

# Estimation of fingerprint orientation field by weighted 2D fourier expansion model

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**Abstract**—Accurate estimation of fingerprint orientation field is an essential module in fingerprint recognition. This paper proposes a novel technique for improving fingerprint orientation field estimation by fingerprint orientation model based on weighted 2D fourier expansion(W-FOMFE). The motivation for the proposed method can be found by: 1)the original FOMFE is sensitive to abrupt changes in orientation field; 2) blocks of different quality should have different impacts on FOMFE. Thus, we take into account the information of the Harris-corner strength (HCS) for orientation field estimation. In our method, we first calculate the fingerprint' HCS; then use the HCS to remove abrupt changes in orientation field; finally, incorporate the normalized HCS as weighted value into original FOMFE. We test our method on FVC2004DB1. Experimental results show that our method (W-FOMFE) has better orientation field estimation than FOMFE.

**Keywords**-fingerprint orientation; weighted 2D fourier expansion; Harris-corner strength;

## I. INTRODUCTION

Fingerprint recognition is one of the most popular and reliable biometric techniques and is widely used in many important applications such as electronic personal identification card, e-commerce and so on. It has been studied for many years and much progress has been made. Nevertheless, there still exist some critical research issues such as the long processing time in large database and low matching rate on poor quality images. To solve the two problems above, improvements to fingerprint classification and identification are required. As a global feature, orientation field which is defined as the local direction of the ridge-valley structures, plays a very important role in both topics.

During the past decades, several methods have been proposed for the estimation of fingerprint orientation field, which can be broadly categorized as gradients-based, methods [1], [2] filtering-based methods,[3], [4] and model-based [5], [6], [7], [8], [9]. Gradient-based methods are the most simple and popular methods for ridge orientation estimation. But they are very sensitive to noisy. Filtering-based methods improve the orientation at a small local region, but they are not able to solve large noisy or missed patches in the poor quality images.

Therefore the mathematical modeling of fingerprint orientation field was studied in an attempt to predict orientation field for the large noise areas and resolve ambiguities in

extracting ridge orientation. Several model-based methods have been proposed in the literature. Sherlock and Monro [5] proposed a so-called zero-pole method, which took the core as zero and delta as pole in a complex plane, to model the fingerprint orientation topology. Zhou and Gu [7] developed a combination model for the estimation of orientation field of fingerprints. Li et.al [8] proposed a piece-wise first-order phase portrait model to estimate the orientation.

But there exists one common limitation in above mentioned fingerprint orientation models: Prior knowledge of singular points is required in order to refine orientation model descriptions. In fact, accurate detection of singular points depends on a good estimation of the orientation field, which often requires, in turn, accurate singular points. This tedious and error-prone recursive process limits the orientation models that depend on singular points in providing accurate descriptions for fingerprint orientation field. Wang et.al [9] proposed a fingerprint orientation model based on 2D fourier series expansions of two nonlinear differential equations(FOMFE) to solve this problem. The FOMFE does not need the third party to detect singular points.

However, the FOMFE has two drawbacks. Firstly, the FOMFE is sensitive to abrupt changes in orientation field, as shown in Figure. 2(b) and Figure. 3(b). Secondly, The FOMFE does not consider that blocks of different quality should have different impacts on the model. Thus, we propose a novel technique for dealing with the two drawbacks by fingerprint orientation model based on weighted 2D fourier Expansion(W-FOMFE). In our method, we first calculate the Harris-corner strength( $HCS$ )[10]; then use the  $HCS$  to remove abrupt changes in orientation field; finally, incorporate the normalized Harris-corner strength( $N_{HCS}$ ) as weighted value into original FOMFE. our statistical experiments on FVC2004DB1[12] show that the W-FOMFE can significantly improve the accuracy of orientation field estimation, even in poor-quality fingerprint images.

The remainder of this paper is organized as follows: Section II outlines the FOMFE, the associated method for orientation field estimation. In section III, due to the FOMFE drawbacks, we proposed our method to improve the orientation field estimation. Experiment results are presented in section IV. Finally, we summarize our work in section V.

## II. MATHEMATICAL APPROACH TO THE FOMFE

An orientation field can effectively summarize the information contained in a fingerprint pattern and is a rich information resource for fingerprint feature retrieval and processing. In order to avoid the difficult problem created by orientation discontinuity ( $\pi \longleftrightarrow 0$ ), a popular approach is to map the orientation field into a new vector field where each orientational element is denoted as a 2D vector  $v = (v_s, v_c)$  with  $v_c, v_s$  being the phase functions of  $\cos 2\theta$  and  $\sin 2\theta$ , respectively, and  $\theta$  is the orientation angle [2].

Since ridges and valleys alternate throughout fingerprint areas, it is nature to assume that fingerprint orientation will behave in a periodic manner. Therefore, it is reasonable to use FOMFE which is a set of cosine and sine functions to represent fingerprint orientation field.

For a bivariate function  $f(x, y)$  in a restricted area in  $R^2 (-l \leq x \leq l, -h \leq y \leq h)$ , its 2D Fourier expansion can be expressed in the following form:

$$f(x, y) = \sum_{m=0}^k \sum_{n=0}^k \{ \lambda_{mn} [ a_{mn} \cos(mvx) \cos(nwy) + b_{mn} \sin(mvx) \cos(nwy) + c_{mn} \cos(mvx) \sin(nwy) + d_{mn} \sin(mvx) \sin(nwy) ] \} + \varepsilon(x, y), \quad (1)$$

where,  $k \in N$  is the Fourier expansion order and  $\varepsilon(x, y)$  is the residual,  $v = \pi/l, w = \pi/h$ ,  $(a_{mn}, b_{mn}, c_{mn}, d_{mn})$  are the Fourier coefficients,  $\lambda_{mn}$  is a constant scalar which can be found in [9].

We can use two bivariate trigonometric polynomial functions  $f_c(v_c)$  and  $f_s(v_s)$ , each in a form of (1), to represent the 2D modeling function  $f$ . As it is understood from (1), the specific problem in the FOMFE is to find the coefficient matrices  $(a_{mn}, b_{mn}, c_{mn}, d_{mn})$ , so that the resulting FOMFE can generate a phase portrait that best fits the given input  $(v_c, v_s)$ . The problem can be formulated as a classical linear least square (LSQ) problem [13]:

$$\min_{\alpha} \|\Lambda \alpha - b\|^2. \quad (2)$$

We solve the LSQ problem based on *QR Factorization* [14].

## III. THE PROPOSED METHOD (W-FOMFE)

The proposed W-FOMFE algorithm works in multiple stages and details of each stage are described below with the output images in Fig. 2.

### A. calculate the original orientation field

In this stage, we adopted a classic gradient-based method [2] to compute the original orientation field  $\theta$ .

1) : Divide the input image into blocks of size  $W \times W$  ( $W = 16$  in our paper) and compute the gradients  $G_x(i, j)$  and  $G_y(i, j)$  at each pixel  $(i, j)$  using *Sobel* operator or the more complex *Marr – Hildreth* operator.

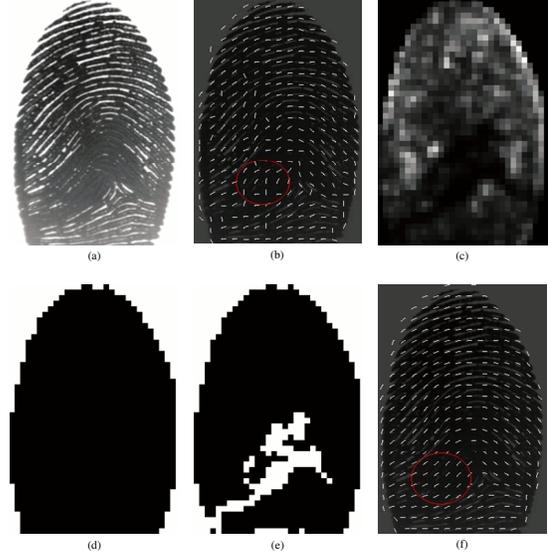


Figure 1. (a) Original fingerprint image from 2004DB1(99\_8.tif); (b) coarse orientation field based on (a); (c) Harris-corner strength image; (d) original foreground image; (e) the foreground image that has removed the abrupt change blocks in orientation; (f) reconstructed orientation field from the proposed M-FOMFE.

2) : Calculate the average squared gradient  $[\overline{G_{sx}}, \overline{G_{sy}}]$  by the following formulas:

$$[\overline{G_{sx}}, \overline{G_{sy}}] = \left[ \sum_{i=1}^W \sum_{j=1}^W G_{sx}(i, j), \sum_{i=1}^W \sum_{j=1}^W G_{sy}(i, j) \right]. \quad (3)$$

In this expression,

$$G_{sx}(i, j) = G_x^2(i, j) - G_y^2(i, j) \quad (4)$$

$$G_{sy}(i, j) = 2G_x(i, j)G_y(i, j) \quad (5)$$

3) : Compute the average ridge-valley orientation  $\theta$  which is perpendicular to gradient direction by (7):

$$\theta = \frac{1}{2}\pi + \frac{1}{2}\angle \left( \sum_W \sum_W (G_x^2 - G_y^2), \sum_W \sum_W (2G_x G_y) \right), \quad (7)$$

where  $\angle(a, b)$  is defined as:

$$\theta = \begin{cases} \tan^{-1}(b/a) & \text{if } a \geq 0 \\ \tan^{-1}(b/a) + \pi & \text{if } a < 0, b \geq 0 \\ \tan^{-1}(b/a) - \pi & \text{if } a < 0, b < 0 \end{cases} \quad (8)$$

The original orientation field of Fig. 1(a) is shown in Fig. 1(b).

### B. Remove the abrupt changes in original orientation

From Figure. 1(b), we can see that the original orientation field of fingerprint is quite affected by the noise. To get more smooth and accurate orientation field, we propose a novel method based on the Harris-corner strength (*HCS*) [10] to remove the abrupt changes in orientation. First, a set of

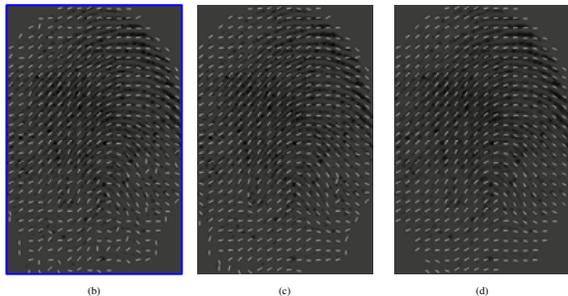


Figure 2. (a) coarse orientation fields based on the image from 2004DB1 88\_6.tif; (b) orientation fields calculated by FOMFE(k=4); (c) orientation results estimated by using our method.

$HCS$  ( $R_{HCS}$ ) of the input image(Fig. 1(a)) are calculated for each  $W \times W$  ( $W = 16$ ) block by (9).

$$R_{HCS} = \frac{\sum_W \sum_W (G_x^2 G_y^2 - G_{xy}^2)}{\sum_W \sum_W (G_x^2 + G_y^2)} \quad (9)$$

We find that the value in smooth areas is larger than in noise areas. The resulting corner strength image is shown in Fig. 1(c). Then a appropriate threshold value( $T$ ) is selected by using OSTU[11]. If the current block  $R_{HCS}$  is less than threshold( $T$ ), we will set the current block  $R_{HCS} = 0$ . Fig.1(e) is the foreground image that has removed the abrupt change blocks in orientation. Fig. 1(d) is the original foreground image.

### C. The proposed W-FOMFE model

Not all blocks are of equal value in determining FOMFE solution. Blocks of different quality should have different impacts on FOMFE. In this stage, we apply  $R_{HCS}$  as weighted value to FOMFE. First, we normalize  $R_{HCS}$  to  $N_{HCS} \in [0, 1]$ ; and then apply  $N_{HCS}$  as a weighted value to FOMFE model. Using this convention, we can rewrite (2) to (10):

$$\min_{\alpha} |(\Delta\alpha - b) \cdot R_{HCS}|^2. \quad (10)$$

The reconstructed orientation field using W-FOMFE model is shown in Fig. 1(f). Compared with the original orientation field Fig. 1(b), our W-FOMFE model can give more smooth and accurate orientation field.

## IV. EXPERIMENTAL RESULTS

Two experiments are carried to evaluate the performance of our method on FVC2004 DB1\_A [12]. Experiment *I* aims to directly test the performance of our method in qualitative way; experiment *II* is designed to evaluate the influence of our method on matching performance. In both of the experiment, we compare the results of our method with the FOMFE.

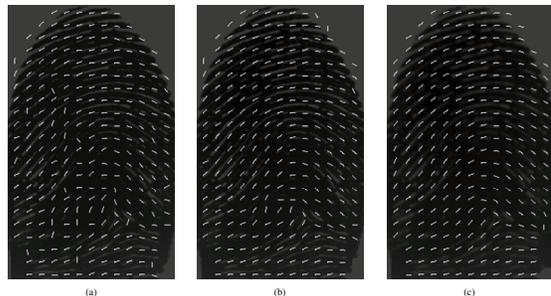


Figure 3. (a) coarse orientation fields based on the image from 2004DB1 99\_8.tif; (b) orientation fields calculated by FOMFE(k=4); (c) orientation results estimated by using our method.

### A. Experiment I

For comparing the two methods qualitatively, we randomly give out some comparative results between our method and the FOMFE (as shown in Fig.2 - Fig.3). In the following, we will display the estimation results to demonstrate that the performance of our method is superior to the FOMFE.

Fig.2 and Fig.3 are experimental results for arch, and loop respectively. These two fingerprints are dry or wet, which causes the original orientation field of some blocks too disorder. We then superimpose the estimated orientation fields image and display the results in Fig.2(a),2(b),2(c) and Fig.3(a),3(b),3(c) respectively. Fig.2(b) and Fig.2(b) are generated by FOMFE. Although the orientation fields estimation is improved by FOMFE, the orientation fields are not yet smooth as shown in Fig.2(c) and Fig.2(c).

### B. Experiment II

In this experiment, we are going to evaluate the influence of our method on matching performance. The matching system is implemented according to [15] which describes a matching algorithm based on minutiae. Since minutiae extraction from enhanced ridges depends heavily on the orientation field, the matching results can reflect the accuracy of orientation field estimation. In order to validate the performance of the proposed method, we compare with FOMFE. We use the same matching system except the difference of orientation field estimation method(using our method and FOMFE respectively).

The proposed method has been evaluated on FVC2004 DB1\_A[12] which contains 800 fingerprints from 100 different fingers. The total number of genuine matches for calculating false nonmatch rate (FNMR) is  $100C_8^2 = 2800$ . The total number of imposter matches for calculating false match rate (FMR) is  $C_{100}^2 = 4950$ . More details about the databases and the protocol are available in the web site [12]. Fig. 4 illustrates the matching results by using the orientation fields estimated by our method and the FOMFE method respectively. The performance indices (EER, FMR100,FMR1000

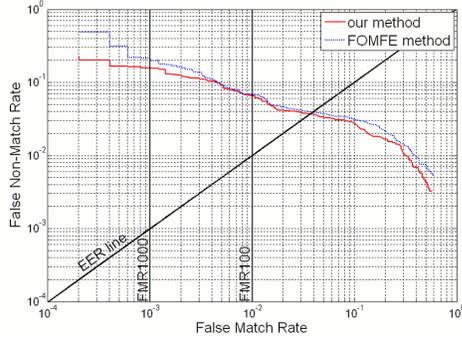


Figure 4. ROC curves: performances of the matching system by using the orientation fields calculated by our method(W-FOMFE) and the FOMFE method respectively.

and ZeroFMR) are also reported in Table. I. From the results, we can find that the matching performance based on the orientation fields calculated by our method is better than FOMFE obviously.

Table I  
EER RESULTS BY OUR METHOD AND FOMFE METHOD.

Algorithm	EER	FMR100	FMR1000	ZeroFMR
FOMFE method	3.92%	6.78%	21.32%	48.89%
our method	3.68%	6.57%	15.79%	22.96%

## V. CONCLUSION

In this paper, we propose a novel technique for improving the coarse fingerprint orientation field estimation using fingerprint orientation model based on weighted 2D fourier expansion(W-FOMFE). Firstly, the coarse field is computed by using the gradient-based algorithm, and the error from the noise can be eliminated using the Harris-corner strength. Then we use the normalized  $R_{HCS}$ -based weighted 2D fourier expansion to estimate the orientation. Experimental results show that our algorithm has a better performance on robustness, accuracy for orientation field estimation than the FOMFE model.

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## REFERENCES

- [1] A.R. Rao, and R.C. Jain. Computerized flow field analysis: oriented texture fields. *IEEE Trans. Pattern Anal. Mach. Intell.*, 693–709, 1992.
- [2] A.M. Bazen, and S.H. Gerez. Systematic methods for the computation of the directional fields and singular points of fingerprints. *IEEE Trans. Pattern Anal. Mach. Intell.*, 905–919, 2002.
- [3] A.K. Jain, L. Hong, S. Pankanti, and R. Bolle. An identity-authentication system using fingerprints. *Proc. IEEE.*, 1365–1388, 1997.
- [4] D. Maio, and D. Maltoni. Direct gray-scale minutiae detection in fingerprints. *IEEE Trans. Pattern Anal. Mach. Intell.*, 27–40, 1997.
- [5] B. Sherlock, and D. Monro. A model for interpreting fingerprint topology. *Pattern Recognition.*, 1047–1095, 1993.
- [6] P. Vizcaya, and L. Gerhardt. A nonlinear orientation model for global description of fingerprints. *Pattern Recognition.*, 1221–1231, 1996.
- [7] J. Gu, J. Zhou and D. Zhang. A combination model for orientation field of fingerprints. *Pattern Recognition.*, 543–553, 2004.
- [8] J. Li, W.Y. Yau and H. Wang. Constrained nonlinear models of fingerprint orientations with prediction. *Pattern Recognition*, 102–114, 2006.
- [9] Y. Wang, J. Hu and D. Philips. A Fingerprint Orientation Model. Based on 2D Fourier Expansion (FOMFE) and Its Application to Singular-Point Detection and Fingerprint Indexing. *IEEE Trans. Pattern Anal. Mach. Intell.*, 573–585, 2007.
- [10] C. Wu, S. Tulyakov, and V. Govindaraju. Robust Point-Based Feature Fingerprint Segmentation Algorithm. *ICB07*, 1095–1103, 2007.
- [11] N. Ostu. A Threshold selection method from gray-level histogram. *IEEE Trans. System Man and Cybernet, SMC-B.*, 62–66, 1978.
- [12] <http://bias.csr.unibo.it/fvc2004/databases.asp>
- [13] W. Press, S. Teukolsky, W. Vetterling, and B. Flannery. *Numerical Recipes in C: The Art of Scientific Computing*. Cambridge Univ. Press, 1992.
- [14] G. Golub and C. Loan. *Matrix Computations*. The Johns Hopkins Univ. Press, 1996 (third ed).
- [15] Y.L. He, J. Tian, L. Li, H. Chen, and X. Yang. Fingerprint Matching Based on Global Comprehensive Similarity. *IEEE Trans. Pattern Anal. Mach. Intell.*, 850–862, 2006.