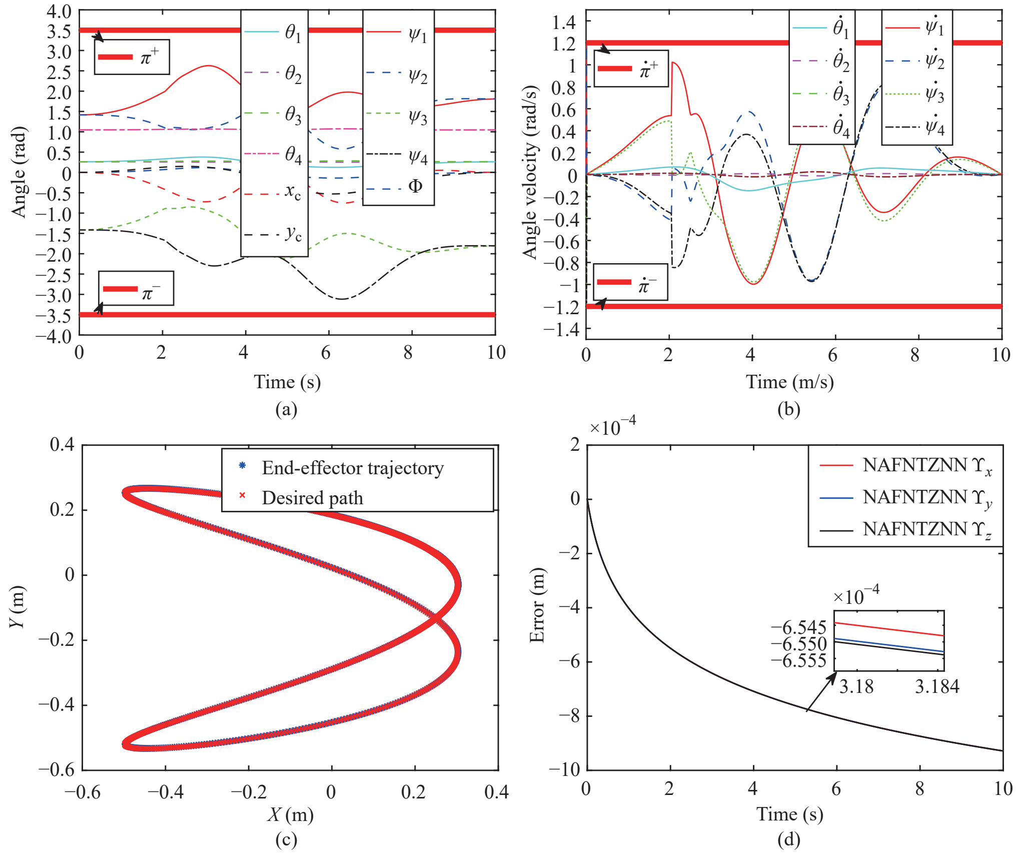

Fig. 1. Numerical results of the NAFNTZNN model with MN $\chi(t) = [1, 1, 1, \ldots, n+m]^T$ for RMC of the OMMs. (a) Joint angles and wheels of the OMMs; (b) Joint velocities and wheel velocities of the OMMs; (c) Tracking results of the OMMs; (d) Position error $\Upsilon = [\Upsilon_x, \Upsilon_y, \Upsilon_z]^T$.

each joint is set as 1.2 rad/s and $\dot{\pi}^+ = -\dot{\pi}^-$. As illustrated in Figs. 1(a) and 1(b), joint angles, wheel angles, joint velocities, and wheel velocities eventually fluctuate back to the initial state and always limited within the predetermined limit boundaries, which ensure repeated motion of the OMMs and show the effectiveness of the proposed NAFNTZNN (7) in the presence of noise. Moreover, from Figs. 1(c) and 1(d), it observes that the actual trajectory of the end-effector is very close to the desired Lissajous trajectory [5]. Therefore, it verifies that the NAFNTZNN model (7) is globally convergent to the theoretical solution of TVQPs (1) with EAICs in the presence of noises. Numerical simulations verify the superiority of the NAFNTZNN model for the TVQPs with EAICs with noises.

Conclusion: The NAFNTZNN model with NAF and FB function was developed for TVQPs with EAICs under different noises. It demonstrated that the NAFNTZNN model has global convergence and robustness to external disturbances. A numerical example demonstrated the effectiveness of the NAFNTZNN model for real-time solution of TVQPs with EAICs. The motion planning of the OMMs affected by moving obstacles will be mainly considered in the future.

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