

A Maneuver Controller with Disturbance Attenuation for Aircraft

Ruyi Yuan¹, Jianqiang Yi¹, Guoliang Fan¹, Wensheng Yu^{1,2}, Liwei Qiu¹

Abstract—In this paper, a maneuver controller for a nonlinear six-degree-of-freedom aircraft is designed, which makes angle-of-attack, sideslip angle, and roll rate asymptotically track the reference trajectories respectively. The design procedures for angle-of-attack and sideslip angle are based on backstepping method. Appropriate cost functions for angle-of-attack and sideslip angle are constructed to satisfy Hamilton-Jacobi-Isaacs (HJI) equations, thereby the tracking of the angle-of-attack and the sideslip angle attains a desired disturbance attenuation level. Simulation results on a nonlinear aircraft model are given to illustrate the performance of the maneuver controller.

I. INTRODUCTION

With the increasing performance requirements for modern aircraft, conventional flight control designs are difficult to build a well performed controller because they lack the ability to deal with the existence of high nonlinearity, strong couple between longitudinal and lateral motion, model uncertainty. Nonlinear dynamic inversion (NDI) is a widely used modern control method in flight control system. NDI cancels system nonlinearity using coordinate transformation and feedback linearization, decouples the system into a linear form. NDI assumes that full information of system model is perfectly known, otherwise this cancelation and decouple are approximate. This shortcoming is more obvious when NDI is used in flight control, so a lot of efforts have been paid to develop robust flight control methods based on NDI [1–4]. There also exists the problem called non-minimum phase or unstable zero dynamics when NDI is applied to some nonlinear systems [5]. For these nonlinear systems, straightforward application of NDI may cause systems with linear input-output form but unstable zero dynamics. When NDI is used to design a flight controller, the non-minimum phase phenomenon results from either the choice of output vector or the small aerodynamic forces generated by control surfaces deflections [6], [7]. This makes the stable tracking design a challenge problem.

Backstepping is a Lyapunov-based recursive design method proposed by Kokotović etc. in 1991 [8], which has attracted a great deal of attentions from then on. A lot of research papers have been published to investigate the fundamental theory and application of backstepping in nonlinear systems [9–11]. Backstepping design provides a systematic method to constructive Lyapunov for nonlinear

systems with low-triangular form, and doesn't require the precise system model, so many papers have already discussed using backstepping method to design robust flight control law [12–18]. [14] designed a backstepping controller for aircraft longitudinal control with the existence of control input saturations. [15] used a command backstepping to design flight controller, which removed the low-triangular form restriction and simultaneously taken account of the rate and magnitude constraints of control surfaces. Block backstepping designs for flight control were considered in [16–18].

In the presence of model or parameters uncertainty and extern disturbances in a nonlinear system, how to design a controller that makes the system stable and simultaneously attains the desired level of disturbance attenuation, motivates the study of nonlinear H^∞ control theory [19]. The difficulty of applying nonlinear H^∞ comes from the difficulty to solve the Hamilton-Jacobi-Bellman (HJB) or the more general Hamilton-Jacobi-Isaacs (HJI) partial differential equation [10], [19]. So a lot of researches have focused on finding a solution to HJI equation without solving it directly. By using the freedom in the choice of the cost function in backstepping, [20] gave a tracking and disturbance attenuation controller for a class of parametric strict-feedback nonlinear systems. Under this controller, the closed-loop system satisfied the HJI equation associated with a cost function, thereby guaranteeing tracking and a desired level disturbance attenuation performance.

In this paper, based on the controller proposed in [19], a maneuver controller is designed for a nonlinear six-degree-of-freedom aircraft. Backstepping approach is used in the design procedure. This controller makes the angle-of-attack, sideslip angle, and roll rate track the reference trajectories in the presence of disturbance. Appropriate cost functions for angle-of-attack and sideslip angle are constructed to satisfy HJI equations, thereby the tracking of angle-of-attack and sideslip angle attains a desired level disturbance attenuation. The validity of the proposed controller is evaluated using a nonlinear aircraft model. The rest of this paper is organized as follows: The dynamic model of the aircraft is described in section II. In section III, we give the design procedure of the maneuver controller. Simulation results are given in section IV. Conclusions are drawn in section V.

II. AIRCRAFT MODEL DESCRIPTION

The dynamic equations for a six-degree-of-freedom aircraft are as follows [21]

1. Institute of Automation, Chinese Academy of Sciences, Beijing, China {ruyi.yuan, jianqiang.yi}@ia.ac.cn

2. Shanghai Key Laboratory of Trustworthy Computing, East China Normal University, Shanghai, China

$$\begin{aligned}
\dot{V}_T &= \frac{1}{m}(-D + F_T \cos \alpha \cos \beta + mg_1) \\
\dot{\alpha} &= q - (p \cos \alpha + r \sin \alpha) \tan \beta \\
&\quad + \frac{1}{mV_T \cos \beta}(-L - F_T \sin \alpha + mg_3) \\
\dot{\beta} &= p \sin \alpha - r \cos \alpha \\
&\quad + \frac{1}{mV_T}(Y - F_T \cos \alpha \sin \beta + mg_2) \\
J\dot{\omega} &= -\omega \times J\omega + M \\
J &= \begin{pmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{xz} & 0 & J_z \end{pmatrix}
\end{aligned} \tag{1}$$

where m is the mass of the aircraft, V_T, α, β are velocity, angle-of-attack, sideslip angle respectively, D, Y, L, F_T are drag force, side force, lift force and thrust respectively, $\omega \stackrel{\text{def}}{=} (p, q, r)^T$ is the angular velocity vector expressed in the body axes frame, J is the inertia matrix of the aircraft, $M = (M_x, M_y, M_z)^T$ is the external torque vector in body frame. g_1, g_2, g_3 can be calculated as

$$\begin{aligned}
g_1 &= g(-\cos \alpha \cos \beta \sin \theta + \sin \beta \sin \phi \cos \theta \\
&\quad + \sin \alpha \cos \beta \cos \phi \cos \theta) \\
g_2 &= g(\cos \alpha \sin \beta \sin \theta + \cos \beta \sin \phi \cos \theta \\
&\quad - \sin \alpha \sin \beta \cos \phi \cos \theta) \\
g_3 &= g(\sin \alpha \sin \theta + \cos \alpha \cos \phi \cos \theta)
\end{aligned} \tag{2}$$

where g is the gravitational constant, θ, ϕ are the pitch angle and the roll angle respectively. D, Y, L are defined as

$$\begin{aligned}
D &= -\bar{X} \cos \alpha \cos \beta - \bar{Y} \sin \beta - \bar{Z} \sin \alpha \cos \beta \\
Y &= -\bar{X} \cos \alpha \sin \beta + \bar{Y} \cos \beta - \bar{Z} \sin \alpha \sin \beta \\
L &= \bar{X} \sin \alpha - \bar{Z} \cos \alpha
\end{aligned} \tag{3}$$

where $\bar{X}, \bar{Y}, \bar{Z}$, are the aerodynamic forces in the body frame. The aerodynamic coefficients of the aircraft are described as follows

$$\begin{aligned}
C_{xtot} &= C_x(\alpha, \delta_e) + \frac{\bar{c}q}{2V_T} C_{xq}(\alpha) \\
C_{ytot} &= -0.02\beta + 0.021 \frac{\delta_a}{21.5} + 0.086 \frac{\delta_r}{30} + \frac{br}{2V_T} C_{yr}(\alpha) \\
&\quad + \frac{bp}{2V_T} C_{yp}(\alpha) \\
C_{ztot} &= C_z(\alpha, \beta, \delta_e) + \frac{\bar{c}q}{2V_T} C_{zq}(\alpha) \\
C_{ltot} &= C_l(\alpha, \beta) + C_{l\delta_a}(\alpha, \beta) \frac{\delta_a}{21.5} \\
&\quad + C_{l\delta_r}(\alpha, \beta) \frac{\delta_r}{30} + \frac{br}{2V_T} C_{lr}(\alpha) + \frac{bp}{2V_T} C_{lp}(\alpha) \\
C_{mtot} &= C_m(\alpha, \delta_e) + C_{ztot}(x_{cgr} - x_{cg}) + \frac{\bar{c}q}{2V_T} C_{mq}(\alpha) \\
C_{ntot} &= C_n(\alpha, \beta) - \frac{\bar{c}}{b} C_{ytot}(x_{cgr} - x_{cg}) + C_{n\delta_a}(\alpha, \beta) \frac{\delta_a}{21.5} \\
&\quad + C_{n\delta_r}(\alpha, \beta) \frac{\delta_r}{30} + \frac{br}{2V_T} C_{nr}(\alpha) + \frac{bp}{2V_T} C_{np}(\alpha)
\end{aligned} \tag{4}$$

where b is the reference wing span of the aircraft, \bar{c} is the mean aerodynamic chord of the aircraft, x_{cg} is the gravity location center, x_{cgr} is the gravity reference location center. $\delta_e, \delta_a, \delta_r$ are the available aerodynamic control surfaces. The aerodynamic coefficients $C_x(\alpha, \delta_e), C_z(\alpha, \beta, \delta_e)$ etc. are obtained from low-speed static and dynamic wind-tunnel tests and have been stored in lookup tables as a function of the current flight condition (e.g., α, β , and δ_e) in the form of 1-D, 2-D, or 3-D.

The aerodynamic forces and external torques are obtained as $\bar{X} = \bar{q}SC_{xtot}, \bar{Y} = \bar{q}SC_{ytot}, \bar{Z} = \bar{q}SC_{ztot}, M_x = \bar{q}SbC_{ltot}, M_y = \bar{q}S\bar{c}C_{mtot}, M_z = \bar{q}SbC_{ntot}$, where $\bar{q} = \frac{1}{2}\rho V_T^2$ is the aerodynamic pressure, S is the reference wing area.

III. DISTURBANCE ATTENUATION MANEUVER CONTROLLER DESIGN

[20] gives a tracking and disturbance attenuation controller for a class of parametric strict-feedback nonlinear systems using the freedom in the choice of the cost function in backstepping. Based on this design procedure, we design a general maneuver controller for aircraft in this section. For general parametric strict-feedback nonlinear systems, how to design an adaptive controller which guarantees the tracking performance and attains a required level of disturbance attenuation, please refer to [20].

The common commands for general maneuver are angle-of-attack, sideslip angle, and roll rate [12], [13]. The sideslip angle always should be kept at zero in the whole maneuver process. The control objectives are summarized as follows:

$$\alpha = \alpha^{\text{ref}}, \beta = 0, p_s = p_s^{\text{ref}} \tag{5}$$

Because this controller is based on backstepping, which can only handle such systems that can be transformed into a low-triangle form, transformation and some assumptions are needed.

Firstly, the angular velocity vector $\omega_s \stackrel{\text{def}}{=} (p_s, q_s, r_s)^T$ in the stability-axes can be used to get a convenient form for controller design [12], [13]. The relationship of ω_s and ω is $\omega_s = S_\alpha \omega$, where

$$S_\alpha = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix}$$

Secondly, we neglect the forces produced by control surfaces deflection, and angular rates. Then the dynamics equations of p_s, α, β can be given as follows

$$\dot{p}_s = u_1 \tag{6}$$

$$\begin{aligned}
\dot{\alpha} &= q_s - p_s \tan \beta \\
&\quad + \frac{1}{mV_T \cos \beta}(-L(\alpha) - F_T \sin \alpha + mg_2) + w_\alpha
\end{aligned} \tag{7}$$

$$\dot{q}_s = u_2 \tag{8}$$

$$\begin{aligned}
\dot{\beta} &= -r_s + \frac{1}{mV_T}(Y(\beta) - F_T \cos \alpha \sin \beta + mg_3) + w_\beta \\
&\quad + w_\beta
\end{aligned} \tag{9}$$

$$\dot{r}_s = u_3 \tag{10}$$

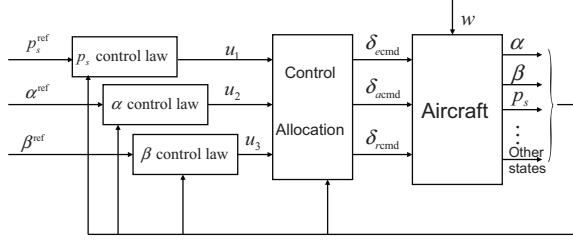


Fig. 1. Block diagram of maneuver controller.

where $u_1 = \dot{p}_s, u_2 = \dot{q}_s, u_3 = \dot{r}_s, w_\alpha, w_\beta$ are the terms due to the neglecting of aerodynamic effects, the unmodeled dynamics and the extern disturbances.

Equations (7)-(8) and (9)-(10) are suitable for backstepping design. So we first view u_1, u_2, u_3 as our control inputs, that is we design control law u_1 for roll rate p_s , control law u_2 for angle-of-attack α , control law u_3 for sideslip angle β . After $u \stackrel{\text{def}}{=} (u_1, u_2, u_3)^T$ is designed, we solve the control surfaces deflection from u using control allocation.

The block diagram of this maneuver controller is depicted in Fig.1.

A. Angle-of-attack α control law

For the control of angle-of-attack, consider equations (7)-(8)

$$\begin{aligned} \dot{\alpha} &= q_s + f_\alpha + w_\alpha \\ \dot{q}_s &= u_2 \end{aligned} \quad (11)$$

where $f_\alpha = -p_s \tan \beta + \frac{1}{mV_T \cos \beta} (-L(\alpha) - F_T \sin \alpha + mg_2)$

Step 1: Define the control error

$$z_1 = \alpha - \alpha^{\text{ref}} \quad (12)$$

The derivative of z_1 is

$$\begin{aligned} \dot{z}_1 &= \dot{\alpha} - \dot{\alpha}^{\text{ref}} \\ &= q_s - \dot{\alpha}^{\text{ref}} + f_\alpha + w_\alpha \end{aligned}$$

Define

$$a_1 = k_1 z_1 + f_\alpha \quad (13)$$

$$z_2 = q_s - \dot{\alpha}^{\text{ref}} + a_1 \quad (14)$$

where k_1 is a positive constant to be designed. Then, the derivative for z_1 can be rewritten as

$$\dot{z}_1 = z_2 - k_1 z_1 + w_\alpha$$

Step 2: Consider the derivative of z_2

$$\begin{aligned} \dot{z}_2 &= \dot{q}_s - \ddot{\alpha}^{\text{ref}} + \dot{a}_1 \\ &= u_2 - \ddot{\alpha}^{\text{ref}} + \frac{\partial a_1}{\partial z_1} (z_2 - k_1 z_1 + w_\alpha) + \frac{\partial a_1}{\partial \alpha^{\text{ref}}} \dot{\alpha}^{\text{ref}} \\ &\quad + \frac{\partial a_1}{\partial f_\alpha} \dot{f}_\alpha \end{aligned}$$

Define

$$\begin{aligned} a_2 &= k_2 z_2 + z_1 + \frac{\partial a_1}{\partial z_1} (z_2 - k_1 z_1) + \frac{\partial a_1}{\partial \alpha^{\text{ref}}} \dot{\alpha}^{\text{ref}} \\ &\quad + \frac{1}{2\gamma_\alpha^2} \frac{\partial a_1}{\partial z_1} z_1 + \frac{\partial a_1}{\partial f_\alpha} \dot{f}_\alpha \end{aligned} \quad (15)$$

where k_2 is a positive constant to be designed. Then

$$\begin{aligned} \dot{z}_2 &= u_2 - \ddot{\alpha}^{\text{ref}} - k_2 z_2 - z_1 + a_2 - \frac{1}{2\gamma_\alpha^2} \frac{\partial a_1}{\partial z_1} z_1 \\ &\quad + \frac{\partial a_1}{\partial z_1} w_\alpha \end{aligned}$$

The control law for angle-of-attack can be designed as

$$u_2 = \ddot{\alpha}^{\text{ref}} - a_2 \quad (16)$$

then

$$\dot{z}_2 = -k_2 z_2 - z_1 - \frac{1}{2\gamma_\alpha^2} \frac{\partial a_1}{\partial z_1} z_1 + \frac{\partial a_1}{\partial z_1} w_\alpha$$

The closed-loop system under the control law is described by the following differential equations

$$\begin{aligned} \dot{z}_1 &= z_2 - k_1 z_1 + w_\alpha \\ \dot{z}_2 &= -k_2 z_2 - z_1 - \frac{1}{2\gamma_\alpha^2} \frac{\partial a_1}{\partial z_1} z_1 + \frac{\partial a_1}{\partial z_1} w_\alpha \end{aligned}$$

which can be rewritten as

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = F + H w_\alpha$$

where

$$F = \begin{pmatrix} z_2 - k_1 z_1 \\ -k_2 z_2 - z_1 - \frac{1}{2\gamma_\alpha^2} \frac{\partial a_1}{\partial z_1} z_1 \end{pmatrix}, H = \begin{pmatrix} 1 \\ \frac{\partial a_1}{\partial z_1} \end{pmatrix}$$

Define a cost function $V_\alpha = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2$. We assume that before the maneuver, the aircraft is in an equilibrium state, which implies $\dot{\alpha}(t)|_{t=0} = 0$. Chosen reference trajectory α^{ref} satisfies $\alpha^{\text{ref}}(0) = \alpha(0), \dot{\alpha}^{\text{ref}}(t)|_{t=0} = 0$, then $V_\alpha(0) = 0$. Consider the Hamilton-Jacobi-Isaacs equation

$$\frac{\partial V_\alpha}{\partial z} F + \frac{1}{4\gamma_\alpha^2} \frac{\partial V_\alpha}{\partial z} H H^T \left(\frac{\partial V_\alpha}{\partial z} \right)^T + \sum_{l=1}^2 b_l z_l^2 = 0 \quad (17)$$

where $b_1 \geq 1, b_2 \geq 1$ are design parameters. This implies that

$$z_1^2 (4\gamma_\alpha^2 b_1 - 4\gamma_\alpha^2 k_1 + 1) + z_2^2 (k_1^2 + 4\gamma_\alpha^2 b_2 - 4\gamma_\alpha^2 k_2) = 0$$

If we choose $k_1 = b_1 + \frac{1}{4\gamma_\alpha^2}$ and $k_2 = b_2 + \frac{k_1^2}{4\gamma_\alpha^2}$, then the HJI equation can be satisfied. The derivative of V_α is

$$\begin{aligned} \dot{V}_\alpha &= -b_1 z_1^2 - b_2 z_2^2 + \gamma_\alpha^2 w_\alpha^2 \\ &\quad - \gamma_\alpha^2 (w_\alpha - \frac{1}{2\gamma_\alpha^2} z_1 (1 + k_1))^2 \end{aligned} \quad (18)$$

Integrating both sides of (18), we get

$$\begin{aligned} V_\alpha(t) &= - \int_0^t (b_1 z_1^2 + b_2 z_2^2) d\tau + \gamma_\alpha^2 \int_0^t w_\alpha^2 d\tau + V_\alpha(0) \\ &\quad - \int_0^t \gamma_\alpha^2 (w_\alpha - \frac{1}{2\gamma_\alpha^2} z_1 (1 + k_1))^2 d\tau \\ &\leq - \int_0^t (b_1 z_1^2 + b_2 z_2^2) d\tau + \gamma_\alpha^2 \int_0^t w_\alpha^2 d\tau + V_\alpha(0) \end{aligned}$$

which means

$$\begin{aligned} \int_0^t (z_1^2 + z_2^2) d\tau &\leq \int_0^t (b_1 z_1^2 + b_2 z_2^2) d\tau \\ &\leq \gamma_\alpha^2 \int_0^t w_\alpha^2 d\tau + V_\alpha(0) - V_\alpha(t) \\ &\leq \gamma_\alpha^2 \int_0^t w_\alpha^2 d\tau \end{aligned} \quad (19)$$

It implies that the control law u_2 makes α asymptotically track the reference trajectory with disturbance attenuation level γ_α . If $w_\alpha = 0$, the closed-loop system of α is asymptotically stable. The derivative of f_α can't be calculated analytically, we can use a Tracking Differentiator (TD) [22] to get its derivative in simulation. The tracking error of f_α 's derivative can be incorporated in w_α in the control law design analysis.

B. Sideslip angle β control law

For the control of sideslip angle, consider equations (9)-(10)

$$\begin{aligned} \dot{\beta} &= -r_s + f_\beta + w_\beta \\ \dot{r}_s &= u_3 \end{aligned} \quad (20)$$

where $f_\beta = \frac{1}{mV_T}(Y(\beta) - F_T \cos \alpha \sin \beta + mg_3)$. Because (20) has a similar form with (11), so similarly to the design procedure for α , we get the control law for β :

$$u_3 = -(\ddot{\beta}^{\text{ref}} - a_4) \quad (21)$$

where

$$\begin{aligned} a_4 &= k_4 z_4 + z_3 + \frac{\partial a_3}{\partial z_3} (z_4 - k_4 z_3) + \frac{\partial a_3}{\partial \beta^{\text{ref}}} \dot{\beta}^{\text{ref}} \\ &\quad + \frac{1}{2\gamma_\beta^2} \frac{\partial a_3}{\partial z_3} z_3 + \frac{\partial a_3}{\partial f_\beta} \dot{f}_\beta \\ k_3 &= b_3 + \frac{1}{4\gamma_\beta^2} \\ z_3 &= \beta - \beta^{\text{ref}} \\ a_3 &= k_3 z_3 + f_\beta \\ k_4 &= b_4 + \frac{k_3^2}{4\gamma_\beta^2} \\ z_4 &= -r_s - \dot{\beta}^{\text{ref}} + a_3 \end{aligned}$$

$b_3 \geq 1, b_4 \geq 1$ are positive constants corresponding to b_1, b_2 in the α control law design respectively. It is easy to verify that the closed-loop of β satisfies a HJJ function associated with cost function $V_\beta = \frac{1}{2}(z_3^2 + z_4^2)$, thereby the control law u_2 makes β asymptotically track the reference trajectory with disturbance attenuation level γ_β . The derivative of f_β can also be got by using a tracking differentiator in simulation.

C. Roll rate p_s control law

For the control of roll rate, we just use a proportional control law. Given the roll rate command, the control law for p_s is designed as

$$u_1 = k_P(p_s^{\text{ref}} - p_s) \quad (22)$$

where $k_P > 0$.

D. Control allocation

In order to obtain control surfaces deflection, all aerodynamic coefficients need to be affine in the control surfaces deflection, but in the above aircraft model, δ_e doesn't appear affine in $C_m(\alpha, \delta_e)$, $C_z(\alpha, \beta, \delta_e)$. To solve this problem, $C_m(\alpha, \delta_e)$, $C_z(\alpha, \beta, \delta_e)$ can be approximated as

$$\begin{aligned} C_m(\alpha, \delta_e) &= C_{m0}(\alpha) + C_{m\delta_e}(\alpha)\delta_e \\ C_z(\alpha, \beta, \delta_e) &= C_{z0}(\alpha, \beta) + C_{z\delta_e}(\alpha, \beta)\delta_e \end{aligned}$$

In the following simulation, this approximation is achieved by the Least Squares method.

As long as u is known, the external torque vector M can be calculated as

$$M = JS_\alpha^{-1}(u - \dot{S}_\alpha \omega) + \omega \times J\omega \quad (23)$$

Then, we get the control surfaces deflection

$$\begin{pmatrix} \delta_{ecmd} \\ \delta_{acmd} \\ \delta_{rcmd} \end{pmatrix} = \mathcal{D}^\dagger \left[M - \begin{pmatrix} M_{x0} \\ M_{y0} \\ M_{z0} \end{pmatrix} \right] \quad (24)$$

where

$$\begin{aligned} M_{x0} &= \bar{q}Sb \left(C_l(\alpha, \beta) + \frac{br}{2V_T} C_{lr}(\alpha) + \frac{bp}{2V_T} C_{lp}(\alpha) \right) \\ M_{y0} &= \bar{q}S\bar{c} \left(C_{m0}(\alpha) + (x_{cgr} - x_{cg}) C_{z0}(\alpha, \beta) \right. \\ &\quad \left. + \frac{\bar{c}q}{2V_T} C_{zq}(\alpha) \right) + \frac{\bar{c}q}{2V_T} C_{mq}(\alpha) \\ M_{z0} &= \bar{q}Sb \left(C_n(\alpha, \beta) - (x_{cgr} - x_{cg}) \frac{\bar{c}}{b} \left(\frac{br}{2V_T} C_{yr}(\alpha) \right. \right. \\ &\quad \left. \left. + \frac{bp}{2V_T} C_{yp}(\alpha) - 0.02\beta \right) + \frac{br}{2V_T} C_{nr}(\alpha) \right. \\ &\quad \left. + \frac{bp}{2V_T} C_{np}(\alpha) \right) \end{aligned}$$

\mathcal{D}^\dagger is the pseudo-inversion of $\mathcal{D} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}$

$$\begin{aligned} D_{11} &= D_{22} = D_{23} = D_{31} = 0 \\ D_{12} &= \frac{1}{21.5} C_{l\delta_a}(\alpha, \beta) \bar{q}Sb \quad D_{13} = \frac{1}{30} C_{l\delta_r}(\alpha, \beta) \bar{q}Sb \\ D_{21} &= (C_{m\delta_e}(\alpha) + C_{z\delta_e}(\alpha, \beta)(x_{cgr} - x_{cg})) \bar{q}S\bar{c} \\ D_{32} &= \frac{1}{21.5} (C_{n\delta_a}(\alpha, \beta) - 0.021(x_{cgr} - x_{cg}) \frac{\bar{c}}{b}) \bar{q}Sb \\ D_{33} &= \frac{1}{30} (C_{n\delta_r}(\alpha, \beta) - 0.086(x_{cgr} - x_{cg}) \frac{\bar{c}}{b}) \bar{q}Sb \end{aligned}$$

IV. SIMULATIONS

In this section, some simulation results based on a non-linear six-degree-of-freedom aircraft model are presented to illustrate the performance of the maneuver controller.

The control surfaces of this aircraft are modeled as first-order low-pass filters with magnitude and rate limits. These limits can be found in Table I. The dynamic model of the control surfaces are $\dot{\delta}_e = S_R(\omega_e(S_M(\delta_{ecmd}) - \delta_e))$, $\dot{\delta}_a = S_R(\omega_a(S_M(\delta_{acmd}) - \delta_a))$, $\dot{\delta}_r = S_R(\omega_r(S_M(\delta_{rcmd}) - \delta_r))$, where δ_{ecmd} , δ_{acmd} , δ_{rcmd} are control surface commands, δ_e , δ_a , δ_r are the real control surface deflections, $\omega_e = \omega_a = \omega_r = 20.5$ rad/s are the control surface bandwidths,

TABLE I
THE CONTROL INPUT UNITS AND MAXIMUM VALUES

Control	units	MIN	MAX	rate limit
Elevator	deg	-25	25	± 60 deg/s
Ailerons	deg	-21.5	21.5	± 80 deg/s
Rudder	deg	-30	30	± 120 deg/s

$S_M(\cdot), S_R(\cdot)$ represent the magnitude and rate limit functions. The function

$$S_M(x) = \begin{cases} M & \text{if } x \geq M \\ x & \text{if } |x| < M \\ -M & \text{if } x \leq -M \end{cases}$$

and $S_R(\cdot)$ is defined similarly. The aerodynamic datum are valid when $-10 \leq \alpha \leq 45$ degrees and $-30 \leq \beta \leq 30$ degrees. The aircraft is trimmed at $h = 15000$ ft, $V_T = 500$ ft/s. The controller parameters are chosen as $b_1 = 1, b_2 = 5, \gamma_\alpha = 3, b_3 = 2, b_4 = 1, \gamma_\beta = 3, k_P = 5$. Fig.2. gives the results that the angle-of-attack α responses to a reference trajectory. The reference trajectory α^{ref} is generated by letting a square wave pass through a command filter. The dash lines represent the reference trajectories and the solid lines the responses. From these curves we can see that angle-of-attack α has a good tracking performance while the sideslip angle and the roll rate are kept at zero though there exists rate saturation in the deflection of the control surface δ_e . Fig.3. shows the aircraft response to both lateral and longitudinal commands. Both α and p_s have good tracking performance, and only a small sideslip angle appears. In all these simulations, variations are added to aerodynamic coefficients as disturbances. The variations of $C_{xtot}, C_{ztot}, C_{mtot}$ are +20% and $C_{ytot}, C_{ltot}, C_{ntot}$ are -20%. These simulations show that the designed maneuver controller has a good tracking and disturbance attenuation performance.

V. CONCLUSION

In this paper, a maneuver controller based on backstepping is designed for a nonlinear aircraft, which makes the angle-of-attack and roll rate asymptotically track the reference trajectories respectively while keeping sideslip angle zero. To obtain satisfied disturbance attenuation performance for the tracking of angle-of-attack and sideslip angle, appropriate cost functions are constructed to satisfy HJI equations. Simulation results show the good tracking and disturbance attenuation performance of this controller.

ACKNOWLEDGMENT

The authors would like to acknowledge the support from CAS Innovation Projects (Grant No. ZKYGC09A02) and the National Natural Science Foundation of China (Grant No. 60904006, 60874010).

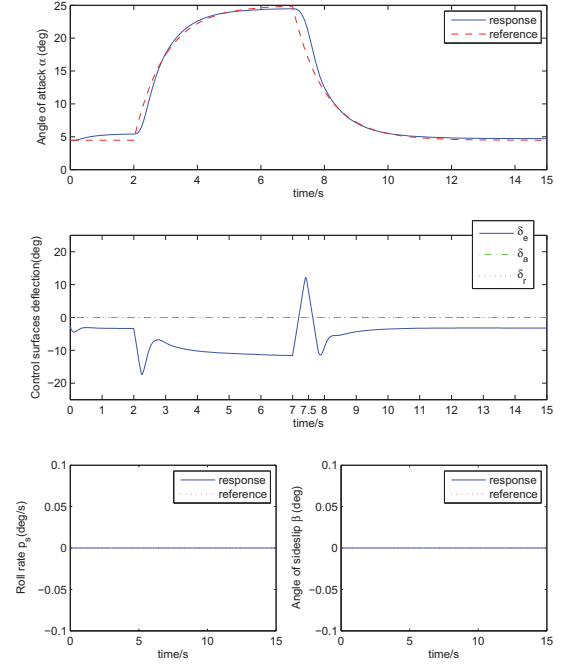


Fig. 2. Aircraft responses to angle-of-attack command.

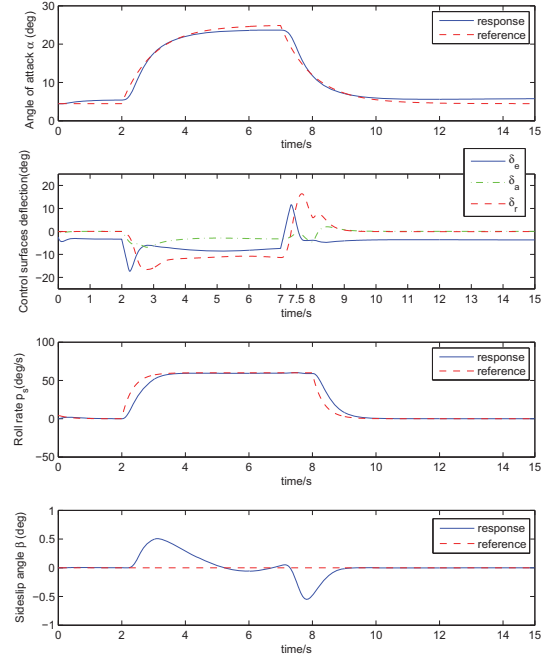


Fig. 3. Aircraft responses to angle-of-attack and roll rate commands.

REFERENCES

- [1] Justin Teo, Jonathan P. How, Eugene Lavretsky, "On Approximate Dynamic Inversion and Proportional-Integral Control," Proceedings of the American Control Conference, Hyatt Regency Riverfront, St. Louis, MO, USA June 10-12, 2009.
- [2] Christakis Papageorgious, Keith Glover, "Robustness analysis of nonlinear dynamic inversion control laws with application to flight control," the 43rd IEEE Conference on Decision and Control, Atlantis, Paradies Island, Bahamas, December 14-17, 2004.
- [3] W. Siwakosit, A. Snell, and R. A. Hess, "Robust Flight Control Design with Handling Qualities Constrains Using Scheduled Linear Dynamic Inversion and Loop-Shaping," IEEE Transactions on control systems technology, Vol.8, No.3, 2000.
- [4] A. Snell, "Decoupling control design with applications to flight," Journal of Guidance, Control, and Dynamics, Vol.12, No.4, 1998.
- [5] A. Isidori, "Nonlinear Control Systems," Springer, 1995.
- [6] Saif A. Al-Hiddabi, N. Harris McClamroch, "Tracking and Maneuver Regulation Control for Nonlinear Nonminimum Phase Systems: Application to Flight Control," IEEE Transactions on Control Systems Technology, Vol.10, No.6, November, 2002.
- [7] Saif A. Al-Hiddabi, "Design of a Flight Control System for Nonminimum Phase 5DOF Aircraft Model," The 11th Mediterranean Conference on Control and Automation, June 18-20, 2003.
- [8] Ioannis Kanellakopoulos, Petar Kokotović, A. Stephen Morse, "Systematic design of adaptive controllers for feedback linearizable systems," IEEE Transactions on Automatic Control, Vol.36, No.11, 1991.
- [9] Miroslav Krstić, Ioannis Kanellakopoulos, Petar Kokotović, "Nonlinear and Adaptive Control Design," John Wiley & Sons, Inc, 1995.
- [10] Kenan Ezal, Zigang Pan, Petar Kokotović, "Locally Optimal and Robust Backstepping Design," IEEE Transactions on Automatic Control, Vol.45, No.2, February 2000.
- [11] Jing Zhou, Changyun Wen, "Adaptive Backstepping Control of Uncertain Systems," Lecture Notes in Control and Information Sciences, Springer Berlin, Volume 372, 2008.
- [12] Härkegård, O., "Flight Control Using Backstepping," M.S.Thesis, Linköping University, Linköping, Sweden, 2001.
- [13] Härkegård, O., "Backstepping and Control Allocation with Application to Flight Control," Ph.D.Thesis, Linköping University, Linköping, Sweden, 2003.
- [14] Marios Polycarpou, Jay Farrell, Manu Sharma, "On-Line Approximation Control of Uncertain Nonlinear Systems: Issues with Control Input Saturation," Proceedings of the American Control Conference, Denver, Colorado June 4-6, 2003.
- [15] L.Sonneveldt, Q. P. Chu, J. A. Mulder, "Nonlinear Flight Control Design Using Constrained Adaptive Backstepping," Journal of Guidance, Control, and Dynamics, Vol.30, No.2, March-April, 2007.
- [16] John W. C. Robinson, Ulrik Nilsson, "Design of a Nonlinear Autopilot for Velocity and Attitude Control using Block Backstepping," AIAA Guidance, Navigation, and Control Conference and Exhibit, San Francisco, California, August 15-18, 2005.
- [17] Härkegård, O., Glad T., "Vector backstepping design for flight control," AIAA Guidance, Navigation, and Control Conference and Exhibit, Hilton Head, South Carolina, August 20-23, 2007.
- [18] John W. C. Robinson, "Nonlinear Analysis and Synthesis Techniques for Aircraft Control," Lecture Notes in Control and Information Sciences, Springer Berlin, 2007.
- [19] A. J. van der Schaft, " \mathcal{L}_2 -Gain and Passivity Techniques in Nonlinear Control," Springer London, 1996.
- [20] Zigang Pan, Tamer Basar, "Adaptive Controller Design for Tracking and Disturbance Attenuation in Parametric Strict-Feedback Nonlinear Systems," IEEE Transactions on Automatic Control, Vol.43, No.8, August, 1998.
- [21] Lewis, B. L., Stevens, F. L., "Aircraft Control and Simulation," John Wiley & Sons, 1992.
- [22] Jingqing Han, Wei Wang, "Nonlinear Tracking-Differentiator[in Chinese]," Journal of Systems Science and Mathematical Sciences, Vol.14, No.2, 1994.