Subspace Identification for Closed-Loop Systems With Unknown Deterministic Disturbances

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***Abstract*—This paper presents a subspace identification method for closed-loop systems with unknown deterministic disturbances. To deal with the unknown deterministic disturbances, two strategies are implemented to construct the row space that can be used to approximately represent the unknown deterministic disturbances using the trigonometric functions or Bernstein polynomials depending on whether the disturbance frequencies are known. For closed-loop identification, CCF-N4SID is extended to the case with unknown deterministic disturbances using the oblique projection. In addition, a proper Bernstein polynomial order can be determined using the Akaike information criterion (AIC) or the Bayesian information criterion (BIC). Numerical simulation results demonstrate the effectiveness of the proposed identification method for both periodic and aperiodic deterministic disturbances.**

***Index Terms*—Bernstein polynomial, closed-loop system, subspace identification, unknown deterministic disturbances.**

I. Introduction

SYSTEM identification has been widely investigated over the past several decades, which serves as an important tool for monitoring and control purpose [1]–[6]. In particular, subspace identification methods (SIMs) can be advantageous over the identification of the state-space model using the process data available, which has been demonstrated to be quite useful for the identification of multivariate systems. Recently, many achievements have been made regarding complex systems and conditions on the subspace identification. In [7], subspace identification for structured state-space models is proposed with the user-defined linear or polynomial parameterization structure. In [8], a unified subspace identification framework for linear parameter-varying (LPV) systems is established in innovation form, which gives an in-depth view on the LPV subspace identification problem. In [9], subspace identification for large-scale interconnected systems with heterogeneous network is investigated with no constrains on the sparse interconnection.

In practice, most industrial processes are required to be operated in practice under closed-loop conditions due to the concern for system safety and product quality. In some cases, the process cannot be separated for open-loop identification tests, for instance, the process needs to be stabilized for bounded outputs or requires online identification. This motivates the subspace identification work for closed-loop systems using the available process data. The main challenge is to deal with the inherited correlation between the control inputs and noise under the feedback control in order to get unbiased estimation results [10]. Until now, many various methods have been developed for closed-loop subspace identification, such as innovation estimation method (IEM) [11], predictor-based subspace identification (PBSID) [12], closed-loop subspace identification method via principal component analysis (CSIMPCA) [13] and nuclear norm subspace identification method (N2SID) [14]. In addition, a novel closed-loop identification method called CCF-N4SID was recently proposed in [15] to integrate the prior knowledge of the controller into the subspace identification framework, based on which the closed-loop identification framework will be used in this work.

On the other hand, practical industrial processes may suffer from various unknown deterministic disturbances, for instance, the flatness of the strip can be greatly influenced due to the roll eccentricities as a typical external unknown disturbance during the rolling process [16] and wind turbines can be prone to unknown periodic disturbances due to the air turbulence [17], [18]. These unknown deterministic disturbances can lead to the biased or even totally incorrect identification results if they cannot be well handled. However, less attention has been paid to the in-depth investigation of subspace identification with unknown deterministic disturbances. A subspace identification method with deterministic disturbances is proposed in [19] based on the multiple-input-multiple-output error state-space model identification (MOESP) algorithm and it is derived under the assumption that disturbances can be parametrisable. The base space of the disturbances is not explicitly constructed though it is mentioned that a proper

Manuscript received September 18, 2022; accepted October 12, 2022. This work was partially supported by National Key Research and Development Program of China (2019YFC1510902), National Natural Science Foundation of China (62073104), Natural Science Foundation of Heilongjiang Province (LH2022F024), and China Postdoctoral Science Foundation (2022M710965). Recommended by Associate Editor Daoyi Dong. *(Corresponding author: Kuan Li.)*

Citation: K. Li, H. Luo, Y. C. Jiang, D. J. Tang, and H. Y. Yang, "Subspace identification for closed-loop systems with unknown deterministic disturbances," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 12, pp. 2248–2257, Dec. 2023.

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Digital Object Identifier 10.1109/JAS.2023.123330

base space can be helpful to the subspace identification under the strong periodic disturbances. A biased-eliminated subspace identification method with constant load or periodic load is proposed in [20], [21] for the consistent estimation where the output responses are decomposed into the disturbed part and the undisturbed part. In addition, the above results have also been extended to the identification of the Hammerstein nonlinear system with periodic or slowly varying disturbances [22], [23]. However, few studies have been dedicated to the subspace identification with aperiodic deterministic disturbances.

In this paper, a closed-loop subspace identification method is proposed to deal with the unknown deterministic disturbances under standard feedback control. The influence of unknown deterministic disturbances can be alleviated via the projection onto the constructed row space, which can easily adapt to aperiodic deterministic disturbances with unknown frequencies using the row space constructed by Bernstein polynomials. The main contributions can be summarized as:

1) The row space that can be used to approximately represent the unknown deterministic disturbances is respectively designed using the trigonometric functions and Bernstein polynomials.

2) CCF-N4SID is extended to the closed-loop subspace identification with unknown deterministic disturbances using the oblique projection.

3) A proper Bernstein polynomial order is determined to approximate the unknown deterministic disturbances via the Akaike information criterion (AIC) or Bayesian information criterion (BIC).

The rest of the paper is organized as follows. Section II briefly introduces the preliminaries and formulates the problem. Section III presents the proposed closed-loop subspace identification algorithm with unknown deterministic disturbances. Section IV discusses the choice of the Bernstein polynomial order. Section V verifies the effectiveness of the proposed algorithm via the simulation study. Section VI concludes this work.

II. Preliminaries and Problem Formulation

A. System Description

Consider the following state space model $G(z)$ with periodic disturbances under the standard feedback control:

$$x(k+1) = Ax(k) + Bu(k) + E_d d(k) + w(k) \tag{1}$$

$$y(k) = Cx(k) + Du(k) + F_d d(k) + v(k) \tag{2}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^l$, $y(k) \in \mathbb{R}^m$ and $d(k) \in \mathbb{R}^{n_d}$ denote the system state, control input, measurement output and periodic disturbances, respectively; $w(t) \in \mathbb{R}^n$ and $v(k) \in \mathbb{R}^m$ are the process noise and measurement noise, respectively; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times l}$, $C \in \mathbb{R}^{m\times n}$, $D \in \mathbb{R}^{m\times l}$, $E_d \in \mathbb{R}^{n\times n_d}$ and $F_d \in \mathbb{R}^{m\times n_d}$ are parameter matrices with appropriate dimensions.

Given the controller $K(z)$ in the form of the state space representation as

$$x_c(k+1) = A_c x_c(k) + B_c(r(k) - y(k)) \tag{3}$$

$$u(k) = C_c x_c(k) + D_c(r(k) - y(k)) \tag{4}$$

where $x_c(k) \in \mathbb{R}^{n_c}$ denotes the state vector of the controller, $r(k) \in \mathbb{R}^m$ denotes the reference signal; $A_c \in \mathbb{R}^{n_c\times n_c}$, $B_c \in \mathbb{R}^{n_c\times m}$, $C_c \in \mathbb{R}^{l\times n_c}$ and $D_c \in \mathbb{R}^{l\times m}$ are parameter matrices of the controller with appropriate dimensions.

B. Definition and Lemma

Definition 1 (Bernstein Polynomial [24]): For a bounded function $f(x) \in C[0,1]$, for $n \in N_+$, the n-order Bernstein polynomial $B_n(f,x)$ is defined as

$$B_n(f,x) = \sum_{l=0}^{n} f\left(\frac{l}{n}\right)\begin{pmatrix} n \\ l \end{pmatrix} x^l (1-x)^{n-l} \tag{5}$$

where $C[0,1]$ denotes the continuous function on [0, 1], $(nl) = \frac{n!}{l!(n-l)!}$.

Lemma 1 (Approximation Theorem [24]): Let a continuous function $f(x)$ be bounded on $C[0,1]$, for any point $x \in [0,1]$, then

$$\lim_{n\to\infty} B_n(f,x) = f(x) \tag{6}$$

holds uniformly in [0, 1].

Definition 1 and Lemma 1 indicate that the deterministic disturbances in analytic form can be approximated by the Berstein polynomial, which serves as the foundation for the design of the row space that approximately represents the unknown deterministic disturbances in Section III-B.

C. Problem Formulation

In this paper, the identification problem is formulated as determining the matrices A, B, C and D from the process $G(z)$ with the unmeasurable deterministic disturbances under the feedback control. The main focus is the solution to two key problems: 1) How to approximate the unknown deterministic disturbances in the subspace context. 2) How to identify the process model with unknown deterministic disturbances under closed-loop conditions.

To identify the process model, several assumptions have to be made:

Assumption 1: The standard feedback control loop is well-posed and internally stabilized by the controller.

Assumption 2: $G(z)$ is minimal, i.e., the pair (A,C) is observable and the pair (A,B) is reachable.

Assumption 3: The reference $\{r(k)\}$ is an ergodic, quasi-stationary random process and can be persistently exciting of any order.

Assumption 4: $\{w(k)\}$ and $\{v(k)\}$ are assumed to be zero-mean white noise sequences and independent of the reference $\{r(k)\}$ with covariance matrix

$$\mathbb{E}\left\{\begin{bmatrix} w(k_1) \\ v(k_1) \\ r(k_1) \end{bmatrix}\begin{bmatrix} w(k_2) \\ v(k_2) \\ r(k_2) \end{bmatrix}^T\right\} = \begin{bmatrix} R_w & 0 & 0 \\ 0 & R_v & 0 \\ 0 & 0 & R_r \end{bmatrix} \tag{7}$$

and

$$R_w = \mathbb{E}\left\{w(k_1)w^T(k_2)\right\} = \sigma_w^2 I \times \delta(k_1 - k_2) \tag{8}$$

$$R_v = \mathbb{E}\left\{v(k_1)v^T(k_2)\right\} = \sigma_v^2 I \times \delta(k_1 - k_2) \tag{9}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator, σ_w^2 and σ_v^2 denote the variances of the process noise and measurement noise, respectively; δ denotes the Dirac function.

III. Closed-Loop Subspace Identification With Unknown Deterministic Disturbances

A. Data Equation

Define the stacked vectors $u_{s,k}$ with length s as follows:

$$u_{s,k}=\begin{bmatrix} u(k-s) \\ u(k-s+1) \\ \vdots \\ u(k) \end{bmatrix} \in \mathbb{R}^{(s+1)\times l}. \tag{10}$$

By arranging the stacked vectors at different instants, define

$$\begin{cases} U_{k,s}=\begin{bmatrix} u_{s,k-N+1} & \cdots & u_{s,k} \end{bmatrix} \in \mathbb{R}^{(s+1)l\times N} \\ Y_{k,s}=\begin{bmatrix} y_{s,k-N+1} & \cdots & y_{s,k} \end{bmatrix} \in \mathbb{R}^{(s+1)m\times N} \\ D_{k,s}=\begin{bmatrix} d_{s,k-N+1} & \cdots & d_{s,k} \end{bmatrix} \in \mathbb{R}^{(s+1)n_d\times N} \\ X_{k,N}=\begin{bmatrix} x(k-N+1) & \cdots & x(k) \end{bmatrix} \in \mathbb{R}^{n\times N} \end{cases} \tag{11}$$

where N is a large integer, $y_{s,k}$ and $d_{s,k}$ have the same structure as $u_{s,k}$.

To split the data into past and future horizons, define

$$\begin{cases} U_p=U_{k-s_f-1,s_p}, \quad U_f=U_{k,s_f} \\ Y_p=Y_{k-s_f-1,s_p}, \quad Y_f=Y_{k,s_f} \\ D_p=D_{k-s_f-1,s_p}, \quad D_f=D_{k,s_f} \end{cases} \tag{12}$$

where s_p and s_f denote the past horizon and the future horizon, respectively.

Iterating on (1), it leads to

$$Y_f=\Gamma_f X_{k,N}+H_{u,f}U_f+H_{d,f}D_f+\Phi_f \tag{13}$$

where Φ_f denotes the noise term, which can refer to [15] for details. Γ_f, $H_{u,f}$, and $H_{d,f}$ are

$$\Gamma_f=\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s_f} \end{bmatrix} \tag{14}$$

$$H_{u,f}=\begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s_f-1}B & CA^{s_f-2}B & \cdots & D \end{bmatrix} \tag{15}$$

$$H_{d,f}=\begin{bmatrix} F_d & 0 & \cdots & 0 \\ CE_d & F_d & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s_f-1}E_d & CA^{s_f-2}E_d & \cdots & F_d \end{bmatrix}. \tag{16}$$

Consider the predictor model of (1) as follows:

$$\hat{x}(k+1)=A_K\hat{x}(k)+B_Ku(k)+E_Kd(k)+Ky(k) \tag{17}$$

$$y(k)=C\hat{x}(k)+Du(k)+F_dd(k)+e(k) \tag{18}$$

where K is the Kalman filter gain matrix, $A_K=A-KC$, $B_K=B-KD$, $E_K=E_d-KF_d$.

Iterating on (17), $X_{k,N}$ can be rewritten as

$$X_{k,N}\approx\begin{bmatrix} \Delta_u & \Delta_y & \Delta_d \end{bmatrix} Z_{p,d} \tag{19}$$

where $Z_{p,d}=\begin{bmatrix} U_p \\ Y_p \\ D_p \end{bmatrix}$, $\Delta_u=\begin{bmatrix} A_K^{s_p-1}B_K & \cdots & B_K \end{bmatrix}$, $\Delta_y=\begin{bmatrix} A_K^{s_p-1}K & \cdots & K \end{bmatrix}$, $\Delta_d=\begin{bmatrix} A_K^{s_p-1}E_K & \cdots & E_K \end{bmatrix}$.

In this way, the data equation of $G(z)$ can be given as

$$Y_f=\Xi_{z,p,d}Z_{p,d}+H_{u,f}U_f+H_{d,f}D_f+\Phi_f \tag{20}$$

where $\Xi_{z,p,d}=\Gamma_f\begin{bmatrix} \Delta_u & \Delta_y & \Delta_d \end{bmatrix}$.

B. Dealing With Unknown Deterministic Disturbances

1) Dealing With Deterministic Disturbances With Known Frequencies: Assume that the deterministic disturbances that can be approximately described by the superposition of finite number of sine functions as follows:

$$d_i(k)\approx a_{i,0}+\sum_{j=1}^{m_i}a_{i,j}\sin\left(\omega_{i,j}k+\varphi_{i,j}\right) \tag{21}$$

where $d_i(k)$ denotes the ith scalar consisting of the disturbance vector, m_i is the number of distinct frequencies, $i=1,2,\ldots,n_d$. $a_{i,0}$ is the bias of $d_i(k)$. a_j, ω_j and φ_j denote the amplitude, frequency and phase for jth sine components, respectively.

Based on the triangle identity, we have

$$a_{i,j}\sin(\omega_{i,j}k+\varphi_{i,j})=\beta_{i,j,1}\sin(\omega_{i,j}k)+\beta_{i,j,2}\cos(\omega_{i,j}k) \tag{22}$$

where $\beta_{i,j,1}=a_{i,j}\cos(\varphi_{i,j})$, $\beta_{i,j,2}=a_{i,j}\sin(\varphi_{i,j})$.

According to (22), (21) can be reformulated as

$$d_i(k)\approx a_{i,0}+\beta_i^T\rho_{i,k} \tag{23}$$

where

$$\beta_i^T=\begin{bmatrix} \beta_{i,1,1} & \beta_{i,1,2} & \cdots & \beta_{i,m_i,1} & \beta_{i,m_i,2} \end{bmatrix} \tag{24}$$

$$\rho_{i,k}^T=\begin{bmatrix} \sin(\omega_{i,1}k) & \cos(\omega_{i,1}k) & \cdots & \sin(\omega_{i,m_i}k) & \cos(\omega_{i,m_i}k) \end{bmatrix}. \tag{25}$$

Therefore, we have

$$\underbrace{\begin{bmatrix} d_1(k) \\ \vdots \\ d_i(k) \\ \vdots \\ d_{n_d}(k) \end{bmatrix}}_{d(k)} \approx \underbrace{\begin{bmatrix} a_{1,0} & \beta_1^T & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{i,0} & 0 & \cdots & \beta_i^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{n_d,0} & 0 & \cdots & 0 & \cdots & \beta_{n_d}^T \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 \\ \rho_{1,k} \\ \vdots \\ \rho_{i,k} \\ \vdots \\ \rho_{n_d,k} \end{bmatrix}}_{\rho_k}. \tag{26}$$

Note that

$$\begin{bmatrix} \sin\left(\omega_{i,j}(k+1)\right) \\ \cos\left(\omega_{i,j}(k+1)\right) \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\left(\omega_{i,j}\right) & \sin\left(\omega_{i,j}\right) \\ -\sin\left(\omega_{i,j}\right) & \cos\left(\omega_{i,j}\right) \end{bmatrix}}_{T_{i,j}} \begin{bmatrix} \sin\left(\omega_{i,j}k\right) \\ \cos\left(\omega_{i,j}k\right) \end{bmatrix} \tag{27}$$

it follows that:

$$\rho_{i,k+1} = \underbrace{\begin{bmatrix} T_{i,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & T_{i,m_i} \end{bmatrix}}_{T_i} \rho_{i,k}. \tag{28}$$

Based on (28), it leads to

$$\underbrace{\begin{bmatrix} 1 \\ \rho_{1,k+1} \\ \vdots \\ \rho_{i,k+1} \\ \vdots \\ \rho_{n_d,k+1} \end{bmatrix}}_{\rho_{k+1}} = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & T_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T_{n_d} \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} 1 \\ \rho_{1,k} \\ \vdots \\ \rho_{i,k} \\ \vdots \\ \rho_{n_d,k} \end{bmatrix}}_{\rho_k}. \tag{29}$$

According to (29), $d_{s,k}$ can be represented in the compact form as follows:

$$\underbrace{\begin{bmatrix} d(k-s) \\ d(k-s+1) \\ \vdots \\ d(k) \end{bmatrix}}_{d_{s,\vec{k}}} \approx \begin{bmatrix} P\rho_{k-s} \\ P\rho_{k-s+1} \\ \vdots \\ P\rho_k \end{bmatrix} = \underbrace{\begin{bmatrix} P \\ PT \\ \vdots \\ PT^{s-1} \end{bmatrix}}_{P_s} \rho_{k-s} \tag{30}$$

and

$$D_{k,s} = \begin{bmatrix} d_{s,k-N+1} & \cdots & d_{s,k} \end{bmatrix} \approx P_s D_{b,tri} \tag{31}$$

where $D_{b,tri} = \begin{bmatrix} \rho_{k-N+1-s} & \cdots & \rho_{k-s} \end{bmatrix} \in \mathbb{R}^{(2n_d m_d+1)\times N}$, $m_d = \sum_{i=1}^{n_d} m_i$. Note that (31) implies that $D_{b,tri}$ can be used to approximately describe the row space to which the deterministic disturbances belongs.

Remark 1: Note that $D_{b,tri}$ is constructed by the trigonometric functions with distinct frequency components. In some cases, the disturbance frequencies can be estimated using the fast fourier transformation (FFT) or other related signal processing techniques. In addition, the calculation of $\omega_{i,j}k$ is needed to construct $D_{b,tri}$ and it can observed that the frequency estimation error may lead to the deviation of $D_{b,tri}$ especially when N is large. Therefore, an empirical suggestion is that the sampling frequency f should satisfy $\frac{f}{N} \geq 1$.

2) Dealing With Deterministic Disturbances With Unknown Frequencies: In practice, the frequency estimation of the deterministic disturbance can be inaccurate, and it is even impossible to estimate the frequency components of the deterministic disturbance in some cases. In the following, the focus will be on how to approximately describe the row space where the deterministic disturbances belong to when disturbances frequencies are unavailable.

The underlying idea is to approximate the trigonometric function in $D_{b,tri}$ based on the Bernstein polynomial and try to construct the row space that only contains the time sequences, which can be used to address the aperiodic deterministic disturbances as well.

Note that $\sin(\omega k) \in C[-1,1]$, $\cos(\omega k) \in C[-1,1]$, which implies that the Lemma 1 can not be directly used. Therefore, the following transformations are made first.

Let $g_{\sin}(k) = \frac{1}{2}(\sin(\omega k)+1) \in C[0,1]$, $g_{\cos}(k) = \frac{1}{2}(\cos(\omega k)+1) \in C[0,1]$, $k \in [a,b]$, based on Definition 1 and Lemma 1, we have

$$g_{\sin}(k) = \lim_{n_b\to\infty} \sum_{l=0}^{n_b} \frac{1}{2}(s_{a,b,\omega}(l)+1)\begin{pmatrix} n_b \\ l \end{pmatrix}\phi_{a,b}(k,l) \tag{32}$$

$$g_{\cos}(k) = \lim_{n_b\to\infty} \sum_{l=0}^{n_b} \frac{1}{2}(c_{a,b,\omega}(l)+1)\begin{pmatrix} n_b \\ l \end{pmatrix}\phi_{a,b}(k,l) \tag{33}$$

where $s_{a,b,\omega}(l) = \sin(\omega(\frac{b-a}{n_b}l+a)$, $c_{a,b,\omega}(l) = \cos(\omega(\frac{b-a}{n_b}l+a)$, $\phi_{a,b}(k,l) = q_{a,b}(k)^l(1-q_{a,b}(k))^{n_b-l}$, $q_{a,b}(k) = \frac{k-a}{b-a} \in [0,1]$, n_b is the order of the Bernstein polynomial.

Note that $\sin(\omega k) = 2g_{\sin}(k)-1$, $\cos(\omega k) = 2g_{\cos}(k)-1$, we have

$$\sin(\omega k) = \lim_{n_b\to\infty} \sum_{l=0}^{n_b} s_{a,b,\omega}(l)\begin{pmatrix} n_b \\ l \end{pmatrix}\phi_{a,b}(k,l) \tag{34}$$

$$\cos(\omega k) = \lim_{n_b\to\infty} \sum_{l=0}^{n_b} c_{a,b,\omega}(l)\begin{pmatrix} n_b \\ l \end{pmatrix}\phi_{a,b}(k,l). \tag{35}$$

Let $a = k_0$, $b = k_0+N-1$, $k \in [k_0, k_0+N-1]$, it follows that:

$$\begin{bmatrix} \sin(\omega_{i,j}k) \\ \cos(\omega_{i,j}k) \end{bmatrix} = Q_{i,j} \begin{bmatrix} \phi_{k_0,k_0+N-1}(k,0) \\ \phi_{k_0,k_0+N-1}(k,1) \\ \vdots \\ \phi_{k_0,k_0+N-1}(k,n_b) \end{bmatrix} \tag{36}$$

where the entries in the rth row and cth column of $Q_{i,j}$ can be given as

$$Q_{i,j}(r,c) = \begin{cases} s_{k_0,k_0+N-1,\omega_{i,j}}(c-1), & \text{if } \quad r=1 \\ c_{k_0,k_0+N-1,\omega_{i,j}}(c-1), & \text{if } \quad r=2. \end{cases} \tag{37}$$

Therefore, we have

$$\underbrace{\begin{bmatrix} \sin(\omega_{i,1}k) \\ \cos(\omega_{i,1}k) \\ \vdots \\ \sin(\omega_{i,m_i}k) \\ \cos(\omega_{i,m_i}k) \end{bmatrix}}_{\rho_{i,k}} = \underbrace{\begin{bmatrix} Q_{i,1} \\ Q_{i,2} \\ \vdots \\ Q_{i,m_i} \end{bmatrix}}_{Q_i} \begin{bmatrix} \phi_{k_0,k_0+N-1}(k,0) \\ \phi_{k_0,k_0+N-1}(k,1) \\ \vdots \\ \phi_{k_0,k_0+N-1}(k,n_b) \end{bmatrix}. \tag{38}$$

Consider the property of the Bernstein basis, i.e.,

$$\sum_{l=0}^{n_b} \begin{pmatrix} n_b \\ l \end{pmatrix} k^l(1-k)^{n_b-l} = 1 \tag{39}$$

and based on (38), it leads to

$$\rho_k = \begin{bmatrix} 1 \\ \rho_{1,k} \\ \vdots \\ \rho_{i,k} \\ \vdots \\ \rho_{n_d,k} \end{bmatrix} = \underbrace{\begin{bmatrix} \varrho \\ Q_1 \\ Q_2 \\ \vdots \\ Q_{n_d} \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} \phi_{k_0,k_0+N-1}(k,0) \\ \phi_{k_0,k_0+N-1}(k,1) \\ \vdots \\ \phi_{k_0,k_0+N-1}(k,n_b) \end{bmatrix}}_{\mathcal{K}_k} \tag{40}$$

where $\varrho = \left[\begin{pmatrix} n_b \\ 0 \end{pmatrix} \ \begin{pmatrix} n_b \\ 1 \end{pmatrix} \ \cdots \ \begin{pmatrix} n_b \\ n_b \end{pmatrix} \right]$.

In fact, (40) implies that

$$D_{b,tri} = QD_{b,\text{bern}} \tag{41}$$

where $D_{b,\text{bern}} = \left[\mathcal{K}_{k_0} \ \ \mathcal{K}_{k_0+1} \ \ \cdots \ \ \mathcal{K}_{k_0+N-1} \right]$.

To facilitate the following description, $D_{b,tri}$ and $D_{b,\text{bern}}$ are unified as D_b.

C. Closed-Loop Subspace Identification

For closed-loop subspace identification, the CCF-N4SID algorithm is extended to the case with unknown deterministic disturbances.

Note that (20) can be rewritten as

$$Y_f = \Xi_{z,p} Z_p + H_{u,f} U_f + \tilde{H}_{d,f} D_b + \Phi_f \tag{42}$$

where $Z_p = \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$, $\Xi_{z,p} = \Gamma_f \left[\Delta_u \ \ \Delta_y \right]$, and it holds that $\tilde{H}_{d,f} D_b = \Gamma_f \Delta_d D_p + H_{d,f} D_f$ based on the fact D_p and D_f can be both represented by D_b.

Given the left coprime factorization of $K(z)$ as

$$K(z) = \hat{V}_c(z)^{-1} \hat{U}_c(z) \tag{43}$$

where $\hat{V}_c(z) = \left[\begin{array}{c|c} A_v & B_v \\ \hline C_v & D_v \end{array} \right]$, $\hat{U}_c(z) = \left[\begin{array}{c|c} A_u & B_u \\ \hline C_u & D_u \end{array} \right]$.

Define the instrumental variable M_f [15],

$$M_f = K^c_{v,f} U_f + K^c_{u,f} Y_f \tag{44}$$

where

$$K^c_{v,f} = \begin{bmatrix} D_v & 0 & \cdots & 0 \\ C_v B_v & D_v & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_v A_v^{s_f-1} B_v & C_v A_v^{s_f-2} B_v & \cdots & D_v \end{bmatrix} \tag{45}$$

$$K^c_{u,f} = \begin{bmatrix} D_u & 0 & \cdots & 0 \\ C_u B_u & D_u & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_u A_u^{s_f-1} B_u & C_u A_u^{s_f-2} B_u & \cdots & D_u \end{bmatrix}. \tag{46}$$

The following Theorem 1 gives a feasible solution to the estimation on the extended observability matrix for closed-loop systems with unknown deterministic disturbances.

Theorem 1: For a sufficiently large N, under the assumption that $\begin{bmatrix} Z_p \\ D_b \\ M_f \\ Y_f \end{bmatrix}$ is of full row rank, perform the following LQ factorization:

$$\underbrace{\begin{bmatrix} Z_p \\ D_b \\ M_f \\ Y_f \end{bmatrix}}_{Z_c} = \underbrace{\begin{bmatrix} L_{c,11} & 0 & 0 & 0 \\ L_{c,21} & L_{c,22} & 0 & 0 \\ L_{c,31} & L_{c,32} & L_{c,33} & 0 \\ L_{c,41} & L_{c,42} & L_{c,43} & L_{c,44} \end{bmatrix}}_{L_c} \underbrace{\begin{bmatrix} Q_{c,1} \\ Q_{c,2} \\ Q_{c,3} \\ Q_{c,4} \end{bmatrix}}_{Q_c} \tag{47}$$

and if $I - L_{c,43} L_{c,33}^{-1} K^c_{u,f}$ is invertible, it follows that:

$$\Gamma_f \check{X}_{k,N} = \lim_{s_p \to \infty} (I - L_{c,43} L_{c,33}^{-1} K^c_{u,f})^{-1} L_{c,z_p} Z_p \tag{48}$$

$$H_{u,f} = (I - L_{c,43} L_{c,33}^{-1} K^c_{u,f})^{-1} L_{c,43} L_{c,43}^{-1} K^c_{v,f} \tag{49}$$

where $L_{c,z_p} = (L_{c,41} - L_{c,42} L_{c,22}^{-1} L_{c,21} - L_{c,43} L_{c,33}^{-1} L_{c,31} + L_{c,43} \times L_{c,33}^{-1} L_{c,32} L_{c,22}^{-1} L_{c,21}) L_{c,11}^{-1}$, $Z_{c,p} = \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$, $\check{X}_{k,N} = \left[\Delta_u \ \ \Delta_y \right] \times \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$.

Proof: See Appendix. ■

Remark 2: It has been proven that M_f is uncorrelated with noise term Φ_f under Assumption 4 in [15], which can be used to eliminate the identification bias under closed-loop conditions.

Based on Theorem 1, perform the singular value decomposition (SVD) on $\Gamma_f \check{X}_{k,N}$ as follows:

$$\Gamma_f \check{X}_{k,N} = \check{U}_{svd} \check{\Sigma}_{svd} \check{V}_{svd} \tag{50}$$

where $rank(\check{\Sigma}_{svd}) = n$.

For a balanced realization based on (50), we have

$$\Gamma_f = \check{U}_{svd} \check{\Sigma}_{svd}^{1/2} \tag{51}$$

$$\check{X}_{k,N} = \check{\Sigma}_{svd}^{1/2} \check{V}_{svd}. \tag{52}$$

Since Γ_f has been derived, the matrices A, B, C and D can be calculated based on subspace identification.

The proposed closed-loop subspace identification method with unknown deterministic disturbances is summarized in Algorithm 1.

IV. Discussion on the Bernstein Polynomial Order

Based on the information entropy theory, the Akaike information criterion (AIC) is widely used to deal with the trade-off between the model complexity and the model fitting goodness. The purpose is to determine a model that can well interpret the data with the least parameters. Therefore, the following AIC indicators can be helpful to determine the proper order of the Bernstein polynomial:

$$AIC = N \ln(\det(\Sigma_e)) + N(n_y \ln(2\pi) + 1) + 2h \tag{53}$$

where $\det(\cdot)$ denotes the determinant operator, N is the sample length, $\Sigma_e = \frac{1}{N} L_{c,44} L_{c,44}^T$, $n_y = (s_f + 1)m$ is the dimension of the model output, $h = (s_f + 1)m[(s_p + 1)(l + m) + (s_f + 1)l + n_b]$ is the model parameter numbers.

Algorithm 1 Closed-loop subspace identification with unknown deterministic disturbances

Input: $u(k)$, $y(k)$.
Output: A, B, C and D.
S1: Set s_p, s_f and N, construct Hankel matrices U_p, U_f Y_p and Y_f according to (12).
S2: Construct the row space D_b that approximates the unknown deterministic disturbances as follows,
a) When frequencies of deterministic disturbances are known, construct $D_{b,tri}$ as D_b,
b) When frequencies of deterministic disturbances are unknown, construct $D_{b,\text{bern}}$ as D_b.
S3: Do left coprime factorization on $K(z)$, obtain $\hat{V}_c(z)$ and $\hat{U}_c(z)$, construct $K^c_{v,f}$ and $K^c_{u,f}$ according to (45) and (46).
S4: Construct the instrumental variable $M_f = K^c_{v,f} U_f + K^c_{u,f} Y_f$.
S5: Do LQ factorization as (47), obtain $\Gamma_f \check{X}_{k,N}$ and $H_{u,f}$.
S6: Identify the matrices A, B, C and D.

Therefore, an appropriate order of the Bernstein polynomial should satisfy

$$n_b = \min_{n_b} AIC. \tag{54}$$

Note that the penalty coefficient of model parameter numbers is set to be 2 for AIC. To avoid the overfitting, the penalty coefficient of model parameter numbers can be properly increased. As a special case, when the penalty coefficient is set to be $ln(N)$, it leads to the Bayesian information criterion (BIC), i.e.,

$$BIC = N\ln(\det(\Sigma_e)) + N(n_y \ln(2\pi) + 1) + h\ln(N). \tag{55}$$

In terms of performance indicators, it is suggested that AIC and BIC can be both analyzed for performance evaluation to determine a proper Bernstein polynomial order in most cases.

V. Simulation Study

Consider a linear invariant system $G(z)$, the parameter matrices are given as

$$A = \begin{bmatrix} -0.0939 & 0.1241 & 0.9861 & 0 \\ -0.3214 & -0.3926 & 0.0941 & 0.8193 \\ 0.3287 & 0.6784 & -0.1096 & 0.3902 \\ -0.3217 & -0.1356 & 0.0817 & -0.4200 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.3257 & 0.3232 \\ 0.6647 & -0.7841 \\ 0.0852 & -1.8054 \\ 0.8810 & 1.8586 \end{bmatrix}, \; C = \begin{bmatrix} 0.4600 & -0.6817 \\ 0 & 0.5932 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$$

$$D = \begin{bmatrix} -0.6045 & 0.5632 \\ 0.1034 & 0.1136 \end{bmatrix}, \; E_d = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The controller $K(z)$ can be designed arbitrarily such that the system is stabilized.

During the simulation, the noise powers of two reference signals are set to be $P_n = 0.2$ and $P_n = 0.3$, respectively. The noise powers of $w(k)$ and $v(k)$ are both set to be $P_n = 0.0002$. The sampling time is set to be $t_s = 0.002$ s. The parameter settings are $N = 5000$, $s_p = 30$ and $s_f = 8$, respectively.

Assume that there exists a deterministic disturbance in the closed-loop system with five distinct frequencies as follows:

$$d(kt_s) = \sum_{j}^{5} a_{1,j} \sin(\omega_{1,j} k t_s) \tag{56}$$

where the amplitude $a_{1,j}$ and the frequency $\omega_{1,j}$ can be referred to Table I for simulation settings.

TABLE I
PARAMETERS OF DETERMINISTIC DISTURBANCE

Amplitude	Value	Frequency	Value
$a_{1,1}$	0.6	$\omega_{1,1}$	3 (rad/s)
$a_{1,2}$	0.8	$\omega_{1,2}$	2 (rad/s)
$a_{1,3}$	0.6	$\omega_{1,3}$	5 (rad/s)
$a_{1,4}$	0.8	$\omega_{1,4}$	7 (rad/s)
$a_{1,5}$	0.6	$\omega_{1,5}$	13 (rad/s)

To verify the effectiveness of the proposed algorithm for closed-loop identification with unknown deterministic disturbances, Fig. 1 compares the method in [15] while ignoring the deterministic disturbance (i.e., CCF-N4SID), the proposed algorithm using $D_{b,tri}$ (i.e., CCF-N4SID-D1) and the proposed algorithm using $D_{b,\text{bern}}$ (i.e., CCF-N4SID-D2) with $n_b = 8$ and $n_b = 9$.

From Fig. 1, it can be observed that CCF-N4SID delivers the wrong pole estimation results due to the fact that it is directly implemented while ignoring the influence of the unknown deterministic disturbances, which implies that the dynamics of the identified model cannot be consistent with the real one. In contrast, the proposed algorithm using $D_{b,tri}$ or $D_{b,\text{bern}}$ can both obtain a relatively reliable pole estimation. It should be mentioned that the pole estimation results are less accurate with obviously biased estimation for poles located on the left half of the unit circle when $n_b = 8$. However, the pole estimation performance improves a lot when $n_b = 9$, which is competitive to the pole estimation results via the proposed algorithm using $D_{b,tri}$. Note that the discussion above in fact indicates the importance of the proper choice of the Bernstein polynomial order.

To determine the proper Bernstein polynomial order, AIC and BIC are respectively tested as shown in Fig. 2, from which it can be observed that the AIC curve and BIC curve both reach the minimum when $n_b = 9$ with the range for the integer $n_b \in [1, 12]$. In addition, the two curves begin to climb up with the increase of n_b due to the possible overfitting and the BIC curve shows more obvious increasing trend when $n_b > 9$ due to the larger penalty coefficient of model parameter numbers. The above analysis implies that $n_b = 9$ can be indeed a proper choice for the construction of $D_{b,\text{bern}}$.

In addition, Fig. 3 compares the estimation error of Markov parameters CA^iB under different Bernstein polynomial orders n_b. The evaluation indicator for the estimation error of CA^iB is

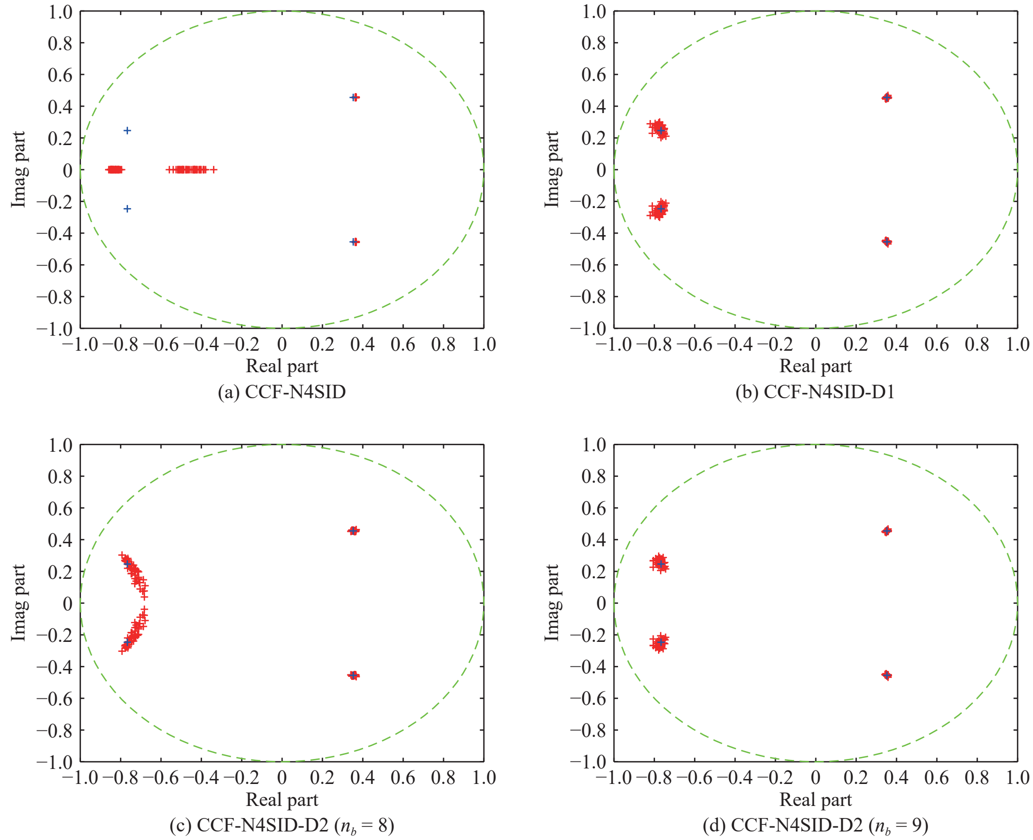

Fig. 1. Comparison of pole estimation with deterministic disturbance ($s_p = 30,\ s_f = 8$).

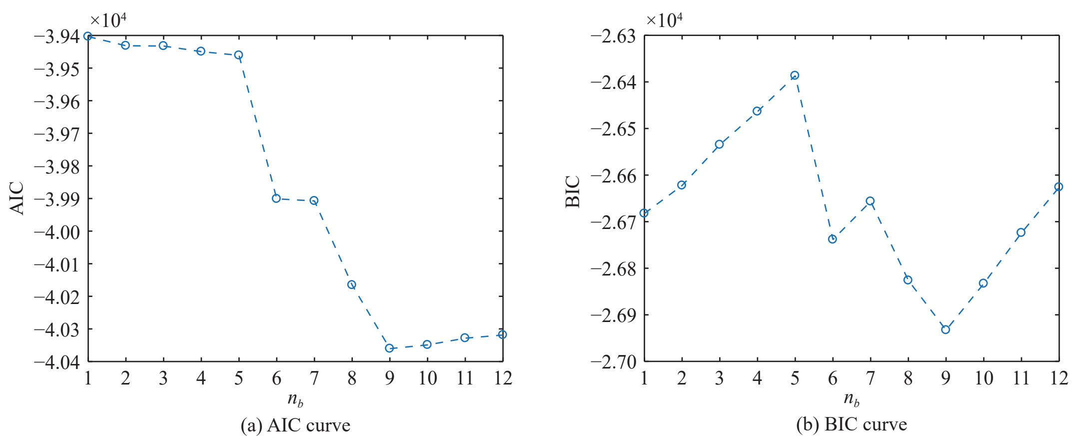

Fig. 2. The determination of the Bernstein polynomial order.

given as follows:

$$E_i = 20\log\left(\frac{\| CA^iB - \hat{C}\hat{A}^i\hat{B} \|_F}{\| CA^iB \|_F}\right)$$

where $\|\cdot\|_F$ denotes the Frobenius norm. Note that the logarithm function is used to magnify the estimation error for better visualization.

From Fig. 3, it can be observed that the estimation error of CA^iB gradually decreases with the increase of n_b until it reaches the minimum when $n_b = 9$. Note that the estimation error shows a slight increase when $n_b = 10$ compared with the result when $n_b = 9$, which again verifies the reasonability of the choice of the Bernstein polynomial order using AIC or BIC.

Fig. 4 shows the structure of the identified $H_{u,f}$ when $n_b = 9$. In Fig. 4, the light color means that the entries in the identified $H_{u,f}$ are close to 0. Therefore, it can be observed that the identified $H_{u,f}$ holds the lower triangular Topelitz structure well though no extra structural constraints are applied, which verifies the effectiveness of the proposed algorithm.

Table II shows the comparison of identification performance in case of different disturbance types including constant, signal in (56), ramp and chirp signals, which involves the periodic and aperiodic signals during the test. The disturbance types in the above four cases are described as

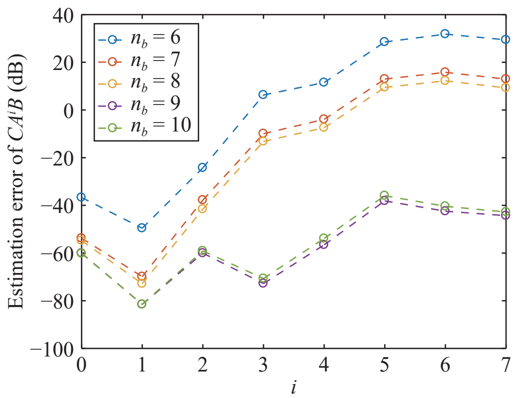

Fig. 3. Comparison of the estimation error of CA^iB under different n_b.

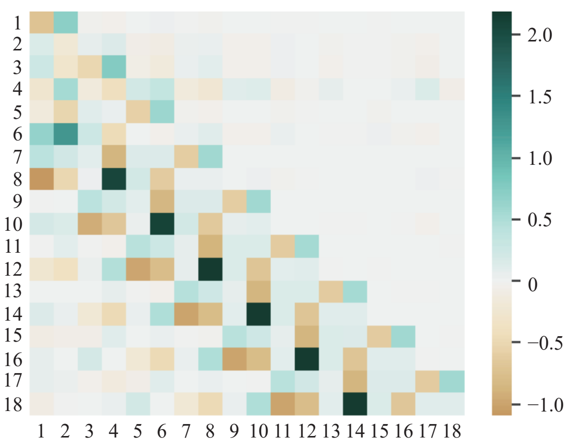

Fig. 4. The structural properties of the identified $H_{u,f}$ ($n_b = 9$).

Case 1 (Periodic): The deterministic disturbance is a constant signal with

$$d(kt_s) = 1.2.$$

Case 2 (Periodic): The deterministic disturbance is the superposition of five distinct frequencies as in (56), i.e.,

$$d(kt_s) = \sum_{j}^{5} a_{1,j} \sin(\omega_{1,j} k t_s).$$

Case 3 (Aperiodic): The deterministic disturbance is a ramp signal with the slope $0.2/s$, i.e.,

$$d(kt_s) = 0.2kt_s.$$

Case 4 (Aperiodic): The deterministic disturbance is a chirp signal varying from 0.1 Hz to 20 Hz in 100 s with amplitude 1, i.e.,

$$d(kt_s) = \cos(2\pi f(kt_s)kt_s)$$

where $f(kt_s) = 0.1 + 0.199kt_s$.

To quantify the accuracy of estimated poles, the indicator $Err_{\text{pole},i}$ is defined as the error between the true poles and the average of estimated poles,

$$Err_{\text{pole},i} = \left|\lambda_i - \overline{\lambda}_i\right|$$

where λ_i and $\overline{\lambda}_i$ denote the ith true pole and the average of ith estimated poles, respectively.

In addition, the determined Bernstein polynomial order via AIC and BIC is shown in Table II. It should be noted that the constant signal can be described by a row vector with all elements being 1, i.e., the polynomial order is zero, and the ramp signal is linear that can be described by the first-order polynomial.

From Table II, it can be observed that CCF-N4SID delivers the wrong estimation of the poles and Markov parameter E_7 with quite large errors for all four disturbance types. However, CCF-N4SID-1 and CCF-N4SID-2 can deliver the correct identification performance with a small estimation error of the poles and E_7, which demonstrates the effectiveness of the proposed identification methods for both periodic and aperiodic deterministic disturbances.

VI. Conclusion

In this paper, a subspace identification method is proposed for closed-loop systems with unknown deterministic disturbances in order to improve the estimation performance. To overcome the influence of the unknown deterministic disturbances, the row space that can be used to approximately represent the unknown deterministic disturbances is constructed using the trigonometric functions or Bernstein polynomials depending on whether the disturbance frequencies are known, which can be used to address the aperiodic deterministic disturbances. CCF-N4SID is then extended to the subspace identification with unknown deterministic disturbances using the oblique projection under feedback control. Moreover, the Bernstein polynomial order can be properly determined using AIC or BIC. The numerical example demonstrates that the proposed method can effectively alleviate the influence of the deterministic disturbances with reliable identification results.

Appendix
Proof of Theorem 1

The LQ factorization in (47) can be interpreted by decomposing Y_f into the following four parts [25]:

$$Y_f = Y_f \Bigg| \begin{bmatrix} Z_p \\ D_b \\ M_f \end{bmatrix} + Y_f \Bigg| \begin{bmatrix} M_f \\ D_b \\ Z_p \end{bmatrix} + Y_f \Bigg| \begin{bmatrix} D_b \\ M_f \\ Z_p \end{bmatrix} + L_{c,44} Q_{c,4} \tag{57}$$

where $Y_f \Big| \begin{bmatrix} Z_p \\ D_b \\ M_f \end{bmatrix}$, $Y_f \Big| \begin{bmatrix} M_f \\ D_b \\ Z_p \end{bmatrix}$ and $Y_f \Big| \begin{bmatrix} D_b \\ M_f \\ Z_p \end{bmatrix}$ denote the projection of Y_f onto $Z_{c,p}$ along $\begin{bmatrix} D_b \\ M_f \end{bmatrix}$, the projection of Y_f onto M_f along $\begin{bmatrix} D_b \\ Z_p \end{bmatrix}$ and the projection of Y_f onto D_b along $\begin{bmatrix} M_f \\ Z_p \end{bmatrix}$, respectively.

By unfolding the LQ factorization, we have

$$Y_f \Bigg| \begin{bmatrix} Z_p \\ D_b \\ M_f \end{bmatrix} = L_{c,z_p} Z_p, \quad Y_f \Bigg| \begin{bmatrix} M_f \\ D_b \\ Z_p \end{bmatrix} = (L_{c,43} L_{c,33}^{-1}) M_f \tag{58}$$

TABLE II
COMPARISON OF IDENTIFICATION PERFORMANCE IN CASE OF DIFFERENT DISTURBANCE TYPES

Disturbance type	Method	$Err_{\text{pole},1}$	$Err_{\text{pole},2}$	$Err_{\text{pole},3}$	$Err_{\text{pole},4}$	Order via AIC	Order via BIC	E_7
Constant	CCF-N4SID-2	0.0007	0.0007	0.0037	0.0037	0	0	−49.90 dB
Constant	CCF-N4SID	1.3233	0.7241	1.0579	1.3545	–	–	26.65 dB
Signal in (56)	CCF-N4SID-1	0.0028	0.0028	0.0032	0.0032	–	–	−47.63 dB
Signal in (56)	CCF-N4SID-2	0.0025	0.0025	0.0010	0.0010	9	9	−47.59 dB
Signal in (56)	CCF-N4SID	1.3200	0.7306	1.0460	1.2916	–	–	26.85 dB
Ramp	CCF-N4SID-2	0.0532	0.0536	0.0840	0.0814	1	1	−49.22d B
Ramp	CCF-N4SID	1.1035	0.6481	1.0567	1.0567	–	–	26.28 dB
Chirp	CCF-N4SID-2	0.0377	0.0268	0.0518	0.0573	3	3	−39.68 dB
Chirp	CCF-N4SID	0.9957	0.6350	1.0352	1.0108	–	–	27.44 dB

Notes: $Err_{\text{pole},i}$ denotes the error between the true poles and the average of estimated poles, $i = 1,2,3,4$; E_7 is the estimation error of CA^7B.

$$Y_f \left| \begin{array}{c} D_b \\ \left[\begin{array}{c} M_f \\ Z_p \end{array}\right] \end{array} \right. = (L_{c,42}L_{c,22}^{-1} - L_{c,43}L_{c,33}^{-1}L_{c,32}L_{c,22}^{-1})D_b \tag{59}$$

where $L_{c,z_p} = (L_{c,41} - L_{c,42}L_{c,22}^{-1}L_{c,21} - L_{c,43}L_{c,33}^{-1}L_{c,31} + L_{c,43}\times L_{c,33}^{-1}L_{c,32}L_{c,22}^{-1}L_{c,21})L_{c,11}^{-1}$.

Substitute (44) into (42), it leads to

$$T_f^c Y_f = \Xi_{z,p}Z_p + H_{u,f}K_{v,f}^{c-1}M_f + \tilde{H}_{d,f}D_b + \Phi_f \tag{60}$$

where

$$T_f^c = I + H_{u,f}K_{v,f}^{c-1}K_{u,f}^c. \tag{61}$$

In addition, (60) can be rewritten as

$$Y_f = T_f^{c-1}\Xi_{z,p}Z_p + T_f^{c-1}H_{u,f}K_{v,f}^{c-1}M_f + T_f^{c-1}\tilde{H}_{d,f}D_b + T_f^{c-1}\Phi_f. \tag{62}$$

Recall that

$$Y_f = L_{c,z_p}Z_p + L_{c,43}L_{c,33}^{-1}M_f + (L_{c,42}L_{c,22}^{-1} - L_{c,43}L_{c,33}^{-1}L_{c,32}L_{c,22}^{-1})D_b + L_{c,44}Q_{c,4}. \tag{63}$$

Comparing (62) with (63), we have

$$H_{u,f}K_{v,f}^{c-1} = T_f^c L_{c,43}L_{c,33}^{-1}. \tag{64}$$

Substituting (64) into (61), it leads to

$$T_f^c = (I - L_{c,43}L_{c,33}^{-1}K_{u,f}^c)^{-1}. \tag{65}$$

Therefore, when $s_p \to \infty$, we have

$$\Gamma_f \check{X}_{k,N} = \lim_{s_p\to\infty}(I - L_{c,43}L_{c,33}^{-1}K_{u,f}^c)^{-1}L_{c,z_p}Z_p \tag{66}$$

and

$$H_{u,f} = T_f^c L_{c,43}L_{c,33}^{-1}K_{v,f}^c. \tag{67}$$

■

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