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Robust Attitude Controller for Unmanned Aerial Vehicle Using Dynamic Inversion and Extended State Observer

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***Abstract*— A robust feedback linearization controller is presented for attitude control of an Unmanned Aerial Vehicle (UAV). The objective of this controller is to make the roll angle, pitch angle, and yaw angle track the given trajectories(commands) respectively. This design is developed using dynamic inversion and extended state observer (ESO). Firstly, dynamic inversion is used to linearize and decouple UAV attitude system into three single-input-single-output (SISO) systems, then three proportional-derivative (PD) controllers are designed for these linearized systems. Extended state observers are used to estimate and compensate unmodeled dynamics and extern disturbances. Simulation results show that the proposed controller is effective and robust.**

Keywords*— *attitude control; dynamic inversion; extended state observer; robustness.

I. Introduction

Attitude tracking is the purpose of UAV's inner loop control which determines the handling qualities of UAV. The major problem in the design of attitude control system comes from high nonlinearity and undesired strong coupling between axes of UAV. With traditional methods it is difficult to design high-precision attitude controller. Because of UAV's nonlinearity, using nonlinear method can better meet the nature of the problem. Feedback linearization is a very important nonlinear control method. The main idea of feedback linearization is to cancel system's nonlinearity directly using nonlinear state feedback transformation or coordinate transformation. Dynamic inversion is one of the widely used feedback linearization methods in engineering fields and has been applied successfully in flight control [1], [2]. However, to perform exact linearization, the precise system model is needed for dynamic inversion, and this requirement usually can not be satisfied because of the existence of unmodeled dynamics and extern disturbances. Some robust control methods combined with dynamic inversion have been proposed in order to improve robustness in flight control, such as neural network [3–5], loop shaping [7], fuzzy control [6], and other adaptive control methods. The main idea of most of these methods is to estimate the uncertain factors and eliminate them. Extended state observer (ESO) [8–10] can be used to estimate uncertainties and disturbances, which is the key part of active disturbance rejection controller (ADRC) [8–10]. ESO takes the disturbances that can affect the system outputs as a new state variable, and uses a special feedback mechanism to establish the extended state. This observer doesn't depend on the mathematical model of disturbances.

In this work, we use the well-known dynamic inversion combined with extended state observer to establish an attitude controller for UAV. Simulation results are presented to show good performance and robustness of this controller.

II. UAV Dynamic Model Description

The dynamic model of an unmanned aerial vehicle is as follows [11]

$$\begin{aligned} &\dot\phi = p + \tan\theta(q\sin\phi + r\cos\phi) && \dot\theta = q\cos\phi - r\sin\phi \\ &\dot\psi = \frac{q\sin\phi + r\cos\phi}{\cos\theta} && \dot p = (c_1 r + c_2 p)q + c_3 L + c_4 N \\ &\dot q = c_5 pr - c_6(p^2 - r^2) + c_7 M && \dot r = (c_8 p - c_2 r)q + c_4 L + c_9 N \end{aligned}$$

where $c_1 = \frac{(I_y - I_z)I_z - I_{xz}^2}{\Sigma}$, $c_2 = \frac{(I_x - I_y + I_z)I_{xz}}{\Sigma}$, $c_3 = \frac{I_z}{\Sigma}$, $c_4 = \frac{I_{xz}}{\Sigma}$, $c_5 = \frac{I_z - I_x}{I_y}$, $c_6 = \frac{I_{xz}}{I_y}$, $c_7 = \frac{1}{I_y}$, $c_8 = \frac{I_x(I_x - I_y) + I_{xz}^2}{\Sigma}$, $c_9 = \frac{I_x}{\Sigma}$, $\Sigma = I_x I_z - I_{xz}^2$. ϕ, θ, ψ are roll, pitch, yaw angles respectively. p, q, r are body-axis roll, pitch, yaw rates respectively. L, M, N are total roll, pitch, yaw moments produced by aerodynamic control surface or thrust respectively. I_x, I_y, I_z are roll, pitch, yaw moments of inertia respectively. I_{xy}, I_{xz}, I_{yz} are product moments of inertia.

Because of the undesired strong coupling between axes of UAV and the existence of unmodeled dynamics and extern disturbances, a robust attitude controller is needed to make the UAV's attitude track the given commands quickly and steady.

III. Dynamic Inversion and Extended State Observer

A. Dynamic Inversion

Consider the multivariable affine nonlinear system with state vector $x \in \mathbb{R}^n$, input vector $u \in \mathbb{R}^m$, and output vector $y \in \mathbb{R}^m$ described by following equations

$$\dot x = f(x) + \sum_{i=1}^{m} g_i(x)u_i \triangleq f(x) + G(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

where $f, g_1, \cdots, g_m$ are smooth vector fields in an open set of $\mathbb{R}^n$, $G(x) = (g_1(x), \cdots, g_m(x))$ is a $n \times m$ matrix, $h(x) = col(h_1(x), \cdots, h_m(x))$ is a smooth m-vector.

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Let $L_f h_i$ stand for Lee derivative of h_i along the vector field f and $L_f^k h_i$ stand for taking k times iteration Lee derivative of h_i along the vector field f. Considering the i-th output of the above affine nonlinear system, differentiating with respect to time, we get $\dot{y}_i = \sum_{j=1}^{n} \frac{\partial h_i}{\partial x_j}\dot{x_j}$. Substituting the i-th equation of (1) into it yields

$$\begin{aligned}\dot{y}_i &= \sum_{j=1}^{n} \frac{\partial h_i}{\partial x_j} f_j(x) + \sum_{k=1}^{m}\sum_{j=1}^{n} \frac{\partial h_i}{\partial x_j} g_{jk}(x) u_k \\ &= L_f h_i(x) + \sum_{k=1}^{m} L_{g_k} h_i(x) u_k\end{aligned}$$

If $\sum_{k=1}^{m} L_{g_k} h_i(x) u_k$ equals to zero, that is u doesn't appear in $\dot{y}_i$, then differentiating $\dot{y}_i$ with respect to time successively until the input appears in the derivative expression, we get

$$y_i^{(\gamma_i)} = L_f^{\gamma_i} h_i(x) + \sum_{k=1}^{m} L_{g_k} L_f^{\gamma_i - 1} h_i(x) u_j$$

where γ_i is the smallest times we must derivative the expression until the input appears in $y_i^{(\gamma_i)}$. Then we can rewrite the output equation (2) as follows [1], [12]

$$y^{(\gamma)} = A(x) + B(x)u \tag{3}$$

where

$$y^{(\gamma)} = \begin{pmatrix} y_1^{(\gamma_1)} \\ \vdots \\ y_m^{(\gamma_m)} \end{pmatrix} \quad A(x) = \begin{pmatrix} L_f^{\gamma_1} h_1 \\ \vdots \\ L_f^{\gamma_m} h_m \end{pmatrix}$$

$$B(x) = \begin{pmatrix} L_{g_1} L_f^{\gamma_1 - 1} h_1 & \dots & L_{g_m} L_f^{\gamma_1 - 1} h_1 \\ \vdots & \vdots & \vdots \\ L_{g_1} L_f^{\gamma_m - 1} h_m & \dots & L_{g_m} L_f^{\gamma_m - 1} h_m \end{pmatrix}$$

Therefore, the decoupling and linearizing control law can be set as

$$u = B^{\dagger}(x)(v - A(x)) \tag{4}$$

where $B^{\dagger}(x)$ is the pseudo-inversion of matrix $B(x)$, v is the pseudo-input. Substituting (4) into (3) yields $y^{(\gamma)} = v$.

However, the system equation (3) is only an approximation of the actual nonlinear system. Supposing the real model of the system is described by $y^{(\gamma)} = \hat{F}(x, u)$, then it can't be decoupled and linearized exactly by the control law $\hat{u} = B^{\dagger}(x)(v - A(x))$. Letting $\Delta(x, \hat{u}) = \hat{F}(x, \hat{u}) - (A(x) + B(x)\hat{u})$, then $y^{(\gamma)} = F(x, \hat{u}) + \Delta(x, \hat{u})$. $\Delta(x, \hat{u})$ stands for unmodeled dynamics or extern disturbances.

The effect of $\Delta(x, \hat{u})$ to system needs to be estimated and eliminated. ESO is suitable to play this role, which can be used to deal with unmodeled dynamics and extern disturbances [8–10].

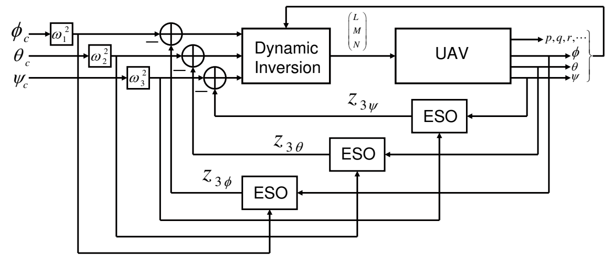

Fig. 1. Block diagram of attitude controller based on dynamic inversion and ESO

B. Extended State Observer

Consider a second order SISO system

$$\ddot{y} = f(t, y, \dot{y}, w) + bu$$

where f represents the real system dynamics, w represents the unmodeled dynamics and disturbances. Letting $x_3 \triangleq f(t, y, \dot{y}, w)$ and $\dot{x_3} = a(t)$, we call x_3 an extended state of this system. Then the system can be described as

$$\begin{cases} \dot{x_1} &= x_2 \\ \dot{x_2} &= x_3 + bu \\ \dot{x_3} &= a(t) \\ y &= x_1 \end{cases} \tag{5}$$

A nonlinear observer of the from

$$\begin{cases} e &= z_1 - y \\ \dot{z}_1 &= z_2 - \beta_1 e \\ \dot{z}_2 &= z_3 - \beta_2 fal(e, \alpha_1, \delta) + bu \\ \dot{z}_3 &= -\beta_3 fal(e, \alpha_2, \delta) \end{cases} \tag{6}$$

can be designed for system (5), where

$$fal(e, \alpha, \delta) \triangleq \begin{cases} |e|^{\alpha} sign(e) & |e| > \delta \\ \frac{e}{\delta^{1-\alpha}} & |e| \le \delta \end{cases} \quad \delta > 0, 0 < \alpha < 1$$

If the parameters $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \delta$ are properly choosed, then this observer can estimate real-time values of $x_1(t), x_2(t)$ and the extended state $x_3(t)$ of system (5), that is, $z_1(t) \to x_1(t), z_2(t) \to x_2(t), z_3(t) \to x_3(t)$. This observer is called the extended state observer (ESO) [8–10] of system (5). If $f(t, y, \dot{y}, w) = f_0(t, y, \dot{y}, w) + f_1(t, y, \dot{y}, w)$, where f_0 and f_1 are the known and unknown parts of f respectively, then f_1 can be estimated as $f_1 = z_3 - f_0$. Usually the unknown parts are unmodeled dynamics or extern disturbances, which can be dynamically estimated and compensated by ESO.

IV. UAV Attitude Controller Design

Now consider the attitude control problem described in section II. The block diagram of our proposed attitude controller is described in Fig.1. Letting

$$x = col(\phi, \theta, \psi), y = x, u = col(L, M, N) \tag{7}$$

and differentiating y with respect to time twice, we get

$$\ddot{y} = A(x) + B(x)u$$

where $A(x) = col(a_1, a_2, a_3)$,

$$B(x) = \begin{pmatrix} c_3 + c_4\cos\phi\tan\theta & c_7\sin\phi\tan\theta & c_4 + c_9\cos\phi\tan\theta \\ -c_4\sin\phi & c_7\cos\phi & -c_9\sin\phi \\ c_4\cos\phi\sec\theta & c_7\sin\phi\sec\theta & c_9\cos\phi\sec\theta \end{pmatrix}$$

The expressions of a_1, a_2, a_3 are omitted because of their complexity. Choosing $u = \hat{u} = B^{\dagger}(x)(v - A(x))$, then

$$\ddot{y} = v \tag{8}$$

It can be clearly seen that system (7) has been linearized and decoupled into three SISO systems, which are simple to deal with.

However, because of the existence of unmodeled dynamics and extern disturbances in system model, the real system under the control $\hat{u}$ is

$$\ddot{y} = v + \Delta(x, \hat{u}) \tag{9}$$

where $\Delta(x, \hat{u})$ stands for unmodeled dynamics and extern disturbances. System (8) also can be viewed as three SISO systems with disturbances. Supposing the attitude commands are given by $(\phi_c, \theta_c, \phi_c)$, we design three proportional-derivative (PD) controllers for each SISO channel of system (8), that is, we set pseudo-input v as

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} k_{p1}(\phi_c - \phi) - k_{d1}\dot{\phi} \\ k_{p2}(\theta_c - \theta) - k_{d2}\dot{\theta} \\ k_{p3}(\psi_c - \psi) - k_{d3}\dot{\psi} \end{pmatrix}$$

Then (9) becomes

$$\begin{array}{rcl} \ddot{\phi} & = & k_{p1}(\phi_c - \phi) - k_{d1}\dot{\phi} + \Delta_1(x, \hat{u}) \\ \ddot{\theta} & = & k_{p2}(\theta_c - \theta) - k_{d2}\dot{\theta} + \Delta_2(x, \hat{u}) \\ \ddot{\psi} & = & k_{p3}(\psi_c - \psi) - k_{d3}\dot{\psi} + \Delta_3(x, \hat{u}) \end{array} \tag{10}$$

Because of the similarity in the form of the three systems in (10), we just take the first equation of (10) into consideration. Viewing ϕ_c as the control input and letting $x_1 = \phi, x_2 = \dot{\phi}, x_3 = -k_{p1}\phi - k_{d1}\dot{\phi} + \Delta_1(x, \hat{u}), \dot{x}_3 = a(t)$, the first equation of (10) can be expressed as follows

$$\left\{ \begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & x_3 + k_{p1}\phi_c \\ \dot{x}_3 & = & a(t) \\ y & = & x_1 \end{array} \right.$$

According to (6), an extended state observer (ESO) for this system can be designed as follows

$$\left\{ \begin{array}{rcl} e & = & z_{1\phi} - y \\ \dot{z}_{1\phi} & = & z_{2\phi} - \beta_1 e \\ \dot{z}_{2\phi} & = & z_{3\phi} - \beta_2 fal(e, \alpha_1, \delta) + k_{p1}\phi_c \\ \dot{z}_{3\phi} & = & -\beta_3 fal(e, \alpha_2, \delta) \end{array} \right.$$

If the parameters are choosed appropriately, then $z_{1\phi}(t) \to x_1(t), z_{2\phi}(t) \to x_2(t), z_{3\phi}(t) \to x_3(t)$. We can get an estimate of $\Delta_1(x, \hat{u})$

$$\Delta_1(x, \hat{u}) = z_{3\phi} + k_{p1}\phi + k_{d1}\dot{\phi}$$

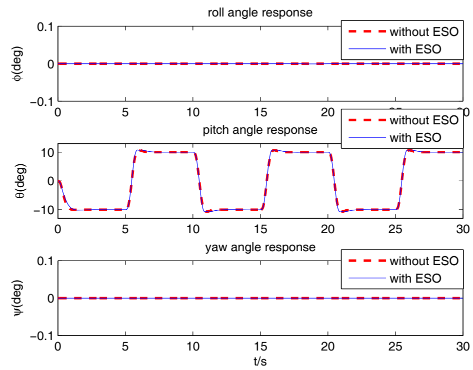

Fig. 2. Attitude angle response of single channel without unmodeled dynamics. Attitude angle command θ_c is a square wave with amplitude 10deg and frequency 0.1Hz, $\phi_c = 0$deg, $\psi_c = 0$deg.

Suppose the required damping and the frequency of ϕ channel are ξ_1, ω_1 respectively. If we set $v_1 = k_{p1}(\phi_c - \phi) - k_{d1}\dot{\phi} - \Delta_1(x, \hat{u})$ instead of $v_1 = k_{p1}(\phi_c - \phi) - k_{d1}\dot{\phi}$ and choose PD gains as $k_{p1} = \omega_1^2, k_{d1} = 2\xi_1\omega_1$, then the closed loop transfer function of the first SISO system of system (9) becomes $\frac{\phi}{\phi_c} = \frac{\omega_1^2}{s^2 + 2\xi_1\omega_1 s + \omega_1^2}$. This is a second order system that has the desired frequency and damping. We design corresponding ESO for the other two SISO systems respectively to estimate the unmodeled dynamics and extern disturbances

$$\begin{array}{c} \Delta_2(x, \hat{u}) = z_{3\theta} + k_{p2}\theta + k_{d3}\dot{\phi} \\ \Delta_3(x, \hat{u}) = z_{3\psi} + k_{p3}\psi + k_{d3}\dot{\psi} \end{array}$$

where $z_{3\theta}, z_{3\psi}$ are the output of corresponding ESO. k_{p2}, k_{p3}, k_{d2}, k_{d3} also can be choosed as $k_{p2} = \omega_2^2, k_{d2} = 2\xi_2\omega_2, k_{p3} = \omega_3^2, k_{d3} = 2\xi_3\omega_3$, where ξ_2, ω_2 are the desired frequency and dumping of θ channel respectively and ξ_3, ω_3 are the desired frequency and dumping of ψ channel respectively. We also set $v_2 = k_{p2}(\theta_c - \theta) - k_{d2}\dot{\theta} - \Delta_2(x, \hat{u})$, $v_3 = k_{p3}(\psi_c - \psi) - k_{d3}\dot{\psi} - \Delta_3(x, \hat{u})$. Finally, under the control of v_1, v_2, v_3, system (7) is equivalent to $\frac{\phi}{\phi_c} = \frac{\omega_1^2}{s^2 + 2\xi_1\omega_1 s + \omega_1^2}$, $\frac{\theta}{\theta_c} = \frac{\omega_2^2}{s^2 + 2\xi_2\omega_2 s + \omega_2^2}$, $\frac{\psi}{\psi_c} = \frac{\omega_3^2}{s^2 + 2\xi_3\omega_3 s + \omega_3^2}$. The real control can be expressed as follows

$$\begin{pmatrix} L \\ M \\ N \end{pmatrix} = B^{\dagger}(x)(v - A(x)) = B^{\dagger}(x)\left[\begin{pmatrix} \omega_1^2\phi_c - z_{3\phi} \\ \omega_2^2\theta_c - z_{3\theta} \\ \omega_3^2\psi_c - z_{3\psi} \end{pmatrix} - A(x)\right]$$

V. Simulation Result

In this section, some examples of attitude tracking are given to demonstrate the proposed method. Suppose the three channels of UAV have the same frequency and damping requirements $\omega = 4$ rad/s, $\xi = 0.8$. The aerodynamic control surfaces that provide moments are modeled as first-order inertial systems. Set $k_{p1} = k_{p2} = k_{p3} = 4^2, k_{d1} = k_{d2} = k_{d3} = 2 * 4 * 0.8$. The parameters of three ESOs can be set to be the same, that is, $\beta_1 = 100, \beta_2 = 3000, \beta_3 = 5000, \delta =$

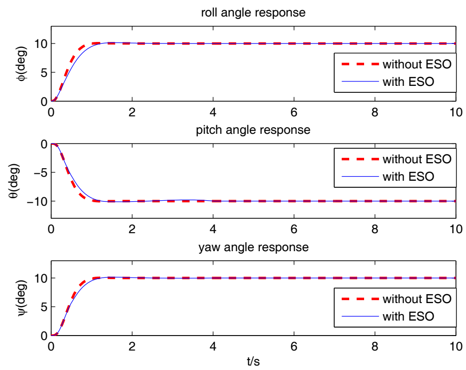

Fig. 3. Attitude angle response of three channels without unmodeled dynamics. Attitude angle command ϕ_c = 10deg, $\theta_c = -10$deg, ψ_c = 10deg.

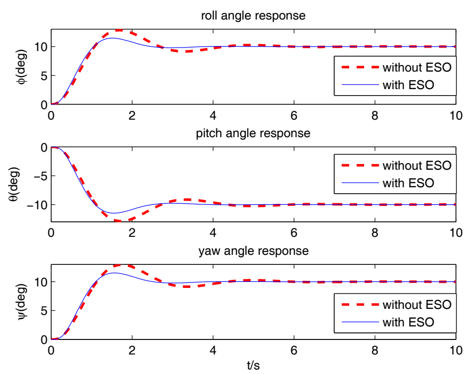

Fig. 4. Attitude angle response and control moment with unmodeled dynamics. Attitude angle command ϕ_c = 10deg, $\theta_c = -10$deg, ψ_c = 10deg.

$0.1, \alpha_1 = 0.9, \alpha_2 = 0.3$. Fig.2 and Fig.3 are obtained in the situation of no unmodeled dynamics. In this situation, the response of the controller with ESOs and without ESOs are nearly the same. Fig.2 shows the attitude angle response of single channel, where the attitude angle command θ_c is a square-wave with amplitude 10deg and frequency 0.1Hz, $\phi_c = \psi_c = 0$. From these response curves, we can see that the pitch angle θ tracks the command quickly and steady without overshot, and the other two attitude angles hold on 0deg. Fig.3 shows the attitude angle response of three channels, where the attitude angle command ϕ_c = 10deg, $\theta_c = -10$deg, $\psi_c = 10$ deg. It can be seen that individual angle command can be well tracked in each channel. The nonlinear system is fully decoupled.

However, if there exist unmodeled dynamics in system model, the importance of ESO is demonstrated. Fig.4 gives us an example, which is obtained in the presence of unmolded dynamics by more than 10% perturbation in each moment of inertia of UAV. The dash lines show the attitude response without using ESOs to estimate and compensate model errors. The solid lines show the attitude response with model errors estimating and compensating using ESOs. It is obvious that the effects of the controller with ESOs are much better than that without ESOs. The controller with ESO has robust performance.

VI. Conclusions

An attitude controller for UAV using dynamic inversion and extended state observer is presented. This controller uses dynamic inversion to linearize UAV's dynamic equation. The unmodeled dynamics and extern disturbances are estimated and compensated using extended state observer. Numerical simulations are performed for an UAV. Simulation results show that the proposed controller is effective and robust.

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