Prior Knowledge Based Systematic Design of SIRMs Connected Interval Type-2 Fuzzy Logic Controller

Chengdong Li, Jianqiang Yi

Institute of Automation
Chinese Academy of Sciences, Beijing 100190, China
E-mail: {chengdong.li, jianqiang.yi}@ia.ac.cn

Abstract: Single input rule modules (SIRMs) connected fuzzy logic control scheme can greatly reduce the number of rules and has found lots of applications. In this paper, using the information from prior knowledge, we present a systematic method for constructing the SIRMs connected fuzzy logic controllers (SIRM-FLCs), and, interval type-2 fuzzy sets are adopted for SIRM-FLCs to achieve better performance. Three kinds of prior knowledge -- odd symmetry, monotonicity, and local stability of closed-loop systems -- are utilized in our study. Such prior knowledge can help us to design SIRM-FLCs systematically and reduce the scale of the feasible parameter space. At last, stabilization control of an inverted pendulum system is presented to show how to use the proposed control strategy and to demonstrate the usefulness of prior knowledge for control synthesis.

Key words: single input rule module (SIRM); type-2 fuzzy; prior knowledge; inverted pendulum

1 Introduction

Single input rule modules (SIRMs) connected fuzzy logic control scheme is first proposed by J. Yi, et al.[1-4] to simplify the design process of fuzzy logic controllers (FLCs), and then studied by H. Seki, et al.[5, 6]. Compared with the other kinds of FLCs, the number of rules in the SIRMs connected fuzzy logic controllers (SIRM-FLCs) can be reduced greatly. And, this control scheme has been applied to many control problems, such as, stabilization control of different kinds of inverted pendulum systems [1-3], positioning control of overhead traveling crane [4], etc.

In recent years, a number of extensions to conventional fuzzy logic (type-1 fuzzy logic) are attracting interest. One of the most widely used extensions is interval type-2 fuzzy logic [7-10]. Generally speaking, compared with type-1 fuzzy logic, interval type-2 fuzzy logic can give better performance, as interval type-2 fuzzy logic utilizes interval type-2 fuzzy sets (IT2FSs) which can provide additional degrees of freedom and more parameters [7-10]. Recently, we have extended the SIRMs connected fuzzy logic control scheme to the interval type-2 case and utilized this strategy to stabilize the TORA system [11]. However, we still lack systematic methods to design the SIRM-FLCs (e.g. set up the fuzzy rules in all the SIRMs), and the large searching space of parameters increases the complexity of the design problems.

In this study, we will try to solve these problems by utilizing the information from prior knowledge. Here, ***the prior knowledge represents the desired properties of the controller or the closed-loop system***. And, three kinds of prior knowledge -- odd symmetry, monotonicity, and local stability of closed-loop systems -- will be explored. Such prior knowledge can help us to set up the fuzzy rules in all the SIRMs intuitively and reduce the scale of the feasible parameter space. At last, to show how to use the proposed control strategy and to demonstrate the usefulness of prior knowledge, an example on the stabilization control of an inverted pendulum system is presented. From the example, we can see that 1) the proposed controller is easy to design and understand 2) satisfactory performance can be achieved.

2 SIRMs Connected Fuzzy Logic Controller [1-6, 11]

For simplicity, we only consider the SIRM-FLC (interval type-2 and type-1) with n input items and 1 output item. Figure 1 shows a block diagram of the SIRMs connected fuzzy logic control system [1-6, 11]. The inputs of the SIRM-FLC are the variables $x_1, \cdots, x_n$ which are the normalized values of the state variables $z_1, \cdots, z_n$ of the plant, and $x_i = \lambda_i z_i$ $(i = 1,$

$2,\cdots,n)$. The output of the SIRM-FLC can be used to control the plant. A SIRM-FLC with n input items $x_1,\cdots,x_n$ is composed of n SIRMs. The SIRMs for input items $x_1,\cdots,x_n$ can be expressed as [1-6, 11]

$$\begin{aligned} &\text{SIRM-1: } \{R_1^j : x_1 = \tilde{A}_1^j \rightarrow y_1 = \tilde{C}_1^j\}_{j=1}^{m_1}, \\ &\qquad\vdots \\ &\text{SIRM-}n\text{: } \{R_n^j : x_n = \tilde{A}_n^j \rightarrow y_n = \tilde{C}_n^j\}_{j=1}^{m_n}, \end{aligned} \tag{1}$$

where $\tilde{A}_i^j$ s are fuzzy sets of type-1 or interval type-2, $\tilde{C}_i^j$ s are weighting factors (denoted as c_i^j) for the type-1 case or interval weighting factors (denoted as $[\underline{c}_i^j, \overline{c}_i^j]$) for the type-2 case, and m_i is the number of fuzzy rules in SIRM-i.

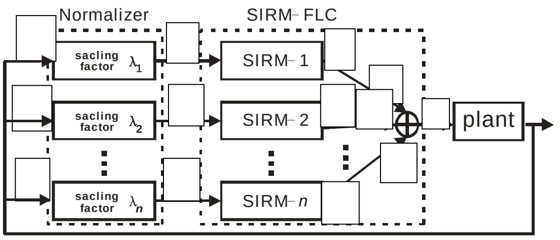

Fig. 1 Block diagram of the SIRMs connected fuzzy logic control system [11]

A SIRM with type-1 fuzzy sets (T1SIRM) can be seen as a type-1 fuzzy logic system while a SIRM with interval type-2 fuzzy sets (IT2SIRM) can be seen as an interval type-2 fuzzy logic system. The inference processes of T1SIRM and IT2SIRM can be found in detail in [11]. Here, let us suppose that the fuzzy inference result of SIRM-i is u_i.

Once the fuzzy inference result u_i of SIRM-i is already calculated, to express clearly the different role of each input item on system performance, the output u of the SIRM-FLC will be computed as the summation of the products of the fuzzy inference result u_i of SIRM-i and the importance degree w_i of the i th input item x_i. In other words, the output u of the SIRM-FLC can be calculated as $u = \sum_{i=1}^{n} w_i u_i$.

3 Design of SIRM-FLCs Using Prior Knowledge

In this section, we will present how to systematically design SIRM-FLCs using prior knowledge. Here, we consider three kinds of prior knowledge: odd symmetry, monotonicity, and local stability of closed-loop systems. Such knowledge is always encountered and needs to be satisfied in fuzzy logic controllers. As a T1SIRM-FLC can be seen as a special IT2SIRM-FLC, hence, in the following, we will only take IT2SIRM-FLCs into account. First, let us consider the prior knowledge of odd symmetry.

3.1 Odd Symmetry

For many control problems, the IT2SIRM-FLCs designed should be odd symmetric. The following theorem will show how to constrain the parameters of IT2SIRMs to ensure that the prior knowledge of odd symmetry can be satisfied.

Theorem 1: An IT2SIRM-FLC is odd symmetric, i.e., $u(\boldsymbol{x}) = -u(-\boldsymbol{x})$, if, for IT2SIRM-$i$ $(i=1,2,\cdots,n)$, $\forall j \in \{1,2,\cdots,m_i\}$, $\exists j^* \in \{1,2,\cdots,m_i\}$ such that

1) the antecedent IT2FSs $\tilde{A}_i^j$ and $\tilde{A}_i^{j^*}$ are distributed symmetrically with respect to 0;

2) $[\underline{c}_i^j, \overline{c}_i^j] = [-\overline{c}_i^{j^*}, -\underline{c}_i^{j^*}]$.

Proof: the proof of this theorem is similar to the proof of theorem 1 in [12].

From Theorem 1, it is easy to show that, $u(\boldsymbol{0}) = 0$, if the IT2SIRM-FLC is odd symmetric. This property always needs to be satisfied for control problems.

3.2 Monotonicity

In [13], we have addressed how to incorporate the monotonicity property into single-input interval type-2 fuzzy logic systems. Here, we will rewrite the relevant results for IT2SIRMs.

Theorem 2 [13]: IT2SIRM-i is monotonically increasing (decreasing), if the antecedent IT2FSs $\tilde{A}_i^1, \tilde{A}_i^2, \cdots, \tilde{A}_i^{m_i}$ form fuzzy partition as shown in Fig. 2, and the consequent interval weights satisfy that $\underline{c}_i^1 \le \underline{c}_i^2 \le \cdots \le \underline{c}_i^{m_i}$ and $\overline{c}_i^1 \le \overline{c}_i^2 \le \cdots \le \overline{c}_i^{m_i}$ ($\underline{c}_i^1 \ge \underline{c}_i^2 \ge \cdots \ge \underline{c}_i^{m_i}$ and $\overline{c}_i^1 \ge \overline{c}_i^2 \ge \cdots \ge \overline{c}_i^{m_i}$).

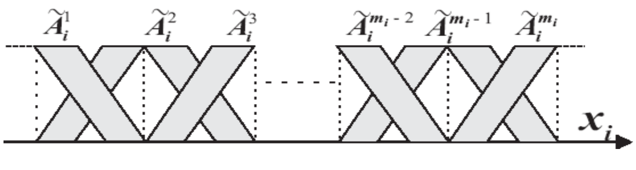

Fig. 2 Interval type-2 fuzzy partition for the input domain of x_i

It is obvious that, if all the IT2SIRMs for an IT2SIRM-FLC are monotonically increasing (decreasing), then the IT2SIRM-FLC will be monotonically increasing (decreasing).

3.3 Local Stability

In control applications, closed-loop systems must be stable. In this subsection, we will discuss how to use the prior knowledge of local stability to constrain the parameter space of IT2SIRM-FLCs.

Now, consider the following system

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{g}((\boldsymbol{z}(t))u(\boldsymbol{z}(t)), \tag{2}$$

where $\boldsymbol{z}(t) = (z_1(t), z_2(t), \cdots, z_n(t))^{\mathrm{T}} \in \mathrm{R}^n$ is the state of the system, $\boldsymbol{f}(\boldsymbol{z}(t)) = (f_1(\boldsymbol{z}(t)), f_2(\boldsymbol{z}(t)), \cdots, f_n(\boldsymbol{z}(t)))^{\mathrm{T}} \in \mathrm{R}^{n\times 1}$, $\boldsymbol{g}(\boldsymbol{z}(t)) = (g_1(\boldsymbol{z}(t)), g_2(\boldsymbol{z}(t)), \cdots, g_n(\boldsymbol{z}(t)))^{\mathrm{T}} \in \mathrm{R}^{n\times 1}$, and $u(\boldsymbol{z}(t)) \in \mathrm{R}$ is the control input from an IT2SIRM- FLC.

The following theorem shows us the most useful result about local stability of the closed-loop systems.

Theorem 3 [14]: Suppose that $\mathbf{0}$ is an equilibrium of the autonomous system $\dot{\boldsymbol{z}}(t) = \boldsymbol{f}(\boldsymbol{z}(t))$, i.e. $\boldsymbol{f}(\mathbf{0}) = \mathbf{0}$, and that $\boldsymbol{f}(.), \boldsymbol{g}(.), u(.)$ are differentiable. And, assume that the Jacobian matrix $\mathbf{J} = \left[\dfrac{\partial(\boldsymbol{f}(\boldsymbol{z}(t)) + \boldsymbol{g}((\boldsymbol{z}(t))u(\boldsymbol{z}(t)))}{\partial \boldsymbol{z}}\right]_{\boldsymbol{z}=0}$ is bounded. Under these conditions, $\mathbf{0}$ is an exponentially stable equilibrium of (2), if:

1) $u(\mathbf{0}) = 0$, which assures that $\boldsymbol{z} = \mathbf{0}$ is an equilibrium of (2);

2) $\mathbf{J}$ is exponentially stable.

To design locally stable IT2SIRMs connected fuzzy control systems, the first condition in Theorem 3 can be realized easily, e.g., the odd symmetric IT2SIRM-FLCs. Then, the remaining task is to determine the parameters of IT2SIRM-FLCs to guarantee the stability of the Jacobian matrix $\mathbf{J}$.

In the following, we will discuss how to obtain the Jacobian matrix at the equilibrium $\mathbf{0}$ for the IT2SIRMs connected fuzzy logic control systems.

Denote the Jacobian matrix $\mathbf{J}$ as $\mathbf{J} = [a_{ij}]_{n\times n}$, where

$$a_{ij} = \frac{\partial f_i(\boldsymbol{z}(t))}{\partial z_j}\Big|_{\boldsymbol{z}=0} + \frac{\partial(g_i((\boldsymbol{z}(t))u(\boldsymbol{z}(t)))}{\partial z_j}\Big|_{\boldsymbol{z}=0}. \tag{3}$$

Note that $u(\mathbf{0}) = 0$ and $u(\boldsymbol{z}(t)) = \Sigma_{i=1}^{n} w_i u_i(x_i) = \Sigma_{i=1}^{n} w_i u_i(\lambda_i z_i)$, where λ_i is the scaling factor for the variable z_i, and the ith input item for IT2SIRM-i becomes $\lambda_i z_i$.

Hence

$$\begin{aligned} &\frac{\partial(g_i((\boldsymbol{z}(t))u(\boldsymbol{z}(t)))}{\partial z_j}\Big|_{\boldsymbol{z}=\mathbf{0}} \\ &= u(\mathbf{0})\frac{\partial g_i((\boldsymbol{z}(t))}{\partial z_j}\Big|_{\boldsymbol{z}=\mathbf{0}} + g_i(\mathbf{0})\frac{\partial u(\boldsymbol{z}(t)}{\partial z_j}\Big|_{\boldsymbol{z}=\mathbf{0}} \\ &= g_i(\mathbf{0}) w_j \frac{\partial u_j(\lambda_j z_j)}{\partial z_j}\Big|_{z_j=0} \end{aligned} \tag{4}$$

Therefore,

$$a_{ij} = \frac{\partial f_i(\boldsymbol{z}(t))}{\partial z_j}\Big|_{\boldsymbol{z}=\mathbf{0}} + g_i(\mathbf{0}) w_j \frac{\partial u_j(\lambda_j z_j)}{\partial z_j}\Big|_{z_j=0} \tag{5}$$

For specific IT2SIRMs, $\partial u_j(\lambda_j z_j)/\partial z_j|_{z_j=0}$ can be determined. In the following, we will determine $\partial u_j(\lambda_j z_j)/\partial z_j|_{z_j=0}$ for the IT2SIRMs shown in Table I which are called IT2SIRM-I and IT2SIRM-II, respectively. In this table, **N**, **Z**, and **P** are fuzzy sets (interval type-2 or type-1) for the input item $x_i = \lambda_i z_i$. Figure 3 shows the membership functions of **N**, **Z**, and **P**, where triangular IT2FSs are used and the changeable width Δ can reflect the uncertainties contained in IT2FSs. When Δ =0, IT2FSs become T1FSs, and IT2SIRM becomes T1SIRM. For IT2SIRM-I and IT2SIRM-II, we can obtain the following results (details are omitted due to page limitation)

$$\frac{\partial u_{\mathbf{I}}(\lambda_j z_j)}{\partial z_j}\Big|_{z_j=0} = \begin{cases} \lambda_j, & \Delta = 0 \\ \dfrac{1}{2}\dfrac{\lambda_j}{1-\Delta}, & \Delta \neq 0 \end{cases} \tag{6}$$

$$\frac{\partial u_{\mathbf{II}}(\lambda_j z_j)}{\partial z_j}\Big|_{z_j=0} = \begin{cases} -\lambda_j, & \Delta = 0 \\ -\dfrac{1}{2}\dfrac{\lambda_j}{1-\Delta}, & \Delta \neq 0 \end{cases} \tag{7}$$

TABLE I Settings of IT2SIRM-I and IT2SIRM-II for input item x

Input Item	Consequent Part of IT2SIRM-I	Consequent Part of IT2SIRM-II
N	-1	1
Z	0	0
P	1	-1

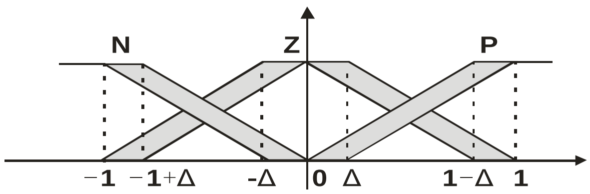

Fig. 3 Interval type-2 fuzzy sets for N, Z, and P with changeable width Δ

From the above discussions, we can obtain the Jacobian matrix at the equilibrium **0** for the IT2SIRMs connected fuzzy logic control systems. And, it is obvious that, for the IT2SIRM-I/II connected FLCs, the scaling factors λ_i, the changeable width Δ and the importance degrees w_i all affect the stability of the equilibrium **0** of the IT2SIRMs connected fuzzy logic control systems. Therefore, to design a desirable IT2SIRM-FLC for the system (2), the parameters of the IT2SIRM-FLC should be chosen to satisfy that all the eigenvalues of the Jacobian matrix **J** have negative real parts.

4 Simulations

In this section, we will use an inverted pendulum system as the example to show how to design IT2SIRMs connected fuzzy logic control systems using prior knowledge. The dynamic equations of the inverted pendulum system can be found in [1]. Here, due to page limitation, we omit these equations.

For the inverted pendulum system, the three kinds of prior knowledge will be discussed as follows:

1) Considering the odd symmetry of states of the inverted pendulum system, all the IT2SIRMs for z_1, z_2, z_3, z_4 should be odd symmetric.

2) From our experience, to make $z_i \to 0$, the larger z_i is, the larger u_i is needed. Hence, all the IT2SIRMs for z_1, z_2, z_3, z_4 should be monotonically increasing. From the above discussion, both IT2SIRM-I and IT2SIRM-II are odd symmetric, but IT2SIRM-I is monotonically increasing, while IT2SIRM-II is monotonically decreasing. Therefore, in this simulation, all the IT2SIRMs for z_1, z_2, z_3, z_4 are chosen to be IT2SIRM-I.

3) The Jacobian matrix **J** for the inverted pendulum system controlled by the IT2SIRM-Is connected FLC can be obtained as

$$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{3(M+m)g}{(4M+m)l} + t_2 k_1 & t_2 k_2 & t_2 k_3 & t_2 k_4 \\ 0 & 0 & 0 & 1 \\ \frac{-3mg}{(4M+m)l} + t_4 k_1 & t_4 k_2 & t_4 k_3 & t_4 k_4 \end{bmatrix}.$$

where $t_2 = -3/(4M+m)l$, $t_4 = 4/(4M+m)$, $k_j = w_j \lambda_j$ $(j = 1, 2, 3, 4)$, if $\Delta = 0$; and $k_j = w_j \lambda_j / 2(1-\Delta)$, if $\Delta \neq 0$.

In this simulation, the parameters are chosen to be $\lambda_1 = 3.82$, $\lambda_2 = 1.90$, $\lambda_3 = 0.5$, $\lambda_4 = 1$, $w_1 = 50$, $w_2 = 50$, $w_3 = 20$, $w_4 = 20$. Then, it is easy to verify that **J** is stable no matter whether $\Delta = 0$ or not.

Simulation results for the initial state $(\pi/6, 0, 0, 0)$ are depicted in Fig. 4 (a) and (b). Fig. 4 (a) demonstrates the time responses of the pole angle controlled by the IT2SIRM-FLCs ($\Delta = 0$ and $\Delta = 0.2$), while Fig 4 (b) displays the time responses of the cart position. Moreover, comparisons between the IT2SIRM-FLCs (Δ =0 and Δ =0.2) are shown in Table II with respect to ISE, IAE, ITAE for different initial states $(\pi/6, 0, 0, 0)$ and $(\pi/6, \pi/6, -1, 0)$.

TABLE II Comparisons between the IT2SIRM-FLCs (Δ = 0 and Δ = 0.2)

initial	**ISE**		**IAE**		**ITAE**	
states	$\Delta = 0$	$\Delta = 0.2$	$\Delta = 0$	$\Delta = 0.2$	$\Delta = 0$	$\Delta = 0.2$
$(\pi/6, 0, 0, 0)$	43.1007	41.2418	8.2003	8.0031	62.2832	55.0349
$(\pi/6, \pi/6, -1, 0)$	64.3305	60.7752	12.0617	11.6667	141.6144	120.4052

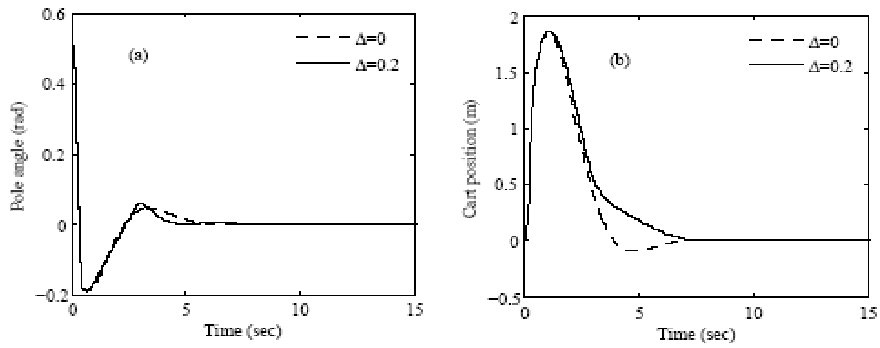

Fig. 4 Time responses of the inverted pendulum system: (a) pole angle, (b) cart position

From Fig. 4 and Table II, we can see that, the proposed control scheme can achieve the control objective, but, from ISE, IAE and ITAE's point of view, the controller with $\Delta = 0.2$ gives better performance. What is more, the controllers can be easily designed and we only need 12 fuzzy rules (3 rules for each IT2SIRM) in our control scheme.

5 Conclusions

In real-world control applications, prior knowledge can provide important information for control synthesis. This paper has focused on utilizing three kinds of prior knowledge -- odd symmetry, monotonicity, and local stability of closed-loop systems -- to alleviate the difficulty of designing interval type-2 fuzzy logic controllers. As shown in our study, with the help of prior knowledge, the design of the SIRMs connected FLCs becomes much easier. And, simulation results have also shown the effectiveness of the proposed control scheme and the usefulness of prior knowledge.

Acknowledgement

This work was partly supported by the NSFC Projects under Grant No. 60975060 and 60621001, National 863 Program (No.2007AA04Z239), China.

References

[1] J. Yi, N. Yubazaki, "Stabilization Fuzzy Control of Inverted Pendulum Systems," Artificial Intelligence in Engineering, 14-2, 153-163, (2000).

[2] J. Yi, N. Yubazaki, K. Hirota, "Stabilization Control of Series-type Double Inverted Pendulum Systems using the SIRMs Dynamically Connected Fuzzy Inference Model," Artificial Intelligence in Engineering, 15-3, 297-308, (2001).

[3] J. Yi, N. Yubazaki, K. Hirota, "A New Fuzzy Controller for Stabilization of Parallel-type Double Inverted Pendulum System," Fuzzy Sets and Systems, 126-1, 105-119, (2002).

[4] J. Yi, N. Yubazaki, K. Hirota, "Anti-Swing and Positioning Control of Overhead Traveling Crane," Information Sciences, 155-1, 19-42, (2003).

[5] H. Seki, H. Ishii, M. Mizumoto, "On the Monotonicity of Single Input Type Fuzzy Reasoning Methods," IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, E90A-7, 1462-1468, (2007).

[6] H. Seki, H. Ishii, M. Mizumoto, "On the Generalization of Single Input Rule Modules Connected Type Fuzzy Reasoning Method," IEEE Transactions on Fuzzy Systems, 16-5, 1180-1187, (2008).

[7] Q. Liang, J. M. Mendel, "Interval Type-2 Fuzzy Logic Systems: Theory and Design," IEEE Transactions on Fuzzy Systems, 8-5, 535-550, (2000).

[8] J. M. Mendel, "Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New directions," Upper Saddle River, NJ: Prentice-Hall, (2001).

[9] J. M. Mendel, "Advances in Type-2 Fuzzy Sets and Systems," Information Sciences, 177-1, 84-110, (2007).

[10] J. M. Mendel, D. Wu, "Perceptual reasoning for perceptual computing," IEEE Transactions on Fuzzy Systems, 16-6, 1550-1564, (2008).

[11] C. Li, J. Yi, D. Zhao, "Control of the TORA System Using SIRMs Based Type-2 Fuzzy Logic," 2009 IEEE International Conference on Fuzzy Systems, 694-699, (2009).

[12] C. Li, J. Yi, D. Zhao, "Design of Interval Type-2 Fuzzy Logic System Using Sampled Data and Prior Knowledge," ICIC Express Letters, 3-3, 695-700, (2009).

[13] C. Li, J. Yi, D. Zhao, "Analysis and Design of Monotonic Type-2 Fuzzy Inference Systems," 2009 IEEE International Conference on Fuzzy Systems, 1193-1198, (2009).

[14] M. Vidyasagar, "Nonlinear Systems Analysis," Englewood Cliffs, NJ: Prentice-Hall, (1993).