

Run-Length and Edge Statistics Based Approach for Image Splicing Detection

Jing Dong¹, Wei Wang¹, Tieniu Tan¹, and Yun Q. Shi²

¹ National Laboratory of Pattern Recognition, Institute of Automation,
Chinese Academy of Sciences, P.O.Box 2728, Beijing

² Dept. of Electrical and Computer Engineering
New Jersey Institute of Technology, Newark NJ, USA
{jdong, wwang, tnt}@nlpr.ia.ac.cn, shi@nuit.edu

Abstract. In this paper, a simple but efficient approach for blind image splicing detection is proposed. Image splicing is a common and fundamental operation used for image forgery. The detection of image splicing is a preliminary but desirable study for image forensics. Passive detection approaches of image splicing are usually regarded as pattern recognition problems based on features which are sensitive to splicing. In the proposed approach, we analyze the discontinuity of image pixel correlation and coherency caused by splicing in terms of image run-length representation and sharp image characteristics. The statistical features extracted from image run-length representation and image edge statistics are used for splicing detection. The support vector machine (SVM) is used as the classifier. Our experimental results demonstrate that the two proposed features outperform existing ones both in detection accuracy and computational complexity.

Keywords: image splicing, run-length, edge detection, characteristic functions, support vector machine (SVM).

1 Introduction

Digital images are a powerful and widely used medium in our society. For example, newspapers and magazines depend on digital images to represent the news and information every day. However, the availability of powerful digital image processing tools and editing software packages, such as PhotoShop, also makes it possible to change the information represented by an image and create forgeries without leaving noticeable visual traces. Since digital images play a crucial role and have an important impact, the authenticity of images is significant in our social and daily life. How much we can believe in seeing is becoming an intractable problem [1]. The need of authenticity assurance and detection of image forgery (tampering) makes image forensics a very important research issue.

Generally speaking, existing image forgery detection approaches are described as active [2],[3],[4] and passive (blind) [5],[6] techniques. Active approaches are usually related to digital signature and watermarking. In these approaches, certain data (such as signature or proprietary information) for multimedia digital

rights protection and content tempering authentication are embedded into images. If the content of image has been changed, the embedded information will also be changed consequently. However, either signature-based or watermark-based methods require pre-processing (e.g. watermark embedding) to generate the labeled images for distribution. Unless all digital images are required to be embedded with watermarks before presented in the Internet, it will be unlikely to detect image alteration using active approaches. In contrast to active approaches, passive approaches look into the problem of image tampering from a different angle. These approaches analyze images without requiring prior information (such as embedded watermarks or signatures) and make blind decision about whether these images are tampered or not. Passive techniques are usually based on supervised learning by extracting certain features to distinguish the original images from tampered ones. The practicality and wider applicability of passive approaches make them a hot research topic.

Image splicing [7] is a common operation for generating a digital image forgery, defined as a simple cut-and-paste of image regions from one image onto the same or another image without performing post-processing such as matting and blending in image compositing. The wide availability of image processing software makes the creation of a tampered image using splicing operation particularly easy. The artificial region introduced by image splicing may be almost imperceptible by human eyes. The detection of image splicing is a preliminary but desirable study for image forensics. In this paper we focus on this issue. We present two kinds of simple but efficient statistical features for splicing detection in terms of image run-length representation and sharp image characteristics. The analysis of the performance of the proposed features are made in our experiments. Also, we analyze the comparison of the proposed features and related features proposed in the literature as well as their combinations to evaluate their performance on splicing detection in the terms of both detection accuracy and computational complexity.

The remainder of this paper is organized as follows. Section 2 contains an introduction of related work on splicing detection. Section 3 introduces our proposed two kinds of statistical features for splicing detection. In Section 4, we carry out a number of experiments and analyze on the performance of the proposed features both in detection accuracy and computational complexity. Discussions and conclusions are presented in Section 5.

2 Related Work

Ng et al. [8],[9] detect the presence of the abrupt discontinuities in an image or the absence of the optical low-pass property as a clue for identifying spliced images. For detecting the abrupt splicing discontinuity, a higher order moment spectrum, bicoherence, is used as features. An important property of bicoherence is its sensitivity to a phenomena called quadratic phase coupling (QPC), while the splicing discontinuity leads to a variant of quadratic phase coupling which induces a $\pm\frac{1}{2}\pi$ phase. However, the detection accuracy evaluated on the

Columbia Image Splicing Detection Evaluation Dataset [10] by using only bicoherence features is reported as 62%. To improve the detection performance, they designed a functional texture decomposition method to decompose an image into a gross-structure component and a fine-texture component. With the aid of the decomposition, the detection rate improves from 62% to 71%.

In [11], *Johnson and Farid* developed a technique of image splicing detection by detecting the inconsistency in lighting in an image. It is often difficult to match the lighting conditions from the individual photographs when creating a digital composite, and large inconsistencies in lighting direction may be obvious. Lighting inconsistencies can therefore be a useful tool for revealing traces of digital tampering. At least one reason for this is that image tampering, especially the manipulation of object or people in an image may require the creation or removal of shadows and lighting gradients. The direction of the light source can be estimated for different objects or people in an image, and the presence of inconsistencies in lighting can be used as evidence of digital tampering.

Hsu and Chang take advantages of geometric invariants and camera characteristic consistency to detect spliced images in [12]. They proposed an authentic vs. spliced image classification method by making use of geometric invariants in a semi-automatic manner. For a given image, they identify suspicious splicing areas, compute the geometric invariants from the pixels within each region, and then estimate the camera response function (CRF) from these geometric invariants. If there exists CRF abnormality, the image is regarded as a spliced one. However, this method needs to label the suspicious region of the images before making decisions. It maybe unrealistic for real applications.

In a series of papers [13],[14],[15], *Shi et al.* studied the splicing detection based on statistical features of characteristic functions within a pattern recognition framework. In [13], Hilbert-Huang transform (HHT) is utilized to generate features for splicing detection due to the high non-linear and non-stationary nature of image splicing operation. The moments of characteristic functions with wavelet decomposition is then combined with HHT features to distinguish the spliced images from the authentic ones. The support vector machine (SVM) is utilized as the classifier. A detection accuracy of 80.15% is reported. In the following work of [14], phase congruency has been introduced as features for splicing detection by making use of its sensitivity to sharp transitions such as lines, edges and corners caused by splicing operation. The moments of wavelet characteristic functions form the second part of features for splicing detection in their method. A 82% detection accuracy is reported on Columbia Image Splicing Detection Evaluation Dataset.

3 Proposed Approach for Image Splicing Detection

It is meaningless to talk about the authenticity of a random pixel image, as it has no meaningful characteristics. Creation of a natural-scene forgery image by splicing often introduces abrupt changes around certain objects or regions such as lines, edges and corners. These changes may be much sharper and rougher

compared to regular lines, edges and corners due to the unskillful cut-and-paste operation. Meanwhile, splicing may also introduce inconsistency in image statistics by replacing or bringing extra image content to its original content. Hence, the features for splicing detection should capture these variations. In this section, we introduce our proposed features which consist of two parts, namely run-length based statistic moments which are extracted in terms of the global disturbance of correlation of image pixels caused by splicing, and image edge statistic moments which focus on local intensity discontinuity due to splicing.

3.1 Run-Length Based Statistic Moments

The motivation of using run-length based statistic moments for splicing detection is due to a recent study by *Shi et al.* [15]. It is reported that these approaches used in steganalysis can promisingly make progress in splicing detection applications if appropriately applied. The conclusion was demonstrated by their analytical research and extensive experiments. Steganalysis is a counter technique of image steganography [16]. Image steganography is the art and science of invisible communication, by concealing the existence of hidden messages in images. The secret messages are usually embedded in the image by means of slight alterations in image content, which could not be observed by human eyes. The goal of steganalysis is to detect these alteration, in another word, to detect if there is any secret message hidden in the image. Since image splicing definitely alters image content and brings extra information to its original version, it is reasonable to make use of effective features developed for steganalysis to splicing detection as both bring changes on image characteristics and cause some statistical artifacts. The run-length based statistic moments were first proposed in our previous work on blind image steganalysis in [17]. The run-length based features outperform the state-of-art steganalysis approaches as very effective features. Inspired by the conclusion in [15], we employ our proposed efficient run-length based statistic moments in [17] for splicing detection in this section.

The concept of run-length was proposed in the 1950s [18]. A run is defined as a string of consecutive pixels which have the same gray level intensity along a specific linear orientation θ (typically in 0° , 45° , 90° , and 135°). The length of the run is the number of repeating pixels in the run. For a given image, a run-length matrix $p(i, j)$ is defined as the number of runs with pixels of gray level i and run length j . For a run-length matrix $p_\theta(i, j)$, let M be the number of gray levels and N be the maximum run length. We can define the image run-length histogram (RLH) as a vector:

$$H_\theta(j) = \sum_{i=1}^M p_\theta(i, j). \quad 1 < j < N \quad (1)$$

This vector represents the sum distribution of the number of runs with run length j in the corresponding image. The length of the runs reflects the size of image structure and texture. For example, smooth image often contains more long runs while an image with finer details usually consists of much more short runs.

The image run-length representation reflects the information of image structure and texture. However, splicing operation, which creates a tampered image by putting several different image regions together in one image, will cause the discontinuity and incoherency on image structure and pixel correlation. It will also be reflected by a variance on their image run-length representation. Hence we take this observation as a clue for splicing detection.

The statistical moments of the characteristic function (denoted as CF) of a histogram are claimed to be very effective features to detect the slight modification of image artifacts [14],[13],[17],[19]. Here we utilize the multi-order moments of the characteristic function of image run-length histograms (in four directions) as features for splicing detection, defined as:

$$M_n = \sum_{j=1}^{L/2} f_j^n |F(f_j)| / \sum_{j=1}^{L/2} |F(f_j)|. \quad (2)$$

where F is the characteristic function of image run-length histogram H (i.e. the Discrete Fourier Transform (DFT) of H), $F(f_j)$ is the component of F at frequency f_j , and L is the DFT sequence length.

In our experiment, we extracted the first three moments of the characteristic functions of image run-length histograms in four directions as features for splicing detection. The 12-D feature vector consists of run-length based features for splicing detection. Fig.1 demonstrates the extraction of the proposed run-length based statistic moment features. The feature extraction procedure is simple and fast, which makes it suitable for splicing detection on large-scale analysis and real applications.

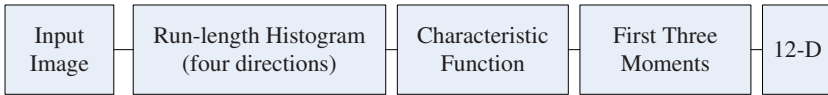


Fig. 1. Diagram of the extraction of the proposed run-length based statistic moment features for splicing detection

3.2 Edge Based Statistic Moments

In addition to the inconsistency of global pixel correlations caused by splicing, there is another change introduced by splicing operation: sharp image intensity variations. Simple copy-and-paste operations introduce extra edges, corners or blobs into images no matter they are visible or not. Such edges, corners or blobs are much sharper compared to natural ones of image content due to the blunt splicing. Hence, the detection of sharp image intensity variations may be served as significant cues for splicing. In [14], *Chen and Shi* focused on Fourier phase and made analysis on 2-D phase congruency to extract features for splicing detection. The presence of sharp edges in images dose cause variations in phase information. However, their method on 2-D phase congruency feature extraction

is time consuming. For real applications, it requires more efficient methods. Since we notice that the 2-D phase congruency is usually regarded as a method for edge detection [20], here we also make attempts on analysis of related features extracted from edge, corner and blobs detection, in a relative simple but more efficient way, for image splicing detection.

The Sobel operator [21] is a well known first-order approach to edge detection. Technically, it is a discrete differentiation operator by computing an approximation of the gradient of the image intensity function. At each point in the image, the result of the Sobel operator is either the corresponding gradient vector or the norm of this vector. The Sobel operator is based on convolving the image with a small, separable, and integer valued filter in horizontal and vertical directions and is therefore relatively inexpensive in terms of computations.

Mathematically, the operator uses two 3×3 kernels which are convolved with the original image to calculate approximations of the derivatives for horizontal and vertical changes. If we define A as the source image, and G_x and G_y are two 2-D arrays which at each point contain the horizontal and vertical derivative approximations, the computations are as follow [21]:

$$G_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} * A \quad \text{and} \quad G_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * A \quad (3)$$

where $*$ denotes the 2-dimensional convolution operation. The x-coordinate is defined as increasing in the right-direction, and the y-coordinate is defined as increasing in the down-direction. At each point in the image, the resulting gradient approximations can be combined to give the gradient magnitude, using:

$$G = \sqrt{G_x^2 + G_y^2} \quad (4)$$

we can also calculate the gradient's direction:

$$\Theta = \arctan(G_y/G_x) \quad (5)$$

Besides Sobel operator, Laplacian of Gaussian (LoG)[22] is regarded as a common and effective detector for image corners and blobs. Since the Laplacian is very sensitive to noise, usually it was applied after the pre-smoothing of Gaussian filter. Given an input image $I(x, y)$, the LoG is defined as:

$$\nabla^2 L(x, y, t) = L_{xx} + L_{yy} \quad (6)$$

where $L(x, y, t) = g(x, y, t) * I(x, y)$ and $g(x, y, t) = e^{-(x^2+y^2)/2t}/2t\pi$.

In the proposed approach, we take the 2-D array of image gradient's direction Θ from the result of Sobel operator and the result after LoG detector as the base of our proposed feature for splicing detection. Note the computation of the two edge detection operations is much more faster than phase congruency. Then we calculate the statistical moments of the two base features. We also extract the same features on their prediction-error image and wavelet reconstruction image

as proposed in [14]. The prediction-error image I' is calculated by predicting each pixel gray-scale value in the original input image I using its neighboring pixels' gray-scale values. The prediction-error image also removes low frequency information and keeps high frequency information, which makes the splicing detection more efficient. To generate the reconstructed image $I_i (i = 1, 2, 3)$ from the input image I , we erase the information (set the wavelet coefficients to be zero) in sub-band LL_i of I after its Daubechies wavelet transform. Identical procedure is conducted for each reconstructed image $I'_i (i = 1, 2, 3)$. It is claimed that the splicing usually introduces the disturbance in high frequency components of spliced images, and the wavelet-based image by zeroing the approximation sub-band could enhance the difference between the authentic and spliced images. This conclusion has been proved to be reasonable by other's previous successful experience [13],[14]. Fig.2 demonstrates our proposed feature extraction for splicing detection based on sharp image intensity variations.

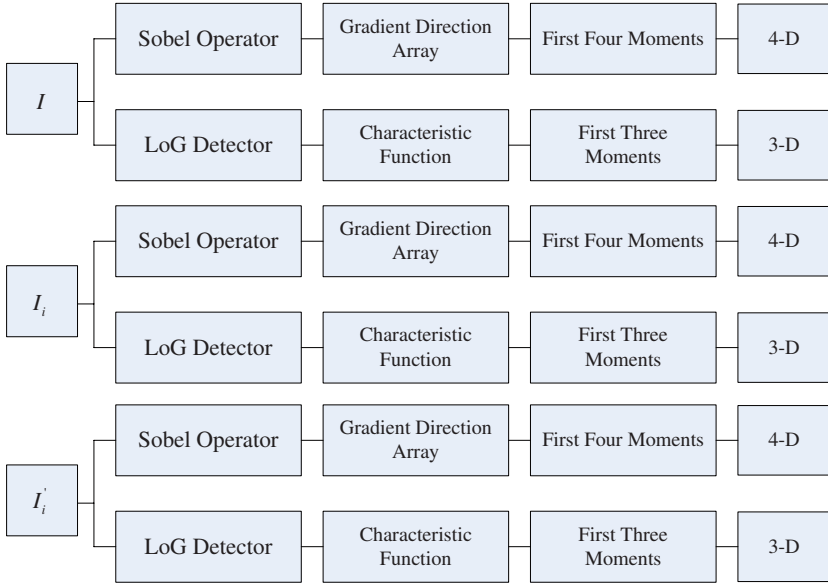
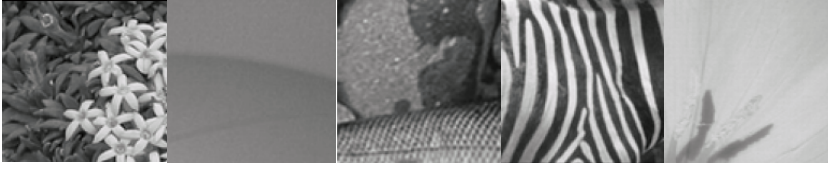


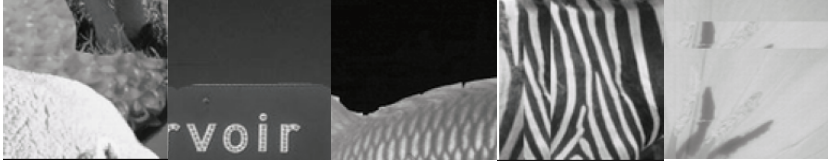
Fig. 2. Framework of edge based feature extraction from the reconstructed images

3.3 Feature Vector for Splicing Detection

Finally we obtain a 61-D feature vector for splicing detection, 12-D from run-length based statistic moments (denoted as RL features), and 49-D from edge detection statistic moments (denoted as SP features). As the feature extraction is implemented at image pixel level and based on simple filtering operation, the computation of the whole feature vector is fairly inexpensive. We will make further analysis on the performance and computation complexity of our proposed features for splicing detection in the following section.



(a) Some examples of authentic images



(b) Some examples of spliced images

Fig. 3. Image examples from the Columbia Image Splicing Detection Evaluation Dataset

4 Experimental Results

In this section, we present a set of experiments to demonstrate the effectiveness and efficiency of the proposed features.

4.1 Database Description

The only public available image database for splicing detection is provided by DVMM, Columbia University [10]. The Columbia Image Splicing Detection Evaluation Dataset has 933 authentic and 912 spliced image blocks of size 128 x 128 pixels. The authentic category refers to those images which are original without any splicing operation. This category consists of image blocks of an entirely homogenous textured or smooth region and those having an object boundary separating two textured regions, two smooth regions, or a textured regions and a smooth region. The location and the orientation of the boundaries are random. The spliced category has the same subcategories as the authentic one. For the spliced subcategories, splicing boundary is either straight or arbitrary object boundaries. The image blocks with arbitrary object boundaries are obtained from images with spliced objects. For the spliced subcategories with an entirely homogenous texture or smooth region, image blocks are obtained from the corresponding authentic subcategories by copying a vertical or a horizontal strip of 20 pixels wide from one location to another location within the same image block. More details about this database may be found in [10]. Example images from this database are shown in Fig.3.

4.2 Classifier

Support Vector Machine (SVM) is an optimal and efficient classifier which is commonly used for machine learning systems. Since our work in this paper only

focuses on feature extraction rather than the design of classifier, we utilize the SVM^{light} as the classifier in our experiment and a non-linear kernel is chosen. All the experiments and comparisons are tested on the same database and the same classifier in this paper.

4.3 Detection Performance

To evaluate the performance of the proposed scheme, all experimental results are obtained under similar conditions, and the average rate of 5 repeating tests is recorded. In each run, the training samples were randomly selected from the whole image dataset to train the classifier. The training samples are 5/6 of whole database in size (i.e, 776 authentic and 760 spliced images) to make sure that the training model are well learned. The remaining images were used for testing.

The average detection rates of our proposed feature sets as well as the comparison with similar feature sets proposed in [13] are shown in Table 1, where the TP (true positive)represents the detection rate of spliced images, TN (ture negative)represents the detection rate of authentic images, and accuracy is the average detection rate. The corresponding ROC curves are shown in Fig.4 The denoted CF feature set and PC feature set are proposed in [13]. The CF feature set represents a 78-D feature vector which is calculated from the first three moments of the characteristic function of a three level DWT decomposition of test images as well as their prediction-error images. The PC feature set represents a 24-D feature vector extracted from four higher-order statistics of image 2-D phase congruency after image DWT LL_i sub-band zeroing reconstruction at level $i = 1, 2$ and 3. As the CF features are commonly used for image splicing detection and the PC features are also edge based features, we compared the performance of our proposed 12-D run-length based feature vector (denoted as RL feature set) and 49-D edge based feature vector (denoted as SP feature set) with these two feature sets both in detection accuracy and computational cost. The feature extraction time in Table 1 is computed under MATLAB7.0 code run time calculation.

From Table 1, we can see tat the detection accuracy of the SP feature set is 74.27%, higher than its similar feature set PC at 70.68%. However, the feature extraction of SP features takes only $\frac{1}{8}$ computational time of PC feature set. Since both of the two kinds of feature sets are extracted in terms of the sharper image characteristic caused by splicing operation, we can expect that these simple edge detectors are better and more efficient features for splicing detection. Moreover, although the detection results of PC feature set and RL feature set are close, their feature extraction cost differs by 200 times. We also list the detection rates achieved by applying different feature combinations in Table 1 to examine how effective the combination is for splicing detection. Although the detection performance of RL feature set and SP feature set are not as good as CF feature set in our experiments, the combination of them does improve the detection accuracy. Compared with the combination of CF and PC feature sets proposed in [13] which achieved a detection accuracy of 81.76%, the combination of CF and the proposed SP and RL feature sets could also achieve a detection

Table 1. Comparison of detection results and computation time between our proposed feature sets, similar feature sets in [13] and their combinations for splicing detection

Feature Set	TP	TN	Accuracy	Extraction Time (Second)
<i>CF</i> (78 <i>D</i>)	78.06%	75.00%	76.22%	0.1056
<i>PC</i> (24 <i>D</i>)	65.16%	76.32%	70.68%	5.2526
<i>RL</i> (12 <i>D</i>)	65.81%	69.74%	69.75%	0.0245
<i>SP</i> (49 <i>D</i>)	78.71%	69.74%	74.27%	0.6424
<i>CF+PC</i> (102 <i>D</i>)	81.29%	82.24%	81.76%	5.3582
<i>CF+RL</i> (90 <i>D</i>)	80.00%	75.00%	77.52%	0.1301
<i>SP+RL</i> (61 <i>D</i>)	76.13%	76.32%	76.52%	0.6708
<i>SP+RL+CF</i> (139 <i>D</i>)	83.87%	76.97%	80.46%	0.7480
<i>ALL</i> (163 <i>D</i>)	83.23%	85.53%	84.36%	6.0251

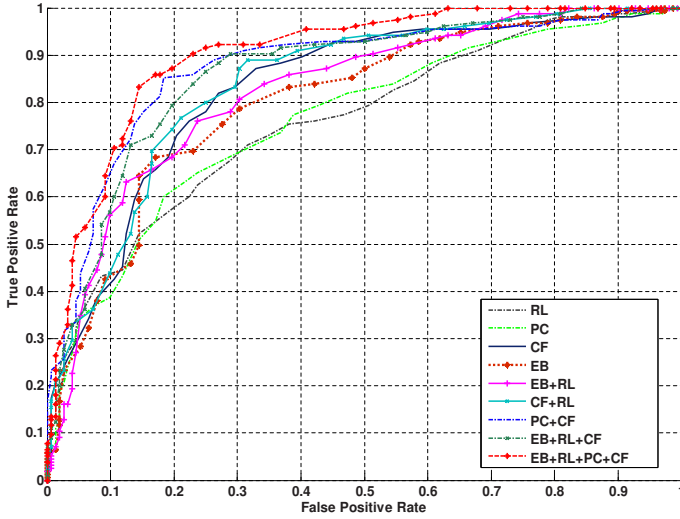


Fig. 4. Comparison of the several feature sets and their combinations for splicing detection

accuracy of 80.46% whereas the computational complexity is just about $\frac{1}{7}$ of the former. Also, when combining all the listed feature sets together, a detection rate as high as of 84.36% is achieved.

5 Conclusion

In this paper, a simple but efficient splicing detection scheme has been proposed. To capture the global inconsistency of pixel correlation caused by splicing, we generated a 12-D feature set based on the statistic moments of characteristic function of image run-length histograms. These features are fast to compute and

prove to be valid for splicing detection. To detect the shaper edges introduced by splicing, we utilize a Sobel operator and LoG detector to obtain the local sharp image intensity variations. Since the two sets of features are very simple to extract, the computation complexity of the proposed scheme is very low. Also, we have demonstrated the performance of our proposed edge based approach is better than phase congruency approach, while providing a much lower computational cost. Our study in this paper is also a verification of the validity of edge information for splicing detection. Surely other methods related to image intensity variation, such as edges, could be investigated for splicing detection. Also, the experimental results indicated that the combination of our proposed features with the state-of-art features further improve the detection accuracy for splicing detection without any significant extra costs.

Acknowledgments. This work is funded by research grants from the National Fundamental Research Program of China (Grant No.2004CB318110), and the National Key Technology R&D Program (Grant No.2006BAH02A13).

References

1. Gloe, T., Kirchner, M., Winkler, A., Behme, R.: Can we trust digital image forensics? In: Proceedings of the 15th international conference on Multimedia, pp. 78–86 (2007)
2. Rey, C., Dugelay, J.L.: A survey of watermarking algorithms for image authentication. *EURASIP J. Appl. Signal Process.* 2002(1), 613–621 (2002)
3. Yeung, N.M.: Digital watermarking introduction. *CACM* 41(7), 31–33 (1998)
4. Fridrich, J.: Methods for tamper detection in digital images. In: Proceedings of the ACM Workshop on Multimedia and Security, pp. 19–23 (1999)
5. Fridrich, J., Soukal, D., Lukas, J.: Detection of copy-move forgery in digital images. In: Proceedings of Digital Forensic research Workshop (August 2003)
6. Luo, W., Qu, Z., Pan, F., Huang, J.: A survey of passive technology for digital image forensics. In: *Front. Comput. Sciences of China*, vol. 1(2)
7. Ng, T.T., Chang, S.F.: A model for image splicing. In: 2004 International Conference on Image Processing (ICIP 2004), pp. 1169–1172 (2004)
8. Ng, T.T., Chang, S.F., Sun, Q.: Blind detection of photomontage using higher order statistics. In: *IEEE International Symposium on Circuits and Systems* (2004)
9. Ng, T.T., Chang, S.F.: Blind detection of photomontage using higher order statistics. In: ADVENT Technical Report 201-2004-1. Columbia University (June 8, 2004)
10. Ng, T.T., Chang, S., Sun, Q.: A data set of authentic and spliced image blocks. In: ADVENT Technical Report 203-2004-3. Columbia University (June 2004), <http://www.ee.columbia.edu/trustfoto>
11. Johnson, M.K., Farid, H.: Exposing digital forgeries by detecting inconsistencies in lighting. In: *ACM Multimedia and Security Workshop* (2005)
12. Hsu, Y., Chang, S.: Detecting image splicing using geometry invariants and camera characteristics consistency. In: *IEEE ICME* (July 2006)
13. Fu, D., Shi, Y.Q., Su, W.: Detection of image splicing based on hilbert-huang transform and moments of characteristic functions with wavelet decomposition. In: Shi, Y.Q., Jeon, B. (eds.) *IWDW 2006*. LNCS, vol. 4283, pp. 177–187. Springer, Heidelberg (2006)

14. Chen, W., Shi, Y.Q.: Image splicing detection using 2-d phase congruency and statistical moments of characteristic function. In: *Imaging: Security, Steganography, and Watermarking of Multimedia Contents* (January 2007)
15. Shi, Y.Q., Chen, C., Xuan, G.: Steganalysis versus splicing detection. In: Shi, Y.Q., Kim, H.-J., Katzenbeisser, S. (eds.) *IWDW 2007*. LNCS, vol. 5041, pp. 158–172. Springer, Heidelberg (2008)
16. Johnson, N.F., Jajodia, S.: Exploring steganography: Seeing the unseen. In: *Computer*, vol. 31, pp. 26–34. IEEE Computer Society, Los Alamitos (1998)
17. Dong, J., Tan, T.: Blind image steganalysis based on run-length histogram analysis. In: *2008 IEEE International Conference on Image Processing (ICIP 2008)* (2008)
18. Galloway, M.M.: Texture analysis using gray level run lengths. In: *Cornput. Graph. Image Proc.*, vol. 4, pp. 171–179 (1975)
19. Shi, Y.Q., Chen, C., Chen, W.: A natural image model approach to splicing detection. In: *ACM Workshop on Multimedia and Security, ACM MMSEC 2007* (2007)
20. Kovess, P.: Phase congruency: A low-level image invariant. *Psych. Research* 64, 136–148 (2000)
21. Sobel, I., Feldman, G.: A 3x3 isotropic gradient operator for image processing. In: Duda, R., Hart, P. (eds.) *Pattern Classification and Scene Analysis*, pp. 271–272. John Wiley and Sons, Chichester (1973)
22. Arfken, G.: *Mathematical methods for physicists*, 3rd edn. Academic Press, Orlando (1985)