Tomographic reconstruction of Cerenkov photons in tissues through approximate message-passing

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ABSTRACT

Solution with adjustable sparsity to tomographic imaging of Cerenkov photons is presented in this work. The sparsity of radionuclides' distribution in tissues is an objective but unknown fact, and the inverse model of qualitative data is an ill-posed problem. Based on the optimization technique, the uniqueness of numerical solution to the ill-conditioned compact operator can be guaranteed by use of sparse regularization with the approximate message-passing (AMP) method. After absorbing formulations with the AMP, we analyzed the behavior of the hard thresholding operator. Iteratively numerical solutions were used to approximate the real light source by assuming the number of non-zero solution in manual mode. This modified AMP algorithm was performed in numerical simulation and physical experiments with 2-[18F]fluoro-2-deoxy-D-glucose. Experimental results indicated that the proposed method was a kind of low-complexity iterative thresholding algorithms for reconstructing 3D sparse distribution from a small set of optical measurements.

Keywords: Tomography, Luminescence, Hard thresholding operator, Approximate message-passing, ^{18}F-FDG

1. INTRODUCTION

Recent research progresses of Cerenkov luminescence and tomography imaging has established a class of *in vivo* molecular imaging methods[1–5] with optical techniques and radiolabeled biomarkers.[6] Although only small amounts were injected with tumor-specific and tumor cell signal-specific positron emission tomography/single-photon emission computed tomography (PET/SPECT) radiopharmaceuticals in preclinical and clinical areas for diagnosis and/or therapy, the number of radiated photons was larger enough to be detected with the scientific-grade charge-coupled device (CCD) after the energy conversion. This transforming relies on the known quantum Vavilov-Cerenkov radiation.[7–14] With the temporary-spatial information, two-dimensional planar or three-dimensional tomographic images clearly give a quantitative representation of targeting distribution.[15–21] Compared planar imaging, Cerenkov luminescence tomography (CLT) has the following outstanding benefits and advantages: achieved depth information of the source; possibility and interface of three-dimensional integration for functional, molecular and structural information. The reconstruction accuracy and cost mutate into an open and challenging problem for CLT imaging because of its ill-condition.

Considering linear ill-conditioned systems, sparse regulation provides an effective routine for the mobility of radionuclides and uptake by organisms. Although the L_p minimization has attracted attention for its success in solving such underdetermined systems, their sparsity-undersampling tradeoff needs a further improvement.[22] For this propose, an approximate message-passing (AMP) method was proposed by applying sum-product belief propagation for a suitable joint distribution over the unknown variables. The AMP-style iterations were practical for large scale use, of which the soft threshold recursion increased computational complexity.[23, 24] In this paper, we introduced a improved AMP algorithm using iterative hard thresholding,[25–27] which could converge to a k-sparse vector. The experiment and results were followed at details by use of digital mouse and physical phantoms. A discussion and conclusion was presented in the conclusion section.

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2. METHODOLOGY

CLT is a medical imaging modality that calculates three-dimensional (3D) maps of radioisotopes' distribution with optical absorption and scattering coefficients in biological tissue by using a radiative transfer model for visible Cerenkov light.

Considering both computational cost and accuracy, the third-ordered simplified spherical harmonics equations yield analytically a suitable solution to the radiative transfer equation (RTE).[28–31]The phenomenological foundations and radiative transfer theory were developed for optical tomography.[32–37]We get the RTE approximated fundamental integro-differential equation for the radiance ϕ from Maxwell's equations:

$$\begin{aligned} \{\hat{s}\cdot\nabla+\mu_a+\mu_s+\frac{\partial}{\partial t}\}\phi(r,\hat{s},t) \; &= \; q(r,\hat{s},t)+ \\ & \mu_s\int(p(\hat{s}',\hat{s})\phi(r,\hat{s}',t))d\Omega'. \end{aligned} \tag{1}$$

After the finite element discretization and derivation, the inverse problem can be defined as an underdetermined linear equations: $AS = B$. A was a $m \times n$ dimensional matrix; S was an n-dimensional unknown vector denoting the isotope distribution in the medium; and B is the m-dimensional vector denoting the measurements. In order to obtain a reasonable solution to S, the modified AMP algorithm[22] was described in the following form

$$\begin{cases} S^{k+1} &= H^k(S^k+\lambda A^T D^k) \\ D^{k+1} &= B - AS^{k+1} + \frac{\|H^k\|_0}{m} \end{cases}, \tag{2}$$

where H^k was the k-th iteratively nonlinear operator that set all but the largest (in magnitude) N elements of to zero. The 0-norm operator, $\|H^k\|_0$, is the total number of nonzero entries in H^k. The initial values of S were set to zeros before the iterative calculation. The number of iterations should be set in advance using empirical data or be determined dynamically depending on $\eta := \|S^{k+1}-S^k\|_2^2/\|S^k\|_2^2$, additionally.

3. RESULTS

The proposed CLT algorithm was coded in C++, and performed with the simulation and physical data on a personal computer (Intel^R Core^{TM} i5 CPU 650 @ 3.2 GHz and 3 GB RAM). The initial value of the solution was set to a zero vector, and the iterative parameter settings was unchanged in the simulation and verification processing; e.g., $N/n = 2.5\%$; maximum number of iterations $k = 500$; $\lambda = 1$; $\eta = 10^{-7}$.

3.1 Simulation experiment

The digimouse[20] was discretized into a tetrahedral mesh (4245 points, 26859 edges, 44085 triangles and 214740 tetrahedrons) for CLT reconstruction as shown in Fig. 1A. A spherical light sources of 1 mm in radius with a total power of 1 nW was placed at (17 mm, 31.6 mm, 38.4 mm) in the heart, and the same kind of light source was at the point of (21.3 mm, 23.6 mm, 50.8 mm) in the stomach. The number of photons was set to 10^6 for each one light source with the shortest distance 3 mm from the surface. The elapsed time for the whole body CLT reconstruction was 380.0 s (m = 1147, n = 21470). The locations of the local peak values from reconstructed intensity were at the point of (16.9 mm, 31.4 mm, 38.0 mm) and (21.3 mm, 23.4 mm, 50.7 mm), which fell within the unit sphere (Fig. 1B and 1C).

3.2 Physical experiment

A real experiment was performed with 2-deoxy-2-(18F)fluoro- D-glucose (^{18}F-FDG). A phantom was designed to simulate the abdomen with muscle, heart, lungs, and bone of small animals (Fig. 2A). The phantom was a cylinder (diameter, 30 mm; height, 30 mm), containing two cylinder holes (diameter, 2 mm; height, 3 mm; center, (23.5 mm, 14.0 mm, 18.5 mm) and (29.5 mm, 14.9 mm, 18.5 mm)). The left hole was filled with a solution containing a total of 2.2 MBq ^{18}F-FDG, and the right hole is with injection of 1.6 MBq ^{18}F-FDG (Fig. 2A). The elapsed time for the whole body CLT reconstruction was 308.1 s (m = 1578, n = 21598). The locations of the local peak values from reconstructed intensity were at the point of (23.8 mm, 15.1 mm, 17.8 mm) and (30.0 mm, 23.4 mm, 17.8 mm), which fell within two cylinders (Fig. 2B).

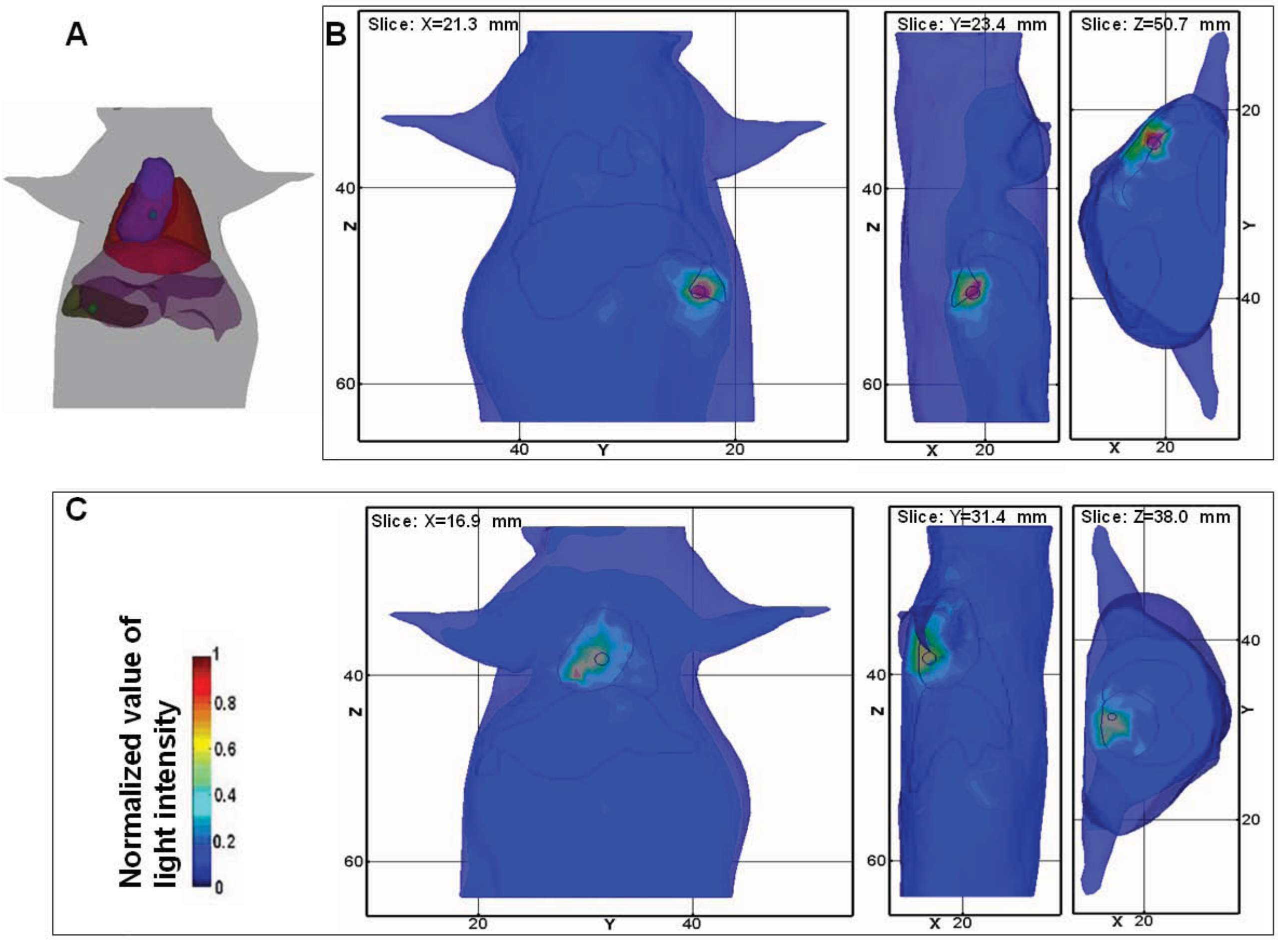

Figure 1. Numerical case for CLT reconstruction. A is the tetrahedral mesh including five organs with different colors; B and C show results with two light sources via different slices in the same experiment, where the black cures stand for the peripheral contour of tissues. The maximum value of CLT reconstruction was normalized to 1.

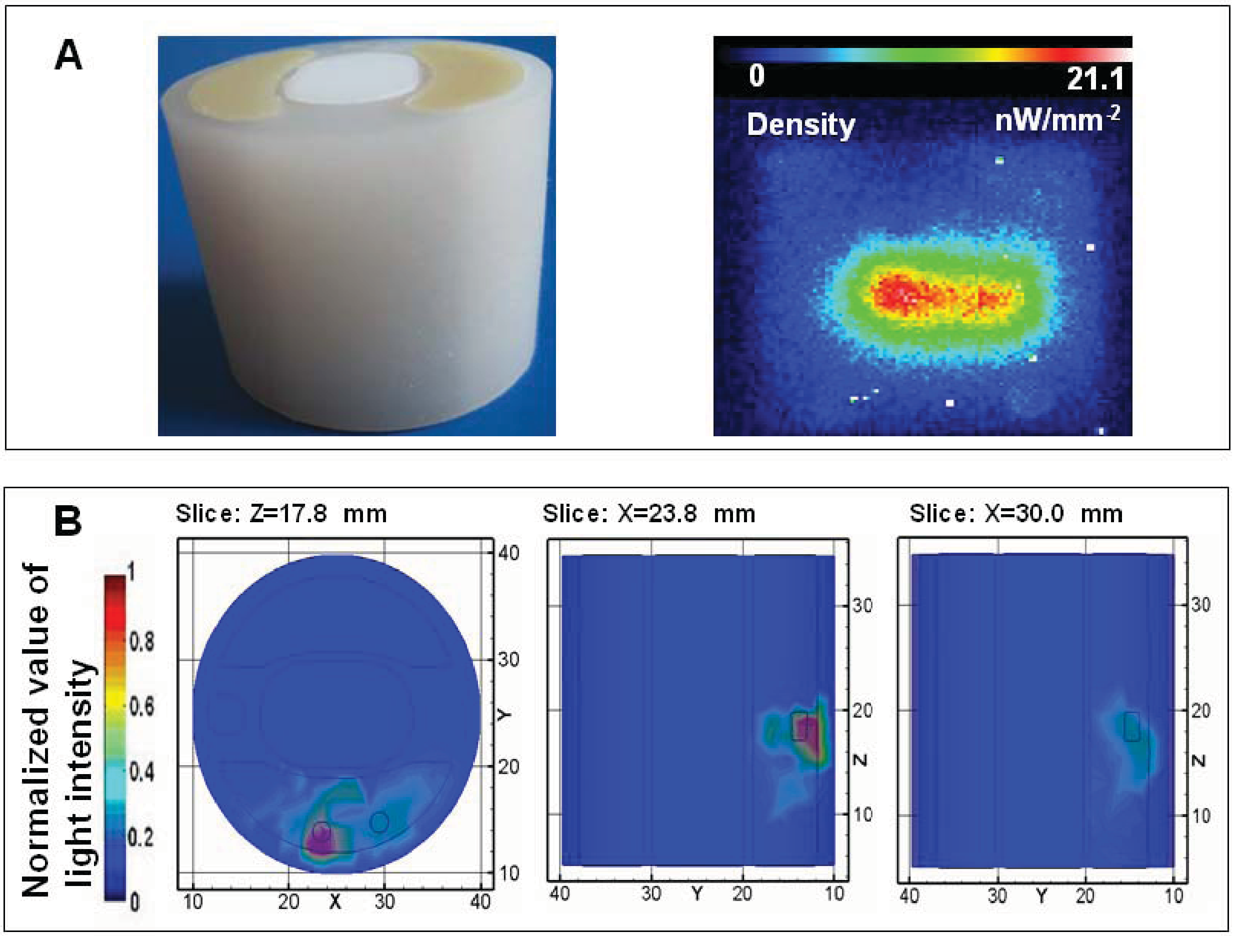

Figure 2. Physical case for CLT reconstruction. A is the phantom with two cylinder holes and Cerenkov luminescence imaging; B shows results with two light sources via different slices in the same experiment, where the black cures stand for the peripheral contour of tissues. The maximum value of CLT reconstruction was normalized to 1.

4. CONCLUSION

CLI and CLT are applied in basic research and preclinical imaging, and attempts to clinical application are under way.[38,39] All of these calls for the further development of imaging technology and date method. Our observation is the effectiveness and convergence of the iteratively hard thresholding algorithm with the approximate message-passing method for CLT problem. Although prof. David L. Donoho has provided a complete analysis and validation of AMP method, we again performed it with Cerenkov optical data. When the sparsity of the source vector $N/n = 2.5\%$, the dual-source reconstruction results were accurate in both numerical and physical experiments. When the sparsity was preset to 0.1%, the elapsed time for CLT reconstruction separately became 362.8 s in the inverse simulation and 280.1 s in the phantom experiment. When the sparsity ratio N/n was set to the smaller, the iteration time was shorter.

In conclusion, we demonstrated the validity of the modified AMP regulation method to inversely approximate the radioisotopes distribution with Cerenkov photons. The tomographic reconstruction can converge to a unique and sparse solution by use of a adjustable hard-thresholding, which should expand the application scope of the message passing algorithms into optical molecular imaging.

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