A Parallel Control Method For Zero-Sum Games With Unknown Time-varying System

Qinglai Wei, Zhenhua Zhu, Jie Zhang, Feiyue Wang

Abstract—In this paper, based on ACP approach, a parallel control method is proposed for zero-sum games of unknown time-varying systems. The process of constructing a sequence of artificial systems, implementing the computational experiments and conducting the parallel execution is presented. The artificial systems are constructed to model the real system. Computational experiments adopting adaptive dynamic programming (ADP) are shown to derive control laws for the a sequence of artificial systems. The purpose of the parallel execution step is to derive the control laws for the real system. Finally, simulation experiments are provided to show the effectiveness of the proposed method.

***Index Terms*—Zero-sum games, parallel control, ACP, adaptive dynamic programming (ADP)**

1. INTRODUCTION

Zero-sum game (ZSG) is a classical problem in the game theory. For the linear systems, the algebraic Riccati equation is solved for ZSG. For the nonlinear systems, the Hamilton-Jacobi-Isaacs equation (HIJE) is solved, which is difficult to solve directly. Hence, in [1]–[3], utilizing the successive approximation method, the HJIE is converted to a series of linear differential equations for known systems.

With the research for the ZSG developing, model predictive control (MPC) methods are used to solve the ZSG with known dynamics. In [4], MPC is used for solving the ZSG between two heterogeneous players, which are an unmanned aerial vehicle and an unmanned ground vehicle respectively. Besides, ADP is introduced to solve ZSG. In [5], two structures about ADP are proposed for solving the ZSG of known systems. In [6], the value iteration with history information is proposed for solving the ZSG of known discrete-time systems. For the unknown systems, in [7], by using the neural networks to approximate the systems, the policy iteration is proposed for solving the HJIE. In [8], utilizing the fuzzy models to identify the unknown systems, ADP is used to solve the ZSG.

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In most of above papers, the systems are time-invariant and known. For the unknown systems, the traditional model-based methods utilize neural networks and fuzzy systems to identify the unknown systems. However, for the unknown time-varying systems, the accurate physical models are difficult to obtain for whole time horizon. Inspired by [9], the ACP-based parallel control [10] is introduced to handle the ZSG for unknown time-varying systems.

The ACP approach [10] is originally proposed for the purpose of modeling, analysis and contrl of complex systems. It consists of the following three steps: 1) Construct artificial systems. As the accurate physical model for a nonlinear system is difficult to obtain, the artificial systems are necessary to model the dynamic of the real system. Note that the artificial systems are not meant to be accurate to the real system but for the purpose of analysis. 2) Implement the computational experiments on the artificial systems. Because of the complexity of the real system, it is difficult to carry out computational experiments on the real system. Therefore, it is necessary to find the results of the equivalent problems in the real system from the artificial systems. 3) Implement the parallel execution. Because an artificial system does not approximate the real system, the interation is necessary to control and manage the real system. The control policies for the real system are derived through the parallel execution.

In this paper, an ACP-based parallel control method is proposed to solve the ZSG of the unknown time-varying systems. Using the ACP approach, the method obtains the control laws for the real system through constructing artificial systems, implementing computational experiments and conducting parallel execution.

This paper is organized as follows. In Section II, the problem about ZSG is formulated. In Section III, the details of the proposed method are presented. In Section IV, simulation results and analysis are presented to demonstrate the effectiveness of the proposed method.

2. PROBLEM FORMULATION

Consider a class of unknown time-varying systems:

$$s_{k+1} = \mathscr{F}(s_k, u_{1k}, u_{2k}, k), \quad k = 0, 1, 2, ..., \tag{1}$$

where $\mathscr{F}(\cdot)$ is the system function, $u_{1k} \in \mathbb{R}^m$ and $u_{2k} \in \mathbb{R}^q$ are the control vectors of two players, $s_k \in \mathbb{R}^n$ is the state vector of the system.

The performance index function of the system (1) is defined as follows:

$$\mathcal{J}(s_k, u_{1k}, u_{2k}, k) = \sum_{i=k}^{\infty} U(s_i, u_{1i}, u_{2i}, i) \tag{2}$$

where $U(s_i, u_{1i}, u_{2i}, i)$ is the utility function.

The goal of the ZSG is to obtain the control pair (u_{1k}^*, u_{2k}^*) such that

$$\mathcal{J}^*(s_k, k) = \min_{u_{1k}} \max_{u_{2k}} \mathcal{J}(s_k, u_{1k}, u_{2k}, k), \tag{3}$$

According to the Bellman optimality principle [11], the optimal performance index $\mathcal{J}^*(s_k, k)$ is written as:

$$\mathcal{J}^*(s_k, k) = \min_{u_{1k}} \max_{u_{2k}} (U(s_k, u_{1k}, u_{2k}, k) + \mathcal{J}^*(s_{k+1}, k+1)). \tag{4}$$

It is difficult to solve the equation (4) directly. To overcome the difficulty, we propose a parallel control method to derive the control laws for the real system (1).

3. METHOD

In this section, the proposed method is presented in detail. First, a sequence of artificial systems are constructed to model the system (1). Second, we implement computational experiments via ADP to obtain the control laws for the artificial systems. Third, through the interaction parallel execution step, the suboptimal control laws for the real system are derived.

3.1. Artificial Systems

In many real world scenarios, precise physical models are difficult to obtain. Hence, artificial systems are introduced to model the real system (1). Since the real system (1) is time-varying, a sequence of artificial systems are constructed to model the real system. The artificial systems are written in the following form:

$$\begin{aligned} \hat{s}_{k+1} &= \mathcal{F}(s_k, u_{1k}, u_{2k}, k) \\ &= \begin{cases} \mathcal{F}_0(s_k, u_{1k}, u_{2k}) & k_0 \le k < k_1, \\ ... & \\ \mathcal{F}_\tau(s_k, u_{1k}, u_{2k}) & k_\tau \le k < k_{\tau+1}, \\ ... & \end{cases} \end{aligned} \tag{5}$$

where $\mathcal{F}_\tau(\cdot), \tau = 0, 1, ...$ is an artificial system function in time horizon $[k_\tau, k_{\tau+1})$, $k_0, k_1, ...$ are positive integers.

In this paper, multilayer perceptrons(MLPs) are used to construct the artificial systems, an artificial system function in time horizon $[k_\tau, k_{\tau+1})$ is expressed as:

$$\hat{s}_{k+1}^\tau = \mathcal{F}_\tau(s_k, u_{1k}, u_{2k}) = W_\tau^\top \phi_\tau(s_k, u_{1k}, u_{2k}), \tag{6}$$

where W_τ are the weight matrix of MLPs, $\phi_\tau(\cdot)$ is the activation functions of MLPs.

In the process of constructing artificial systems, the gradient descend algorithm [12] is used to update the weights of MLPs. When the weights of MLPs converge, the real system (1) is modelled by the artificial system. The process of constructing artificial systems is shown in Algorithm 1.

Algorithm 1 The process of constructing artificial systems

Initialization:
1: Collect observational data from the real system
2: Construct MLPs like (6)
3: Give the modeling precision $\epsilon > 0$ and learning rate α
Iteration:
4: Using the gradient descend algorithm to update the weights of MLPs
5: If $\|W_{\tau(i)} - W_{\tau(i-1)}\|_2 > \epsilon$ holds, go to step 4
Return: the weight matrix $W_{\tau(i)}$

3.2. Computational Experiments

Since the real system is complex and unknown, carrying out computational experiments on the real system is difficult. Therefore, computational experiments are implemented on the artificial systems via ADP to evaluate and derive control laws for the artificial systems.

To distinguish from the state of the real system (1), we introduce the artificial system in time horizon $[k_\tau, k_{\tau+1})$:

$$z_{k+1} = \mathcal{F}_\tau(z_k, u_{1k}, u_{2k}), \tag{7}$$

where $z_k \in \mathbb{R}^n$ denotes the state in the artificial systems.

Assume saddle point exists [13], computational experiments via ADP is used to derive the control laws for the artificial system in certian time horizon. According to ADP method [14], the value function and control laws are obtained. For a sequence of artificial systems, there are a sequece of value funxtions and control laws written as:

$$\mathcal{V}(z_k, k) = \begin{cases} \mathscr{V}^0(z_k) & k_0 \le k < k_1, \\ ..., & \\ \mathscr{V}^\tau(z_k) & k_\tau \le k < k_{\tau+1}, \\ .... & \end{cases} \tag{8}$$

$$\mathcal{U}_1(z_k, k) = \begin{cases} \mathscr{U}_1^0(z_k) & k_0 \le k < k_1, \\ ..., & \\ \mathscr{U}_1^\tau(z_k) & k_\tau \le k < k_{\tau+1}, \\ ..., & \end{cases} \tag{9}$$

and

$$\mathcal{U}_2(z_k, k) = \begin{cases} \mathscr{U}_2^0(z_k) & k_0 \le k < k_1, \\ ..., & \\ \mathscr{U}_2^\tau(z_k) & k_\tau \le k < k_{\tau+1}, \\ ..., & \end{cases} \tag{10}$$

In practical applications for computational experiments, MLPs are used to approximate the control laws and the value functions. The structure diagram of the computational experiments is given in Fig. 1. The critic module approximates the value functions, two actor modules approximate the control laws of two players and the artificial system module is the artificial systems constructed in III-A.

For $k \in [k_\tau, k_{\tau+1})$, $\tau = 0, 1, 2, ...$, the approximation functions of the control laws and the value functions are expressed as:

$$\hat{\mathscr{V}}^\tau(z_k) = W_c^{\top(\tau)} \phi_c(z_k), \tag{11}$$

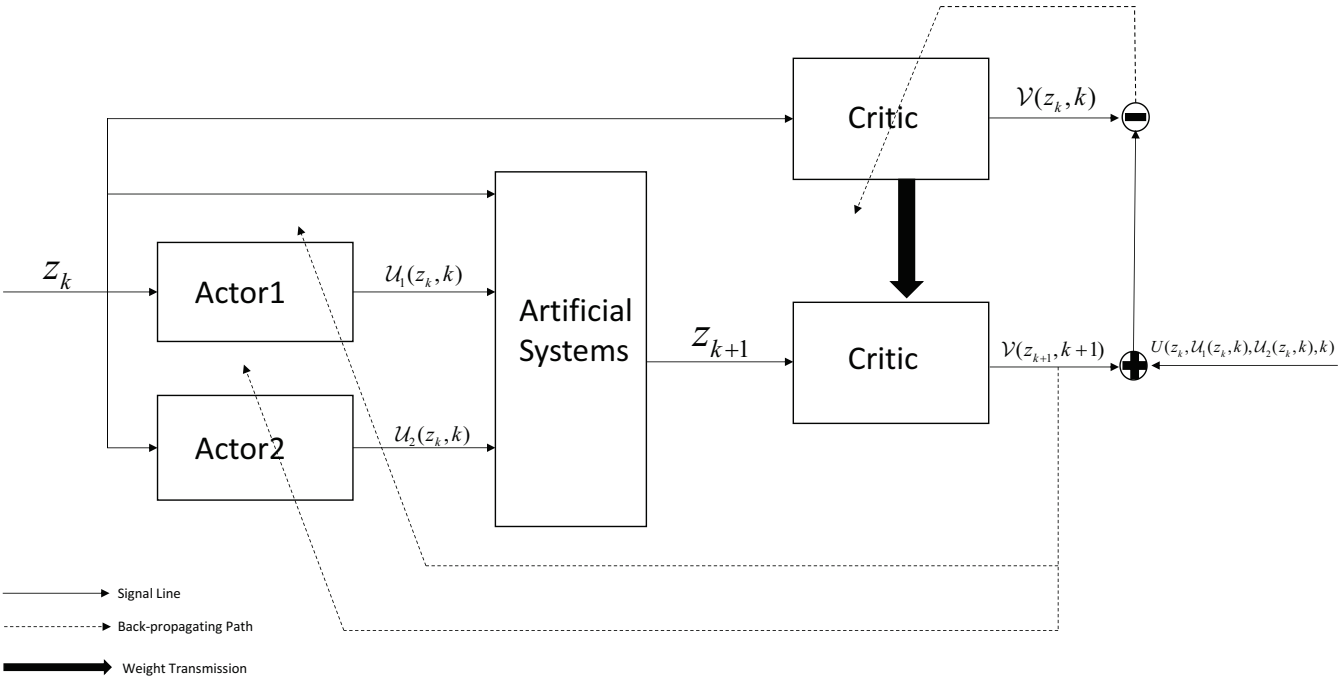

Fig. 1. The structure diagram of the computational experiments for a sequence of artificial systems

$$\hat{\mathscr{U}}_1^{\tau}(z_k) = W_{u1}^{\top(\tau)}\phi_{u1}(z_k), \tag{12}$$

and

$$\hat{\mathscr{U}}_2^{\tau}(z_k) = W_{u2}^{\top(\tau)}\phi_{u2}(z_k), \tag{13}$$

where W_c^{τ},W_{u1}^{τ} and W_{u2}^{τ} are the weight matrix of MLPs, $\phi_c(\cdot)$,$\phi_{u1}(\cdot)$ and $\phi_{u2}(\cdot)$ are the activation functions of MLPs.

In the process of iteration for computational experiments, the weights of MLPs are tuned by the the gradient descend algorithm. When the weights of MLPs converge, the control laws for the artificial systems are derived. The details about the process of computational experiments are presented in Algorithm 2.

Algorithm 2 Computational Experiments Algorithm

Initialization:

1: Give an initial value function $\mathscr{V}_0^{\tau}(z_k) = 0$
2: Give the computation precision $\epsilon > 0$
3: Construct three MLPs for approximating the value function and control laws

Iteration:

4: Compute the iterative value functions and control laws by using ADP
5: Using the gradient descend algorithm to update the weights of three MLPs
6: If $\|W_{c(i+1)} - W_{c(i)}\|_2 > \epsilon$ holds, go to step 4

Return: $W_{u(i)}^{\tau}, W_{w(i)}^{\tau}, W_{c(i)}^{\tau}$

3.3. Parallel Execution

After constructing the artificial systems and implementing the computational experiments, the control laws for the artificial systems are derived. As we mention above, an artificial system in certain time horizon does not approximate the real system, hence, feedback and interaction are necessary to obtain the control laws for the real system.

In parallel execution step, the control laws derived from computational experiments are implemented not only on the artificial systems but also on the real system, then the difference about performance of the artificial systems and the real system is evaluated. If the diffirence is not large, the control laws derived from computational experiments continue to be applied to the real system. Otherwise, it is necessary to update the artificial systems and conduct the computational experiments again.

Through constructing the artificial systems, implementing the computational experiments and conducting the parallel execution, the control laws for the real system are obtained until the state of the real system is stable.

We present a detailed process of the parallel execution in Algorithm 3

4. Simulation Study

In this section, to show the effectiveness of the proposed method, we choose an example obtained from [9] with some modifications, the dynamics of the system is as follows.

Algorithm 3 Parallel Execution Algorithm

Initialization:

1: Give the computation precision ϵ_1 and ϵ_2
2: Let $\tau = 0$, $k = 0$, $k_\tau = k$
3: Give initial state s_0 and z_0, let z_0 = s_0

Iteration:

4: Implement Algorithm 1 to construct the artificial system (6) under ϵ_1
5: Obtain the weights of two action modules W_u^τ, W_w^τ and the weights of the critic module W_c^τ by using Algorihtm 2
6: Implement the control laws on the artificial system (6) and the real system (1) and evaluate the difference:
if the difference is larger than a threshold
go to step 4
else
if $||s_k|| > \epsilon_2$, go to step 6

$$\begin{pmatrix} s_{1(k+1)} \\ s_{2(k+1)} \end{pmatrix} = \begin{pmatrix} s_{1k} + 0.1s_{2k} \\ -0.1s_{1k} + 1.1s_{2k} - 0.1s_{1k}^2 s_{2k} \end{pmatrix} + B_k u_{1k} + C_k u_{2k} \tag{14}$$

where

$$B_k = \begin{bmatrix} -0.5 + 0.2\sin(k) & 0 \\ 0 & -0.5 + 0.2\sin(k) \end{bmatrix} \tag{15}$$

and

$$C_k = \begin{bmatrix} 0.1 + 0.05\cos(k) & 0 \\ 0 & 0.1 + 0.05\cos(k) \end{bmatrix}. \tag{16}$$

Let utility function $U(s_i, u_{1i}, u_{2i}) = s_i^\top Q s_i + u_{1i}^\top R u_{1i} - \gamma^2 u_{2i}^\top u_{2i}$, where $Q = I_1 \in \mathbb{R}^{2\times 2}$, $R = I_2 \in \mathbb{R}^{2\times 2}$.

In the Artificial Systems, multilayer perceptrons (MLPs) are used model the real system. For the MLPs, The structure is 6–8–2 and the activation functions of the output layer and the hidden layer are purelin functions and tansig functions respectively. The bias of the output layer and the hidden layer are zero. The training process of weights for MLPs is shown in Algorithm 1 with model precision $\epsilon = 0.001$ and the learning rate $\alpha = 0.02$.

Next, In the Computational Experiments, three MLPs, which are two action networks and critic networks, are used as basis functions to approximate the control laws of two players and the value function. The structures of two action networks and critic networks are chosen as 2–8–2, 2–8–2, and 2–8–1. For all three MLPs,the activation functions of the output layer and the hidden layer are purelin functions and tansig functions respectively. The bias of the output layer and the hidden layer are zero. Implement computational experiments by Algorithm 2 with the computation precision $\epsilon = 0.001$ and the learning rate $\alpha = 0.02$ to evaluate and derive the control laws for multiple artificial systems.

Then, in the parallel execution, we implement Algorithm 3 to derive the control laws for the real system. After implementing the proposed method, the trajectory of the states for the real system is shown in Fig. 2. According to the Fig. 2, the states of system converge to the equilibrium, showing the effectiveness of the proposed method.

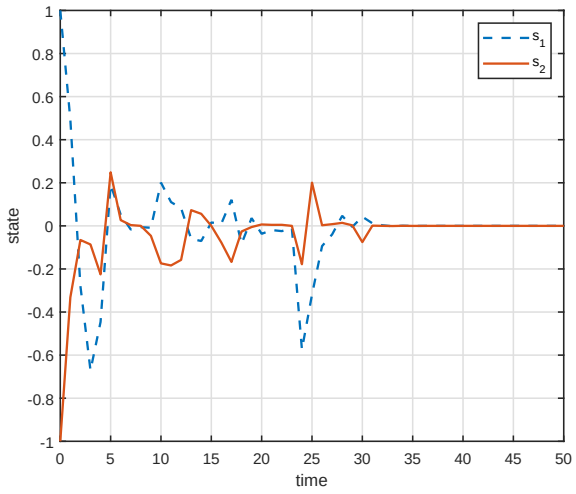

Fig. 2. The trajectory of the states for the real system

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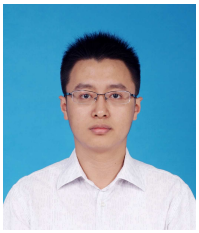

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