Letter

Exponential Synchronization of Delayed Stochastic Complex Dynamical Networks via Hybrid Impulsive Control

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Dear Editor,

This letter addresses the synchronization problem of a class of delayed stochastic complex dynamical networks consisting of multiple drive and response nodes. The aim is to achieve mean square exponential synchronization for the drive-response nodes despite the simultaneous presence of time delays and stochastic noises in node dynamics. Toward this aim, a hybrid impulsive controller, featuring both delayed and non-delayed impulses, is developed. Sufficient conditions of mean square exponential stability of the delayed stochastic synchronization error system are then derived. It is further shown that exponential synchronization can still be preserved even if the impulsive controller involves only delayed impulses. An illustrative example is finally presented to demonstrate the effectiveness of the obtained theoretical results.

Complex dynamical networks (CDNs) are networks consisting of multiple interconnected nodes (or subsystems) where each node has its own dynamical behavior. Examples of such networks include power grids [1], multi-agent systems [2], connected vehicles [3], [4], and distributed games [5]. The concept of synchronization in CDNs refers to the ability of the nodes within the network to reach a coordinated objective. So far, several effective synchronization control methods, such as adaptive synchronization control [6], sampled-data synchronization control [7], intermittent synchronization control [8], impulse synchronization control [9]–[11], have been developed for various CDNs. Among these methods, the impulse control strategy has attracted a great deal of attention due to its simple structure and discrete nature of implementation, and finds potential applications in various energy efficiency scenarios owing to its low energy consumption at impulse instants.

Impulsive control for CDNs relies on the feedback information transmitted from the drive nodes to the response nodes at discrete instants. However, due to measurement failures, restricted transmission rates over communication networks and other uncertainties at the sampling instants, the feedback data may suffer from time delays as well. Given this fact, how to explore delayed impulses and develop effective impulsive control approach to synchronization of CDNs is of great significance. However, it should be noted that the delayed impulses are treated as a kind of interference in many existing results, such as [9]–[11]. This may introduce certain conservatism into the analysis and design criteria. On the other hand, although many impulsive synchronization control results have been available to CDNs, there has been a few concerning about the simultaneous presence of time delays and stochastic noises in system dynamics. It is well acknowledged that delays and/or stochastic noises can cause oscillation and instability of the CDN, and further make the impulsive synchronization control issue quite challenging.

Motivated by the above discussion, we revisit the exponential synchronization problem of a class of delayed stochastic CDNs. Our focus is laid on the inclusion of both time delay and stochastic noise in the system dynamics and further the development of an effective hybrid impulsive controller. The main contributions are twofold. 1) A hybrid impulsive controller incorporating both delayed and non-delayed impulses is designed to account for the phenomenon that the transient behavior of impulses depends on both the current and historical information of the synchronization error system subject to time delay and stochastic noise. 2) Numerically tractable criteria in both hybrid and delayed impulsive control cases are derived to preserve the mean square exponential stability of the delayed stochastic synchronization error system. The explicit relationship between the inherent nonlinearities in the node dynamics and the proposed impulsive controller parameters can be established. To deal with delayed impulses, different from [9]–[11], we provide a better impulse estimation in the proof of our main result, based on which we also show that the stability criterion in the case of delayed impulsive control only can be readily derived.

Problem formulation: Consider a class of drive-response system whose dynamics can be described by a stochastic CDN of the following stochastic delayed differential equations:

$$\begin{aligned} dx_i(t) = \Big(&f(x_i(t), x_i(t-\tau)) + c\sum_{j=1}^{N} b_{ij}\Gamma x_j(t) \\ &+ \tilde{c}\sum_{j=1}^{N} d_{ij}\Gamma x_j(t-\tau)\Big)dt + g(x_i(t), x_i(t-\tau))d\omega(t) \end{aligned} \tag{1}$$

$$\begin{aligned} dy_i(t) = \Big(&f(y_i(t), y_i(t-\tau)) + u_i(t) + c\sum_{j=1}^{N} b_{ij}\Gamma y_j(t) \\ &+ \tilde{c}\sum_{j=1}^{N} d_{ij}\Gamma y_j(t-\tau)\Big)dt + g(y_i(t), y_i(t-\tau))d\omega(t) \end{aligned} \tag{2}$$

for any $i \in \{1,2,\ldots,N\}$, where $x_i(t) \in \mathbb{R}^n$ is the state vector of the i-th drive node of the CDN with $x_i(s) = \phi_i(s)$ for any $s \in [-\tau, 0]$ being the initial condition; $y_i(t) \in \mathbb{R}^n$ is the state vector of the i-th response node of the CDN with $y_i(s) = \varphi_i(s)$ for any $s \in [-\tau, 0]$ being the initial condition; $f(\cdot,\cdot), g(\cdot,\cdot): \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ are nonlinear vector-valued functions; $u_i(t) \in \mathbb{R}^n$ denotes the desired control input to the i-th response node; $\tau > 0$ denotes the constant delay; $\omega(t) \in \mathbb{R}^n$ denotes an n-dimensional Brownian motion; $\Gamma = (\gamma_{ij}) \in \mathbb{R}^{n\times n}$ is a positive inner coupling matrix; c and $\tilde{c}$ denote the prescribed coupling strengths of the non-delayed and delayed state terms, respectively; $b_{ij} \geq 0$ and $d_{ij} \geq 0$, $i \neq j$, denote the connection weights among the interacting nodes.

Our primary objective for the drive-response system above is to design an admissible control law $u_i(t)$, $i \in \{1,2,\ldots,N\}$ such that under the co-existence of the time delay and stochastic noise, all the response nodes in (2) can exponentially synchronize with the corresponding drive nodes in (1) in a mean square sense; namely, there exist constants $\lambda_1 > 0$ and $\lambda_2 > 1$ such that [12]: $\mathbb{E}\|e(t)\|^2 \leq \lambda_2 e^{-\lambda_1 t} \mathbb{E}\|\xi\|_\tau^2$ holds for any $t \geq 0$ and $\xi \in PC^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$, where $e(t)$ denotes the stacked form of $e_i(t)$, i.e., $e(t) = [e_1(t), e_2(t), \ldots, e_N(t)]^T$ with $e_i(t) = y_i(t) - x_i(t)$ representing the synchronization error of the i-th node.

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In this study, we propose a hybrid impulsive controller of the following form for each response node i, $i \in \{1,2,\ldots,N\}$:

$$u_i(t) = \sum_{k=1}^{\infty}\Big(Le_i(t) + Ke_i(t-\tau)\Big)\delta(t-t_k^-) \tag{3}$$

where $L, K \in \mathbb{R}^{n\times n}$ denote the control gain matrices; $\{t_k | \, k \in \mathbb{N}\}$ denotes the impulse time sequence satisfying $t_k < t_{k+1}$ and $\lim_{k\to\infty} t_k = +\infty$; and $\delta(\cdot)$ is the Dirac function. Let $e_i(t_k) = e_i(t_k^+) = \lim_{k\to t_k^+} e_i(t)$ and $e_i(t_k^-) = \lim_{k\to t_k^-} e_i(t)$. To facilitate the subsequent analysis, we let $\beta_0 \le t_k - t_{k-1} \le \beta_1$ for any $k \in \mathbb{N}$, where β_0,β_1 denote the bounds of the impulsive intervals.

Remark 1: In particular, if $K = \mathbf{0}$, the controller (3) reduces to a non-delayed impulse controller which has been intensively employed in the literature; see, e.g., [13], [14]. On the other hand, if $L = \mathbf{0}$, then the controller becomes a completely delayed impulse controller, which seems more practical. This is because the drive nodes are often remotely located from the response nodes, meaning that the drive nodes' states $\{x_i(t)\}$ may experience transmission delays (say τ) when arriving at the corresponding response nodes. As a result, the delayed error $e_i(t-\tau)$ should not be neglected during the construction of an impulsive synchronization controller.

Substituting (3) into (2) and combining (1) yield the following synchronization error system:

$$\begin{aligned} de_i(t) &= \Big(F(t) + c\sum_{j=1}^{N} b_{ij}\Gamma e_j(t) + \tilde{c}\sum_{j=1}^{N} d_{ij}\Gamma e_j(t-\tau)\Big)dt \\ &\quad + G(t)d\omega(t) \\ e_i(t_k) &= (L+I_n)e_i(t_k^-) + Ke_i(t_k^- - \tau), \;\; k \in \mathbb{N} \end{aligned} \tag{4}$$

for any $i \in \{1,2,...,N\}$, where $e_i(s) = \xi(s) = \phi_i(s) - \varphi_i(s)$, $s \in [-\tau, 0]$, $F(t) = f(y_i(t), y_i(t-\tau)) - f(x_i(t), x_i(t-\tau))$, and $G(t) = g(y_i(t), y_i(t-\tau)) - g(x_i(t), x_i(t-\tau))$.

Main results: The following theorem states a sufficient condition for the mean square exponential stability of the delayed stochastic synchronization error system (4).

Theorem 1: Given scalars $\beta_1 > \beta_0 > 0$, $\alpha_1 \in \mathbb{R}$, $\alpha_2 \ge 0$, $q > 1$, $\nu \in (0,1)$, $\varepsilon_1 > 0$, positive Lipschitz constants λ_1, λ_2, γ_1, γ_2, and matrices L, K, if there exist positive definite matrices P and Q such that the following inequalities hold:

$$\begin{bmatrix} \Phi & I_n\otimes P & \tilde{c}D\otimes P\Gamma \\ * & -I_n\otimes Q & 0 \\ * & * & \Omega \end{bmatrix} \le 0 \tag{5}$$

$$[(L+I_n)+K]^T P[(L+I_n)+K] - \nu P \le 0 \tag{6}$$

where $\Phi = cB\otimes P\Gamma + c(B\otimes P\Gamma)^T + I_N\otimes Y_1QY_1 + I_N\otimes L_1QL_1 - \alpha_1 I_N \otimes P$, $B = (b_{ij})$, $\Omega = I_N\otimes L_2QL_2 + I_N\otimes Y_2QY_2 - \alpha_2 I_N\otimes P$, $D = (d_{ij})$, $L_1 = \lambda_1 I_N$, $L_2 = \lambda_2 I_N$, $Y_1 = \gamma_1 I_N$, $Y_2 = \gamma_2 I_N$, and further the following condition is satisfied:

$$\begin{aligned} \mathbf{e}^{\alpha\beta_1} < \min\{q, [(1+\varepsilon)\nu + 6\|K\|^2\mathbf{e}^{2\lambda\tau}(1+1/\varepsilon) \\ (c_2/c_1)(K_1\tau + K_2 + l^2\bar{h})]^{-1}\} \end{aligned} \tag{7}$$

where $c_1 = \lambda_{\min}(P), c_2 = \lambda_{\max}(P)$, $\alpha = \alpha_1 + \alpha_2 q < 0$, $K_1 = \|\lambda_1 I_n + cB\otimes\Gamma\|^2 + \|\lambda_2 I_n + cD\otimes\Gamma\|^2$, $K_2 = \gamma_1^2 + \gamma_2^2$, $\bar{h} = \max\{\|L\|^2, \|K\|^2\}$, l is a positive constant satisfying $l\beta_0 \le \tau < (l+1)\beta_0$, then the synchronization error system (4) is mean square exponentially stable.

Proof: Set $\zeta(t) = [e^T(t), e^T(t-\tau)]^T$ and choose the Lyapunov function candidate as $V(t) = e^T(t)Pe(t)$. For $\forall t \in [t_{k-1}, t_k)$, $k \in \mathbb{N}$, by using Itô formula, we have $\mathcal{L}V(t) = 2\sum_{i=1}^N e_i^T(t)P(F(t) + c\sum_{j=1}^N b_{ij}\Gamma e_j(t) + \tilde{c}\sum_{j=1}^N d_{ij}\Gamma e_j(t-\tau)) + Tr(G^T(t)PG(t)) \le \sum_{i=1}^N [e_i^T(t)PQ^{-1}Pe_i(t) + e_i^T(t) L_1QL_1e_i(t) + e_i^T(t-\tau)L_2QL_2e_i(t-\tau) + 2ce_i(t)^T\sum_{j=1}^N b_{ij}P\Gamma e_j(t) + \tilde{c}\times e_i^T(t)\sum_{j=1}^N d_{ij}P\Gamma e_j(t-\tau) + e_i^T(t)Y_1QY_1e_i(t) + e_i^T(t-\tau)Y_2QY_2e_i(t-\tau)]$, where $Tr(\cdot)$ denote the matrix trace. Then by (5), we have $\mathbb{E}\mathcal{L}V(e(t), e(t-\tau)) \le \zeta^T(t)\Lambda\zeta(t) + \alpha_1\mathbb{E}e^T(t)Pe(t) + \alpha_2\mathbb{E}e^T(t-\tau)Pe(t-\tau) \le \alpha_1\mathbb{E}V(e(t)) + \alpha_2\mathbb{E}V(e(t-\tau)) \le \alpha\mathbb{E}V(e(t))$ whenever $\mathbb{E}V(e(t+\theta)) \le q\mathbb{E}V(e(t))$, $\theta \in [-\tau, 0]$, where $\Lambda = (\Lambda_{ss})$ is a symmetric 2-by-2 block matrix with its entries given by $\Lambda_{11} = PQ^{-1}P\otimes I_n + cB\otimes P\Gamma + c(B\otimes P\Gamma)^T + I_n\otimes L_1QL_1 + I_n\otimes Y_1QY_1 - \alpha_1 I_n\otimes P$, $\Lambda_{12} = \tilde{c}D\otimes P\Gamma$ and $\Lambda_{22} = I_n\otimes L_2QL_2 + I_n\otimes Y_2QY_2 - \alpha_2 I_n\otimes P$.

If the condition (7) holds, then there exist λ, $\sigma > 0$ such that

$$\mathbf{e}^{(\lambda+\alpha)\beta_1} < \sigma < \min\{q\mathbf{e}^{-\lambda\tau}, \eta\} \tag{8}$$

where $\eta = [(1+\varepsilon_2)\nu + 6\|D_k\|^2\mathbf{e}^{2\lambda\tau}(1+1/\varepsilon_2)(\frac{c_2}{c_1})(K_1\tau + K_2\tau + l^2\bar{h})]^{-1}$.

Define a function as $W(t) = \mathbf{e}^{\lambda t}V(t)$, $t \ge -\tau$. Choose a small enough $\Delta t > 0$ such that for $\forall t \in [t_{k-1}, t_k)$, we have $t + \Delta t \in (t_{k-1}, t_k)$. Then by Dynkin's formula, one has $\mathbb{E}W(t+\Delta t) - \mathbb{E}W(t) = \int_t^{t+\Delta t}\mathbb{E}\mathcal{L}W(s)ds$, where $\mathcal{L}W(s) = \mathbf{e}^{\lambda s}[\lambda V(s) + \mathcal{L}V(s)]$, which implies that $D^+\mathbb{E}W(t) = \mathbb{E}\mathcal{L}W(t, e(t), e(t-\tau))$, where $D^+\mathbb{E}W(t) = \limsup_{h\to 0^+}\frac{\mathbb{E}W(t+h) - \mathbb{E}W(t)}{h}$. For given $\xi \in PC_{\mathcal{F}_0}^b([-\tau, 0]; \mathbb{R}^n)$, the following inequality holds: $\mathbb{E}W(t) \le c_2\mathbb{E}\|\xi\|_\tau^2 < \sigma c_2\mathbb{E}\|\xi\|_\tau^2$ for any $t \in [-\tau, 0]$, where $c_2 = \lambda_{\max}(P)$.

Next, we shall show that $\mathbb{E}W(t) \le \sigma c_2\mathbb{E}\|\xi\|_\tau^2$, $t \ge 0$. We first prove that $\mathbb{E}W(t) \le \sigma c_2\mathbb{E}\|\xi\|_\tau^2$ holds for any $t \in [0, t_1)$. The proof is conducted by contradiction. Suppose that there exists a $t \in (0, t_1)$ such that $\mathbb{E}W(t) \ge \sigma c_2\mathbb{E}\|\xi\|_\tau^2$. Set $t^* = \inf\{t \in [0, t_1) : \mathbb{E}W(t) \ge \sigma c_2\mathbb{E}\|\xi\|_\tau^2\}$. Due to the continuity of $\mathbb{E}W(t)$ on $t \in [0, t_1)$, it can be deduced that $t^* \in (0, t_1)$ and $\mathbb{E}W(t^*) = \sigma c_2\mathbb{E}\|\xi\|_\tau^2$, $\mathbb{E}W(t) < \sigma c_2\mathbb{E}\|\xi\|_\tau^2$, $t \in [-\tau, t^*)$. Set $t^{**} = \sup\{t \in [0, t^*) : \mathbb{E}W(t) \ge c_2\mathbb{E}\|\xi\|_\tau^2\}$, it can be obtained $\mathbb{E}W(t^{**}) = c_2\mathbb{E}\|\xi\|_\tau^2$, and $\mathbb{E}W(t) > c_2\mathbb{E}\|\xi\|_\tau^2$, $t \in (t^{**}, t^*]$. From above mentioned, for $t \in [t^{**}, t^*]$, $\theta \in [-\tau, 0]$, we have $\mathbb{E}W(t+\theta) \le \sigma c_2\mathbb{E}\|\xi\|_\tau^2 \le \sigma\mathbb{E}W(t)$, which means that $\mathbb{E}V(t+\theta) \le \sigma\mathbf{e}^{\lambda\tau}\mathbb{E}V(e(t)) \le q\mathbb{E}V(e(t))$. Then for $t \in [t^{**}, t^*]$, we get $D^+\mathbb{E}W(t) = \mathbb{E}\mathcal{L}W(t) \le (\lambda+\alpha)\mathbb{E}W(e(t))$, which implies that $\mathbb{E}W(t^*) \le \mathbb{E}W(t^{**})\mathbf{e}^{(\lambda+\alpha)(t^*-t^{**})} \le \mathbf{e}^{(\lambda+\alpha)\beta_1}c_2\mathbb{E}\|\xi\|_\tau^2 < \sigma c_2\mathbb{E}\|\xi\|_\tau^2$. This is a contradiction, thus $\mathbb{E}W(t) \le \sigma c_2\mathbb{E}\|\xi\|_\tau^2$ holds for any $t \in [0, t_1)$. Assume for $\forall m \ge 1$, $m \in \mathbb{N}$,

$$\mathbb{E}W(t) \le \sigma c_2\mathbb{E}\|\xi\|_\tau^2, \quad t \in [-\tau, t_m). \tag{9}$$

We next prove that $\mathbb{E}W(t) < \sigma c_2\mathbb{E}\|\xi\|_\tau^2$, $t \in [t_m, t_{m+1})$. Due to $\{t_k - t_{k-1}\} \ge \beta_0$, there exists $l \in \mathbb{N}$ such that $\tau \in [l\beta_0, (l+1)\beta_0)$. Assume there exists impulsive instant sequence t_{m_i}, $i = 1,2,\ldots,i_0$, $i_0 \le l$ on $[t_m - \tau, t_m)$. By (9), we can obtain $\mathbb{E}|e_i(t_m^-) - e_i((t_m-\tau))^-)|^2 = \mathbb{E}|\int_{t_m-\tau}^{t_m} [F(e_i(s), e_i(s-\tau)) + c\sum_{j=1}^N b_{ij}\Gamma e_j(s) + \tilde{c}\sum_{j=1}^N d_{ij}\Gamma e_j(s-\tau)]ds + \int_{t_m-\tau}^{t_m}[G_i (e_i(s), e_i(s-\tau))]\,dw(s) - \sum_{i=1}^{i_0}\Delta e(t_{m_i})|^2 \le 6\mathbf{e}^{2\lambda\tau}(K_1\tau + K_2 + l^2\bar{h})(\frac{\sigma c_2}{c_1}) \mathbb{E}\|\xi\|_\tau^2\mathbf{e}^{-\lambda t_m}$.

Denote $\varsigma_m(x,y) = Lx + Ky$ for any $x, y \in \mathbb{R}^n$ and let $\Delta\varsigma_m = \varsigma_m(e(t_m^-), e((t_m-\tau)^-)) - \varsigma_m(e(t_m^-), e(t_m^-))$. Then one has $\mathbb{E}|\Delta\varsigma_m|^2 = \mathbb{E}|Ke((t_m-\tau)^-) - Ke(t_m^-)|^2 \le \|K\|^2\mathbb{E}|e(t_m^-) - e((t_m-\tau)^-)|^2 \le 6\|K\|^2\mathbf{e}^{2\lambda\tau}(K_1\tau + K_2 + l^2\bar{h}) (\sigma c_2/c_1)\mathbb{E}\|\xi\|_\tau^2\mathbf{e}^{-\lambda t_m}$. By (6), we obtain that $\mathbb{E}V(\varsigma_m(e(t_m^-), e(t_m^-))) = \mathbb{E}e^T(t_m^-)(L + I_n + K)^T P(L + I_n + K)e(t_m^-) \le \nu\mathbb{E}e^T(t_m^-)Pe(t_m^-) = \nu\mathbb{E}\times V(e(t_m^-))$.

Recalling (8) and summarizing the above yield that $\mathbb{E}W(t_m) = \mathbf{e}^{\lambda t_m}\mathbb{E}V(e(t_m)) = \mathbf{e}^{\lambda t_m}\mathbb{E}V(\varsigma_m(e(t_m^-), e(t_m^-)) + \Delta\varsigma_m) \le \mathbf{e}^{\lambda t_m}(1+\varepsilon)\mathbb{E}V(\varsigma_m (e(t_m^-), e(t_m^-))) + \mathbf{e}^{\lambda t_m}(1+1/\varepsilon)\mathbb{E}V(\Delta\varsigma_m) \le c_2\mathbb{E}\|\xi\|_\tau^2 < \sigma c_2\mathbb{E}\|\xi\|_\tau^2$. Then, we obtain that $\mathbb{E}W(t) \le \sigma c_2\mathbb{E}\|\xi\|_\tau^2$, $t \in [t_m, t_{m+1})$. By induction, we have $\mathbb{E}W(t) \le \sigma c_2\mathbb{E}\|\xi\|_\tau^2$, $t \ge 0$, which means that $\mathbb{E}\|e(t)\|^2 \le \frac{\sigma c_2}{c_1}\mathbb{E}\|\xi\|_\tau^2\mathbf{e}^{-\lambda t}$ for any $t \ge 0$. ■

Remark 2: Two notable features of our main result Theorem 1 are stated below. First, it discloses the explicit relationship between the nonlinear dynamics (Lipschitz constants λ_1, λ_2, γ_1, γ_2) and impulsive controllers (gains L, K and impulsive interval bounds β_0,β_1). Second, different from [9]–[11] where delayed impulses are handled as "interference", a better impulse estimation is provided in the proof, through which the stability criterion in the case of delayed impulsive control only can be easily established.

As a byproduct, we next consider a completely delayed impulsive synchronization controller $u_i(t) = \sum_{k=1}^{\infty}(Ke_i(t-\tau))\delta(t-t_k^-)$. The following conclusion can be made where the proof follows a similar procedure as that in Theorem 1 and is thus omitted.

Theorem 2: Given scalars $\beta_1 > \beta_0 > 0$, $\alpha_1 \in \mathbb{R}$, $\alpha_2 \ge 0$, $q > 1$,

$\nu \in (0,1)$, $\varepsilon_1 > 0$, positive Lipschitz constants λ_1, λ_2, γ_1, γ_2, and matrices L, K, if there exist positive definite matrices P and Q such that (5), $(I_n+K)^T P(I_n+K) - \nu P \le 0$, and further (7) are satisfied, where $\bar{h}$ therein is modified as $\bar{h} = \max\{\|I_n\|^2, \|K\|^2\}$, then the synchronization error system (4) is mean square exponentially stable.

An illustrative example: We next present a numerical example to demonstrate the effectiveness of the proposed theoretical results. Assume that there are two nodes, i.e., $N = 2$, and the nonlinear functions take the following forms: $f(x,y) = A_1\tanh(x) + A_2\tanh(y)$, $g(x,y) = B_1\tanh(x) + B_2\tanh(y)$ with $A_1 = [0.1, 0; 0.1, 1]$, $A_2 = [0.2, -0.3; 0.1, 0.7]$, $B_1 = [0.2, 0.5; -0.5, -0.3]$, $B_2 = [0.1, -0.3; 0.4, 1.8]$. The Lipschitz constants are set as $\lambda_1 = 0.7$, $\lambda_2 = 0.8$, $\gamma_1 = 0.61$, $\gamma_2 = 0.5$. Set the parameters $c = 0.5$, $\tilde{c} = 1$, $\tau = 0.01$, $B = (b_{ij}) = [-1, 1; 1, -1]$, $D = (d_{ij}) = [-2, 2; 1, -1]$, $\Gamma = I_2$, $L = 0.05 I_2$, $K = -0.2 I_2$, $\alpha_1 = 7.5, \alpha_2 = 1.5$, and $\nu = 0.75$. Solving (5) and (6) in Theorem 1, we get that $P = [0.2245, 0.0622; 0.0622, 0.2146]$ and $Q = [0.8784, 0.0577; 0.0577; 0.8752]$. Furthermore, choosing $q = 2$, $\varepsilon = 0.1$, $\beta_0 = 0.01$, $\beta_1 < 0.0791$, it is easily verified that (7) in Theorem 1 holds. We next arbitrarily select an initial state $\xi = [8.2, 3.6]^T$, set the impulse time interval $t_{k+1} - t_k = 0.02$ s, the synchronization errors of the drive-response network without control and with the proposed hybrid impulsive controller (3) are shown in Figs. 1(a) and 1(b), respectively. For a comparison purpose, Theorem 2 is also explored under the same K above and yields $P = [0.1936, 0.0580; 0.0580; 0.1936]$ and $Q = [0.8605, 0.0547; 0.0547, 0.8605]$. The generated synchronization error of the drive-response network with the only delayed impulsive controller is depicted in Fig. 1(c). From Fig. 1, it can be seen that the proposed hybrid and delayed only impulsive controllers can both effectively regulate the response nodes to synchronize with the drive nodes in the network.

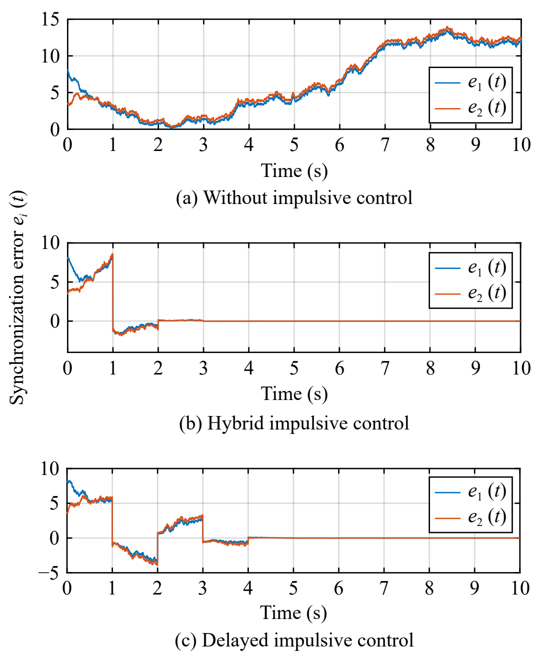

Fig. 1. The resulting synchronization errors: (a) Without impulsive control; (b) With hybrid impulsive control; (c) With only delayed impulsive control.

Conclusion: In this letter, we have presented a hybrid delayed and non-delayed impulsive control method to solve the synchronization problem of a class of delayed stochastic complex networks. We have established explicit sufficient conditions on the exponential stability in mean square for the resulting synchronization error system even in the presence of time delays and stochastic noises. We have also studied the special case of completely delayed impulsive control and derived the corresponding stability criterion. We have finally provided a numerical example to validate the proposed main results.

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