Controller design for flying boats taking off from water with regular waves

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Abstract – As a special airplane, a flying boat can take off from water and land in water, which makes its controller design more complicated than that of other general airplanes. Based on the mathematical model of a flying boat, two controllers applying classical PID (Proportional Integration Differential) method and a compound control method (combining active-disturbance rejection controller (ADRC) and dynamic inversion (DI) methods) are designed respectively. Then the performances of the two controllers are validated by tracking a speed step signal from 15m/s to 16m/s in clam water, holding the speed and the pitch angle of the flying boat in water with regular waves, and taking off from water with and without waves. The results of the simulations show that the controllers can improve the stability and seakeeping quality of the flying boat.

Key Words - Flying boat, ADRC, dynamic inversion, PID

1 Introduction

Although flying boats have been studied and produced for nearly a hundred years, there is few published research works relating to controller designing methods for flying boats, without mention of some systematical validation and verification guidelines for flying boats. However, controllers design based on its mathematical model is one of meaningful research fields on flying boats. When flying boats are taking off from water or landing in water, porpoising (an instable motion) is easily induced due to the longitudinal coupling among heave, pitch angle and speed [1]. This kind of motion may damage flying boats in some severe conditions, such as large waves. It is a hard work for pilots to operate flying boats in such a condition. In order to ease the burden of the pilots, decrease damage effects from waves and protect flying boats from serious water impact, some effective controllers for flying boats are necessary, especially for those unmanned flying boats [2].

As flying boats are special airplanes, the research achievements on flight control have great significances on the designing of flying boats’ controllers. As to flight control, there are three important basic requirements: guaranteeing stability for different flying conditions, holding airplanes to desired states and tracking desired flying trajectories. In order to improve the performances of different airplanes, researchers have been proposing lots of flight control methods: such as gain scheduling methods based on classic PID controllers [3], adaptive control methods [4], nonlinear control methods [6], robust control methods [6], optimal control methods [7], intelligent control methods, and so on. In fact, this is a rough classification method. Most advanced controllers designed for airplanes always belong to more than one kind of control methods, or they may be some hybrids of different control methods.

Regarding to flying boats, the control designing problem will be more difficult due to the impacts from water. When the flying boats are taking off or landing, two abilities are very important. One is the ability to keep itself stable. Although this ability has large relationships to the structure of the flying boats, such as the hull form, center of gravity, and engine position, it is also related to the speed and the trim angle which can be regulated by some controllers. The other one is the seakeeping ability which is the key factor to determine whether the flying boats are appropriately designed. So the controllers on flying boats should have the ability to improve the stability and seakeeping quality.

In this paper, based on the mathematical model of a flying boat, only the planing state of the flying boat is researched. And the article is organized as following: Section 2 describes the mathematical model of the flying boat briefly. In section 3 and 4 two controllers designed by using PID method and a compound control method (based on DI and ADRC) respectively are illustrated in detail. In section 5 the performances of the two controllers are demonstrated by some simulations. And section 6 concludes the paper.

2 The model of the flying boat

When the flying boat planes in water, the forces acting on it can be categorized into weight, forces from water, aerodynamic forces, and engine thrust (Fig. 1).

Fig. 1: The forces acting on the flying boat

The longitudinal model of the flying boat is expressed as Eqs (1)[3].

\[ \text{[Equation]} \]
\[
mV = T \cos(\alpha + \alpha_e) - D_x - N_x \sin \alpha - D_y \cdot \cos \alpha + G_x \\
mV \dot{\alpha} = mVq - T \sin(\alpha + \alpha_e) - L_x - N_x \cdot \cos \alpha + D_y \cdot \sin \alpha + G_x \\
I, \dot{\phi} = M_x + M_y + M_z \\
\theta = q \\
\dot{\theta} = u \cos \theta + w \sin \theta \\
\ddot{\theta} = -u \sin \theta + w \cos \theta
\]

where \(m\) is the mass of the flying boat; \(V\) is the speed of the flying boat; \(T = T(\delta_e)\), the thrust of engine, which is a function of the engine throttle (\(\delta_e\)); \(u, w\) is the speed components of the flying boat along body coordinate system axes \((X, Y, Z)\); \(\alpha\) is the angle between engine force and \(X\); \(\alpha_e\) is the angle between \(X\) and \(Y\); \(D_x = \text{the water pressure} \); \(N_x = \text{the gravity} \); \(g = \text{the acceleration due to gravity}\); \(D_y = \text{the gravity} \); \(G_x, G_y = \text{the gravity} \); \(g = \text{the gravity} \); \(\delta_e = \text{the pitch angle rate}\); \(\delta_e = \text{the pitch angle}\); \(\theta = \text{the pitch angle}\); \(\dot{\theta} = \text{the pitch angle rate}\). The outer loop PID controller is a pitch angle holding controller which can stabilize the pitch angle of the flying boat for some different flying tasks.

The inner loop PID controller can be expressed as:

\[
\delta_e = k_{\alpha p} e + k_{\alpha i} \int e_\delta \, dt + k_{\alpha d} \dot{e}_\delta
\]

The speed controller based on PID

3.1 Attitude controller based on PID

The flying boat has only two control actuators to control its longitudinal motions: an elevator and an engine. By changing deflection angle (\(\delta_e\)) of the elevator, the pitch angle and the pitch angle rate can be stabilized and kept to desired values. And by the changing the magnitude of the engine throttle, the engine thrust can be adjusted to control the speed of the flying boat usually.

3 The controller based on classical PID

PID is the most widely used control method in practical flight control systems due to its simple structure, robustness, and few efforts to adjust its parameters [8].

The flying boat is a multi-input multi-output system (MIMO), so, by applying PID, the controller of the flying boat is divided into attitude PID controller and speed PID controller.

3.1 Attitude controller based on PID

The usual control structure on airplanes can be divided into inner loop PID controller and outer loop PID controller [9]. In this paper, such a structure is applied in the designing of the attitude controller of the flying boat as showed in Fig. 2. The inner loop PID controller seems as a pitch damper which is designed to maintain an appropriate damping ratio for the short period motion of the flying boat (relating to the pitch angle rate). The outer loop PID controller is a pitch angle holding controller which can stabilize the pitch angle of the flying boat for some different flying tasks.

The inner loop PID controller can be expressed as:

\[
\delta_e = k_{\alpha p} e + k_{\alpha i} \int e_\delta \, dt + k_{\alpha d} \dot{e}_\delta
\]

where \(e_\delta = q - q_i\) is the value generated by the outer loop controller, and \(k_{\alpha p}, k_{\alpha i}, k_{\alpha d}\) are the proportional, integral and differential parameters in the inner loop PID controller.

The outer loop PID controller can be expressed as:

\[
q_i = k_{\alpha p} e + k_{\alpha i} \int e_\delta \, dt + k_{\alpha d} \dot{e}_\delta
\]

where \(e_\delta = \theta - \theta_i\), \(\theta_i\) is the control command of pitch angle, and \(k_{\alpha p}, k_{\alpha i}, k_{\alpha d}\) are the proportional, integral and differential parameters in the outer loop PID controller.

4 The compound controller based on ADRC and DI

4.1 Basic theory of DI

Another popular method in dealing with flight control is non-linear dynamic inversion control which is a feedback linearization method [11]. It can provide a simple design process but very good performance on objects with exact mathematic models. However, the exact mathematic models are often hard to get, which hinders the application of DI in industry. In order to improve its robust, researchers use this
method by combining with some robust control methods usually [12].

It is assumed that a general MIMO dynamics can be expressed by the following form:
\[ \dot{x} = f(x) + g(x) \cdot u \quad x \in \mathbb{R}^{n_x}, u \in \mathbb{R}^{n_u} \]
\[ y = h(x) \quad y \in \mathbb{R}^{n_y} \]
where \( x \) is state vector, \( u \) is input vector, \( y \) is output vector, and \( f(x), g(x), h(x) \) are nonlinear matrixes determined by system state.

The desired closed loop dynamics model of the general system is expressed as:
\[ \dot{x}_d = G(x_e - x) \]  
(6)

where \( x_d \) is the state of the desired closed loop dynamics model; \( x_e \) is the state of the desired closed loop dynamics model.

Then, the system’s inputs (the control forces) can be deduced as [13]:
\[ u = g(x)^{-1} (-f(x) + \dot{x}_d) = g(x)^{-1} (-f(x) + G(x_e - x)) \]  
(7)

In order to decouple the system, the desired closed loop dynamics model is usually defined in the following form:
\[ \dot{x}_d = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & k_n \end{bmatrix} (x_e - x) \]  
(8)

### 4.2 Basic theory of ADRC

ADRC, firstly proposed by Han [14], was developed and applied to many industry fields by lots of recent research works widely [15]. As it does not need to know the accurate mathematical models of control objects, it can meet the needs of industrial applications well. The complete ADRC controller has three components as showed in Fig. 4.

![Fig. 4: Complete ADRC control structure](image)

The transient profile generator is used to tracking the differential signals of the inputs. The extended state observer (ESO) can observe the uncertainties of the control objects, such as disturbances, un-modeled parts, structure uncertainties and parameter uncertainties. By feeding back the observed values of the ESO, the systems’ uncertainties can be well compensated. The third part is a nonlinear processing (NP), which is used to deal with the errors between the commands and the observed states of the control object by some nonlinear functions [16][17].

In this paper, only the ESO is used to observe the deviations between the actual flying boat and its nominal model.

### 4.3 Compound controller based on DI and ADRC

According to the longitudinal model of the flying boat, its system matrix is not a square matrix, which means that the DI method can not be adopted on the flying boat directly. So, similar to the designing of the PID controller above, this controller is divided into an inner loop controller and outer loop controller. The states controlled by the inner loop controller are \((V, q)\) and the inner loop controller is designed by the compound control method. The state controlled by the outer loop controller, which is designed by PID control method, is \( \theta \).

The specific control structure is showed in Fig. 5.

![Fig. 5: The hybrid control structure](image)

For the inner loop controller, it is assumed that the general dynamic model of the flying boat can be expressed by Eq(9).\[ J(X) \cdot \dot{X} = F(X) + G(X) \cdot U + w(X) \]
(9)
\[ J(X), G(X) \in \mathbb{R}^{n}\]
\[ X, U, w(X), F(X) \in \mathbb{R}^{n} \]

where \( X \) is the state vector; \( U \) is the input vector; \( w(X) \) is the uncertainty vector; and \( J(X), F(X), G(X) \) are matrixes determined by system states.

The system matrix and vectors are defined as follow for the flying boat:
\[ U = \begin{bmatrix} \delta_{\alpha} \\ \delta_{\beta} \end{bmatrix} \]
\[ X = \begin{bmatrix} V \\ q \end{bmatrix} \]
(10)
\[ J(X) = \begin{bmatrix} m & 0 \\ 0 & I_n \end{bmatrix} \]
\[ F(X) = \begin{bmatrix} G_a & -D_a \\ M_{a\alpha} & \dot{M}_{a\alpha} \end{bmatrix} \]
\[ G(X) = \begin{bmatrix} C_{p \alpha} \cos(\alpha + \alpha_e) & 0 \\ M_{a\theta} \cos \alpha_e & M_{a\theta} \end{bmatrix} \]
\[ w(X) = \begin{bmatrix} -N_e \sin \alpha - D_e \cdot \cos \alpha + \Delta_e \\ M_e + M_{\alpha} \cdot \dot{\alpha} + \Delta_{\alpha} \end{bmatrix} \]

where \( C_{p \alpha} \) is the thrust coefficient; \( M_{\alpha} \) is pitching moment coefficient of the engine; \( \Delta_{\alpha}, \Delta_{\theta} \) are uncertainty parts.

By multiplying \( J(X)^{-1} \) on both sides of Eq(9), the dynamic model is expressed by Eq(11).
\[ X = J(X)^{-1} \cdot F(X) + J(X)^{-1} \cdot G(X) \cdot U + J(X)^{-1} \cdot w(X) \]
(11)
According to the work of Han, the ESO is designed by Eq(12) to compensate the uncertainties \((J(X)^{+} \cdot w(X))\).

\[
Z_a = J(X)^{+} \cdot F(X) + J(X)^{+} \cdot G(X) \cdot U_a + Z_a - \beta_a(Z_a - X) \tag{12}
\]

\[
\dot{Z}_a = -\beta_a \cdot |Z_a - X|^p \cdot \text{sign}(Z_a - X)
\]

where \(Z_a\) is the observed state vector of the flying boat; \(Z_a\) is the observed uncertainty vector; and \(\beta_{a1}, \beta_{a2}\) are the adjustable parameters of the ESO.

By compensating the observed uncertainties at the control inputs of the flying boat, the inputs can be calculated by Eq(13).

\[
U_a = G(X)^{+} \cdot J(X) \cdot Z_a
\]

or

\[
U_a = G(X)^{+} \cdot J(X) \cdot (U_a - Z_a)
\]

where \(U_a\) is the command input vector of the inner loop system which includes inner loop states dynamic model and its ESO. As there are two types to feedback the observed uncertainties, the flying boat’s inputs have two expressions.

Replacing \(U\) in Eq(9) and Eq(12) by Eq(13), the inner loop system can be expressed by Eq(14) and Eq(15):

\[
Z_a = J(X)^{+} \cdot F(X) + J(X)^{+} \cdot G(X) \cdot U_a - \beta_a(Z_a - Y) \tag{14}
\]

or

\[
Z_a = J(X)^{+} \cdot F(X) + J(X)^{+} \cdot G(X) \cdot U_a + J(X)^{+} \cdot \{w(X) - Z_a\}
\]

\[
\dot{X} = J(X)^{+} \cdot F(X) + U_a + J(X)^{+} \cdot \{w(X) - Z_a\} \tag{15}
\]

Assuming \(Z_i\) in the ESO can track \(w(X)\) well, the Eq(15) can be replaced by Eq(16) used as the nominal model in DI.

\[
\dot{X} = J(X)^{+} \cdot F(X) + U_a
\]

The desired dynamic model in Eq(6) is \(K \cdot (X_i - X)\), where \(K\) is an adjustable parameter vector, and \(X_i\) is the command input of DI control. Eq(6) is changed into the form as follow:

\[
K \cdot (X_i - X) = J(X)^{+} \cdot F(X) + J(X)^{+} \cdot G(X) \cdot U_a
\]

or

\[
K \cdot (X_i - X) = J(X)^{+} \cdot F(X) + U_a
\]

(17)

And the control output of the dynamic inversion is calculated by Eq(18).

\[
U_a = G(X)^{+} \cdot J(X) \cdot \{K \cdot (X_i - X) - J(X)^{+} \cdot F(X)\}
\]

or

\[
U_a = K \cdot (X_i - X) - J(X)^{+} \cdot F(X)
\]

Replacing \(U_a\) in Eq(14) and Eq(13) by Eq(18), the ESO can be expressed by Eq(19), and the final control input of the flying boat can be express by Eq(20).

\[
Z_a = K \cdot (X_i - X) - \beta_a(Z_a - X)
\]

or

\[
Z_a = K \cdot (X_i - X) - \beta_a(Z_a - X)
\]

(19)

\[
U = G(X)^{+} \cdot J(X) \cdot \{K \cdot (X_i - X) - J(X)^{+} \cdot F(X) - Z_a\}
\]

or

\[
U = G(X)^{+} \cdot J(X) \cdot \{K \cdot (X_i - X) - Z_a\}
\]

As shown in the final results of Eq(19) and Eq(20), no matter how exactly the mathematical model can be built, the ESO has no connection to the known parts in the model, such as \(J(X), F(X)\) and \(G(X)\). However, the final control forces of this compound controller depend on these known parts.

For the attitude outer sloop controller, the PID controller expressed by Eq(3) is used.

As showed in Eq(10), the influences of water impacts on the flying boat and uncertainty of the mathematical model are observed and compensated by the ESO, which can improve the robustness of the flying boat.

5 The performance of the two controllers

In order to show the performances of the controllers, the model of the flying boat is trimmed to equilibrium status which is the initial state of each simulation. The initial state is: \(\delta_a = 0.51\), \(\delta_x = -0.1122\) (rad.), \(V = 15\) (m/s), \(\alpha = 0.121\) (rad.), \(q = 0\) (rad/s), \(\theta = 0.121\) (rad.), \(x_c = 0\) (m), \(z_c = 0.46\) (m).

5.1 The PID controller

There are nine control parameters needed to be determined \((k_{p}, k_{pd}, k_{pi}, k_{pdr}, k_{pi\delta}, k_{pd\delta}, k_{pd\delta}, k_{pd\delta}, k_{pd\delta})\). In this paper, MATLAB control design tools are used to regulate these parameters. The accessible order to determine these parameters is the speed PID, the inner loop PID and, then, the outer loop PID. At last, the parameters of the PID control are set as: \(k_{p} = 98.1\); \(k_{pd} = 66.8\); \(k_{pi} = -0.63\); \(k_{pdr} = -58.4\); \(k_{pd\delta} = -9.7\); \(k_{pd\delta} = -39.8\); \(k_{pd\delta} = 7.3\); \(k_{pd\delta} = 17.8\); \(k_{pd\delta} = -0.03\).

5.2 The compound controller

The ADRC has been shown to have lots of advantages by lots of practical applications and research works. However, the way to set the parameters in the ADRC controller is a very hard work. By lots of simulations, the parameters are set as follow: \(K = [1.9\ 3]^T\), \(\beta_a = [148.6\ 100.4]^T\), \(\beta_a = [76.9\ 0.118]^T\).

As showed in Fig.6, both the PID controller and the compound controller can control the flying boat well, when the flying boat is expected to increase its speed from 15m/s to 16m/s. Due to the convergence process of the ESO, the flying boat controlled by the compound controller will bounce in a short time at the beginning of the simulation. Fig.7 illustrates the time responses of the flying boat in waves with three operating conditions: the red curve shows the condition with no control effects; the blue dashed curve shows the condition of the flying boat with the compounded control forces; and the green curve shows the condition with PID control forces. From Fig.7, it is easy to find that the time responses of the flying boat with control forces are better than that without control effects. As the height of the flying boat is not expected to hold to a constant value, the time responses of the height are similar among the three conditions. Fig. 8 shows the flying boat taking
off from calm water with the two controllers respectively. In this simulation, the flying boat is expected to be accelerated with a constant acceleration of $2 \text{ m/s}^2$. The initial speed of the flying boat is 15 m/s, and the flying boat is accelerated to 35 m/s when it has enough aerodynamic lift to carry it. In this process, the control command of the pitch angle is constant value, which simplifies the control structure. As we can see from Fig. 8, the flying boat can take off from clam water surface smoothly with controlling forces. With the same initial state conditions and the same control commands, the flying boat takes off from water with regular waves as showed in Fig. 9. Although the flying boat vibrates for the disturbances from waves, the flying boat can take off successfully with a wave height of 0.4 m, which means the controllers have good performances to improve the seakeeping ability of the flying boat.

Although there are some differences of the time responses of the flying boat between the two controllers, it is still hard to say which one is better than the other, because the control performances will be changed with the changing of parameters of the controllers.
6 Conclusion

In this paper, based the mathematical model of a flying boat, PID (Proportional Integration Differential), DI (dynamic inversion) and ADRC (active-disturbance rejection controller) are applied to design controllers for the flying boat. The time responses of the flying boat in clam water and water with regular waves are simulated under the controlling of the PID controller and the compound controller. The taking-off processes from clam water and from water with regular waves are simulated too. Both the controllers show good performances in stabilizing the flying boat and decreasing the influences of waves.

References


