Evolutionary Optimization Methods for High-Dimensional Expensive Problems: A Survey

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***Abstract*—Evolutionary computation is a rapidly evolving field and the related algorithms have been successfully used to solve various real-world optimization problems. The past decade has also witnessed their fast progress to solve a class of challenging optimization problems called high-dimensional expensive problems (HEPs). The evaluation of their objective fitness requires expensive resource due to their use of time-consuming physical experiments or computer simulations. Moreover, it is hard to traverse the huge search space within reasonable resource as problem dimension increases. Traditional evolutionary algorithms (EAs) tend to fail to solve HEPs competently because they need to conduct many such expensive evaluations before achieving satisfactory results. To reduce such evaluations, many novel surrogate-assisted algorithms emerge to cope with HEPs in recent years. Yet there lacks a thorough review of the state of the art in this specific and important area. This paper provides a comprehensive survey of these evolutionary algorithms for HEPs. We start with a brief introduction to the research status and the basic concepts of HEPs. Then, we present surrogate-assisted evolutionary algorithms for HEPs from four main aspects. We also give comparative results of some representative algorithms and application examples. Finally, we indicate open challenges and several promising directions to advance the progress in evolutionary optimization algorithms for HEPs.**

***Index Terms*—Evolutionary algorithm (EA), high-dimensional expensive problems (HEPs), industrial applications, surrogate-assisted optimization.**

Manuscript received October 18, 2023; revised December 21, 2023; accepted February 11, 2024. This work was supported in part by the Natural Science Foundation of Jiangsu Province (BK20230923, BK20221067), the National Natural Science Foundation of China (62206113, 62203093), Institutional Fund Projects Provided by the Ministry of Education and King Abdulaziz University (IFPIP-1532-135-1443), and FDCT (Fundo para o Desenvolvimento das Ciencias e da Tecnologia) (0047/2021/A1). Recommended by Associate Editor Chengdong Li. *(Corresponding authors: MengChu Zhou and Meiji Cui.)*

Citation: M. C. Zhou, M. Cui, D. Xu, S. Zhu, Z. Zhao, and A. Abusorrah, "Evolutionary optimization methods for high-dimensional expensive problems: A survey," *IEEE/CAA J. Autom. Sinica*, vol. 11, no. 5, pp. 1092–1105, May 2024.

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Digital Object Identifier 10.1109/JAS.2024.124320

I. Introduction

MANY real-world complex engineering optimization problems require the computationally expensive evaluation of objective and constraint functions, such as trauma system optimization [1], optimal traffic signal timing [2], aerial vehicle design optimization [3]. Among many reasons that lead to such expensive evaluations are 1) a problem involves complex physical and chemical processes; 2) a problem needs time-consuming computer simulations or other complicated analysis tools, e.g., finite-element analysis [4]. For such expensive problems, a single fitness evaluation may cost hours, days, and even weeks [5], bringing tremendous computational or physical resource burden for traditional algorithms. Moreover, with the advent of big data era, a great deal of optimization problems usually involve hundreds or even thousands decision variables, i.e., high-dimensional problems, leading to a phenomenon called "curse of dimensionality". In other words, search space size exponentially increases with problem size (mainly the number of decision variables), posing a great challenge to traverse the whole search space within reasonable time. Moreover, as problem dimension increases, their characteristics may change and interactions among decision variables become more complicated [6]. As a result, local optimal regions increase as well, making conventional optimization algorithms more likely to fall into local optima. The resulting problems are named high-dimensional expensive problems (HEPs) in this survey paper, a rather popular and challenging research direction from both academia and industry. Since these problems may not have exact math expressions or extremely complex ones, exact solution methods tend to become helpless. Intelligent optimization methods, especially nature-inspired evolutionary optimization methods, have become the dominant ones to solve HEPs and received more and more attention since two decades ago as shown in Fig. 1. We use the following keywords to search IEEE Xplore and Scopus: high-dimensional expensive optimization, high-dimensional expensive evolutionary algorithm, and high-dimensional expensive problem. It is clearly seen that the number of published papers on HEPs increases nearly 100% every 5 years, which confirms it as a promising research direction. Although several survey papers on evolutionary computation for expensive optimization problems have been published [7]–[13], their algorithms mainly focus on expensive problems with less than 30 dimensional decision variables. Moreover, a crescent number of papers focusing on both high-dimensional decision variables (30 and higher dimension) and computationally expensive evaluations have been researched recently. Their solutions are extremely challenging but critically important in many practical applications. It is in great demand to write a specific survey paper to evaluate them in this increasingly important area. Therefore, this work tries to do so to benefit the HEP community.

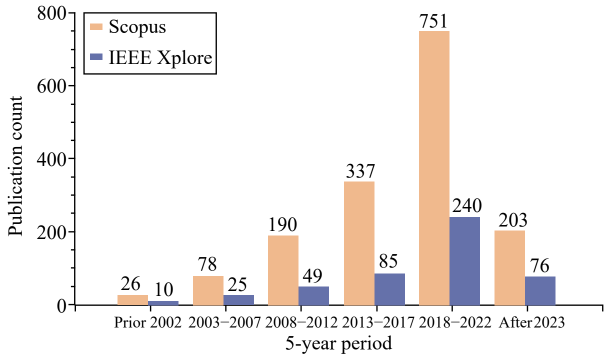


Fig. 1. The number of papers related to HEPs.

The rest of the paper is organized as follows. We present the basic concepts of HEPs in Section II. Section III describes surrogate-assisted evolutionary algorithms for HEPs from four main aspects, i.e., surrogate selection, model construction, model management and base optimizers. Section IV presents comparative results of representative algorithms and application examples. Section V gives open challenges and several promising directions of HEPs. Section VI draws conclusions on this topic.

II. High-Dimensional Expensive Problems

High-dimensional expensive problems (HEPs) are those whose objective functions take much time or resource to evaluate and generally consist of high-dimensional decision variables. Maximization problems can be mathematically solved by transforming them into minimization ones. Therefore, we take minimization problems as HEP examples in this paper. They can be formally expressed as

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & \check{\boldsymbol{x}} \le \boldsymbol{x} \le \hat{\boldsymbol{x}} \end{aligned} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_D) \in \mathbb{R}^D$ denotes a vector containing D decision variables, $f(\cdot)$ represents an objective function, D is the problem dimension, $\mathbb{R}^D$ is the feasible solution space, and $\check{\boldsymbol{x}}$ and $\hat{\boldsymbol{x}}$ are the lower and upper bounds of $\boldsymbol{x}$, respectively. When $f(\cdot)$ is a simple mathematical function or model, its computational cost is limited. When $f(\cdot)$ represents a physical model, complex system, sophisticated design, and simulation model, its computational cost can be tremendously expensive. There is no formal definition of high dimension, i.e., how high is high. After reviewing papers dedicated for HEPs, we find that most of them test their problems with dimensions varying from 30 to 200. Therefore, we regard optimization problems as high-dimensional expensive problems when D is over 30 in this paper, which is consistent with other papers.

In reality, many optimization problems are subject to some complex linear/non-linear constraints instead of the relatively simple one in (1). A general form can be expressed as

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & \overset{\le}{H}(\boldsymbol{x}) \le 0 \\ & \overset{=}{H}(\boldsymbol{x}) = 0 \end{aligned} \tag{2}$$

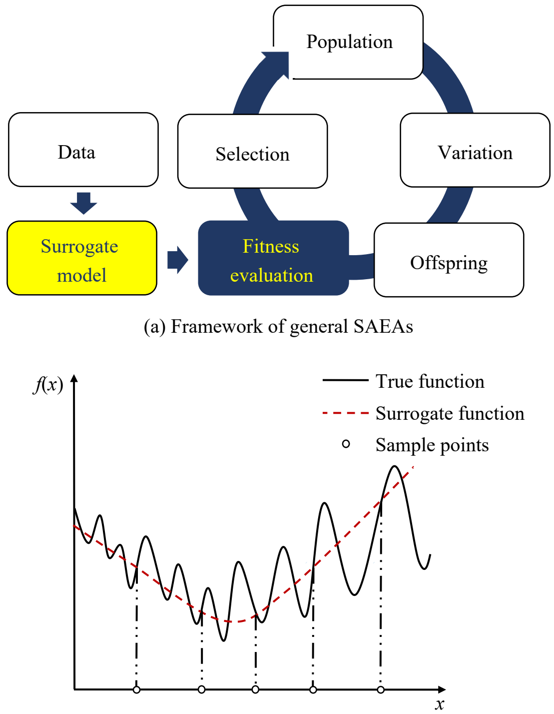
where $\overset{\le}{H}(\boldsymbol{x})$ is a set of inequality constraints, and $\overset{=}{H}(\boldsymbol{x})$ is a set of equality ones. A common way to deal with constrained problems is to use a penalty function to represent any violation of such constraints, therefore converting a constrained one into a non-constrained one [14]. Also, Lagrangian multipliers or fuzzy logic can be adopted to handle constraints [15]. Readers can find more details in [16]. We may also develop a multi-objective or many-objective HEPs if we extend a single objective function into a number of conflicting functions, i.e., $\min\{f_1(x), f_2(x), \ldots, f_m(x)\}$ with $m \ge 2$. The resulting problems are called multi-objective HEPs if $m \in \{2,3\}$, and many-objective ones if $m > 3$ [17], [18].

HEPs are attracting more and more attention from both academia and industry. Bayesian optimization using Gaussian Process, also known as efficient global optimization, has gained some success in the field of expensive problems [19]–[21]. However, as mentioned in [22], Gaussian Process based Bayesian Optimization is rather challenging to deal with high-dimensional problems. Moreover, evolutionary algorithms, as powerful global optimizers, have been widely investigated to solve such challenging problems. Therefore, we focus on using evolutionary algorithms (EAs) to solve HEPs in this survey paper.

III. Evolutionary Algorithms for High-Dimensional Expensive Problems

Surrogate-assisted evolutionary algorithms (SAEAs) are the mainstream methods to deal with expensive optimization problems [23]–[25]. With the idea of using computationally cheap surrogate models to replace part of truly expensive models for fitness evaluations, SAEAs are able to solve computationally expensive problems satisfactorily with limited resource, such as integrated circuit design [26], neural architecture search [27], and trauma system deployment [28]. The basic framework of SAEAs is illustrated in Fig. 2. Obviously, the main difference between conventional EAs and SAEAs rely on fitness evaluations. As seen in Fig. 2, SAEAs can take advantages of surrogate models constructed based on some pre-processed historical data to approximate the true models. Then, surrogate models can be adopted to pre-screen candidate solutions with the aim to reduce unnecessary use of truly expensive models. In other words, only those solutions predicted as promising ones consume highly expensive computational/physical resource while others are assisted by cheap surrogate models.

Although SAEAs have gained some success in dealing with small-scale expensive problems (less than 30 decision vari-



(a) Framework of general SAEAs

(b) Surrogate construction

Fig. 2. The illustration of SAEAs.

ables), their performance degrade to some extent when handling high-dimensional problems (especially those with 100 and higher dimensional decision variables) due to exponentially expanded search space. To be specific, the inefficiency of conventional SAEAs coping with HEPs can be attributed to: 1) dimensionality sensitivity of surrogates; 2) difficulty of surrogate construction; 3) indetermination of model management strategy; 4) low efficiency of base optimizers. When it comes to high-dimensional expensive problems, some surrogate-assisted algorithms have been elaborated designed to adapt to high-dimensional decision variables (shown in Table I). Therefore, we review a series of SAEAs dedicated for HEPs from above-mentioned four issues, namely surrogate selection, surrogate construction, surrogate management, and base optimization algorithms to investigate which strategy they adopt for these challenging problems, as shown in Fig. 3.

A. Model Selection

Generally, machine learning methods which are possessed with prediction ability can be adopted to train approximate models, such as Gaussian process (GP) [29], radial basis function (RBF) [27], [49], and random forest (RF) [28], mostly used in recent papers. We introduce some representative surrogates and analyze their strengths and weaknesses for modeling in the high-dimensional space.

1) Global Surrogates: A GP model, also known as a Kriging model, can be used to approximate an unknown function. Compared with other surrogate models, GP can provide prediction information as well as uncertainty information, i.e., variance values, which is beneficial to enhance surrogate accuracy by investigating less explored areas [29]. However, both time complexity and model accuracy of GP construction are sensitive to problem dimensionality. As problem size increases, large-scale training samples are required to train the hyperparameters of the GP, which inevitably exacerbates computational burden. Moreover, the approximation uncertainty provided by GP becomes less reliable on high-dimensional space. The commonly-used infill criteria of GP that considers performance-based and uncertainty-based information simultaneously tends to lose efficacy. To overcome the above-mentioned challenges encountered by high-dimensional GP models, some researchers try to construct them in the relative low-dimensional space assisted by some dimension reduction tools, such as Sammon mapping [61]. Liu *et al*. [29] adopt Sammon mapping as a dimension reduction technique to transform the training data to a lower dimensional space and then conduct GP modeling in the reduced space. In this way, modeling in the shrunken space can enhance the model accuracy significantly as well as reduce the computational consumption greatly, which is rather attracting for HEP community [36]. Moreover, Cai *et al*. [39] propose a simplified GP model by considering every correlation parameter allocated with the same weight, which can significantly relieve computational burden yet sacrifice model accuracy. Overall, some useful neighbour information can be inevitably lost during the mapping or simplified process, which is harmful for the final optimization performance.

2) Radial Basis Function (RBF): Radial basis function (RBF) takes advantage of a weighted sum of several basis functions with the aim to approximate the search space as far as possible. The history of RBF as an interpolating method can be tracked back to 1971 when it was investigated to approximate irregular data [62]. Since then, RBF are widely adopted as surrogate models for approximation in various fields. For example, Li *et al*. [63] propose a three-level RBF-assisted algorithm to solve computationally expensive problems, where RBF models are adopted for global surrogate, local surrogate and local search, respectively. According to the recent research [64], RBF models still have satisfactory accuracy and efficiency for high-dimensional data while other surrogate models suffer from curse of dimensionality. It is conceivable that RBF are widely selected as surrogate models when dealing with HEPs, as shown in Table I. Although RBF is insensitive to problem dimension to some extent, its accuracy is still hard to guarantee in such huge modelling space with limited available training samples. Therefore, some researchers take advantages of dimension reduction tools, such as random projection, to train RBF models for each subspace with a relative low dimension, and then achieve the final predicted fitness by calculating the average result of constructed models [47], [65]. Unlike the above unsupervised dimensionality reduction methods, Lin *et al*. [54] select several important decision variables based on an adaptive dropout model, and then build a lower dimensional yet higher accuracy RBF based on the chosen data samples for solving high-dimensional expensive multiobjective optimization problems.

3) Random Forest: Random forest (RF) uses multiple decision trees to predict data samples for some tasks, e.g., classification and regression [66]. Subsets are formed by bootstrap

TABLE I
COMPARISONS OF SAEAS FOR HIGH-DIMENSIONAL EXPENSIVE PROBLEMS (HEPS)

Algorithm	Year	Base optimizers	Model selection	Model construction	Model management	$\hat{D}$
GPEME [29]	2014	DE	GP	Global/local	Hybrid	50
SA-COSO [30]	2017	PSO + SL-PSO	RBF + FES	Global+local	Performance-based	200
FAACSO [31]	2018	CSO	FES	Local	Performance-based	500
SHPSO [32]	2018	PSO + SL-PSO	RBF	Global + local	Performance-based	100
RF-CMOCO [28]	2018	NSGA-II	RF	Local	Hybrid	100
MGP-SLPSO [33]	2019	SL-PSO	GP	Global	Hybrid	100
PESPSO [34]	2019	PSO	RBF	Global + local	Performance-based	100
S-JADE [35]	2019	DE	RBF	Global + local	Performance-based	200
TASEA [36]	2019	DE	GP	Global + local	Performance-based	100
ESAO [37]	2019	DE	RBF	Global + local	Performance-based	200
GORS-SSLPSO [38]	2019	SL-PSO	RBF	Global/local	Performance-based	100
GSGA [39]	2020	GA	RBF + GP	Global + local	Hybrid	100
Gr-based SAPSO [40]	2020	SL-PSO	GP + FIS	Global + local	Hybrid	200
SAGWO [41]	2020	GWO	RBF	Global	Performance-based	100
MS-MTO [42]	2020	MS-MTO	RBF	Global + local	Performance-based	200
EHSDE [43]	2021	DE	RBF	Global + local	Hybrid	100
SAMSO [44]	2021	PSO + TLBO	RBF	Global	Performance-based	100
FHSAPPSO [45]	2021	PSO	RBF	Global + local	Performance-based	100
SATLBO [46]	2021	TLBO	RBF	Local	Performance-based	100
RPHSA [47]	2021	DE	RBF	Global + local	Performance-based	200
SA-MPSO [48]	2022	PSO	RBF	Global + local	Performance-based	100
RFMOISR [49]	2022	NSGA-II	RF	Local	Hybrid	750
MSODE [50]	2022	DE	RBF	Global + local	Hybrid	100
SAHSO [51]	2022	PSO + TLBO	RBF	Global	Performance-based	200
RSAEH [52]	2022	DE	RBF	Global + local	Performance-based	100
VSMPSO [53]	2022	PSO	RBF	Global	Performance-based	200
ADSAPSO [54]	2022	PSO	RBF	Local	Performance-based	200
SAEO [55]	2022	TLBO	RBF	Global	Performance-based	500
TS-DDEO [56]	2023	PSO + DE	RBF	Global + local	Performance-based	100
GL-SADE [57]	2023	DE	RBF + GP	Global + local	Hybrid	200
SDAMA-SPS [58]	2023	Monkey algorithm	RBF	Global + local	Hybrid	100
LSADE [59]	2023	DE	Lipschitz + RBF	Global + local	Performance-based	200
STORA [60]	2023	TLBO	RBF	Global	Performance-based	200

Note: $\hat{D}$ is the highest dimension of tested problems; FES means fitness estimation strategy; FIS means fitness inheritance strategy; and GWO means grey wolf optimization.

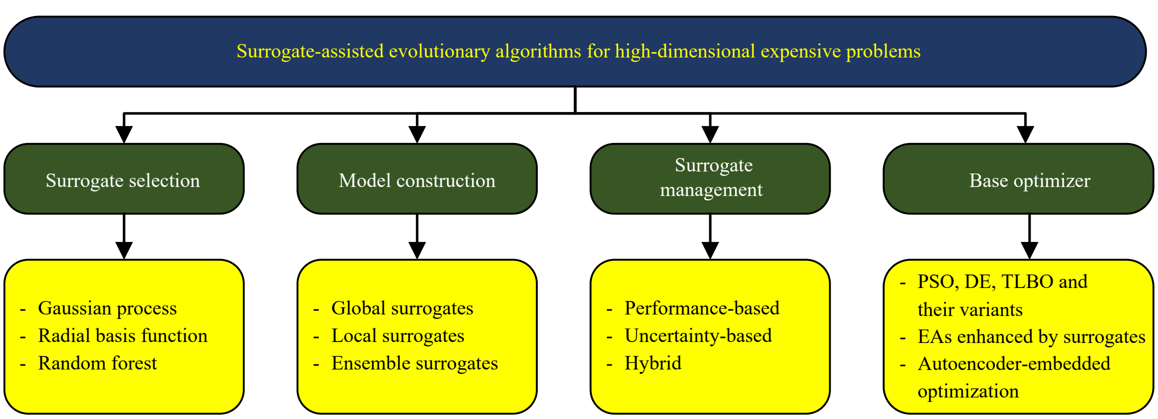


Fig. 3. The main issues of SAEAs for solving HEPs.

sampling from the training data where subset size is much smaller than the original dataset size. Each subset includes different bootstrap data samples and accordingly each decision tree trained by them has different structures. Once decision trees are trained, we can use them to predict the results. Specifically, for the phase of training process, each decision tree selects the same subset from inputs, and then the predicted results can be obtained accordingly. The final result is the average of the outputs of decision trees. RF has been widely adopted as a surrogate model for solving discrete problems since it possesses a binary structure and thus it is easy to realize [28], [49]. To accelerate convolutional neural network architecture design, Sun *et al*. [27] make use of an offline RF model as a predictor to replace part of really expensive fitness evaluations. Although few of research works concentrating on this kind of extremely challenging problems, i.e., high-dimensional expensive discrete/combinatorial problems, RF is a promising and important surrogate model to solve such problems in the future.

B. Surrogate Construction

According to construction scenarios of surrogate models, existing surrogate construction methodologies for HEPs can be classified into three categories: local surrogates, global surrogates, and ensemble surrogates (global and local surrogates) [67], as shown in Fig. 4.

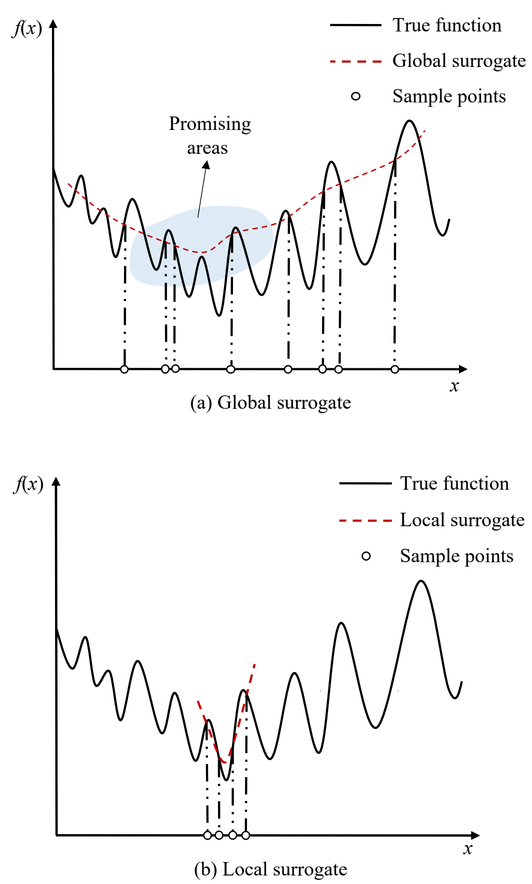


Fig. 4. Types of surrogate construction.

1) Global Surrogates: Global surrogates are trained by all real-evaluated points with the aim to model the whole search space, and then performs a global solution search based on these models. As presented in Fig. 4(a), global surrogate models can smoothen out some local optima and locate promising areas quickly, which is beneficial for further exploitation. Li *et al*. [44] build a global RBF model to capture the profile of landscape and use it to predict fitness values of sub-population produced by two swarms. Dong and Dong [41] construct a global RBF model to assist grey wolf algorithm to search the high-dimensional space and exploit the local trust regions predicted by the RBF model. Tian *et al*. [53] train and update a global RBF model facilitated with a simple sampling strategy to select data points for its training. The constantly updated RBF models can approach different promising areas so as to enhance search diversity. Cui *et al*. [55] propose a bi-population co-evolution optimization strategy where one population is evaluated under the assistance of a global RBF model. Nevertheless, it is difficult to train a reliable surrogate models in high-dimensional space due to “curse of dimensionality”. As seen in Fig. 4(a), the optimum predicted by the global surrogate model is inconsistent with the true optimum, and thus misleading the optimization search direction.

2) Local Surrogates: Different from global ones, local surrogate models are trained by part of real-evaluated data points to capture the landscape of sub-spaces. As presented in Fig. 4(b), local surrogate models trained by some good individuals found so far assist optimization algorithms to exploit the promising areas. Sun *et al*. [31] adopt a fitness estimation strategy based on the positional relationships between individuals and use it in a local manner. Compared with other surrogate models heavily relying on data samples, the fitness estimation strategy is less sensitive to problem dimensions, and thus fitting to deal with HEPs. Moreover, with the help of dimension reduction techniques, multiple local surrogate models are built in the reduced sub-spaces [29], [54]. Dong *et al*. [46] train multiple surrogate models for different sub-spaces to relieve the difficulty of modelling in the high-dimensional space. Li *et al*. [68] build a lightweight and reliable local surrogate model based on newly produced offspring, and a model-free evolutionary algorithm is activated when the former surrogate model does not work.

3) Ensemble Surrogates: As analyzed above, a global surrogate model can ensure the population’s global search ability and smoothen out some local areas, but its use tends to sacrifice accuracy performance in some key regions. Contrarily, a local surrogate model can help the population to exploit some key regions fast and accurately, but may ignore other promising regions and trap into local optima. Hence, some ensemble or multi-fidelity surrogates are investigated to make use of global and local surrogates’ strengths [25]. Sun *et al*. [30] employ a RBF as a global surrogate to capture the whole search space, and adopt a fitness estimation strategy as a local surrogate according to the positional relationship between the particles. Cai *et al*. [39] make full use of a global surrogate model built in the whole design space and a local one built in a neighbor region around the optima found so far to guide efficient search. Chen *et al*. [43] adopt a global surrogate model and two local surrogates, where local ones are trained by the most promising sample points and the sample points around the current best solution, respectively. Along with the same idea, Chu *et al*. [45] propose a global surrogate and two local surrogates, among which a local surrogate is constructed based on the sub-archived data divided by fuzzy clustering algorithm. Yang *et al*. [36] construct a global GP model to locate some promising regions and a local GP model to exploit the space neighboring the best solution found so far. Wang *et al*. [37] alternatively use a global surrogate and a local surrogate to assist the optimization process. The local surrogate is trained by several best data samples to capture the landscape of a sub-space, and a global optimizer is adopted to find the optimum of it. Tian *et al*. [40] employ a GP model as a global surrogate which is trained by coarse-grained individuals, and adopt a fitness inheritance strategy as a local surrogate that is built by fine-grained individuals. Wang *et al*. [57] take advantages of heterogeneous surrogates to balance the ability of exploration and exploitation. Their analysis shows that a RBF model is appropriate to estimate global trend while a GP model is suitable to assist the population to jump out of local optima. Liao *et al*. [42] treat a global surrogate and a local surrogate as two different but related tasks, and thus multi-tasking optimization is adopted to solve HEPs.

C. Model Management

Surrogate models need to be re-constructed based on newly added data pairs, which are re-evaluated by true models, to refine their accuracy. Then optimization assisted by more accurate surrogate models can be more effective. However, how to select candidate individuals for true evaluation is an elaborate issue affecting the overall performance of SAEAs, which is called model management or evolution control [7]. In general, model management criteria can be divided into three categories, i.e., performance-based, uncertainty-based and their hybrid.

1) Performance-Based: The performance-based model management intends to re-evaluate the promising individuals with the aim to exploit the promising areas found so far. The key issue is how to determine whether individuals are promising or not. The natural way is to choose the best predicted individuals for true evaluations [31]. Yu *et al*. [32] conduct true evaluations of all individuals whose predicted fitness values are better than that of their historical bests. Furthermore, Li *et al*. [44] consider the minimum distance between the promising individuals and other real-evaluated points to prevent neighbouring individuals from being over-evaluated. Cai *et al*. [35] select several trial individuals with best approximation values for real fitness evaluations, and the exact number of selected points is tested through experiments. Sun *et al*. [30] re-evaluate the individuals only if their fitness values estimated by two surrogate models separately are better than their historical fitness values. Wang *et al*. [37] carry out true evaluations of the best individual pre-screened by a global surrogate model as well as the optimal one found by a local surrogate model.

2) Uncertainty-Based: The uncertainty-based model management strategy chooses the individuals considered as most indeterminate for true evaluations. It benefits to explore the area with little information, i.e., blessing of uncertainty [7]. However, how to determine the degree of uncertainty is the foremost consideration of using it. Here, we conclude several uncertainty measurements proposed in recently published papers. The variance values provided by GP models can be naturally used as uncertainty information, thereby widely adopted in related papers [29], [39], [40]. However, the uncertainty information provided by GP models is hard to distinguish as problem size increases. To get over the above difficulty, Chen *et al*. [43] take advantage of the Euclidean distance between offspring and existing solutions as the metric to determine uncertainty degree. Guo *et al*. [69] determine the uncertainty information assisted by the discrepancies among predicted fitness values provided by surrogate ensembles. Wang *et al*. [57] consider both the prediction discrepancies of multiple surrogates and the minimum distance of the individual to existing samples as uncertainty information. Nevertheless, few works of HEPs only rely on the uncertainty criterion since convergence speed in this scenario can not be guaranteed in reasonable time.

3) Hybrid: Their hybrid model management, naturally, is capable of keeping a good balance between global exploration and local exploitation. For example, lower confidence bound (LCB) [70] and expected improvement (EI) [71] provided by GP models consider the search both in the promising areas (i.e., best predicted fitness values) and less explored areas (i.e., with high variance) [29], [39]. Wang *et al*. [57] adopt a RBF model as global surrogate and a GP model as a local one, and thus employing different infill criterion considering both performance and uncertainty. Unlike combing them into a scalar function, Tian *et al*. [33] consider approximation fitness and uncertainty information as two objectives, i.e., a multiobjective infill criterion for GP modelling, and then adopt a nondominated sorting strategy for model management.

D. Base Optimizers

Although any optimization algorithm can be employed to solve optimization problems, the selection of appropriate base optitmizer for some complex and challenging problems play a significant role in final performance in terms of both accuracy and efficiency. As presented in Table I, we notice that several evolutionary optimization methods are widely adopted in solving HEPs as basic optimizers, i.e., particle swarm optimization (PSO), differential evolution (DE), teaching-learning-based-optimization (TLBO) and their variants. Traditional evolutionary algorithms are difficult to achieve satisfactory results within limited computational/physical resource, their variants and some newly proposed evolutionary algorithms that dedicated to deal with high-dimensional/large-scale problems are suitable to be employed as base optimizers. As shown in Table I, compared with other algorithms, PSO, DE, TLBO and their variants are absolutely top selection by researchers for solving HEPs. Therefore, we review them for peer reference, and more optimizer selections can refer to [72]–[74] if readers interested in. Also, several efficient strategies used for enhancing evolutionary algorithms are introduced.

1) Particle Swarm Optimization and Its Variants: Particle swarm optimization (PSO) [75] emulates behaviors of a fish shoal or a bird flock. PSO shows outstanding performance when dealing with optimization problems due to its simplicity and fast convergence [76]. Based on the canonical PSO, some variants are proposed to deal with large-scale optimization problems, among which social learning-based PSO (SL-PSO) [77] and competitive swarm optimization (CSO) [78] are two representative variants. SL-PSO is inspired by a social learning mechanism, i.e., an imitator learns the behaviors of different demonstrators [77]. In SL-PSO, the particles are first sorted in an increasing order of their fitness values, i.e., from the worst to the best. Each particle, except the best particle, learns from a randomly chosen particle whose fitness is better than that of particle, known as a demonstrator. Then, the position of the particle is updated. While CSO enhances the global search capability by introducing a pairwise competition strategy on the basis of the standard PSO [78]. To be specific, all particles are randomly divided into two swarms for further pairwise competition. Then particle with a better fitness value is selected into the next generation directly while the inferior one is updated by learning from the better one. Furthermore, Wei *et al*. [13] adopt the level-based learning swarm optimizer as the base optimizer, which is well suited to the adopted surrogate model, namely gradient boosting classifier. No matter PSO or its variants demonstrate high efficiency in handling high-dimensional problems, several research works adopt them as base optimizers and achieved good results [30], [32]–[34]

2) Differential Evolution and Its Variants: Differential evolution (DE) is a competitive evolutionary algorithm with a typical memory characteristic and a global search capability. Its characteristics such as ease of implementation, fewer control parameters, and low space complexity have attracted tremendous attention from the evolutionary computation community [79]. A canonical DE algorithm has four basic steps: initialization, mutation, crossover, and selection. The initial population are randomly generated and each individual in the population is called a target vector. Then, different frequently considered mutation strategies [80] can be chosen according to problems' characteristics. To increase population diversity, a crossover process is conducted by creating a trail vector. At last, the better individuals are selected for next generation in terms of fitness values. DE and its variants have been used to solve HEPs and validated their good performance in this field [29], [35]–[37]. Readers can refer to [79] for more details.

3) Teaching-Learning-Based-Optimization (TLBO): In state-of-the-art research, as a competitive heuristic algorithm, TLBO has attracted an enormous attention in solving high-dimensional/large-scale optimization problems due to its few parameters and fast convergence [81]. It simulates teaching and learning procedures in schools so as to enhance students' knowledge. Specifically, students learn knowledge from the teacher, the best individual, according to the difference between the teacher and the mean of students. Moreover, a student can interact randomly with other students to enhance their knowledge so as to prompt diversity. Due to its excellent performance on solving large-scale optimization problems, TLBO has been widely adopted in HEPs community and regarded as a promising and competitive algorithms for complex problems characterized by high dimension [44], [46], [55], [82]–[84]. Readers interested in TLBO can find more information in [85].

4) Evolutionary Algorithms Enhanced by Surrogates: The intuitive employment of surrogate models is to screen out some promising candidate individuals to avoid unnecessary use of truly expensive evaluations. Furthermore, the effectiveness of surrogate models have been adopted throughout the whole process of evolutionary algorithms, such as surrogate-assisted updating strategy and surrogate-assisted global/local search, which expand the use scenarios of surrogate models and thus advance the conventional algorithms' performance. Cai *et al*. [35] generate trail vectors of DE by taking advantages of optimum information predicted by local surrogates, and thus speeding up the convergence speed. Likewise, they consider the predicted best individuals provided by global surrogates and local surrogates to enhance the updating operators of the traditional PSO [34]. Furthermore, they combine surrogate-assisted local search, surrogate-assisted updating strategy, and surroagte-assisted pre-screening strategy together to form an efficient SAEA framework. However, the extensive training of surrogate models throughout the whole process can inevitably result in improving computational burdens [39].

5) Autoencoder-Embedded Optimization: The obstacle of evolutionary algorithms to solve HEPs lies in the expanded search space and complicated landscape as problem size increases, making it difficult to produce high-quality offspring in the high-dimensional space. However, if high-quality offspring can be generated, then the effect of every fitness evaluations to improve the overall performance can be fully used and HEPs can be handled with limited computational resource. As a result, some works naturally attempt to generate offspring in the shrunken space assisted by some dimension recursion techniques [55], [60], [86], [87]. Cui *et al.* [86] propose an autoencoder-embedded optimization (AEO) framework where they compress the high-dimensional space to the lower one with the assistance of trained autoencoders. Then, variations are conducted in the reduced yet informative space which benefits to generate promising offspring. Also, a bi-population cooperative optimization strategy is adopted to make a trade-off between exploration and exploitation, where one sub-population is assisted by autoencoder-embedded evolution and the other one is optimized by a baseline optimizer (i.e., conventional evolution). Their experimental results show its superiority over traditional evolutionary algorithms. Different from constructing surrogate models in the reduced space, AEO framework enables offspring generated in the compressed yet informative space under the assistance of dimension reduction techniques. It is worth stressing that AEO framework is compatible with other frameworks, such as SAEAs, which has been validated in [55]. Overall, the incorporation AEO with other frameworks can further enhance their efficacy on some complex optimization problems [60], [87].

Accordingly, researchers or practitioners can design/select their algorithms by means of considering above-mentioned four aspects, as shown in Fig. 3. After reviewing them, we have several observations for how to design/choose an algo-

TABLE II
BENCHMARK FUNCTIONS

Function	Name	Design space	$f^{*\dagger}$	Property
F1	Ellipsoid	$[-5,5]^D$	0	Unimodal
F2	Rosenbrock	$[-2,2]^D$	0	Multimodal with narrow valley
F3	Ackley	$[-32,32]^D$	0	Multimodal
F4	Griewank	$[-600,600]^D$	0	Multimodal
F5	Rastrigin	$[-5,5]^D$	0	Multimodal
F6	Shifted rotated F5	$[-5,5]^D$	−330	Multimodal and shifted
F7	F19 in [92]	$[-5,5]^D$	10	Multimodal and non-separable

^{D} means problem dimension.
† f^* means global optimum.

TABLE III
COMPARISONS OF DIFFERENT ALGORITHMS FOR HEPS

	GPEME	SA-COSO	SHPSO	ESAO	GSGA	SAMSO	TS-DDEO	SAEO	GL-SADE	RSAEH
GPEME	×	/	<	<	<	/	/	/	/	<
SA-COSO	/	×	<	<	<	<	<	<	<	/
SHPSO	>	>	×	<	<	<	<	<	<	<
ESAO	>	>	>	×	/	<	<	<	<	<
GSGA	>	>	>	/	×	/	/	<	/	/
SAMSO	/	>	>	>	/	×	<	<	/	/
TS-DDEO	/	>	>	>	/	>	×	/	/	/
SAEO	/	>	>	>	>	>	/	×	/	/
GL-SADE	/	>	>	>	/	/	/	/	×	/
RSAEH	>	/	>	>	/	/	/	/	/	×

Note: > means that the algorithm in the column performs better than that in the row; < means that the algorithm in the column performs worse than that in the row; / means that two algorithms are not compared; × means that an algorithm does not compare itself.

rithm for HEPs as follows.

a) Surrogate selection: Random forests perform well on discrete HEPs thanks to their binary structures. Radial basis functions are less sensitive to problem dimensions and they are suitable for high-dimensional problems. Gaussian Processes can enhance their accuracy by taking account of uncertainty information.

b) Model construction: Global model construction can capture the main profile of the HEPs and thus benefiting to smoothening out local optima and locate the promising areas quickly. Although local surrogate models can focus on exploiting local areas, they can hardly capture the landscape of high-dimensional space. Therefore, surrogate ensembles should be used to make full use of multiple surrogates with different advantages, which is widely adopted in recent research.

c) Surrogate management: Performance-based strategy intends to exploit promising areas found so far, which are beneficial to quickening the convergence speed. The uncertainty one can explore the area with less information so as to enhance surrogate accuracy. Naturally, the combination of performance and uncertainty strategies can strike a good balance between global exploration and local exploitation.

d) Base optimizer: Compared with conventional algorithms, some enhanced algorithms with different strategies, such as surrogate-based search and autoencoder-embeded frameworks, show promising results in solving HEPs.

IV. COMPARISON RESULTS AND APPLICATION EXAMPLES

A. Benchmark Functions

After reviewing HEPs-related papers, we identify seven widely used benchmark problems as shown in Table II where D is problem dimension [39], [44], [88]−[91]. They can be divided into three classes:

1) Unimodal Function F1: Continuous, convex and unimodal.

2) Multimodal Functions F2–F6: A huge number of local optima.

3) Hybrid Composition Function F7: Rotated hybrid composition function with a narrow basin global optimum.

B. Comparison of Results of Representative Algorithms

To show the effectiveness of different algorithms for solving HEPs, we review the most popular ten related algorithms for solving HEPs, as shown in Table III. We compare the average best performance of algorithms according to the results from their original papers. Functions varying from D = 50 to D = 200 listed in Table II are adopted if algorithms are tested on them. Since most algorithms were not tested in F5 with D = 200, and a total of 20 functions excluding F5 are adopted and reported in this survey for the case of D = 200. If two algorithms are not compared, the corresponding entry is noted with "/". Otherwise, if the algorithm in the column is better than that in the row, the corresponding entry is represented as ">"; and otherwise "<". SA-COSO, SHPSO, ESAO, and SAMSO are the most compared algorithms as they have been published two to five years ago. Among these four algorithms, SAMSO [44] achieves the best overall performance facilitated by two swarms, dynamic swarm size, and powerful search algorithms, i.e., social learning-PSO and TLBO. GSGA [34], as a contemporaneous algorithm, is seldom adopted as a compared algorithm although experimental results have validated its high performance in solving benchmark problems. As stated in [34], too many times of re-training surrogate models cost large amounts of computational resource as a result of three kinds of surrogate-assisted strategies used throughout the evolutionary process. In addition, some recently proposed algorithms, i.e., TS-DDEO [56], SAEO [55], GL-SADE [57], and RSAEH [52] can obtain overall better performance than the previous algorithms [30], [32], [37]. However, the comparison results among them are not available yet, which can be investigated as important future work.

After reviewing them, we can conclude that the trends of algorithms for HEPs are 1) surrogate-assisted strategies adopted throughout the whole process, i.e., sampling, search, crossover/mutation, and fitness prescreening; and 2) high-quality offspring generated in reduced space assisted by both dimension reduction tools and surrogate models. Overall, newly proposed algorithms for HEPs and their improved performance can be regarded as the evidence of advancement of the HEP research field.

C. Application Examples

Real-world high-dimensional expensive optimization problems are existing extensively in various fields. We select several representative application examples for readers who are interested in.

1) Shape Design Optimization: The shape design optimization problems usually contain tens or even hundreds of variables and the calculation of objectives involves time-consuming CFD simulations, resulting in the typical high-dimensional computationally expensive problems [93]. For example, airfoil shape design optimization problems involve tens of decision variables and a single simulation may take several hours or even days [94]. Several algorithms have been proposed to deal with these problems and achieved the promising results [37], [48], [57]. In their works, the airfoil geometry is parameterized by the class shape transformation method [95] and NACA0012 airfoil is adopted as the baseline. Likewise, some other shape design optimization problems are researched as well, such as all-direction propellers [35], [36], [51], [96] and blended-wing-body underwater vehicles shape design [46], [93], [97]. The goal of handling shape design optimization problems is to find the optimal design parameters with as less computational resource as possible.

2) Resource Allocation Optimization: No matter medical resource or computational resource are limited in the real world. How to allocate intensive resource to meet the requirement is an important research direction. As reported in [1], the objective and constraints of the trauma system optimization can be evaluated only by using incidents, which is a typical resource-restricted high-dimensional expensive problem. To obtain a high-quality solution in a reasonable computation time, Wang *et al*. [1] present a surrogate management scheme by establishing a regression model that can estimate the number of clusters required based on the maximum acceptable approximation error. Moreover, Bi *et al*. [60] propose an evolutionary algorithm assisted by autoencders to solve computation offloading problem in mobile edge computing, which intends to migrate a part of data processing from resource-constrained smart mobile devices (SMDs) to high-performing platforms.

3) Production Scheduling Optimization: Production scheduling problems with different characteristics are existing in different fields, such as flexible job shop scheduling [98]–[101] and order scheduling [102], [103]. The increasing scale of scheduling problems usually accounts for an exponentially increased search space [104]. Sun *et al*. [105], [106] test their algorithms on a large-scale scheduling problem of 80 jobs with 600 operations processed on 50 machines, which is a typical high-dimensional problem. Zhang *et al*. [100] regard the allocation of individuals for different tasks as computationally expensive problems. They build surrogates for each task by considering the behavior of individuals and their fitness in dynamic flexible job shop scheduling.

V. Open Issues

Although some promising algorithms have been proposed to solve HEPs, their solution methods are far from being mature. Some open issues and challenges need to be addressed as future research.

A. Surrogate Models for HEPs

As a dominant method for solving expensive problems, how to construct effective and efficient surrogate models in high-dimensional space remains an immense challenge. A high-accuracy surrogate model can be trained by sufficient data samples, however, which can inevitably result in high surrogate construction time consumption. How to balance surrogate accuracy and construction time is a key issue when adopting SAEAs for solving HEPs. The existing surrogate models are mainly selected from GP, RF and RBF. Some novel and effective machine learning methods, such as graph convolutional networks [107] and deep brief networks [108], should be investigated as surrogate models in the evolutionary community, especially for problems with complicated data structures. Moreover, some efficient model-free optimization methods for HEPs need to be researched in the future so as to alleviate the difficulty of building surrogate models in high-dimensional space [68].

B. Hybrid HEPs

Nowadays, optimization problems tend to be ones that are hybridized with different characteristics. As listed in Fig. 5, any optimization problem hybridization can result in a challenging and potential research direction. Although several works intend to solve some complicated HEPs, such as high-dimensional expensive multiobjective optimization [54], there still remains a great demand to investigate how to design efficient algorithms for HEPs characterized by highly constrained [14], [109], multi-objective or many-objective [110],

[111], multi-modal [112], dynamic [113], robust [114], discontinuous [24], and bi-level [115]. As a result, HEPs with more complicated characteristics, especially those arise from real scenarios, are on great demand to be deeper researched.

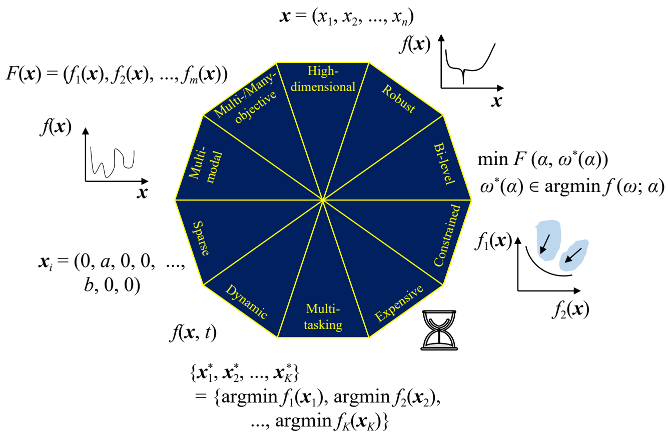


Fig. 5. The main research directions of EAs.

C. Parallel Computing and Distributed Computing for HEPs

Modern advances in computing power enables parallel and distributed computing to some large-scale or expensive optimization problems [116]−[118]. Since the evaluations of candidate individuals are independent of each other in most cases, inherent parallel characteristic of evolutionary algorithms enables parallel computing and distributed computing naturally suitable to deal with such challenging optimization problems and thus speed up the convergence rates [119]. Also, parallel computing and distributed computing can be investigated to accelerate the speed of surrogate constructions in the SAEA framework, especially multiple surrogate retraining process. Therefore, advanced computing techniques are required to be considered to handle HEPs with reasonable time budget [120].

D. Comprehensive Benchmark Suites and Real-World Applications

We notice that most papers adopt only seven basic functions, which are insufficient to validate HEP algorithms' performance. Although it is indeed challenging to solve HEPs with limited computational resource, some complex HEPs with different characteristics, such as rotated and shifted, need to be included as benchmark functions for future research [121]. The existing algorithms are hardly tested on a comprehensive benchmark suite thus resulting in insufficient validation of their overall performance in solving HEPs. In addition, most of the industrial applications, such as airfoil design optimization [57] and propeller design optimization [51], are benchmark applications. We should investigate how to apply the algorithms to solve practical and industrial-size HEPs, e.g., large-scale scheduling problems in intelligent manufacturing [122] and vehicle routing planning in intelligent transportation [123].

VI. CONCLUSION

This paper provides the first comprehensive survey of evolutionary optimization approaches for high-dimensional expensive problems (HEPs). After introducing the basic concepts of HEPs, we discuss the main ideas of surrogate-assisted evolutionary algorithms and summarize the existing SAEAs in solving different HEPs from four main aspects, namely surrogate selection, model construction, model management and base optimizers. Then, we present the commonly-used benchmark suites for HEPs and show the comparative results of several representative algorithms. Some HEPs arising from reality scenarios are presented in our paper for potential applications. We also outline the challenges and issues that need to be addressed as future studies. We expect these introduced evolutionary algorithms to play an increasingly important role in helping engineers solve their particular engineering optimization problems arising from various industrial sectors, e.g., transportation, manufacturing, aerospace, biology, and environment [124]−[126]. Hence, by providing in-depth understanding and useful insights into HEPs and their updated solution methods, we hope this paper is instrumental and helpful to researchers and practicing engineers, especially novices in the area of evolutionary optimization. We also hope that this paper can provide valuable inspirations for them to develop better algorithms for solving challenging HEPs.

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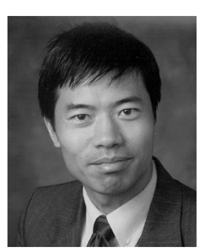

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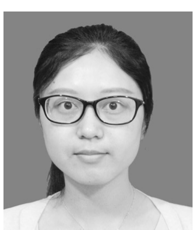

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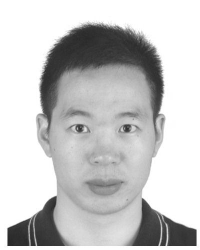

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