

Motion planning for the fast opening of the protection cover based on high order polynomial interpolation

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Abstract-- A high order polynomial interpolation method is proposed for the motion planning of the fast opening of the protection cover, which is based on its characteristics to ensure the steady ability. Fifth-order polynomial is chosen to complete the motion planning, and subsection search method is employed to set its acceleration. Optimal planning path is achieved after several subsection searching, and MATLAB simulation confirms the validity and efficiency of the proposed method.

I. INTRODUCTION

THE robot motion planning is the basic problem in the robot control, which is faced with many important challenges in theory and application [1]. Motion planning includes path planning and motion control. Path planning is to find an optimal trajectory of the path from the start to the end in the robot motion space. The moving speed, rotating angle is subject to the limits of the control signal, that is, to meet the dynamics constraint [2]. The basic problem related to the path planning is the universal model expression and path search strategy. Universal model expression methods include: visibility graph, free space method and grid method. Optimal path search problem is then converted to finding the shortest way from a starting point to the target point via the visible lines. By far, most robots employ trapezoidal speed curves for its motion planning [3]. This method has two shortcomings:

--First, when the acceleration or the velocity is fixed, the acceleration and the velocity must be set to a low value to ensure that they don't exceed the limitation during the whole process. This makes it impossible to optimize acceleration and velocity at other points in the trajectory.

--Second, system oscillation caused by sudden change of acceleration (at velocity transition point) confines the limitation of the acceleration. When the robot moves according to a control method based on its dynamic model, sudden change of the acceleration will certainly cause the system oscillation.

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Robot dynamic control is raised to solve the problems on how to realize large range, high speed and high accuracy trajectory tracking [4]. Robot dynamic control has two main purposes [5]:

--First, how to keep the system stable. Advanced control strategies should be studied and applied to robot control, so that the tracking error can diminish to zero as quickly as possible.

--Second, how to suppress disturbance, how to diminish the influence of the disturbance to the accuracy of the tracking.

If the precise dynamic model of the robot can be achieved, and the disturbance signal can be detected, then controller designed with linear control theory can realize the two purposes. But it is almost impossible to build up the precise and complete dynamic model of the robot, because of the error in measurement and modeling, the changing of the load and disturbance from the environment [6]. So when we build the robot dynamic model, some reasonable approximations need to be made and some unimportant uncertainties should be ignored.

The main purpose of robot motion planning is to find a reasonable polynomial function or other linear function to conduct the interpolation, so that the joint motion can be smooth, stable and within the allowed range. The basic index indicating the quality of the planned motion is the time consumed from the beginning of the motion to the end [7].

In the process of opening the protection cover, we need to open it as fast as possible. Moreover, the vibration of the fast opening process of the protection cover must be smaller than 5um. In this paper, two fifth-order polynomial functions and a first-order polynomial function are used to plan the motion trajectory of the protection cover. In addition, a simple method is proposed to minimize the maximum acceleration value in the acceleration phase and deceleration phase.

II. MOTION PLANNING PRINCIPLE

The main task of the robot motion trajectory planning is to select reasonable polynomial function or other linear function to accomplish interpolation operation, in order to make joint movement smooth, steady ability, and keep joint movement within allowed range [8]. In the process of the robot movement, the joint angle θ_0 at the beginning is known, and the joint angle θ_f at the end can be achieved using the inverse kinematics [9, 10]. Thus, the description of the motion trajectory can be represented by a smooth interpolation function of joint angle $\theta(t)$ from the starting point to the end point. At the time t_0 , $\theta(t)$ is the starting joint angle θ_0 . At the time t_f , $\theta(t)$ is the end joint angle θ_f . In order to realize the

smooth movement of individual joint, trajectory $\theta(t)$ at least needs to meet four constraint conditions [11]:

$$\begin{cases} \theta(0) = \theta_0, \theta(t_f) = \theta_f \\ \theta'(0) = 0, \theta'(t_f) = 0 \end{cases} \quad (1)$$

The above four conditions can confine a unique cubic polynomial

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad (2)$$

The first derivative of the function is the speed of the joint

$$\theta'(t) = a_1 + 2a_2 t + 3a_3 t^2 \quad (3)$$

Using the constraints in (1), the following equations can be obtained from (2) and (3).

$$\begin{aligned} \theta(0) &= a_0 = \theta_0 \\ \theta(t_f) &= a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 = \theta_f \\ \theta'(0) &= a_1 = 0 \\ \theta'(t_f) &= a_1 + 2a_2 t_f + 3a_3 t_f^2 = 0 \end{aligned} \quad (4)$$

Transfer (4) into matrix form, the following equation can be obtained.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2t_f & 3t_f^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \theta_0 \\ \theta_f \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

The following result can be achieved by calculating (5).

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \theta_0 \\ 0 \\ \frac{3}{t_f^2}(\theta_f - \theta_0) \\ -\frac{2}{t_f^3}(\theta_f - \theta_0) \end{pmatrix} \quad (6)$$

A unique cubic order polynomial equation can be determined by (6). Therefore, if the starting angle, starting speed, end angle and end speed are known, a complete motion trajectory can be determined using cubic polynomial interpolation method. When the system's acceleration has limitations, a fifth-order polynomial interpolation method will be needed to plan the motion trajectory of the system. Equation is expressed in the following form.

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \quad (7)$$

First order derivative of (7) is the speed of the motion.

$$\theta'(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 \quad (8)$$

Second order derivative of (7) is the acceleration of the motion.

$$\theta''(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 \quad (9)$$

According to the fast opening process of the protection cover characteristic of small vibration and fast opening, we can divide the whole motion trajectory into three phases: accelerating phase, uniform phase and decelerating phase. The acceleration is very important in the accelerating phase and decelerating phase, so accelerating phase and

decelerating phase use fifth-order polynomial interpolation to plan. Uniform phase uses one order polynomial interpolation to plan. The accelerating phase uses the following equation to plan.

$$\theta(t) = a_{00} + a_{01}t + a_{02}t^2 + a_{03}t^3 + a_{04}t^4 + a_{05}t^5 \quad (10)$$

The uniform phase uses the following equation to plan.

$$\theta(t) = a_{10} + a_{11}t \quad (11)$$

The decelerating phase uses the following equation to plan.

$$\theta(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3 + a_{24}t^4 + a_{25}t^5 \quad (12)$$

Four moments are critical for the whole motion trajectory planning, which are the starting time of the accelerating phase t_0 , the end time of the accelerating phase t_1 , the starting time of the decelerating phase t_2 , and the end time of the decelerating phase t_3 , with the joint angle θ_0 , θ_1 , θ_2 and θ_3 accordingly. In order to minimize the maximum acceleration value both in the accelerating phase and the decelerating phase, t_1 , θ_1 , t_2 , θ_2 must be optimized. Here, we use subsection searching extreme value point method to look for the optimum t_1 , θ_1 , t_2 , θ_2 .

III. MAXIMUM ACCELERATION OPTIMIZATION

In general, we plan the trajectory through the path point. If robot stayed at the path point for a while, polynomial interpolation method can be directly used, that is to say, the whole path is composed of a number of trajectory segments, which initial velocity and the termination velocity is zero. If it does not stop at the path point, inverse kinematics solution can be used to determine polynomial interpolation function, connect the path point smoothly. Fifth-order polynomial interpolation method is used to plan the trajectory of the fast opening process of the protection cover in the accelerating phase. Its starting angle $\theta(0)$, starting velocity $\theta'(0)$ and starting acceleration $\theta''(0)$ are:

$$\theta(0) = \theta_0 = 0, \theta'(0) = \theta'_0 = 0, \theta''(0) = \theta''_0 = 0 \quad (13)$$

At the end of the acceleration phase t_1 , the displacement $\theta(t_1)$, velocity $\theta'(t_1)$ and acceleration $\theta''(t_1)$ are:

$$\theta(t_1) = \theta_1, \theta'(t_1) = \theta'_1 = a_{11}, \theta''(t_1) = \theta''_1 = 0 \quad (14)$$

Substitute (13) and (14) into (10), the following equation can be obtained:

$$\begin{cases} \theta(0) = a_{00} = 0 \\ \theta'(0) = a_{01} = 0 \\ \theta''(0) = 2a_{02} = 0 \\ \theta(t_1) = a_{00} + a_{01}t_1 + a_{02}t_1^2 + a_{03}t_1^3 + a_{04}t_1^4 + a_{05}t_1^5 = \theta_1 \\ \theta'(t_1) = a_{01} + 2a_{02}t_1 + 3a_{03}t_1^2 + 4a_{04}t_1^3 + 5a_{05}t_1^4 = a_{11} \\ \theta''(t_1) = 2a_{02} + 6a_{03}t_1 + 12a_{04}t_1^2 + 20a_{05}t_1^3 = 0 \end{cases} \quad (15)$$

In (15), t_1 is known, a_{02} , a_{03} , a_{04} , a_{05} are unknown. Equation (15) can be rewritten as:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & t_1 & t_1^2 & t_1^3 & t_1^4 & t_1^5 \\ 0 & 1 & 2t_1 & 3t_1^2 & 4t_1^3 & 5t_1^4 \\ 0 & 0 & 2 & 6t_1 & 12t_1^2 & 20t_1^3 \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{03} \\ a_{04} \\ a_{05} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_1 \\ a_{11} \\ 0 \end{pmatrix} \quad (16)$$

a_{04}, a_{05} . Subsection searching method is proposed based on branch and bound method of optimization theory. Branch and bound method needs two types of operations: The first is branch, which divides the solutions into several non-intersect solution sets, according to certain rules [12]. The second is bound, which selects an appropriate algorithm to calculate the bound of the subsection will be conducted again and again, thus, the solution set will become smaller and smaller, and at last, an accurate solution will be achieved.

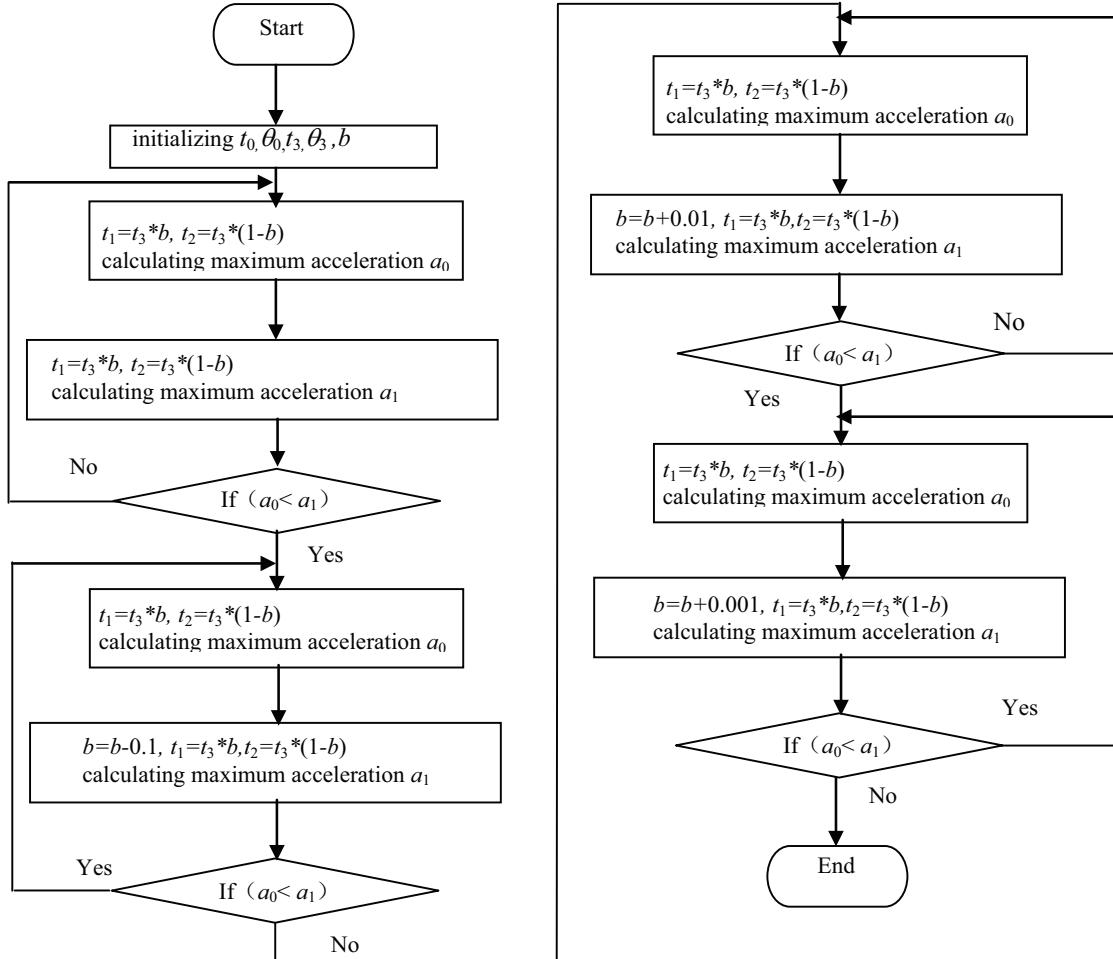


Fig. 1. Subsection searching method optimizing acceleration

In (16) a_{11} is the velocity of the uniform phase, which can be achieved by substituting t_1 , θ_1 , t_2 and θ_2 into equation (11).

$$\begin{cases} \theta(t_1) = a_{10} + a_{11}t_1 = \theta_1 \\ \theta(t_2) = a_{10} + a_{11}t_2 = \theta_2 \end{cases} \quad (17)$$

The acceleration of the accelerating phase is

$$\theta''(t) = 2a_{02} + 6a_{03}t + 12a_{04}t^2 + 20a_{05}t^3 \quad (18)$$

In order to minimize the maximum acceleration value of the nonlinear high order polynomial of the accelerating phase, we use subsection searching method to find the best a_{02} , a_{03} ,

Figure 1 is the process of optimizing t_1 using subsection searching method. The proposed subsection searching method in this paper uses a random point in a solution set as the starting searching point, then, the subsection begins. In the figure, a_0 is last calculating maximum acceleration value based given subset and target function, a_1 is this time calculating maximum acceleration value. Subsection direction depends on a_0 and a_1 value. Search speed depends on b . In order to improve the calculating speed, b can select to increase or decrease. The step can select 0.1, 0.01 or 0.001. After repeated calculation and comparison many times, t_1 can

be obtained. Applying the same method to θ_1 , θ_1 can be obtained, then, t_2 , θ_2 can be obtained. Take t_1 , θ_1 , t_2 , θ_2 into (16) and (17), so a_{00} , a_{01} , a_{02} , a_{03} , a_{04} , a_{05} can be obtained.

IV. SYSTEM MODELING AND CONTROL

A. Modeling

Servo motor is used to control the motion of the fast opening system of the protection cover. Servo motor's job is to transfer the input electric power into the switching system's mechanical energy [13]. The mechanical structure of the protection cover is shown in figure 2.

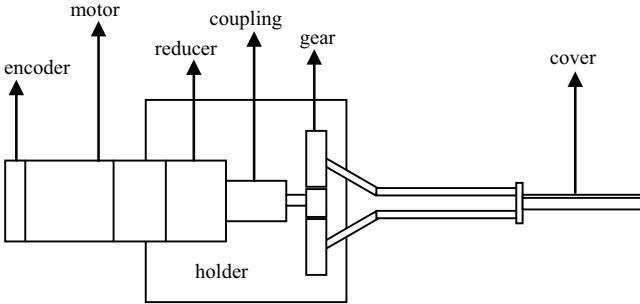


Fig. 2. The mechanical structure of the protection cover

The rotor voltage $u(t)$ induces the rotor current $i(t)$ in the rotor circuit. Then the rotor current and stator magnetic flux interact to produce electromagnetic torque $M(t)$. Its equation is

$$u(t) = L \frac{di(t)}{dt} + Ri(t) + E \quad (19)$$

Where E is counter electromotive force (EMF), $E = C_e \omega(t)$, C_e is EMF constant.

Electromagnetic torque equation is

$$M(t) = C_m i(t) \quad (20)$$

Where C_m is Servo motor torque coefficient, $M(t)$ is the electromagnetic torque produced by servo motor.

Servo motor torque balance equation is

$$J_m \frac{d\omega(t)}{dt} + f_m \omega(t) = M(t) - M_c(t) \quad (21)$$

Where f_m is motor shaft sticky friction coefficient, J_m is motor shaft rotary inertia, $M_c(t)$ is total load torque. Remove the middle variable of (19), (20) and (21), the following motor differential equation can be obtained.

$$\begin{aligned} LJ_m \frac{d^2\omega(t)}{dt^2} + (Lf_m + RJ_m) \frac{d\omega(t)}{dt} + \\ (Rf_m + C_m C_e) \omega(t) = C_m u(t) - L \frac{dM_c(t)}{dt} - RM_c(t) \end{aligned} \quad (22)$$

The inductance L is very small, which can be ignored, so (22) is simplified as

$$T_m \frac{d\omega(t)}{dt} + \omega(t) = K_1 u(t) - K_2 M_c(t) \quad (23)$$

Where

$$T_m = \frac{RJ_m}{Rf_m + C_m C_e}, K_1 = \frac{C_m}{Rf_m + C_m C_e}, K_2 = \frac{R}{Rf_m + C_m C_e}.$$

Let $M_c(t)=0$, (23) becomes (24)

$$T_m \frac{d\omega(t)}{dt} + \omega(t) = K_1 u(t) \quad (24)$$

Applying Laplace transform to (24), the equation (25) can be obtained.

$$G(s) = \frac{\Omega_m(s)}{U(s)} = \frac{K_1}{T_m s + 1} \quad (25)$$

Furthermore, we can obtain the transfer function from voltage $u(t)$ to angular displacement θ :

$$G(s) = \frac{\Theta_m(s)}{U(s)} = \frac{K_1}{s(T_m s + 1)} \quad (26)$$

B. PID Control

The PID controller is composed of proportional, integral and differential, it is a linear controller. It has the advantages of simple structure, convenient application etc. For a simple object, the application of PID control algorithm can achieve good control effect. And the control system has good robustness. In the field of industrial control, The PID control algorithm is still one of the most important control algorithms. It plays a vital role in industrial production. As far as practice is concerned, PID has dominated the scene for over 80 years, during which, few fundamental innovations were produced. While much progress has been made in model based mathematical control science, it has yet to make a significant impact in industry, whose pressures force us to seek alternatives in any place [14]. PID controller is as follows [15, 16]:

$$u(t) = K_p [e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt}] \quad (27)$$

Where $e(t)=r(t)-c(t)$, $r(t)$ is the input, $c(t)$ is the output.

Furthermore, (27) can be rewritten as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (28)$$

$$\text{Where } K_i = \frac{K_p}{T_i}, K_d = K_p T_d.$$

The discrete equation of (28) is as follows:

$$\begin{aligned} u(k) = u(k-1) + K_p [e(k) - e(k-1)] + \\ K_i e(k) + K_d [e(k) - 2e(k-1) + e(k-2)] \end{aligned} \quad (29)$$

V. SIMULATION

Applying the proposed method simulate the fast opening process of the protection cover. Firstly, the motion trajectory is divided into three segments; they are the acceleration phase, uniform phase and deceleration phase. The parameters are set as follows: $t_3=0.2s$, $\theta_3=120$ degree and $b=0.21$. Then, we begin to seek the extreme value point using the multi-segments searching method. After this point is found, two fifth-order polynomials and a first-order polynomial are

used to plan system's trajectory. At last, we obtain the motion trajectory with the minimum acceleration when $b=0.302$.

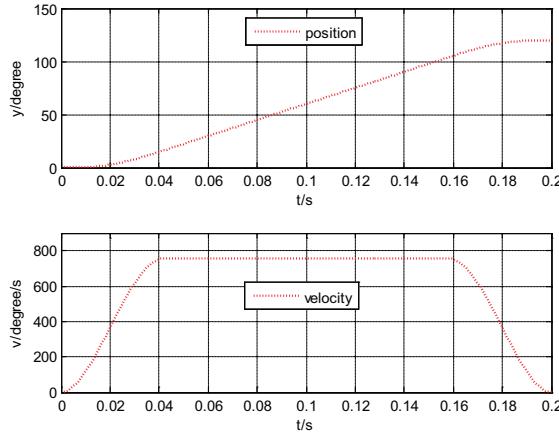


Fig. 3. Position and velocity curve before optimization ($b=0.21$)

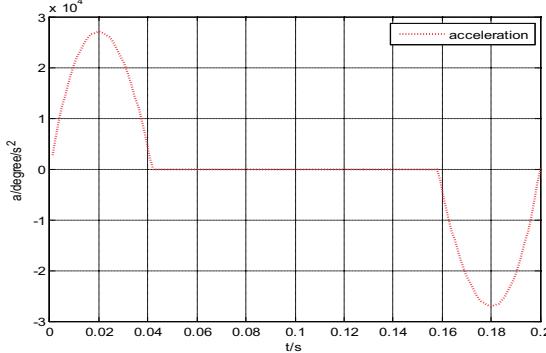


Fig. 4. Acceleration curve before optimization ($b=0.21$)

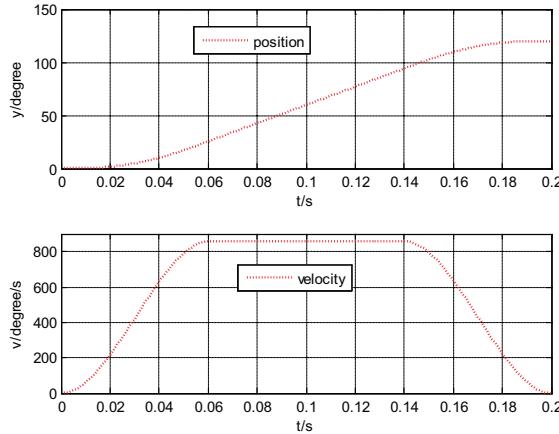


Fig. 5. Position and velocity curve after optimization ($b=0.302$)

Fig. 3-Fig. 6 show the simulation results of the motion planning using MATLAB. As shown these figures, all the curves of position, velocity and acceleration have no singular point. The changes of the acceleration can be seen from Fig. 4 and Fig. 6. Maximum acceleration is 2.8×10^4 degree/s² before optimization, and then it becomes 2.1×10^4 degree/s² after

optimization. Maximum acceleration reduces 33%. Thus, this planning path is more reasonable.

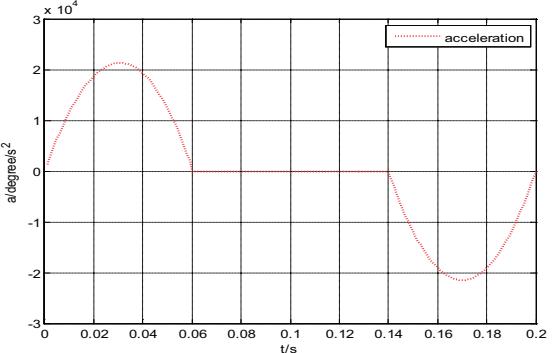


Fig. 6. Acceleration curve after optimization ($b=0.302$)

TABLE I.
THE MECHANICAL PARAMETERS OF THE MOTOR

Rated torque	Torque constant	Rotation inertia	Rated acceleration	Damping coefficient
1.27 N·m	0.512 N·m/A	35.442×10^{-4} kgm ²	28800 rad/s ²	7.403×10^{-5} Nms/rad

The mechanical parameters of the protection cover are shown in table I. According to these parameters, we can obtain: $K_1=1.70$, $T_m=0.0184$. The transfer function is

$$G(s) = \frac{1.7}{0.0184s + 1} = \frac{92.4}{s + 54.4} \quad (30)$$

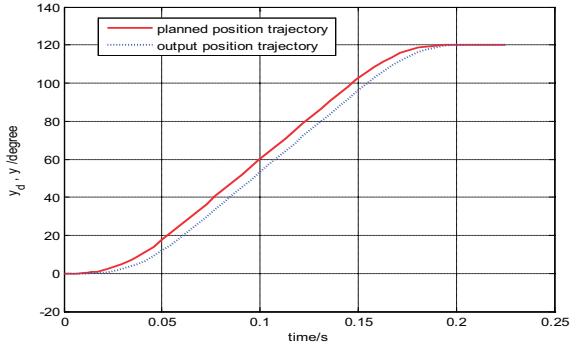


Fig. 7. Position curve of the protection cover

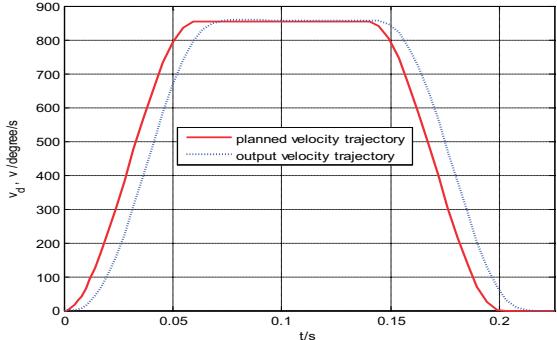


Fig. 8. Velocity curve of the protection cover

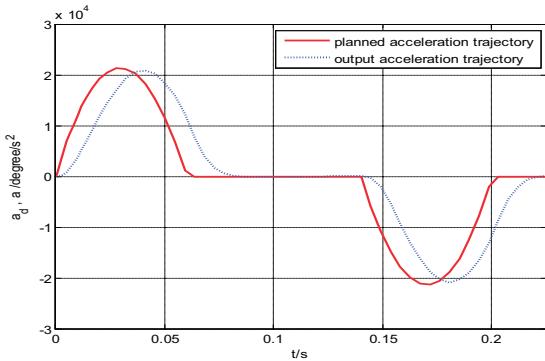


Fig. 9. Acceleration curve of the protection cover

The movement curves of the protection cover are shown in Fig. 7, where the PID controller was used. In Fig. 7, y_d is planned position curve, y is output position curve. In Fig. 8, v_d is planned velocity curve, v is output velocity curve. In Fig. 9, a_d is planned acceleration curve, a is output acceleration curve. From the above curves, we can see that the motion of the protection cover agrees more similar with the planned trajectory. Position curve, velocity curve and acceleration curve are all very smooth, and acceleration is very small.

VI. CONCLUSION

The protection cover should open as fast as possible with small vibration. This article proposes the fifth-order polynomial to plan the motion trajectory for the fast opening process of the protection cover. In order to minimize the maximum acceleration value during the motion, subsection searching method is designed to select the fifth-order polynomial coefficient. The optimized fifth-order polynomial is simulated in MATLAB. The simulation result shows that the motion trajectory, velocity and acceleration are smooth in the whole process. And acceleration is small, which satisfies the design requirements. Simulation shows that the proposed subsection searching method for planning motion trajectory for the fast opening system of the protection cover is very effective.

REFERENCES

- [1] R. C. Arkin "Behavior-based robotics".1st ed. Cambridge USA:MIT Press, 1998.
- [2] L. Liu, P. Xiang, Y. J. Wang, et al. "Trajectory tracking of a nonholonomic wheeled mobile robot". *Journal of Tsinghua University (Science and Technology)*, 2007, 47(S2): 1884-1889.
- [3] Y. Li, B .Tang, Z. Shi, Y. Lu. "Experimental study for trajectory tracking of a two link flexible manipulator". *International Journal of System Science*, 2000, 31(1): 3-9.
- [4] W. Sun, Y. N. Wang. "AN CMAC network model and its application to robotic control". *Journal of dynamics and control* , 2005, 3(3): 56—62.
- [5] Y. C. Li, Y. F. Lu, B. J. Tang. "Robust control for trajectory tracking of a two-link flexible manupulator". *Acta Automatica Sinica*, 1999, 25(3): 330-336.
- [6] X. F. Dai, L. N. Sun, H. G. Cai. "Fuzzy sliding mode control for positioning flexible link manipulators". *Journal of Harbin Institute of Technology*, 2005. 37(2): 148—150.
- [7] Y. H. Zhou, W. Z. Tang, J. X. Zhang. "Arithmetic for multi-joint redundant robot inverse kinematics based on the Bayesian-BP neural network". *Proceedings of the 2008 IEEE International Conference on Intelligent Computation Technology and Automation*. 2008: 173-178.
- [8] M. Reyhanoglu, A. Schaft, N. H. Clamroch, I. Kolmanovsky. "Dynamics and control of a class of underactuated mechanical systems". *IEEE Transactions on Automatic Control*, 1999, 44 (9) : 1663-1671.
- [9] L. Moreno, E. Dapena. "Path quality measures for sensor-based motion planning". *Robotics and Autonomous System*, 2003, 44:131 – 150.
- [10] E. Papadopoulos, I. Tortopidis, K. Nanos. "Smooth planning for free-floating space robots using polynomials". *The IEEE International Conference on Robotics and Automation*, Barcelona, Spain, April 18-22, 2005: 4272 – 4277.
- [11] M. Tan, D. Xu, Z. G. Hou, S. Wang, Z. Q. Cao. "Advanced robot control". Beijing: *Higher Education Press*, 2007.
- [12] Z. J. Diao, H. D. Zheng, J. Z. Liu, G.Z. Liu. "Operations research". Beijing: *Higher Education Press*, 2000.
- [13] S. S. Hu. "Automatic control theory". Beijing: *Science Press*, 2007.
- [14] J. Q. Han. "From PID to active disturbance rejection control". *IEEE Transactions on Industrial Electronics*, 2009, 56(3): 900-906.
- [15] J. K. Liu. "Advanced PID control based on MATLAB". Beijing: *Publishing House of Electronics Industry*, 2011.
- [16] S. S. Ge, T. H. Lee, Z. P. Wang. "Adaptive robust controller design for multi-link flexible robots". *Proceedings of the American Control Conference*, Arlington, 2001: 947-952.