Letter

Relay-Switching-Based Fixed-Time Tracking Controller for Nonholonomic State-Constrained Systems: Design and Experiment

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Dear Editor,

This letter investigates the fixed-time trajectory tracking controller design for nonholonomic chained systems with static state constraints. Firstly, a fixed-time tracking control law is given to carry out relay switching, which divides the controller development process into two stages. For the goal of designing fixed-time tracking controller in the second stage and avoiding the explosion of complexity, fixed-time filters are introduced. Under these preparations, a fixed-time tracking controller is elaborated by applying barrier-Lyapunov-function-based backstepping. In addition, a new tracking controller is also proposed to guarantee that the tracking error system is bounded before switching. It has been proved that the resulting closed-loop tracking error system states converge to a small neighborhood around zero within a fixed time and output states are kept in the predefined constrained regions.

Recently, the control of nonholonomic systems has attracted increasing attention due to their broad application in practice, such as the wheeled mobile cars [1]. Based on scholars' previous works, stabilization control strategies for nonholonomic systems can be divided into inconsecutive method [2], time-varying smooth method [3], and method with hybrid feedback [4]. In terms of tracking control, scholars usually adopt the relay-switching-based method to achieve the effect of tracking [5]. In addition, [6] solved the arbitrary configuration stabilization problem by a trajectory approach.

Generally, the general form for describing state constraints is $-F_{i1}(t) < x_i(t) < F_{i2}(t), (F_{ij}(t) > 0, j = 1,2)$. Up to now, the main methods for solving state constraint problems have invariant set theory [7], model prediction [8], barrier Lyapunov function (BLF) [9], and state-dependent functions (SDF), etc. In view of literature, BLF-based [10] and SDF-based [11] methods are also utilized to manage nonholonomic control systems under state constraints in recent years.

In addition, fixed-time control is more useful in practice than finite-time control [12]. So far, the main methods for solving fixed-time problems have terminal sliding mode control [13], adding one power integrator technique [14], etc. At the same time, a few results have been achieved for the fixed-time study of nonholonomic control systems, such as [15]. Up to now, although many works have been reported on nonholonomic systems, it has seldom been considered how to design a tracking controller in the presence of state constraints and fixed-time control simultaneously.

Motivated by these points, this paper investigates the fixed-time trajectory tracking controller design for nonholonomic chained systems with static state constraints. The main works are summarized as:

1) The fixed-time tracking control is addressed initially for an output-state-constrained chained nonholonomic system. The designed tracking controller can ensure that system tracking error states converge to a bounded set within a fixed time and output states are kept in the pre-specified state constraints regions.

2) Generally, introducing filters in fixed-time control problem will lead to virtual controller in each backstepping step not differentiable. To remedy this difficulty, we have proposed "compound virtual controllers", which further extends the previous results.

Problem statement: The studied nonholonomic system is [2]

$$\dot{x}_0 = u_0,\ \dot{x}_1 = x_2u_0, \dots, \dot{x}_{n-1} = x_nu_0,\ \dot{x}_n = u_1 \tag{1}$$

where x_0 and $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ are system states, u_0 and u_1 are control inputs. The tracking trajectory states x_{0d} and $x_d = (x_{1d}, \dots, x_{nd})^T$ are system states of a virtual system as

$$\dot{x}_{0d} = u_{0d}, \dots, \dot{x}_{(n-1)d} = x_{nd}u_{0d},\ \dot{x}_{nd} = u_{1d} \tag{2}$$

where u_{0d} and u_{1d} are virtual control inputs. In order to formulate the trajectory tracking dynamic equation, let us introduce tracking errors as $x_{0e} = x_0 - x_{0d}$, $x_{1e} = x_1 - x_{1d}$, …, $x_{ne} = x_n - x_{nd}$. Then, the tracking error dynamic equations are described as

$$\begin{aligned} &\dot{x}_{0e} = u_0 - u_{0d}, \dot{x}_{ie} = x_{(i+1)e}u_{0d} + x_{i+1}(u_0 - u_{0d}) \\ &\dot{x}_{ne} = u_1 - u_{1d},\ 1 \le i \le n-1. \end{aligned} \tag{3}$$

The control goal is expressed as: designing tracking controllers u_0, u_1 to ensure that x_{0e}, x_{1e}, …, x_{ne} converge to zero within a fixed time, and output tracking error states always stay in the desired constrained regions $\Omega_{x_{ie}} \triangleq \{x_{ie} : -k_{i1} < x_{ie}(t) < k_{i1}\}$ $(i = 0,1)$ with constants $k_{i1} > 0$.

Assumption 1: $u_{0d}(t)$ is continuous, and there are constants $\bar{u}_{0d} > 0$ and $\underline{u}_{0d} > 0$ such that $\bar{u}_{0d} > u_{0d}(t) > \underline{u}_{0d} > 0$.

Fixed-time tracking control of x_{0e}-subsystem: Now, we consider x_{0e}-subsystem and choose a candidate BLF as $V_0 = \frac{1}{2}\ln\frac{k_0^2}{k_0^2 - x_{0e}^2}$. The fixed-time tracking control signal u_0 is picked as

$$\begin{aligned} u_0 = u_{0d} &- \alpha_0 \mathrm{sig}(x_{0e})^{\frac{m_0}{n_0}} \omega_{01}(k_0^2 - x_{0e}^2) \\ &- \beta_0 \mathrm{sig}(x_{0e})^{\frac{p_0}{q_0}} \omega_{02}(k_0^2 - x_{0e}^2) \end{aligned} \tag{4}$$

where $\mathrm{sig}(\mu)^\sigma = \mathrm{sign}(\mu)|\mu|^\sigma$, $\omega_{01} = [1/(k_0^2 - x_{0e}^2)]^{\gamma_{01}}$, $\omega_{02} = [1/(k_0^2 - x_{0e}^2)]^{\gamma_{02}}$, $\alpha_0, \beta_0 > 0$, $0 < \frac{m_0}{n_0} < 1$, $\frac{p_0}{q_0} > 1$, $\frac{1}{2} < \gamma_{01} = (\frac{m_0}{n_0} + 1)/2 < 1$, $\gamma_{02} = (\frac{p_0}{q_0} + 1)/2 > 1$, m_0, n_0, p_0, and q_0 are odd integers. By computing the time derivative of V_0, in view of (4), one has

$$\dot{V}_0 \le -\alpha_0 x_{0e}^{2\gamma_{01}} \omega_{01} - \beta_0 x_{0e}^{2\gamma_{02}} \omega_{02}. \tag{5}$$

Moreover, utilizing Lemma 1 in [11], one concludes that

$$2^{\gamma_{01}} V_0^{\gamma_{01}} \le \frac{x_{0e}^{2\gamma_{01}}}{k_0^2 - x_{0e}^2},\ 2^{\gamma_{02}} V_0^{\gamma_{02}} \le \frac{x_{0e}^{2\gamma_{02}}}{k_0^2 - x_{0e}^2}. \tag{6}$$

With the help of (6), one has

$$\dot{V}_0 \le -\alpha_0 2^{\gamma_{01}} V_0^{\gamma_{01}} - \beta_0 2^{\gamma_{02}} V_0^{\gamma_{02}}. \tag{7}$$

Let $\kappa_1 = \alpha_0 2^{\gamma_{01}}, \kappa_2 = \beta_0 2^{\gamma_{02}}$. According to the fixed-time stability in [16], we can pick switching time t_1 in such a way that $t_1 \ge \frac{1}{\kappa_1(1-\gamma_{01})} + \frac{1}{\kappa_2(\gamma_{02}-1)}$.

Proposition 1: Assume that $-k_{01} < x_{0e}(0) < k_{01}$, the fixed-time tracking control strategy (4) designed for $x_{0e}-$ subsystem, can guarantee that $x_{0e} = x_0 - x_{0d}$ and $u_0 - u_{0d}$ tend to zero within a fixed time and keep zero afterwards. Meantime, by choosing $k_0 < k_{01}$, then $|x_{0e}(t)| < k_0 < k_{01}$ for $t > 0$, that is, state constraint is realized.

Fixed-time tracking control of $(x_{1e}, \dots, x_{ne})$-subsystem when $t \ge t_1$: Owing to $u_0 - u_{0d} = 0$ when $t \ge t_1$, we can obtain that the dynamic equation of $(x_{1e}, \dots, x_{ne})$-subsystem is written as

$$\dot{x}_{1e} = x_{2e}u_{0d}, \dots, \dot{x}_{(n-1)e} = x_{ne}u_{0d},\ \dot{x}_{ne} = u_1 - u_{1d} \tag{8}$$

for which backstepping design can be applied. For the subsequent design, we introduce the following transformations $z_i = x_{ie} - \epsilon_{id}$ and $y_i = \epsilon_{id} - \epsilon_i^*$ $(2 \le i \le n)$ where ϵ_i^* represent virtual controllers and ϵ_{id} are filter states which are generated by the following fixed-time filters [17] :

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$$\tau_i\dot{\epsilon}_{id} = \text{sign}(\epsilon_i^* - \epsilon_{id})^{\frac{m_1}{n_1}} + \text{sign}(\epsilon_i^* - \epsilon_{id})^{\frac{p_1}{q_1}} \tag{9}$$

where $\tau_i > 0$ denote filter-related small parameters. Next, we provide controller development in detail.

Step 1: In this step, we choose a candidate Lyapunov function as $V_1 = \frac{1}{2}\ln\frac{k_1^2}{k_1^2-z_1^2} + \frac{1}{2}y_2^2$, $z_1 = x_{1e}$. The first virtual controller ϵ_2^* is designed as

$$\epsilon_2^* = -\frac{k_1^2 - z_1^2}{u_{0d}}\frac{z_1\bar{\epsilon}_2^{*2}}{\sqrt{z_1^2\bar{\epsilon}_2^{*2} + \varepsilon^2}} \tag{10}$$

where $\bar{\epsilon}_2^* = \alpha_1\text{sig}(z_1)^{\frac{m_1}{n_1}}\omega_{11} + \beta_1\text{sig}(z_1)^{\frac{p_1}{q_1}}\omega_{12} + \frac{1}{2}\frac{z_1u_{0d}^2}{(k_1^2-z_1^2)^2}$. Moreover, we can get that

$$y_2\dot{y}_2 \leq -\frac{y_2^{2\gamma_1}}{\tau_2} - \frac{y_2^{2\gamma_2}}{\tau_2} + \frac{1}{2}y_2^2 + \frac{1}{2}M_2^2 \tag{11}$$

where $S_2 = -\frac{\partial\epsilon_2^*}{\partial z_1}\dot{z}_1 - \frac{\partial\epsilon_2^*}{\partial u_{0d}}\dot{u}_{0d}$. Since S_2 is a continuous function, in view of Assumption 1, it is bounded in a compact set, which in turn means that $|S_2| \leq M_2$ with $M_2 > 0$ being a constant. Now, by combining (10) and (11) with some deductions, it yields that

$$\begin{aligned}\dot{V}_1 \leq & -\alpha_1 z_1^{2\gamma_1}\omega_{11} - \beta_1 z_1^{2\gamma_2}\omega_{12} - \frac{y_2^{2\gamma_1}}{\tau_2} \\ & - \frac{y_2^{2\gamma_2}}{\tau_2} + y_2^2 + \frac{u_{0d}z_1z_2}{k_1^2 - z_1^2} + \Theta_1\end{aligned} \tag{12}$$

where $\Theta_1 = \frac{1}{2}M_2^2 + \varepsilon$, $\omega_{11} = [1/(k_1^2 - z_1^2)]^{\gamma_1}$, $\omega_{12} = [1/(k_1^2 - z_1^2)]^{\gamma_2}$, $\alpha_1, \beta_1 > 0$, $0 < \frac{m_1}{n_1} < 1$, $\frac{p_1}{q_1} > 1$, $\frac{1}{2} < \gamma_1 = \frac{\frac{m_1}{n_1}+1}{2} < 1$, $\gamma_2 = \frac{\frac{p_1}{q_1}+1}{2} > 1$, m_1, n_1, p_1, and q_1 are odd integers. The term $\frac{u_{0d}z_1z_2}{k_1^2-z_1^2}$ will be handled later.

Step i $(2 \leq i < n)$: In this step, the candidate Lyapunov function is configured as $V_i = V_{i-1} + \frac{1}{2}\ln\frac{k_i{}^2}{k_i{}^2-z_i{}^2} + \frac{1}{2}y_{i+1}^2$. Virtual controller ϵ_{i+1}^* is designed as

$$\epsilon_{i+1}^* = -\frac{k_i^2 - z_i^2}{u_{0d}}\frac{z_i\bar{\epsilon}_{i+1}^{*2}}{\sqrt{z_i^2\bar{\epsilon}_{i+1}^{*2} + \varepsilon^2}} \tag{13}$$

where

$$\begin{aligned}\bar{\epsilon}_{i+1}^* = & -\frac{\dot{\epsilon}_{id}}{k_i^2 + z_i^2} + \alpha_i\text{sig}(z_i)^{\frac{m_1}{n_1}}\omega_{i1} + \beta_i\text{sig}(z_i)^{\frac{p_1}{q_1}}\omega_{i2} \\ & + \frac{u_{0d}z_{i-1}}{k_{i-1}^2 - z_{i-1}^2} + \frac{1}{2}\frac{z_iu_{0d}^2}{(k_i^2 - z_i^2)^2}.\end{aligned} \tag{14}$$

Similar to step 1, after some tedious calculation processing, one can deduce a desired result as

$$\begin{aligned}\dot{V}_i \leq & -\sum_{j=1}^{i}\left(\alpha_j z_j^{2\gamma_1}\omega_{j1} + \beta_j z_j^{2\gamma_2}\omega_{j2}\right) + \sum_{j=1}^{i}y_{j+1}^2 \\ & - \sum_{j=1}^{i}\left(\frac{y_{j+1}^{2\gamma_1}}{\tau_{j+1}} + \frac{y_{j+1}^{2\gamma_2}}{\tau_{j+1}}\right) + \frac{u_{0d}z_iz_{i+1}}{k_i^2 - z_i^2} + \sum_{j=1}^{i}\Theta_i.\end{aligned} \tag{15}$$

Step n: Now we choose the last Lyapunov candidate function as $V_n = V_{n-1} + \frac{1}{2}\ln\frac{k_n^2}{k_n^2-z_n^2}$. After some computations, the actual control input u_1 can be proposed as

$$\begin{aligned}\bar{\epsilon}_{n+1}^* &= \alpha_n\text{sig}(z_n)^{\frac{m_1}{n_1}}\omega_{n1} + \beta_n\text{sig}(z_n)^{\frac{p_1}{q_1}}\omega_{n2} \\ &\quad + \frac{u_{0d}z_{n-1}}{k_{n-1}^2 - z_{n-1}^2} - \frac{u_{1d} + \dot{\epsilon}_{nd}}{k_n^2 - z_n^2}, \\ u_1 &= -\frac{(k_n^2 - z_n^2)z_n\bar{\epsilon}_{n+1}^{*2}}{\sqrt{z_n^2\bar{\epsilon}_{n+1}^{*2} + \varepsilon^2}}.\end{aligned} \tag{16}$$

Similar to (15), we generate the ultimate result as

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n}\left(\alpha_j z_j^{2\gamma_1}\omega_{j1} + \beta_j z_j^{2\gamma_2}\omega_{j2}\right) \\ & - \sum_{j=1}^{n-1}\left(\frac{y_{j+1}^{2\gamma_1}}{\tau_{j+1}} + \frac{y_{j+1}^{2\gamma_2}}{\tau_{j+1}}\right) + \sum_{j=1}^{n-1}y_{j+1}^2 + \Theta\end{aligned} \tag{17}$$

where $\Theta = \sum_{j=1}^{n}\Theta_i$. Like [18], to prove the tracking error system (8) is practically fixed-time stable, let us rearrange the subscript serial number of variable y in the following way: $1 \leq h \leq l$, $|y_{h+1}| \geq 1$, $l+1 \leq d \leq n-1$, $0 \leq |y_{d+1}| < 1$. And for the coming stability analysis presentation, $\frac{1}{\tau_{j+1}}$ are divided into $\frac{1}{\tau_{j+1}} = \bar{\tau}_{j+1} + \hat{\tau}_{j+1}$. Owing to $0 \leq |y_{d+1}| < 1$, we can obtain the following inequalities:

$$\hat{\tau}_{d+1}y_{d+1}^{2\gamma_1} \geq \hat{\tau}_{d+1}y_{d+1}^2,\ \hat{\tau}_{h+1}y_{h+1}^{2\gamma_2} \geq \hat{\tau}_{h+1}y_{h+1}^2. \tag{18}$$

Under these, the following inequality is obtained as:

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n}\left(\alpha_j z_j^{2\gamma_1}\omega_{j1} + \beta_j z_j^{2\gamma_2}\omega_{j2}\right) \\ & - \sum_{d=l+1}^{n-1}\left(\bar{\tau}_{d+1}y_{d+1}^{2\gamma_1} + \hat{\tau}_{d+1}y_{d+1}^2\right) \\ & - \sum_{h=1}^{l}\left(\bar{\tau}_{h+1}y_{h+1}^{2\gamma_1} + \hat{\tau}_{h+1}y_{h+1}^2\right) + \sum_{j=1}^{n-1}y_{j+1}^2 \\ & - \sum_{h=1}^{l}\frac{y_{h+1}^{2\gamma_1}}{\tau_{h+1}} - \sum_{d=l+1}^{n-1}\frac{y_{d+1}^{2\gamma_2}}{\tau_{d+1}} + \Theta.\end{aligned} \tag{19}$$

Note that it is not difficult to choose filter parameters τ_i such that $\min\{\hat{\tau}_{d+1}, \hat{\tau}_{h+1}\} \geq 1$. Under which, the following inequality is further obtained as:

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n}\left(\alpha_j z_j^{2\gamma_1}\omega_{j1} + \beta_j z_j^{2\gamma_2}\omega_{j2}\right) \\ & - \sum_{h=1}^{l}\frac{y_{h+1}^{2\gamma_1}}{\tau_{h+1}} - \sum_{d=l+1}^{n-1}\bar{\tau}_{d+1}y_{d+1}^{2\gamma_1} \\ & - \sum_{d=l+1}^{n-1}\frac{y_{d+1}^{2\gamma_2}}{\tau_{d+1}} - \sum_{h=1}^{l}\bar{\tau}_{h+1}y_{h+1}^{2\gamma_2} + \Theta.\end{aligned} \tag{20}$$

Define $\alpha = \min\{\alpha_j\}$, $\beta = \min\{\beta_j\}$, $\chi_1 = \min\{\frac{1}{\tau_{h+1}}, \bar{\tau}_{d+1}\}$, $\chi_2 = \min\{\frac{1}{\tau_{d+1}}, \bar{\tau}_{h+1}\}$, and let $A = \min\{\alpha, \chi_1\}$, $B = \min\{\beta, \chi_2\}$. Meanwhile, according to Lemma 1 in [11] and Lemmas 3.3 and 3.4 in [16], it yields the following result:

$$\dot{V}_n \leq -aV_n^{\gamma_1} - bV_n^{\gamma_2} + \Theta \tag{21}$$

where $a = An^{1-\gamma_1}2^{\gamma_1}$, $b = B2^{\gamma_2}$.

Proposition 2: Let us study the tracking error system (8). Tracking controller specified by (16) can ensure that $x_{1e}, \ldots, x_{ne}$ converge to a small region around zero within a fixed time. Additionally, $|x_{1e}(t)| < k_{11}$ for $t \geq t_1$, that is, output state constraint is achieved.

Proof: Define $\varpi = \max\{(\Theta/((1-c)a))^{1/\gamma_1}, (\Theta/((1-c)b))^{1/\gamma_2}\}$. Due to (21), it holds that $\dot{V}_n \leq -c\left(aV_n^{\gamma_1} + bV_n^{\gamma_2}\right) \leq 0$ with $0 < c < 1$ when $V_n \geq \varpi$. Therefore $\{V_n \leq \varpi\}$ is an invariant set. In the sequel, two possible cases: $\varpi \geq 1$ or $\varpi < 1$ will be discussed. If $\varpi \geq 1$, since $\dot{V}_n \leq -cbV_n^{\gamma_2}$, it yields that $V_n \leq \varpi$ for $t \geq \varpi/(cb(\gamma_2 - 1))$. If $\varpi < 1$, since $\dot{V}_n \leq -caV_n^{\gamma_1}$, the time that taken to get from $V_n = 1$ to $V_n \leq \varpi$ should satisfy $t \geq 1/(ca(1-\gamma_1)) - \varpi^{1-\gamma_1}/(ca(1-\gamma_1))$, therefore, tracking error system (8) is practically fixed-time stable. Meanwhile, since $|z_i(0)| < k_i$ $(1 \leq i \leq n)$, it is known that $z_i(t)$ $(1 \leq i \leq n)$ are bounded and $|z_i(t)| < k_i$ $(1 \leq i \leq n)$. Further, since $z_1 = x_{1e}$, hence by choosing $k_1 < k_{11}$, we have $|x_{1e}(t)| = |z_1(t)| < k_1 < k_{11}$. ■

Tracking control of $(x_{1e}, \ldots, x_{ne})$**-subsystem for** $[0, t_1]$**:** Further, we will establish that tracking error system (3) is bounded during time interval $[0, t_1]$.

Assumption 2: Except for Assumption 1, it is supposed that $u_{0d} > \alpha_0k_0 + \beta_0k_0$.

Assumption 2 results in $u_0(t) > 0$ for $t \geq 0$. In addition, since u_0

specified by (4), u_{0d}, u_{1d}, and x_{id} $(2 \le i \le n)$ are bounded. As a consequence, $|\phi_n| = |-u_{1d}| < \zeta_n$, $|\phi_i| = |x_{(i+1)d}(u_0 - u_{0d})| < \zeta_i$ $(1 \le i \le n-1)$ with ζ_i $(1 \le i \le n)$ being positive constants. For the simplicity of presentation, system (3) is rewritten as

$$\dot{x}_{1e} = x_{2e}u_0 + \phi_1,\ \dot{x}_{ie} = x_{(i+1)e}u_0 + \phi_i,\ \dot{x}_{ne} = u_1 + \phi_n \tag{22}$$

which has a similar form with system (3). In fact, in Step i, an additional inequality should be added as

$$\frac{z_i\phi_i}{k_i^2 - z_i^2} \le \frac{|z_i\phi_i|}{k_i^2 - z_i^2} \le \frac{\mu_i z_i^2 \zeta_i^2}{(k_i^2 - z_i^2)^2} + \frac{1}{4\mu_i}. \tag{23}$$

Subsequently, system control input u_1 is designed as

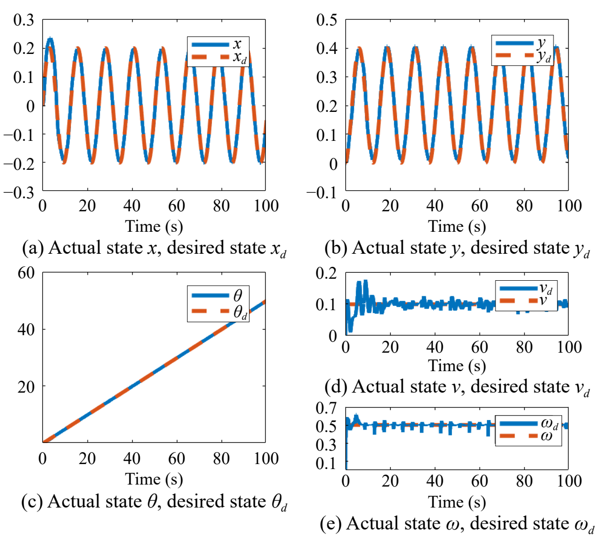
$$\begin{aligned} \bar{\epsilon}_{n+1}^{*} &= \alpha_n \mathrm{sig}(z_n)^{\frac{m_1}{n_1}}\omega_{n1} + \beta_n \mathrm{sig}(z_n)^{\frac{p_1}{q_1}}\omega_{n2} \\ &\quad + \frac{u_0 z_{n-1}}{k_{n-1}^2 - z_{n-1}^2} + \frac{\mu_n z_n \zeta_n^2}{(k_n^2 - z_n^2)^2} - \frac{\dot{\epsilon}_{nd}}{k_n^2 - z_n^2} \\ u_1 &= -\frac{(k_n^2 - z_n^2) z_n \bar{\epsilon}_{n+1}^{*2}}{\sqrt{z_n^2 \bar{\epsilon}_{n+1}^{*2} + \varepsilon^2}}. \end{aligned} \tag{24}$$

Proposition 3: Let us study the resulting closed-loop tracking error system (22) and (24). The designed tracking controller (24) can ensure that the error system (22) is bounded in the operation time interval $[0, t_1]$ and $|x_{1e}(t)| < k_{11}$ for $0 \le t \le t_1$.

Theorem 1: For the closed-loop tracking error system (3), we assume that system initial states $x_{ie}(0) \in \Omega_{x_{ie}} \triangleq \{-k_{i1} < x_{ie}(t) < k_{i1}\}$ $(0 \le i \le n)$ and tracking controllers u_0 and u_1 are actualized as: 1) $u_0 = (4)$ for $t \ge 0$; 2) $u_1 = (24)$ for $0 \le t < t_1$ and $u_1 = (16)$ for $t \ge t_1$, then the following control objectives can be achieved as: 1) $\lim_{t \to t_1} x_{0e}(t) = 0$; 2) $-k_{i1} < x_{ie}(t) < k_{i1}$ $(0 \le i \le 1)$ for any $t \ge 0$; and 3) all signals in closed-loop system (3) converge to a bounded set within a fixed time.

Experiment results analysis: We use the QBot2e wheeled mobile robot manufactured by Quanser (detailed description can be found in [5]) to demonstrate the effectiveness of the presented tracking method. The kinematic model of this robot can be expressed as $\dot{x} = v\cos(\theta)$, $\dot{y} = v\sin(\theta)$, $\dot{\theta} = w$. Assume that (x_d, y_d, θ_d) is generated by $\dot{x}_d = 0.1\cos(0.5t)$, $\dot{y}_d = 0.1\sin(0.5t)$, $\dot{\theta}_d = 0.5$. By applying the following state and control transformations $x_0 = \theta$, $x_1 = x\sin(\theta) - y\cos(\theta)$, $x_2 = x\cos(\theta) + y\sin(\theta)$, $u_0 = \omega$, $u_1 = v - x_1 u_0$ to the kinematic system, and applying some similar transformations to the virtual system, we can obtain a tracking error system like (3).

For experiment, the control parameters are picked as $\alpha_0 = 0.6$, $\beta_0 = 0.6$, $\alpha_1 = 0.4$, $\beta_1 = 0.5$, $\alpha_2 = 0.6$, $\beta_2 = 0.5$, $m_0 = 7$, $n_0 = 9$, $p_0 = 9$, $q_0 = 7$, $m_1 = 5$, $n_1 = 7$, $p_1 = 7$, $q_1 = 5$, $\zeta_1 = \zeta_2 = 0.01$, $\mu_1 = \mu_2 = 1$, $\varepsilon = 0.05$, $\tau_2 = 0.05$, $k_0 = 0.6$, $k_1 = 1.2$, $k_2 = 1.2$. The initial system states are set as $[x_d(0), y_d(0), \theta_d(0)]^T = [0, 0, 0]^T$. In view of Fig. 1, it follows that system trajectory states x, y, and θ can track their desired trajectory states x_d, y_d, and θ_d, respectively.



(a) Actual state x, desired state x_d (b) Actual state y, desired state y_d (c) Actual state θ, desired state θ_d (d) Actual state v, desired state v_d (e) Actual state ω, desired state ω_d

Fig. 1. The state and speed responses of the QBot2e.

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