Dynamic Event-Triggered Quadratic Nonfragile Filtering for Non-Gaussian Systems: Tackling Multiplicative Noises and Missing Measurements

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***Abstract*—This paper focuses on the quadratic nonfragile filtering problem for linear non-Gaussian systems under multiplicative noises, multiple missing measurements as well as the dynamic event-triggered transmission scheme. The multiple missing measurements are characterized through random variables that obey some given probability distributions, and thresholds of the dynamic event-triggered scheme can be adjusted dynamically via an auxiliary variable. Our attention is concentrated on designing a dynamic event-triggered quadratic nonfragile filter in the well-known minimum-variance sense. To this end, the original system is first augmented by stacking its state/measurement vectors together with second-order Kronecker powers, thus the original design issue is reformulated as that of the augmented system. Subsequently, we analyze statistical properties of augmented noises as well as high-order moments of certain random parameters. With the aid of two well-defined matrix difference equations, we not only obtain upper bounds on filtering error covariances, but also minimize those bounds via carefully designing gain parameters. Finally, an example is presented to explain the effectiveness of this newly established quadratic filtering algorithm.**

***Index Terms*—Dynamic event-triggered scheme, missing measurements, multiplicative noises (MNs), non-Gaussian noises, quadratic filter.**

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I. Introduction

PAST several decades have witnessed the enthusiasm towards researching stochastic state estimation or filtering owing to the successful usage in a wide range of engineering areas including target tracking, satellite navigation, industrial automation and so forth. As is well known, for the exact linear Gaussian system, the famed Kalman filter is able to give the minimum-mean-square-error (MMSE) estimate [1], [2]. Unfortunately, parameter uncertainties are inevitable in many real-world applications due to random component failures, environment changes and sensor aging.

Notably, the parameter uncertainties, in many cases, might incur serious performance degradation [3]−[10]. In this context, researchers have tried hard on designing multifarious filtering strategies, see e.g., [11]−[17]. Particularly, robust nonfragile linear filtering has been investigated in [16] under norm-bounded uncertainties. In [14], the optimal H_2 filtering issue has been addressed for continuous-time systems suffering from multiplicative noises (MNs) and multiple sampled delay measurements.

It should be pointed out that in engineering practice, the non-Gaussian noises are quite common as a result of the complicated environments, and some representative examples include the heavy-tailed glint noises [18] and the non-Gaussian Lévy noises [19]. When it comes to the non-Gaussian noises, the Kalman filter no longer works as the optimal MMSE estimator and might produce unsatisfactory state estimates. Accordingly, the non-Gaussian filtering has risen to an active research topic in recent years with many feasible filtering methods available in [19]−[26] and the references therein. Among them, a modified Tobit Kalman filter and a polynomial filter have been designed in [19], [22] so as to cope with non-Gaussian Lévy noises and non-Gaussian singular systems, respectively.

Quadratic filtering, also known as the second-order polynomial filtering, has proved to be an effective filtering technique to deal with the non-Gaussian noises [27], [28]. The pivotal feature of quadratic filtering is to achieve system estimation by taking full advantage of the information contained in the second-order Kronecker powers with respect to original states/measurements. Compared with polynomial filtering schemes, the quadratic counterpart is able to provide a compromise between the filtering performance and computational cost. As such, the non-Gaussian quadratic filtering has attracted a lot of research interest.

Recent literature has reported a number of elegant quadratic filtering results. For example, a least-squares quadratic filter has been novelly proposed in [29] for stochastic systems under random parameter matrices, which can later be extended to multi-sensor cases with MNs/fading measurements. In [12], a novel linear quadratic filter has been devised for non-Gaussian systems under MNs and quantization effects. Nevertheless, different from the relatively mature Kalman filtering theory, the design and analysis problems of the quadratic filter have not been adequately investigated yet.

The phenomenon of missing measurements has been well recognized as one of the major causes for performance loss in a typical networked environment. Consequently, much attention has been deliberately focused on investigating effects from such a phenomenon onto the filtering performance [30], [31]. For instance, an optimal distributed and saturated filter has been devised in [32] for nonlinear systems under the random access protocol and missing measurements, where the theoretical analyses in terms of boundedness and monotonicity were also provided. In existing literature, there have mainly been three models to characterize these missing measurements, namely, arbitrary probability distribution (within the interval $[0,1]$), Bernoulli distribution and Markov chain models. Particularly, the first kind of missing models is customarily referred to as the multiple missing measurements (MMMs) model. It is worth mentioning that, for the linear non-Gaussian systems with MMMs, the corresponding quadratic filtering problem is far from being fully examined, which motivates this current investigation.

For the networked systems, a noticeable fact is that the communication resources are usually constrained and hence it is paramount to explore how to reasonably utilize the limited resources [33]. In such a context, a huge amount of efficient transmission strategies have been developed where the dynamic event-triggered scheme (DETS) has now become a popular choice, thus drawn an ever-increasing research attention [34]−[38]. In comparison with the static transmission scheme, such a DETS (with a dynamically adjustable threshold parameter) has greater potentials in reducing not only resource consumption but also communication burden. So far, some elegant results have been reported on the dynamic event-triggered filtering problems [39]−[42]. For example, this DETS has been used in complex networks [40] to devise filters under sensor failures and switching topologies, and in sensor networks [43] to design set-membership filters under bounded noises.

In most existing literature, successful implementation of the designed filters largely depends on a prerequisite that the desired gain parameters can be exactly realized. Unfortunately, such a prerequisite might not always hold in engineering practice due to gain fluctuations caused by the round-off/programming errors and the finite resolution of instrumentation. Therefore, it is significant to design the filters with certain resilience against the potential gain fluctuations, and this gives rise to an emerging filtering scheme called resilient or non-fragile filter. Roughly speaking, there have been two popular models to describe the phenomenon of gain perturbations, namely, the norm-bounded uncertainty model and the stochastic uncertainty model (governed by zero-mean matrices that have bounded second-order-moment), see e.g., [16] and [44]−[47]. It should be pointed out that, up to now, very few quadratic nonfragile filtering literature has been given in case of non-Gaussian systems, not to mention cases that consider MNs, DETS, and MMMs.

We endeavor to design a linear quadratic nonfragile filtering scheme for non-Gaussian systems subject to MNs, DETS, and MMMs. Three challenging issues that need to be tackled are identified as follows: 1) How to derive the high-order moments for the parameters related to DETS and the random variables describing the phenomenon of MMMs? 2) How to analyze the statistics of augmented noises composed of original noises and second-order Kronecker powers? and 3) How to design a quadratic non-Gaussian nonfragile filter with MNs, DETS, and MMMs?

Corresponding to the identified challenges, the main contributions of this paper lie in: 1) The quadratic non-Gaussian nonfragile filter is devised firstly under MNs, DETS, and MMMs; 2) The statistics of augmented noises and involved random variables are revealed; and 3) A new quadratic nonfragile filter is designed by minimizing upper bounds on filtering error covariances, which yields better accuracy than traditional filters only using original measurements.

Notations: $\circ$ and $\otimes$ represent, respectively, the Hadamard product and the Kronecker power. $z^{[l]}$ represents the lth-order Kronecker power of z, which is denoted by $z^{[l]} = z \otimes z^{[l-1]} (l \geq 1)$ with $z^{[0]} = 1$. $\mathbb{E}\{z\}$ is the mathematical expectation of random variable z. $\phi_z^{(l)}$ is the lth-order moment of z. $\tilde{\Gamma}_{m,n}(x \otimes z)$ denotes $x \otimes z + z \otimes x$. $\mathrm{sti}(\cdot)$ denotes an inverse operation that transfers the vectorized matrix into the original one. $\lambda_{\max}(A)$ and $\mathrm{Sym}\{A\}$ stand for the maximum eigenvalue of matrix A and $A + A^T$, respectively.

II. Problem Formulation

Consider the following stochastic non-Gaussian system with MNs and MMMs:

$$\begin{cases} x_{t+1} = (F_t + \sum_{i=1}^{s} \alpha_{i,t} F_{i,t}) x_t + B_t w_t \\ y_t = \Lambda_t H_t x_t + D_t v_t \end{cases} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the system state and its initial value is x_0, $y_t \in \mathbb{R}^m$ denotes the measurement output, and $\alpha_{i,t} \in \mathbb{R}$ is the multiplicative noise. w_t and v_t represent non-Gaussian noises. $\Lambda_t \triangleq \mathrm{diag}\{\lambda_{1,t}, \ldots, \lambda_{m,t}\}$ is the MMMs related variable, where $\lambda_{j,t}$ $(j = 1, 2, \ldots, m)$ satisfy certain probability distributions within the interval $[0,1]$. F_t, $F_{i,t}$, B_t, H_t, and D_t are known matrices.

Assumption 1: Random sequences x_0, $\alpha_{i,t}$, w_t and v_t are white, zero-mean, and mutually independent. In addition, their second-order, third-order as well as fourth-order moments are known.

Assumption 2: Mutually uncorrelated random sequences $\lambda_{j,t}$ for $j = 1, 2, \ldots, m$ are uncorrelated with x_0, $\alpha_{i,t}$, w_t and v_t. Moreover, the expectations $\mathbb{E}\{\lambda_{j,t}^l\}$ $(l = 1, 2, 3, 4)$ are known.

Remark 1: In this paper, the involved MNs $\alpha_{i,t}$, and noises w_t, v_t are all non-Gaussian sequences with known high-order moments. In this sense, the existing results with respect to the Gaussian filtering problems might be no longer applicable to such a case. On the other hand, the random variables $\lambda_{j,t}$ $(j = 1,2,\ldots,m)$ distributed over $[0,1]$ are used to characterize MMMs phenomenon of the jth sensor at time instant t. Specifically, if $\lambda_{j,t} = 1$, the jth sensor works normally, otherwise the jth sensor suffers from the measurement degradation. Obviously, the common Bernoulli distribution model is one special case with respect to the considered MMMs model.

To reduce transmission, we adopt a DETS whose triggering condition is

$$\|u_t\| - \frac{\eta_t}{\theta} - \sigma \geq 0 \tag{2}$$

where $u_t \triangleq y_{t_i} - y_t$, y_{t_i} and y_t denote, respectively, the latest (time t_i) and current measurements. θ and σ are given positive parameters. Moreover, the auxiliary variable η_t satisfies the following recursion:

$$\eta_{t+1} = \chi\eta_t + \sigma - \|u_t\| \tag{3}$$

with $\eta_0 \geq 0$, $0 < \chi < 1$ and $\theta\chi \geq 1$. Based on the triggering condition (2), the transmitted measurements can be described by

$$\tilde{y}_t = y_{t_i}, \quad t \in \{t_i, t_i+1, t_i+2, \ldots, t_{i+1}-1\}. \tag{4}$$

Remark 2: The threshold in the triggering condition (2) is the time-varying parameter $\frac{\eta_t}{\theta} + \sigma$ rather than the fixed scalar σ, which means that the DETS has greater potentials than its static version in reducing the amount of successfully transmitted measurements [43]. Notably, the threshold $\frac{\eta_t}{\theta} + \sigma \longrightarrow \sigma$ when $\theta \longrightarrow \infty$. In this sense, the considered DETS includes the previous static version.

III. The Quadratic Filtering Problem

This section investigates the problem of quadratic filtering for original system (1). To construct an augmented system, let us first give the second-order Kronecker powers of x_t, y_t and $\tilde{y}_t$, i.e., $x_t^{[2]}$, $y_t^{[2]}$ and $\tilde{y}_t^{[2]}$.

Based on the definition and properties of Kronecker powers, we have

$$\begin{aligned} x_{t+1}^{[2]} &= (F_t + \sum_{i=1}^{s} \alpha_{i,t}F_{i,t})^{[2]} x_t^{[2]} + B_t^{[2]} w_t^{[2]} \\ &\quad + \tilde{\Gamma}_{n,n}[(F_t + \sum_{i=1}^{s} \alpha_{i,t}F_{i,t})x_t \otimes B_t w_t] \\ &= (F_t^{[2]} + \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(2)} F_{i,t}^{[2]}) x_t^{[2]} + B_t^{[2]} \phi_{w_t}^{(2)} + \tilde{w}_t \end{aligned} \tag{5}$$

where

$$\begin{aligned} \tilde{w}_t \triangleq {} & [\tilde{\Gamma}_{n,n}(F_t \otimes \sum_{i=1}^{s} \alpha_{i,t}F_{i,t}) + \sum_{i=1}^{s} (\alpha_{i,t}^2 - \phi_{\alpha_{i,t}}^{(2)}) F_{i,t}^{[2]} \\ & + \sum_{i=1}^{s} \sum_{1=j\neq i}^{s} \alpha_{i,t}\alpha_{j,t} F_{i,t} \otimes F_{j,t}] x_t^{[2]} + B_t^{[2]} (w_t^{[2]} \\ & - \phi_{w_t}^{(2)}) + \tilde{\Gamma}_{n,n}[(F_t + \sum_{i=1}^{s} \alpha_{i,t}F_{i,t})x_t \otimes B_t w_t]. \end{aligned}$$

Recalling the expression of y_t obtains

$$\begin{aligned} y_t^{[2]} &= (\Lambda_t H_t x_t + D_t v_t) \otimes (\Lambda_t H_t x_t + D_t v_t) \\ &= \Lambda_t^{[2]} H_t^{[2]} x_t^{[2]} + \tilde{\Gamma}_{m,m}(\Lambda_t H_t x_t \otimes D_t v_t) + D_t^{[2]} v_t^{[2]} \\ &= \mathbb{E}\{\Lambda_t^{[2]}\} H_t^{[2]} x_t^{[2]} + D_t^{[2]} \phi_{v_t}^{(2)} + \tilde{v}_t \end{aligned} \tag{6}$$

where

$$\begin{aligned} \tilde{v}_t \triangleq {} & (\Lambda_t^{[2]} - \mathbb{E}\{\Lambda_t^{[2]}\}) H_t^{[2]} x_t^{[2]} + D_t^{[2]} (v_t^{[2]} - \phi_{v_t}^{(2)}) \\ & + \tilde{\Gamma}_{m,m}(\Lambda_t H_t x_t \otimes D_t v_t). \end{aligned}$$

Similarly, $\tilde{y}_t^{[2]}$ can be described by

$$\begin{aligned} \tilde{y}_t^{[2]} &= (y_t + u_t) \otimes (y_t + u_t) \\ &= \mathbb{E}\{\Lambda_t^{[2]}\} H_t^{[2]} x_t^{[2]} + D_t^{[2]} \phi_{v_t}^{(2)} + u_t^{[2]} + \tilde{\tilde{v}}_t \end{aligned} \tag{7}$$

where

$$\tilde{\tilde{v}}_t \triangleq \tilde{v}_t + \tilde{\Gamma}_{m,m}(y_t \otimes u_t).$$

In what follows, let us define the augmented state vector $\mathcal{X}_t$ and measurement vectors $\mathcal{Y}_t$ and $\tilde{\mathcal{Y}}_t$:

$$\mathcal{X}_t \triangleq \begin{bmatrix} x_t \\ x_t^{[2]} \end{bmatrix}, \quad \mathcal{Y}_t \triangleq \begin{bmatrix} y_t \\ y_t^{[2]} \end{bmatrix}, \quad \tilde{\mathcal{Y}}_t \triangleq \begin{bmatrix} \tilde{y}_t \\ \tilde{y}_t^{[2]} \end{bmatrix} \tag{8}$$

then system (1) is converted into

$$\begin{cases} \mathcal{X}_{t+1} = \mathcal{F}_t \mathcal{X}_t + \tilde{f}_t + \mathcal{W}_t \\ \tilde{\mathcal{Y}}_t = \mathcal{H}_t \mathcal{X}_t + \tilde{g}_t + \mathcal{U}_t + \mathcal{V}_t \end{cases} \tag{9}$$

where

$$\mathcal{F}_t \triangleq \begin{bmatrix} F_t & 0 \\ 0 & F_t^{[2]} + \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(2)} F_{i,t}^{[2]} \end{bmatrix}$$

$$\mathcal{W}_t \triangleq \begin{bmatrix} \sum_{i=1}^{s} \alpha_{i,t} F_{i,t} x_t + B_t w_t \\ \tilde{w}_t \end{bmatrix}$$

$$\mathcal{H}_t \triangleq \begin{bmatrix} \mathbb{E}\{\Lambda_t\} H_t & 0 \\ 0 & \mathbb{E}\{\Lambda_t^{[2]}\} H_t^{[2]} \end{bmatrix}$$

$$\tilde{f}_t \triangleq \begin{bmatrix} 0 \\ B_t^{[2]} \phi_{w_t}^{(2)} \end{bmatrix}, \quad \tilde{g}_t \triangleq \begin{bmatrix} 0 \\ D_t^{[2]} \phi_{v_t}^{(2)} \end{bmatrix}$$

$$\mathcal{U}_t \triangleq \begin{bmatrix} u_t \\ u_t^{[2]} \end{bmatrix}, \quad \mathcal{V}_t \triangleq \begin{bmatrix} (\Lambda_t - \mathbb{E}\{\Lambda_t\}) H_t x_t + D_t v_t \\ \tilde{\tilde{v}}_t \end{bmatrix}.$$

By resorting to the available measurements $\tilde{\mathcal{Y}}_t$, the nonfragile filter for system (9) is

$$\begin{cases} \hat{\mathcal{X}}_{t+1|t} = \mathcal{F}_t \hat{\mathcal{X}}_{t|t} + \tilde{f}_t \\ \hat{\mathcal{X}}_{t+1|t+1} = \hat{\mathcal{X}}_{t+1|t} + \tilde{\mathcal{L}}_{t+1}[\tilde{\mathcal{Y}}_{t+1} - \mathcal{H}_{t+1} \hat{\mathcal{X}}_{t+1|t} - \tilde{g}_{t+1}] \end{cases} \tag{10}$$

where $\hat{\mathcal{X}}_{t+1|t}$ and $\hat{\mathcal{X}}_{t+1|t+1}$ are, respectively, the prediction and estimate of $\mathcal{X}_{t+1}$. Gain $\tilde{\mathcal{L}}_{t+1} \triangleq \mathcal{L}_{t+1} + \Delta\mathcal{L}_{t+1}$, where $\Delta\mathcal{L}_{t+1}$ describes the gain parameter fluctuations satisfying the following statistical characteristics:

$$\mathbb{E}\{\Delta\mathcal{L}_{t+1}\} = 0, \ \mathbb{E}\{\Delta\mathcal{L}_{t+1} \Delta\mathcal{L}_{t+1}^T\} \leq \delta I \tag{11}$$

where δ is a positive scalar. Moreover, we assume that all random variables, i.e., $\Delta\mathcal{L}_{t+1}$, x_0, $\alpha_{i,t}$, w_t, v_t and $\lambda_{j,t}$, are mutually independent.

Remark 3: It should be pointed out that, many excellent non-fragile filtering algorithms have been proposed in the literature, see e.g., [16], [46]. Different from these two works focusing on the design of the robust nonfragile Kalman filter and the l_2-l_∞ state estimator, respectively, this paper concentrates on the issue of the quadratic nonfragile filtering for non-Gaussian systems. By utilizing the Kronecker powers with respect to state and measurement vectors, the quadratic filtering problem for system (1) has been successfully transformed into a recursive filtering problem for the augmented system (9). The computational or implementation error $\Delta\mathcal{L}_{t+1}$ is modeled as (11), and the parameter δ is dependent on the word-length of the adopted computing device. The distinctive features of (10) lie in: 1) The capability to deal with the effects of the MNs, MMMs, DETS, and stochastic gain fluctuations (SGFs); 2) The full use of information contained in second-order moments of the non-Gaussian random variables (e.g., $\phi^{(2)}_{\alpha_{i,t}}$, $\mathbb{E}\{\Lambda_t^{[2]}\}$, $\phi^{(2)}_{w_t}$, and $\phi^{(2)}_{v_t}$) and the capability of improving the filtering performance; and 3) The recursive form suitable to be implemented online.

Let $\tilde{\mathcal{X}}_{t+1|t} \triangleq \mathcal{X}_{t+1}-\hat{\mathcal{X}}_{t+1|t}$ and $\tilde{\mathcal{X}}_{t+1|t+1} \triangleq \mathcal{X}_{t+1}-\hat{\mathcal{X}}_{t+1|t+1}$. The corresponding covariance matrices can be defined as follows:

$$\mathcal{P}_{t+1|t} \triangleq \mathbb{E}\{\tilde{\mathcal{X}}_{t+1|t}\tilde{\mathcal{X}}^T_{t+1|t}\}$$
$$\mathcal{P}_{t+1|t+1} \triangleq \mathbb{E}\{\tilde{\mathcal{X}}_{t+1|t+1}\tilde{\mathcal{X}}^T_{t+1|t+1}\}.$$

The main purpose is to design filter (10) to ensure there exists an upper bound for $\mathcal{P}_{t+1|t+1}$, and minimize this bound by designing $\mathcal{L}_{t+1}$.

IV. Main Results

This section first presents a few preliminary lemmas, then determines an upper bound for $\mathcal{P}_{t+1|t+1}$, finally minimizes this bound by designing $\mathcal{L}_{t+1}$.

A. Preliminary Lemmas

Lemma 1 [32]: Letting $A \triangleq \mathrm{diag}\{a_1,a_2,\ldots,a_n\}$ and C, respectively, be random and real-valued matrices, we have

$$\mathbb{E}\{ACA^T\} = \begin{bmatrix} \mathbb{E}\{a_1^2\} & \mathbb{E}\{a_1a_2\} & \cdots & \mathbb{E}\{a_1a_n\} \\ \mathbb{E}\{a_2a_1\} & \mathbb{E}\{a_2^2\} & \cdots & \mathbb{E}\{a_2a_n\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{a_na_1\} & \mathbb{E}\{a_na_2\} & \cdots & \mathbb{E}\{a_n^2\} \end{bmatrix} \circ C. \quad (12)$$

Lemma 2 [40]: For any two given matrices A and B,

$$AB^T+BA^T \le \alpha AA^T+\alpha^{-1}BB^T \quad (13)$$

holds where the scalar $\alpha>0$.

Lemma 3: Define

$$\Omega^{(1)}_{t+1} \triangleq \mathbb{E}\{\eta^2_{t+1}\}$$
$$\Omega^{(2)}_{t+1} \triangleq \mathbb{E}\{\eta^4_{t+1}\}$$
$$\Omega^{(u,1)}_{t+1} \triangleq \mathbb{E}\{\|u_{t+1}\|^2\}$$
$$\Omega^{(u,2)}_{t+1} \triangleq \mathbb{E}\{\|u_{t+1}\|^4\}$$

then, the following conditions are satisfied:

$$\Omega^{(1)}_{t+1} \le \bar{\Omega}^{(1)}_{t+1}$$
$$\Omega^{(2)}_{t+1} \le \bar{\Omega}^{(2)}_{t+1}$$
$$\Omega^{(u,1)}_{t+1} \le \bar{\Omega}^{(u,1)}_{t+1}$$
$$\Omega^{(u,2)}_{t+1} \le \bar{\Omega}^{(u,2)}_{t+1} \quad (14)$$

where

$$\bar{\Omega}^{(1)}_{t+1} \triangleq \Big[(1+e_{1,t})(1+e_{2,t})\chi^2+(1+e^{-1}_{1,t})\frac{1+\theta}{\theta^2}\Big]\bar{\Omega}^{(1)}_t + \Big[(1+e_{1,t})(1+e^{-1}_{2,t})+(1+e^{-1}_{1,t})(1+\theta^{-1})\Big]\sigma^2$$
$$\bar{\Omega}^{(2)}_{t+1} \triangleq \Xi_{1,t}\bar{\Omega}^{(2)}_t+\Xi_{2,t}\bar{\Omega}^{(1)}_t+\Xi_{3,t}\sigma^4$$
$$\bar{\Omega}^{(u,1)}_{t+1} \triangleq \frac{1+\theta}{\theta^2}\bar{\Omega}^{(1)}_{t+1}+(1+\theta^{-1})\sigma^2$$
$$\bar{\Omega}^{(u,2)}_{t+1} \triangleq \frac{(1+\theta)^2}{\theta^4}\bar{\Omega}^{(2)}_{t+1}+(1+\theta^{-1})^2\sigma^4 + 2(1+\theta)\frac{(1+\theta^{-1})}{\theta^2}\sigma^2\bar{\Omega}^{(1)}_{t+1}$$

with

$$\Xi_{1,t} \triangleq (1+e_{1,t})^2(1+e_{2,t})^2(1+e_{3,t})\chi^4 + 2(1+e_{1,t})(1+e^{-1}_{1,t})(1+e_{2,t})\chi^2\frac{(1+\theta)}{\theta^2} + (1+e^{-1}_{1,t})^2\frac{(1+\theta)^2}{\theta^4}$$
$$\Xi_{2,t} \triangleq 2(1+e_{1,t})(1+e^{-1}_{1,t})\sigma^2\Big[(1+e_{2,t})(1+\theta^{-1})\chi^2 + (1+e^{-1}_{2,t})\frac{(1+\theta)}{\theta^2}\Big]+2(1+e^{-1}_{1,t})^2\frac{(1+\theta^{-1})}{\theta^2} \times(1+\theta)\sigma^2$$
$$\Xi_{3,t} \triangleq (1+e_{1,t})^2(1+e^{-1}_{2,t})^2(1+e^{-1}_{3,t}) + 2(1+e_{1,t})(1+e^{-1}_{1,t})(1+e^{-1}_{2,t})(1+\theta^{-1}) + (1+e^{-1}_{1,t})^2(1+\theta^{-1})^2.$$

Proof: See Appendix A. ■

Remark 4: In order to deal with the difficulties incurred by the DETS, the second-order and fourth-order Kronecker powers of η_t and u_t have been derived in Lemma 3. Obviously, it is hard to exactly calculate $\Omega^{(1)}_{t+1}$, $\Omega^{(2)}_{t+1}$, $\Omega^{(u,1)}_{t+1}$, and $\Omega^{(u,2)}_{t+1}$ mainly because of the introduced triggering condition (2). To this end, we have established their respective upper bounds (i.e., $\bar{\Omega}^{(1)}_{t+1}$, $\bar{\Omega}^{(2)}_{t+1}$, $\bar{\Omega}^{(u,1)}_{t+1}$, and $\bar{\Omega}^{(u,2)}_{t+1}$) by means of Lemma 2.

Based on the above lemmas, we are ready to analyze statistical properties of noises $\mathcal{W}_t$ and $\mathcal{V}_t$.

Lemma 4: Let us define

$$\mathcal{Q}_{\mathcal{W}_t} \triangleq \mathbb{E}\{\mathcal{W}_t\mathcal{W}_t^T\} = \begin{bmatrix} \mathcal{Q}_{\mathcal{W}_{11,t}} & \mathcal{Q}_{\mathcal{W}_{12,t}} \\ \mathcal{Q}^T_{\mathcal{W}_{12,t}} & \mathcal{Q}_{\mathcal{W}_{22,t}} \end{bmatrix}$$
$$\mathcal{Q}_{\mathcal{V}_t} \triangleq \mathbb{E}\{\mathcal{V}_t\mathcal{V}_t^T\} = \begin{bmatrix} \mathcal{Q}_{\mathcal{V}_{11,t}} & \mathcal{Q}_{\mathcal{V}_{12,t}} \\ \mathcal{Q}^T_{\mathcal{V}_{12,t}} & \mathcal{Q}_{\mathcal{V}_{22,t}} \end{bmatrix}$$
$$\bar{\mathcal{Q}}_{\mathcal{V}_t} \triangleq \begin{bmatrix} \mathcal{Q}_{\mathcal{V}_{11,t}} & \mathcal{Q}_{\mathcal{V}_{12,t}} \\ \mathcal{Q}^T_{\mathcal{V}_{12,t}} & \bar{\mathcal{Q}}_{\mathcal{V}_{22,t}} \end{bmatrix}$$

where

$$\begin{aligned}
Q_{\mathcal{W}_{11,t}} &\triangleq \sum_{i=1}^{s}\phi^{(2)}_{\alpha_{i,t}}F_{i,t}\mathrm{sti}(\phi^{(2)}_{x_t})F^T_{i,t}+B_t\mathrm{sti}(\phi^{(2)}_{w_t})B^T_t\\
Q_{\mathcal{W}_{12,t}} &\triangleq \sum_{i=1}^{s}\phi^{(2)}_{\alpha_{i,t}}F_{i,t}\mathrm{sti}(\phi^{(3)}_{x_t})(F_t\otimes F_{i,t})^T\tilde{\Gamma}^T_{n,n}\\
&\quad+\sum_{i=1}^{s}\phi^{(3)}_{\alpha_{i,t}}F_{i,t}\mathrm{sti}(\phi^{(3)}_{x_t})(F^{[2]}_{i,t})^T+B_t\mathrm{sti}(\phi^{(3)}_{w_t})(B^{[2]}_t)^T\\
Q_{\mathcal{W}_{22,t}} &\triangleq \tilde{\Gamma}_{n,n}\sum_{i=1}^{s}\phi^{(2)}_{\alpha_{i,t}}(F_t\otimes F_{i,t})\mathrm{sti}(\phi^{(4)}_{x_t})(F_t\otimes F_{i,t})^T\tilde{\Gamma}^T_{n,n}\\
&\quad+\sum_{i=1}^{s}(\phi^{(4)}_{\alpha_{i,t}}-(\phi^{(2)}_{\alpha_{i,t}})^2)F^{[2]}_{i,t}\mathrm{sti}(\phi^{(4)}_{x_t})(F^{[2]}_{i,t})^T\\
&\quad+\sum_{i=1}^{s}\sum_{j\neq i=1}^{s}\phi^{(2)}_{\alpha_{i,t}}\phi^{(2)}_{\alpha_{j,t}}(F_{i,t}\otimes F_{j,t})\mathrm{sti}(\phi^{(4)}_{x_t})(F_{i,t}\\
&\quad\otimes F_{j,t})^T+\tilde{\Gamma}_{n,n}\sum_{i=1}^{s}\phi^{(3)}_{\alpha_{i,t}}(F_t\otimes F_{i,t})\mathrm{sti}(\phi^{(4)}_{x_t})(F^{[2]}_{i,t})^T\\
&\quad+\sum_{i=1}^{s}\phi^{(3)}_{\alpha_{i,t}}F^{[2]}_{i,t}\mathrm{sti}(\phi^{(4)}_{x_t})(F_t\otimes F_{i,t})^T\tilde{\Gamma}^T_{n,n}\\
&\quad+\tilde{\Gamma}_{n,n}\Big\{\Big(\sum_{i=1}^{s}\phi^{(2)}_{\alpha_{i,t}}F_{i,t}\mathrm{sti}(\phi^{(2)}_{x_t})F^T_{i,t}\\
&\quad+F_t\mathrm{sti}(\phi^{(2)}_{x_t})F^T_t\Big)\otimes(B_t\mathrm{sti}(\phi^{(2)}_{w_t})B^T_t)\Big\}\tilde{\Gamma}^T_{n,n}\\
&\quad+B^{[2]}_t(\mathrm{sti}(\phi^{(4)}_{w_t})-\phi^{(2)}_{w_t}(\phi^{(2)}_{w_t})^T)(B^{[2]}_t)^T\\
Q_{\mathcal{V}_{11,t}} &\triangleq \Upsilon_t\circ(H_t\mathrm{sti}(\phi^{(2)}_{x_t})H^T_t)+D_t\mathrm{sti}(\phi^{(2)}_{v_t})D^T_t\\
Q_{\mathcal{V}_{12,t}} &\triangleq (1-2\rho_t)(M_t+D_t\mathrm{sti}(\phi^{(3)}_{v_t})(D^{[2]}_t)^T)\\
\bar{Q}_{\mathcal{V}_{22,t}} &\triangleq \tilde{\Gamma}_{m,m}(T_t\circ(H_t\mathrm{sti}(\phi^{(2)}_{x_t})H^T_t))\otimes(D_t\mathrm{sti}(\phi^{(2)}_{v_t})\\
&\quad\times D^T_t)\tilde{\Gamma}^T_{m,m}+N_t\circ(H^{[2]}_t\mathrm{sti}(\phi^{(4)}_{x_t})(H^{[2]}_t)^T)\\
&\quad-\mathbb{E}\{\Lambda^{[2]}_t\}H^{[2]}_t\mathrm{sti}(\phi^{(4)}_{x_t})(H^{[2]}_t)^T(\mathbb{E}\{\Lambda^{[2]}_t\})^T\\
&\quad+D^{[2]}_t(\mathrm{sti}(\phi^{(4)}_{v_t})-\phi^{(2)}_{v_t}(\phi^{(2)}_{v_t})^T)(D^{[2]}_t)^T\\
&\quad+\tilde{\Gamma}_{m,m}[(T_t\circ(H_t\mathrm{sti}(\phi^{(2)}_{x_t})H^T_t)+D_t\mathrm{sti}(\phi^{(2)}_{v_t})D^T_t)\\
&\quad\otimes((1+\theta)\frac{\bar{\Omega}^{(1)}_t}{\theta^2}+(1+\theta^{-1})\sigma^2)I]\tilde{\Gamma}^T_{m,m}\\
&\quad+\mathrm{Sym}\Big\{-2\rho_t[N_t\circ(H^{[2]}_t\mathrm{sti}(\phi^{(4)}_{x_t})(H^{[2]}_t)^T)\\
&\quad-\mathbb{E}\{\Lambda^{[2]}_t\}(H^{[2]}_t\mathrm{sti}(\phi^{(4)}_{x_t})(H^{[2]}_t)^T)(\mathbb{E}\{\Lambda^{[2]}_t\})^T]\\
&\quad-2\rho_t\tilde{\Gamma}_{m,m}(T_t\circ(H_t\mathrm{sti}(\phi^{(2)}_{x_t})H^T_t))\\
&\quad\otimes(D_t\mathrm{sti}(\phi^{(2)}_{v_t})D^T_t)\tilde{\Gamma}^T_{m,m}-2\rho_tD^{[2]}_t(\mathrm{sti}(\phi^{(4)}_{v_t})\\
&\quad-\phi^{(2)}_{v_t}(\phi^{(2)}_{v_t})^T)(D^{[2]}_t)^T\Big\}
\end{aligned}\tag{15}$$

with

$$\begin{aligned}
E_i &\triangleq \mathrm{diag}\{0,0,\ldots,0,1,0,\ldots,0\},\bar{\lambda}_{i,t}\triangleq\mathbb{E}\{\lambda_{i,t}\}\\
\Upsilon_t &\triangleq \mathrm{diag}\{\phi^{(2)}_{\lambda_{1,t}}-\bar{\lambda}^2_{1,t},\ldots,\phi^{(2)}_{\lambda_{m,t}}-\bar{\lambda}^2_{m,t}\}\\
M_t &\triangleq \sum_{i=1}^{m}(\phi^{(3)}_{\lambda_{i,t}}-\bar{\lambda}_{i,t}\phi^{(2)}_{\lambda_{i,t}})E_iH_t\mathrm{sti}(\phi^{(3)}_{x_t})(H^{[2]}_t)^T(E^{[2]}_i)^T\\
&\quad+\sum_{i=1}^{m}\sum_{1=j\neq i}^{m}(\phi^{(2)}_{\lambda_{i,t}}-\bar{\lambda}^2_{i,t})\bar{\lambda}_{j,t}E_iH_t\mathrm{sti}(\phi^{(3)}_{x_t})(H^{[2]}_t)^T\\
&\quad\times(E_i\otimes E_j+E_j\otimes E_i)^T\\
T_t &\triangleq \begin{bmatrix}\phi^{(2)}_{\lambda_{1,t}} & \bar{\lambda}_{1,t}\bar{\lambda}_{2,t} & \cdots & \bar{\lambda}_{1,t}\bar{\lambda}_{m,t}\\ \bar{\lambda}_{2,t}\bar{\lambda}_{1,t} & \phi^{(2)}_{\lambda_{2,t}} & \cdots & \bar{\lambda}_{2,t}\bar{\lambda}_{m,t}\\ \vdots & \vdots & \cdots & \vdots\\ \bar{\lambda}_{m,t}\bar{\lambda}_{1,t} & \bar{\lambda}_{m,t}\bar{\lambda}_{2,t} & \cdots & \phi^{(2)}_{\lambda_{m,t}}\end{bmatrix}\\
N_t &\triangleq \begin{bmatrix}\phi^{(4)}_{\lambda_{1,t}} & \phi^{(3)}_{\lambda_{1,t}}\bar{\lambda}_{2,t} & \cdots & \phi^{(2)}_{\lambda_{1,t}}\phi^{(2)}_{\lambda_{m,t}}\\ \phi^{(3)}_{\lambda_{1,t}}\bar{\lambda}_{2,t} & \phi^{(2)}_{\lambda_{1,t}}\phi^{(2)}_{\lambda_{2,t}} & \cdots & \bar{\lambda}_{1,t}\bar{\lambda}_{2,t}\phi^{(2)}_{\lambda_{m,t}}\\ \vdots & \vdots & \cdots & \vdots\\ \phi^{(2)}_{\lambda_{m,t}}\phi^{(1)}_{\lambda_{1,t}} & \bar{\lambda}_{1,t}\bar{\lambda}_{2,t}\phi^{(2)}_{\lambda_{m,t}} & \cdots & \phi^{(4)}_{\lambda_{m,t}}\end{bmatrix}.
\end{aligned}$$

Note that if the condition (2) is satisfied at time instant t, $\rho_t=0$, otherwise $\rho_t=1$. Then, $\bar{Q}_{\mathcal{V}_t}$ is an upper bound of $Q_{\mathcal{V}_t}$.

Proof: See Appendix B. ■

Remark 5: It is clear to see that great effort has been made on the analysis of statistical properties of noises $\mathcal{W}_t$ and $\mathcal{V}_t$. The essential difficulties result from the co-existence of the high-order moments about non-Gaussian noises and the parameters involved in MMMs and DETS when computing the exact value of $Q_{\mathcal{V}_t}$. To this end, the matrix decomposition technique has been exploited to cope with the cross-terms containing the high-order moments of Λ_t (i.e., Λ_t has been decomposed into $\sum_{i=1}^{m}\lambda_{i,t}E_i$). Considering the DETS, much attention should be devoted to the term $y_t\otimes u_t$ since it equals to zero when t is the triggering time instant, and nonzero otherwise.

Lemma 5: The state covariance matrix defined by $\mathcal{J}_{t+1}\triangleq\mathbb{E}\{\mathcal{X}_{t+1}\mathcal{X}^T_{t+1}\}$ satisfies the following recursion:

$$\mathcal{J}_{t+1}=\mathcal{F}_t\mathcal{J}_t\mathcal{F}^T_t+\tilde{f}_t\tilde{f}^T_t+Q_{\mathcal{W}_t}+\mathcal{F}_t\mathcal{K}_t\tilde{f}^T_t+\tilde{f}_t\mathcal{K}^T_t\mathcal{F}^T_t\tag{16}$$

where $\mathcal{K}_t\triangleq\mathbb{E}\{\mathcal{X}_t\}=[0,(\phi^{(2)}_{x_t})^T]^T$.

Proof: Definition of $\mathcal{J}_{t+1}$ and (9) imply

$$\begin{aligned}\mathcal{J}_{t+1}&=\mathcal{F}_t\mathcal{J}_t\mathcal{F}^T_t+\tilde{f}_t\tilde{f}^T_t+\mathbb{E}\{\mathcal{W}_t\mathcal{W}^T_t\}+\mathrm{Sym}\Big\{\mathcal{F}_t\mathbb{E}\{\mathcal{X}_t\tilde{f}^T_t\}\\&\quad+\mathcal{F}_t\mathbb{E}\{\mathcal{X}_t\mathcal{W}^T_t\}+\mathbb{E}\{\tilde{f}_t\mathcal{W}^T_t\}\Big\}.\end{aligned}\tag{17}$$

Assumption 1 indicates

$$\mathbb{E}\{\mathcal{X}_t\mathcal{W}^T_t\}=0,\mathbb{E}\{\tilde{f}_t\mathcal{W}^T_t\}=0.\tag{18}$$

Then, substituting (18) into (17) leads to (16).

In addition, we further know that $\mathcal{J}_{11,t+1}=\mathrm{sti}(\phi^{(2)}_{x_{t+1}})$, $\mathcal{J}_{12,t+1}=\mathrm{sti}(\phi^{(3)}_{x_{t+1}})$ and $\mathcal{J}_{22,t+1}=\mathrm{sti}(\phi^{(4)}_{x_{t+1}})$. ■

Lemma 6: $\mathcal{P}_{t+1|t}$ satisfies

$$\mathcal{P}_{t+1|t}=\mathcal{F}_t\mathcal{P}_{t|t}\mathcal{F}^T_t+Q_{\mathcal{W}_t}.\tag{19}$$

Proof: According to (9) and (10),

$$\tilde{\mathcal{X}}_{t+1|t} = \mathcal{F}_t \tilde{\mathcal{X}}_{t|t} + \mathcal{W}_t \tag{20}$$

which together with the definition of $\mathcal{P}_{t+1|t}$, yields

$$\mathcal{P}_{t+1|t} = \mathcal{F}_t \mathcal{P}_{t|t} \mathcal{F}_t^T + \mathcal{Q}_{\mathcal{W}_t} + \mathcal{G}_t + \mathcal{G}_t^T \tag{21}$$

where $\mathcal{G}_t \triangleq \mathbb{E}\{\mathcal{F}_t \tilde{\mathcal{X}}_{t|t} \mathcal{W}_t^T\}$.

From Assumption 1, we further have $\mathcal{G}_t = 0$, which implies that (19) holds. ■

Lemma 7: $\mathcal{P}_{t+1|t+1}$ satisfies

$$\begin{aligned}\mathcal{P}_{t+1|t+1} = {} & (I - \mathcal{L}_{t+1}\mathcal{H}_{t+1})\mathcal{P}_{t+1|t}(I - \mathcal{L}_{t+1}\mathcal{H}_{t+1})^T \\ & + \mathcal{L}_{t+1}\mathbb{E}\{\mathcal{U}_{t+1}\mathcal{U}_{t+1}^T\}\mathcal{L}_{t+1}^T + \mathcal{L}_{t+1}\mathcal{Q}_{\mathcal{V}_{t+1}}\mathcal{L}_{t+1}^T \\ & + \mathbb{E}\{\Delta\mathcal{L}_{t+1}(\mathcal{H}_{t+1}\mathcal{P}_{t+1|t}\mathcal{H}_{t+1}^T + \mathcal{U}_{t+1}\mathcal{U}_{t+1}^T \\ & + \mathcal{V}_{t+1}\mathcal{V}_{t+1}^T)\Delta\mathcal{L}_{t+1}^T\} + \mathrm{Sym}\Big\{\mathcal{M}_{1,t+1} \\ & + \mathcal{M}_{2,t+1} + \mathcal{M}_{3,t+1}\Big\}\end{aligned} \tag{22}$$

where

$$\begin{aligned}\mathcal{M}_{1,t+1} &\triangleq \mathbb{E}\{-(I - \tilde{\mathcal{L}}_{t+1}\mathcal{H}_{t+1})\tilde{\mathcal{X}}_{t+1|t}\mathcal{U}_{t+1}^T\tilde{\mathcal{L}}_{t+1}^T\} \\ \mathcal{M}_{2,t+1} &\triangleq \mathbb{E}\{-(I - \tilde{\mathcal{L}}_{t+1}\mathcal{H}_{t+1})\tilde{\mathcal{X}}_{t+1|t}\mathcal{V}_{t+1}^T\tilde{\mathcal{L}}_{t+1}^T\} \\ \mathcal{M}_{3,t+1} &\triangleq \mathbb{E}\{\tilde{\mathcal{L}}_{t+1}\mathcal{U}_{t+1}\mathcal{V}_{t+1}^T\tilde{\mathcal{L}}_{t+1}^T\}.\end{aligned}$$

Proof: Subtracting $\hat{\mathcal{X}}_{t+1|t+1}$ from $\mathcal{X}_{t+1}$ yields

$$\begin{aligned}\tilde{\mathcal{X}}_{t+1|t+1} &= \tilde{\mathcal{X}}_{t+1|t} - \tilde{\mathcal{L}}_{t+1}(\tilde{\mathcal{Y}}_{t+1} - \mathcal{H}_{t+1}\hat{\mathcal{X}}_{t+1|t} - \tilde{g}_{t+1}) \\ &= \tilde{\mathcal{X}}_{t+1|t} - \tilde{\mathcal{L}}_{t+1}(\mathcal{H}_{t+1}\tilde{\mathcal{X}}_{t+1|t} + \mathcal{U}_{t+1} + \mathcal{V}_{t+1}) \\ &= (I - \tilde{\mathcal{L}}_{t+1}\mathcal{H}_{t+1})\tilde{\mathcal{X}}_{t+1|t} - \tilde{\mathcal{L}}_{t+1}\mathcal{U}_{t+1} - \tilde{\mathcal{L}}_{t+1}\mathcal{V}_{t+1}.\end{aligned} \tag{23}$$

Recalling $\mathcal{P}_{t+1|t+1}$'s definition, the recursion (22) can be immediately obtained. ■

It should be mentioned that the covariance recursion provided in Lemma 7 contains cross-terms $\mathcal{M}_{1,t+1}$, $\mathcal{M}_{2,t+1}$, and $\mathcal{M}_{3,t+1}$ (induced by MMMs, DETS, and SGFs), which put extreme difficulties on exactly calculating $\mathcal{P}_{t+1|t+1}$ via (22). We are, thus, going to seek a bound for $\mathcal{P}_{t+1|t+1}$ in the following subsection.

B. Upper Bound

Theorem 1: Let scalars $e_{i,t+1} > 0$ $(i = 4,5,6)$ be given. Assume equations

$$\tilde{\mathcal{P}}_{t+1|t} = \mathcal{F}_t \tilde{\mathcal{P}}_{t|t} \mathcal{F}_t^T + \mathcal{Q}_{\mathcal{W}_t} \tag{24}$$

and

$$\begin{aligned}\tilde{\mathcal{P}}_{t+1|t+1} = {} & (1 + e_{4,t+1} + e_{5,t+1})(I - \mathcal{L}_{t+1}\mathcal{H}_{t+1})\tilde{\mathcal{P}}_{t+1|t} \\ & \times (I - \mathcal{L}_{t+1}\mathcal{H}_{t+1})^T + (1 + e_{4,t+1}^{-1} + e_{6,t+1}) \\ & \times \mathcal{L}_{t+1}(\bar{\Omega}_{t+1}^{(u,1)} + \bar{\Omega}_{t+1}^{(u,2)})\mathcal{L}_{t+1}^T \\ & + (1 + e_{5,t+1}^{-1} + e_{6,t+1}^{-1})\mathcal{L}_{t+1}\bar{\mathcal{Q}}_{\mathcal{V}_{t+1}}\mathcal{L}_{t+1}^T \\ & + \lambda_{\max}\Big\{(1 + e_{4,t+1} + e_{5,t+1})\mathcal{H}_{t+1}\tilde{\mathcal{P}}_{t+1|t}\mathcal{H}_{t+1}^T \\ & + (1 + e_{4,t+1}^{-1} + e_{6,t+1})(\bar{\Omega}_{t+1}^{(u,1)} + \bar{\Omega}_{t+1}^{(u,2)})I \\ & + (1 + e_{5,t+1}^{-1} + e_{6,t+1}^{-1})\bar{\mathcal{Q}}_{\mathcal{V}_{t+1}}\Big\}\delta I\end{aligned} \tag{25}$$

have solutions $\tilde{\mathcal{P}}_{t+1|t}$ and $\tilde{\mathcal{P}}_{t+1|t+1}$ under conditions $\mathcal{P}_{0|0} = \tilde{\mathcal{P}}_{0|0} > 0$. Then,

$$\mathcal{P}_{t+1|t} \le \tilde{\mathcal{P}}_{t+1|t}, \ \mathcal{P}_{t+1|t+1} \le \tilde{\mathcal{P}}_{t+1|t+1}.$$

Proof: This theorem is proved by the mathematical induction method. Evidently, the condition $\mathcal{P}_{0|0} \le \tilde{\mathcal{P}}_{0|0}$ holds. Assuming $\mathcal{P}_{t|t} \le \tilde{\mathcal{P}}_{t|t}$, we will need to show $\mathcal{P}_{t+1|t+1} \le \tilde{\mathcal{P}}_{t+1|t+1}$.

According to Lemma 2,

$$\begin{aligned}& \mathcal{M}_{1,t+1} + \mathcal{M}_{1,t+1}^T \\ & \quad \le e_{4,t+1}\mathbb{E}\{(I - \tilde{\mathcal{L}}_{t+1}\mathcal{H}_{t+1})\tilde{\mathcal{X}}_{t+1|t}\tilde{\mathcal{X}}_{t+1|t}^T(I - \tilde{\mathcal{L}}_{t+1}\mathcal{H}_{t+1})^T\} \\ & \quad + e_{4,t+1}^{-1}\mathbb{E}\{\tilde{\mathcal{L}}_{t+1}\mathcal{U}_{t+1}\mathcal{U}_{t+1}^T\tilde{\mathcal{L}}_{t+1}^T\}\end{aligned} \tag{26}$$

$$\begin{aligned}& \mathcal{M}_{2,t+1} + \mathcal{M}_{2,t+1}^T \\ & \quad \le e_{5,t+1}\mathbb{E}\{(I - \tilde{\mathcal{L}}_{t+1}\mathcal{H}_{t+1})\tilde{\mathcal{X}}_{t+1|t}\tilde{\mathcal{X}}_{t+1|t}^T(I - \tilde{\mathcal{L}}_{t+1}\mathcal{H}_{t+1})^T\} \\ & \quad + e_{5,t+1}^{-1}\mathbb{E}\{\tilde{\mathcal{L}}_{t+1}\mathcal{V}_{t+1}\mathcal{V}_{t+1}^T\tilde{\mathcal{L}}_{t+1}^T\}.\end{aligned} \tag{27}$$

Similarly, the term $\mathcal{M}_{3,t+1} + \mathcal{M}_{3,t+1}^T$ are calculated as

$$\begin{aligned}\mathcal{M}_{3,t+1} + \mathcal{M}_{3,t+1}^T \le {} & e_{6,t+1}\mathbb{E}\{\tilde{\mathcal{L}}_{t+1}\mathcal{U}_{t+1}\mathcal{U}_{t+1}^T\tilde{\mathcal{L}}_{t+1}^T\} \\ & + e_{6,t+1}^{-1}\mathbb{E}\{\tilde{\mathcal{L}}_{t+1}\mathcal{V}_{t+1}\mathcal{V}_{t+1}^T\tilde{\mathcal{L}}_{t+1}^T\}.\end{aligned} \tag{28}$$

Substituting (26)−(28) into (22) leads to

$$\begin{aligned}\mathcal{P}_{t+1|t+1} \le {} & (1 + e_{4,t+1} + e_{5,t+1})(I - \mathcal{L}_{t+1}\mathcal{H}_{t+1})\mathcal{P}_{t+1|t} \\ & \times (I - \mathcal{L}_{t+1}\mathcal{H}_{t+1})^T + (1 + e_{4,t+1}^{-1} + e_{6,t+1}) \\ & \times \mathcal{L}_{t+1}(\bar{\Omega}_{t+1}^{(u,1)} + \bar{\Omega}_{t+1}^{(u,2)})\mathcal{L}_{t+1}^T \\ & + (1 + e_{5,t+1}^{-1} + e_{6,t+1}^{-1})\mathcal{L}_{t+1}\bar{\mathcal{Q}}_{\mathcal{V}_{t+1}}\mathcal{L}_{t+1}^T \\ & + \lambda_{\max}\Big\{(1 + e_{4,t+1} + e_{5,t+1})\mathcal{H}_{t+1}\mathcal{P}_{t+1|t}\mathcal{H}_{t+1}^T \\ & + (1 + e_{4,t+1}^{-1} + e_{6,t+1})(\bar{\Omega}_{t+1}^{(u,1)} + \bar{\Omega}_{t+1}^{(u,2)})I \\ & + (1 + e_{5,t+1}^{-1} + e_{6,t+1}^{-1})\bar{\mathcal{Q}}_{\mathcal{V}_{t+1}}\Big\}\delta I.\end{aligned} \tag{29}$$

Bearing in mind that $\mathcal{P}_{t|t} \le \tilde{\mathcal{P}}_{t|t}$, we can easily obtain $\mathcal{P}_{t+1|t} \le \tilde{\mathcal{P}}_{t+1|t}$. Utilizing the mathematical induction method, we further have

$$\mathcal{P}_{t+1|t+1} \le \tilde{\mathcal{P}}_{t+1|t+1}.$$

■

Now, we are in a position to minimize bound $\tilde{\mathcal{P}}_{t+1|t+1}$.

Theorem 2: $\tilde{\mathcal{P}}_{t+1|t+1}$ is minimized by designing the filter gain matrix as follows:

$$\mathcal{L}_{t+1} = (1 + e_{4,t+1} + e_{5,t+1})\tilde{\mathcal{P}}_{t+1|t}\mathcal{H}_{t+1}^T\Psi_{t+1}^{-1}. \tag{30}$$

Furthermore, the desired minimal upper bound can be expressed by

$$\begin{aligned}\tilde{\mathcal{P}}_{t+1|t+1} = {} & (1 + e_{4,t+1} + e_{5,t+1})\tilde{\mathcal{P}}_{t+1|t} \\ & + \lambda_{\max}(\Psi_{t+1})\delta I - \mathcal{L}_{t+1}\Psi_{t+1}\mathcal{L}_{t+1}^T\end{aligned} \tag{31}$$

where

$$\begin{aligned}\Psi_{t+1} \triangleq {} & (1 + e_{4,t+1} + e_{5,t+1})\mathcal{H}_{t+1}\tilde{\mathcal{P}}_{t+1|t}\mathcal{H}_{t+1}^T \\ & + (1 + e_{4,t+1}^{-1} + e_{6,t+1})(\bar{\Omega}_{t+1}^{(u,1)} + \bar{\Omega}_{t+1}^{(u,2)})I \\ & + (1 + e_{5,t+1}^{-1} + e_{6,t+1}^{-1})\bar{\mathcal{Q}}_{\mathcal{V}_{t+1}}.\end{aligned}$$

Proof: Using the method of completing the square, we rewrite (25) as follows:

$$\begin{aligned}\tilde{\mathcal{P}}_{t+1|t+1} &= (1+e_{4,t+1}+e_{5,t+1})\tilde{\mathcal{P}}_{t+1|t} + \lambda_{\max}(\Psi_{t+1})\delta I \\ &\quad + [\mathcal{L}_{t+1} - (1+e_{4,t+1}+e_{5,t+1})\tilde{\mathcal{P}}_{t+1|t}\mathcal{H}_{t+1}^T\Psi_{t+1}^{-1}]\Psi_{t+1} \\ &\quad \times [\mathcal{L}_{t+1} - (1+e_{4,t+1}+e_{5,t+1})\tilde{\mathcal{P}}_{t+1|t}\mathcal{H}_{t+1}^T\Psi_{t+1}^{-1}]^T \\ &\quad - (1+e_{4,t+1}+e_{5,t+1})^2\tilde{\mathcal{P}}_{t+1|t}\mathcal{H}_{t+1}^T\Psi_{t+1}^{-1}\mathcal{H}_{t+1}\tilde{\mathcal{P}}_{t+1|t}\end{aligned} \tag{32}$$

which indicates that $\tilde{\mathcal{P}}_{t+1|t+1}$ is minimal when

$$\mathcal{L}_{t+1} = (1+e_{4,t+1}+e_{5,t+1})\tilde{\mathcal{P}}_{t+1|t}\mathcal{H}_{t+1}^T\Psi_{t+1}^{-1}. \qquad \blacksquare$$

Remark 6: We have now addressed the quadratic nonfragile filter design issue for linear non-Gaussian systems with MNs, MMMs, SGFs, and DETS. The original design issue has been converted into the filter design issue for an augmented system that stacks not only original vectors but also second-order Kronecker powers. An upper bound on the filtering error covariance and the filter gain matrix have been, respectively, obtained in Theorems 1 and 2. Clearly, the effects from the aforementioned factors on the filter performance have been reflected in the designed quadratic filtering algorithm. To be specific, $\phi_{\alpha_{i,t}}^{(l)}(i=1,2,\ldots,s;\ l=2,3,4)$ in $\mathcal{Q}_{\mathcal{W}_t}$ account for the effect of MNs, $\phi_{\lambda_{j,t}}^{(l)}(j=1,2,\ldots,m;\ l=2,3,4)$ in $\mathcal{Q}_{\mathcal{V}_t}$ reflect the influence from MMMs, δ, $\bar{\Omega}_{t+1}^{(u,1)}$, and $\bar{\Omega}_{t+1}^{(u,2)}$ characterize the impacts of SGFs and DETS, respectively. Moreover, in order to further minimize the upper bound $\tilde{\mathcal{P}}_{t+1|t+1}$, the parameters $e_{4,t+1}$, $e_{5,t+1}$ and $e_{6,t+1}$ can also be selected by means of optimization algorithms in [4], [33] and the famous genetic algorithm in [48]. In addition, the computation complexity of this quadratic filtering algorithm is $O((n+n^2)^3)$.

Remark 7: In comparison with existing literature, the main novelties lie in: 1) A novel design framework of the quadratic nonfragile filter is proposed to handle the complexities caused by non-Gaussian noises, MNs, MMMs, SGFs, and DETS; 2) Statistical properties about the augmented noises and high-order moments of certain involved parameters are discussed in depth; 3) The newly proposed algorithm possesses a recursive form that is suitable to be implemented online; and 4) The designed algorithm has a higher filtering accuracy than the traditional filter only using the measurements $\tilde{y}_t$.

V. An Illustrative Example

Consider system (1) with parameters

$$F_t = \begin{bmatrix} 0.75 & 0.3+0.4\sin(0.1t) \\ 0.15 & 0.29 \end{bmatrix}$$

$$F_{1,t} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix},\quad B_t = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}$$

$$H_t = \begin{bmatrix} 0.2 & 0.18 \\ 0.7+0.5\cos(0.5t) & 0.3 \end{bmatrix}$$

$$\Lambda_t = \begin{bmatrix} \lambda_{1,t} & 0 \\ 0 & \lambda_{2,t} \end{bmatrix},\quad D_t = \begin{bmatrix} 0.15 \\ 0.1 \end{bmatrix}.$$

In this simulation, x_0 is Gaussian distributed where $\mathrm{Cov}(x_0) = 10^{-2}I_2$. Moreover, we set $s=1$, $\eta_0=1$, $\chi=0.2$, $\sigma=0.3$, $\theta=10$, $e_{i,t}=0.5$ $(i=1,2,\ldots,6)$, and $\delta=10^{-2}$. The probabilistic distributions of $\lambda_{1,t}$ and $\lambda_{2,t}$ satisfy

$$\mathbb{P}\{\lambda_{1,t}=1\}=0.9,\ \mathbb{P}\{\lambda_{1,t}=0.7\}=0.1$$
$$\mathbb{P}\{\lambda_{2,t}=1\}=0.8,\ \mathbb{P}\{\lambda_{2,t}=0.8\}=0.2.$$

The non-Gaussian random sequences w_t, v_t, and $\alpha_{1,t}$ are chosen as follows:

$$\begin{aligned} w_t &= -1.4\tau_{w_t} + 0.6(1-\tau_{w_t}) \\ v_t &= 0.7\tau_{v_t} - 1.3(1-\tau_{v_t}) \\ \alpha_{1,t} &= -1.5\tau_{\alpha_{1,t}} + 0.5(1-\tau_{\alpha_{1,t}}) \end{aligned}$$

where τ_{w_t}, τ_{v_t}, and $\tau_{\alpha_{1,t}}$ are independent Bernoulli variables that satisfy

$$\mathbb{P}\{\tau_{w_t}=1\}=0.3,\ \mathbb{P}\{\tau_{v_t}=1\}=0.65,\ \mathbb{P}\{\tau_{\alpha_{1,t}}=1\}=0.25.$$

The corresponding second-order, third-order and fourth-order moments of w_t, v_t, $\alpha_{1,t}$, $\lambda_{1,t}$, and $\lambda_{2,t}$ are provided in Table I.

TABLE I
The 2nd, 3rd and 4th-Order Moments of Random Variables

	$\mathbb{E}\{(\cdot)^2\}$	$\mathbb{E}\{(\cdot)^3\}$	$\mathbb{E}\{(\cdot)^4\}$
w_t	0.8400	−0.6720	1.2432
v_t	0.9100	−0.5460	1.1557
$\alpha_{1,t}$	0.7500	−0.7500	1.3125
$\lambda_{1,t}$	0.9490	0.9343	0.9240
$\lambda_{2,t}$	0.9280	0.9024	0.8819

True states and their respective estimates are plotted in Fig. 1 where it is illustrated that the original states can be well tracked. Fig. 2 shows the trajectories of upper bounds $\tilde{\mathcal{P}}_{t+1|t+1}$, which confirms that trajectories of actual errors stay below the bounds. Fig. 3 depicts the mean square error (MSE) curves of the designed quadratic filter and the recursive filter only using $\tilde{y}_t$. Clearly, the developed quadratic filter is able to improve filtering performance. All simulations have demonstrated the effectiveness of this newly established quadratic filtering algorithm.

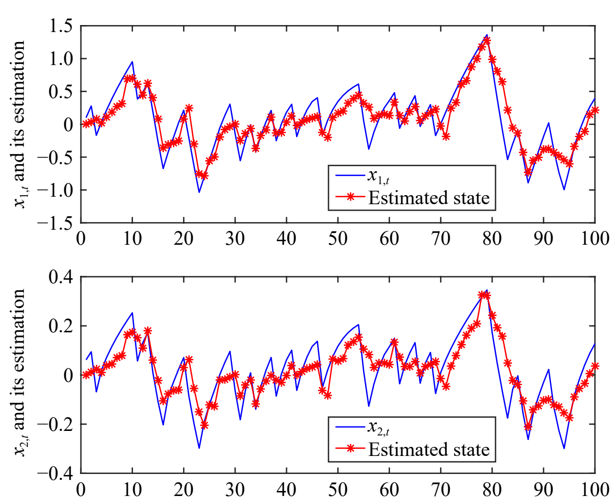

Fig. 1. True state x_t and the estimate $\hat{x}_t$.

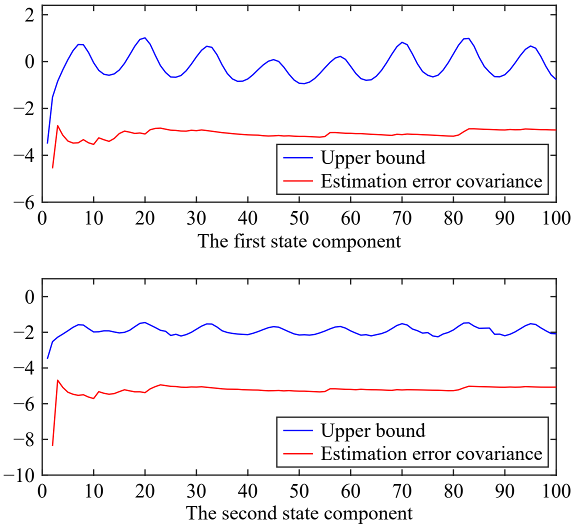

Fig. 2. Actual error covariances and their upper bounds.

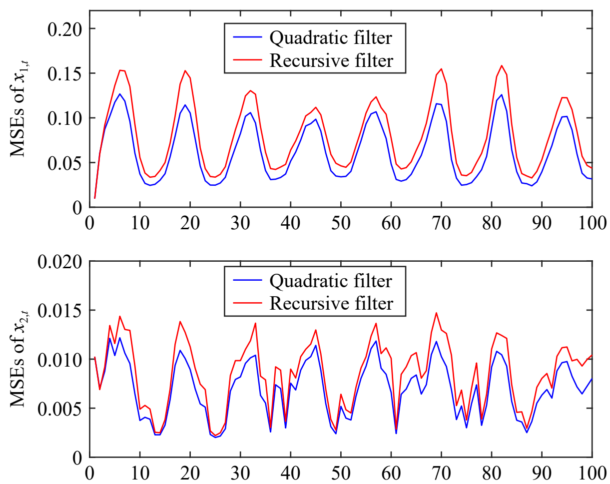

Fig. 3. MSEs of the developed quadratic filter and the traditional filter only using $\tilde{y}_t$.

VI. Conclusions

In this paper, the quadratic nonfragile filtering design issue has been addressed for linear non-Gaussian systems under MNs, MMMs, SGFs, and DETS. An augmented system has been obtained by stacking the original system' state/measurement vectors together with second-order Kronecker powers, thus the original design issue has been reformulated as that of the augmented system. Subsequently, we have analyzed statistical properties of augmented noises as well as high-order moments of certain random parameters. With the aid of two well-defined matrix difference equations, we not only have obtained upper bounds on filtering error covariances, but also have minimized those bounds via appropriate design of gain parameters. Finally, an example has been presented to explain the effectiveness of this newly established quadratic filtering algorithm. Future research topics would include the extension of the proposed quadratic non-fragile filtering scheme to more general systems.

Appendix A
Proof of Lemma 3

Proof: Lemma 2 and (3) imply

$$\begin{aligned}\eta_{t+1}^2 &= (\chi\eta_t+\sigma-\|u_t\|)^2\\ &\le (1+e_{1,t})(\chi\eta_t+\sigma)^2+(1+e_{1,t}^{-1})\|u_t\|^2\\ &\le (1+e_{1,t})[(1+e_{2,t})\chi^2\eta_t^2+(1+e_{2,t}^{-1})\sigma^2]+(1+e_{1,t}^{-1})\|u_t\|^2\end{aligned}\tag{33}$$

and

$$\begin{aligned}\eta_{t+1}^4 \le\ & (1+e_{1,t})^2(\chi\eta_t+\sigma)^4+(1+e_{1,t}^{-1})^2\|u_t\|^4\\ &+2(1+e_{1,t})(1+e_{1,t}^{-1})(\chi\eta_t+\sigma)^2\|u_t\|^2\\ \le\ & (1+e_{1,t})^2[(1+e_{3,t})(1+e_{2,t})^2\chi^4\eta_t^4\\ &+(1+e_{3,t}^{-1})(1+e_{2,t}^{-1})^2\sigma^4]\\ &+2(1+e_{1,t})(1+e_{1,t}^{-1})[(1+e_{2,t})\chi^2\eta_t^2\\ &+(1+e_{2,t}^{-1})\sigma^2]\|u_t\|^2+(1+e_{1,t}^{-1})^2\|u_t\|^4.\end{aligned}\tag{34}$$

On the other hand, when $t\in\{t_i,t_i+1,t_i+2,\ldots,t_{i+1}-1\}$, one has

$$\|u_t\|^2\le(1+\theta)\frac{\eta_t^2}{\theta^2}+(1+\theta^{-1})\sigma^2\tag{35}$$

and

$$\begin{aligned}\|u_t\|^4 &\le [(1+\theta)\frac{\eta_t^2}{\theta^2}+(1+\theta^{-1})\sigma^2]^2\\ &=(1+\theta)^2\frac{\eta_t^4}{\theta^4}+(1+\theta^{-1})^2\sigma^4\\ &\quad+2(1+\theta)(1+\theta^{-1})\frac{\eta_t^2}{\theta^2}\sigma^2.\end{aligned}\tag{36}$$

Substituting (35) and (36) into (33) and (34) leads to

$$\begin{aligned}\eta_{t+1}^2\le&\Big[(1+e_{1,t})(1+e_{2,t})\chi^2+(1+e_{1,t}^{-1})\frac{1+\theta}{\theta^2}\Big]\eta_t^2\\ &+\Big[(1+e_{1,t})(1+e_{2,t}^{-1})+(1+e_{1,t}^{-1})(1+\theta^{-1})\Big]\sigma^2\end{aligned}\tag{37}$$

and

$$\eta_{t+1}^4\le\Xi_{1,t}\eta_t^4+\Xi_{2,t}\eta_t^2+\Xi_{3,t}\sigma^4\tag{38}$$

where $\Xi_{1,t}$, $\Xi_{2,t}$ and $\Xi_{3,t}$ are defined in Lemma 3.

Moreover,

$$\mathbb{E}\{\|u_t\|^2\}\le(1+\theta)\frac{\mathbb{E}\{\eta_t^2\}}{\theta^2}+(1+\theta^{-1})\sigma^2\tag{39}$$

and

$$\begin{aligned}\mathbb{E}\{\|u_t\|^4\}\le&(1+\theta)^2\frac{\mathbb{E}\{\eta_t^4\}}{\theta^4}+(1+\theta^{-1})^2\sigma^4\\ &+2(1+\theta)(1+\theta^{-1})\frac{\mathbb{E}\{\eta_t^2\}}{\theta^2}\sigma^2.\end{aligned}\tag{40}$$

Based on (37)−(40), it can be concluded that conditions of (14) are satisfied. ■

Appendix B
Proof of Lemma 4

Proof: Recalling the definition of $Q_{W_{11,t}}$ and Assumption 1, one has

$$\begin{aligned} Q_{\mathcal{W}_{11,t}} &= \sum_{i=1}^{s} \mathbb{E}\{\alpha_{i,t}^2 F_{i,t} x_t x_t^T F_{i,t}^T\} + B_t \mathbb{E}\{w_t w_t^T\} B_t^T \\ &= \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(2)} F_{i,t} \mathrm{sti}(\phi_{x_t}^{(2)}) F_{i,t}^T + B_t \mathrm{sti}(\phi_{w_t}^{(2)}) B_t^T. \end{aligned} \tag{41}$$

Based on the expression of $\tilde{w}_t$,

$$\begin{aligned} Q_{\mathcal{W}_{12,t}} &= \mathbb{E}\Big\{(\sum_{i=1}^{s} \alpha_{i,t} F_{i,t}) x_t (x_t^{[2]})^T (F_t \otimes \sum_{i=1}^{s} \alpha_{i,t} F_{i,t})^T \tilde{\Gamma}_{n,n}^T\Big\} \\ &\quad + \mathbb{E}\Big\{(\sum_{i=1}^{s} \alpha_{i,t} F_{i,t}) x_t (x_t^{[2]})^T \Big[\sum_{i=1}^{s} (\alpha_{i,t}^2 - \phi_{\alpha_{i,t}}^{(2)}) F_{i,t}^{[2]}\Big]^T\Big\} \\ &\quad + B_t \mathrm{sti}(\phi_{w_t}^{(3)}) (B_t^{[2]})^T \\ &= \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(2)} F_{i,t} \mathrm{sti}(\phi_{x_t}^{(3)}) (F_t \otimes F_{i,t})^T \tilde{\Gamma}_{n,n}^T \\ &\quad + \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(3)} F_{i,t} \mathrm{sti}(\phi_{x_t}^{(3)}) (F_{i,t}^{[2]})^T \\ &\quad + B_t \mathrm{sti}(\phi_{w_t}^{(3)}) (B_t^{[2]})^T \end{aligned} \tag{42}$$

where the facts that $\mathbb{E}\{\alpha_{i,t}(\alpha_{i,t}^2 - \phi_{\alpha_{i,t}}^{(2)})\} = \phi_{\alpha_{i,t}}^{(3)}$ and $\mathbb{E}\{\alpha_{i,t}(\alpha_{j,t}^2 - \phi_{\alpha_{j,t}}^{(2)})\} = 0$ $(i \neq j)$ have been used.

For the sake of simplicity, let us define

$$\begin{aligned} \tilde{F}_t &\triangleq F_t + \sum_{i=1}^{s} \alpha_{i,t} F_{i,t} \\ \Delta\tilde{F}_t &\triangleq \tilde{\Gamma}_{n,n}(F_t \otimes \sum_{i=1}^{s} \alpha_{i,t} F_{i,t}) + \sum_{i=1}^{s} (\alpha_{i,t}^2 - \phi_{\alpha_{i,t}}^{(2)}) F_{i,t}^{[2]} \\ &\quad + \sum_{i=1}^{s} \sum_{1=j\neq i}^{s} \alpha_{i,t} \alpha_{j,t} F_{i,t} \otimes F_{j,t}. \end{aligned} \tag{43}$$

Then, the term $\tilde{w}_t$ can be rewritten as follows:

$$\tilde{w}_t = \Delta\tilde{F}_t x_t^{[2]} + \tilde{\Gamma}_{n,n}(\tilde{F}_t x_t \otimes B_t w_t) + B_t^{[2]}(w_t^{[2]} - \phi_{w_t}^{(2)}). \tag{44}$$

From $Q_{\mathcal{W}_{22,t}}$'s definition, we know that $Q_{\mathcal{W}_{22,t}} = \mathbb{E}\{\tilde{w}_t \tilde{w}_t^T\}$. In what follows, we are going to calculate the terms $\mathbb{E}\{\Delta\tilde{F}_t x_t^{[2]} (x_t^{[2]})^T \Delta\tilde{F}_t^T\}$, $\mathbb{E}\{\tilde{\Gamma}_{n,n}(\tilde{F}_t x_t \otimes B_t w_t)(\tilde{F}_t x_t \otimes B_t w_t)^T \tilde{\Gamma}_{n,n}^T\}$ and $\mathbb{E}\{B_t^{[2]}(w_t^{[2]} - \phi_{w_t}^{(2)})(w_t^{[2]} - \phi_{w_t}^{(2)})^T (B_t^{[2]})^T\}$ one by one.

Based on (43), we have

$$\begin{aligned} &\mathbb{E}\{\Delta\tilde{F}_t x_t^{[2]} (x_t^{[2]})^T \Delta\tilde{F}_t^T\} \\ &= \mathbb{E}\{\tilde{\Gamma}_{n,n} \sum_{i=1}^{s} \alpha_{i,t}^2 (F_t \otimes F_{i,t}) x_t^{[2]} (x_t^{[2]})^T (F_t \otimes F_{i,t})^T \tilde{\Gamma}_{n,n}^T\} \\ &\quad + \mathbb{E}\{\sum_{i=1}^{s} (\alpha_{i,t}^4 - (\phi_{\alpha_{i,t}}^{(2)})^2) F_{i,t}^{[2]} x_t^{[2]} (x_t^{[2]})^T (F_{i,t}^{[2]})^T\} \\ &\quad + \mathbb{E}\Big\{\sum_{i=1}^{s} \sum_{j\neq i=1}^{s} \alpha_{i,t}^2 \alpha_{j,t}^2 (F_{i,t} \otimes F_{j,t}) x_t^{[2]} (x_t^{[2]})^T (F_{i,t} \otimes F_{j,t})^T\Big\} \\ &\quad + \mathrm{Sym}\Big\{\mathbb{E}\{\tilde{\Gamma}_{n,n} \sum_{i=1}^{s} \alpha_{i,t}^3 (F_t \otimes F_{i,t}) x_t^{[2]} (x_t^{[2]})^T (F_{i,t}^{[2]})^T\}\Big\} \\ &= \tilde{\Gamma}_{n,n} \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(2)} (F_t \otimes F_{i,t}) \mathrm{sti}(\phi_{x_t}^{(4)}) (F_t \otimes F_{i,t})^T \tilde{\Gamma}_{n,n}^T \\ &\quad + \sum_{i=1}^{s} [\phi_{\alpha_{i,t}}^{(4)} - (\phi_{\alpha_{i,t}}^{(2)})^2] F_{i,t}^{[2]} \mathrm{sti}(\phi_{x_t}^{(4)}) (F_{i,t}^{[2]})^T \\ &\quad + \sum_{i=1}^{s} \sum_{j\neq i=1}^{s} \phi_{\alpha_{i,t}}^{(2)} \phi_{\alpha_{j,t}}^{(2)} (F_{i,t} \otimes F_{j,t}) \mathrm{sti}(\phi_{x_t}^{(4)}) (F_{i,t} \otimes F_{j,t})^T \\ &\quad + \mathrm{Sym}\Big\{\tilde{\Gamma}_{n,n} \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(3)} (F_t \otimes F_{i,t}) \mathrm{sti}(\phi_{x_t}^{(4)}) (F_{i,t}^{[2]})^T\Big\}. \end{aligned} \tag{45}$$

On the other hand, we can further obtain that

$$\begin{aligned} &\mathbb{E}\{\tilde{\Gamma}_{n,n}(\tilde{F}_t x_t \otimes B_t w_t)(\tilde{F}_t x_t \otimes B_t w_t)^T \tilde{\Gamma}_{n,n}^T\} \\ &\quad = \tilde{\Gamma}_{n,n} \mathbb{E}\{(\tilde{F}_t x_t x_t^T \tilde{F}_t^T) \otimes (B_t w_t w_t^T B_t^T)\} \tilde{\Gamma}_{n,n}^T \\ &\quad = \tilde{\Gamma}_{n,n}\Big\{\Big(F_t \mathrm{sti}(\phi_{x_t}^{(2)}) F_t^T + \sum_{i=1}^{s} \phi_{\alpha_{i,t}}^{(2)} F_{i,t} \mathrm{sti}(\phi_{x_t}^{(2)}) F_{i,t}^T\Big) \\ &\qquad \otimes (B_t \mathrm{sti}(\phi_{w_t}^{(2)}) B_t^T)\Big\} \tilde{\Gamma}_{n,n}^T \end{aligned} \tag{46}$$

and

$$\begin{aligned} &\mathbb{E}\{B_t^{[2]}(w_t^{[2]} - \phi_{w_t}^{(2)})(w_t^{[2]} - \phi_{w_t}^{(2)})^T (B_t^{[2]})^T\} \\ &\qquad = B_t^{[2]}(\mathrm{sti}(\phi_{w_t}^{(4)}) - \phi_{w_t}^{(2)} (\phi_{w_t}^{(2)})^T)(B_t^{[2]})^T. \end{aligned} \tag{47}$$

Then, from (45)–(47), we can acquire the expression of $Q_{\mathcal{W}_{22,t}}$.

Based on Assumption 1,

$$\begin{aligned} Q_{\mathcal{V}_{11,t}} &= \mathbb{E}\{(\Lambda_t - \mathbb{E}\{\Lambda_t\}) H_t x_t x_t^T H_t^T (\Lambda_t - \mathbb{E}\{\Lambda_t\})^T\} \\ &\quad + D_t \mathbb{E}\{v_t v_t^T\} D_t^T \end{aligned} \tag{48}$$

which, together with Lemma 1, implies that

$$Q_{\mathcal{V}_{11,t}} = \Upsilon_t \circ (H_t \mathrm{sti}(\phi_{x_t}^{(2)}) H_t^T) + D_t \mathrm{sti}(\phi_{v_t}^{(2)}) D_t^T. \tag{49}$$

For the term $Q_{\mathcal{V}_{12,t}}$, we can obtain that

$$\begin{aligned} Q_{\mathcal{V}_{12,t}} &= \mathbb{E}\{(\Lambda_t - \mathbb{E}\{\Lambda_t\}) H_t x_t (x_t^{[2]})^T (H_t^{[2]})^T (\Lambda_t^{[2]} - \mathbb{E}\{\Lambda_t^{[2]}\})^T\} \\ &\quad + \mathbb{E}\{D_t v_t (v_t^{[2]} - \phi_{v_t}^{(2)})^T (D_t^{[2]})^T\} \\ &\quad + \mathbb{E}\{(\Lambda_t - \mathbb{E}\{\Lambda_t\}) H_t x_t (y_t \otimes u_t)^T \tilde{\Gamma}_{m,m}^T\} \\ &\quad + \mathbb{E}\{D_t v_t (y_t \otimes u_t)^T \tilde{\Gamma}_{m,m}^T\}. \end{aligned} \tag{50}$$

To handle the difficulties induced by the term $\Lambda_t^{[2]} - \mathbb{E}\{\Lambda_t^{[2]}\}$, the matrix Λ_t is rewritten as $\sum_{i=1}^{m} \lambda_{i,t} E_i$. Then, one has

$$\begin{aligned} \Lambda_t^{[2]} - \mathbb{E}\{\Lambda_t^{[2]}\} &= \sum_{i=1}^{m} (\lambda_{i,t}^2 - \phi_{\lambda_{i,t}}^{(2)}) E_i^{[2]} + \sum_{i=1}^{m} \sum_{1=j\neq i}^{m} (\lambda_{i,t} \lambda_{j,t} \\ &\qquad - \bar{\lambda}_{i,t} \bar{\lambda}_{j,t})(E_i \otimes E_j). \end{aligned} \tag{51}$$

Consequently, for (50), we have

$$\begin{aligned}
&\mathbb{E}\{(\Lambda_t-\mathbb{E}\{\Lambda_t\})H_t x_t (x_t^{[2]})^T (H_t^{[2]})^T (\Lambda_t^{[2]}-\mathbb{E}\{\Lambda_t^{[2]}\})^T\}\\
&\quad=\mathbb{E}\Big\{\sum_{i=1}^{m}(\lambda_{i,t}-\bar{\lambda}_{i,t})E_i H_t x_t (x_t^{[2]})^T (H_t^{[2]})^T\\
&\qquad\times\Big[\sum_{i=1}^{m}(\lambda_{i,t}^2-\phi_{\lambda_{i,t}}^{(2)})E_i^{[2]}+\sum_{i=1}^{m}\sum_{1=j\neq i}^{m}(\lambda_{i,t}\lambda_{j,t}\\
&\qquad-\bar{\lambda}_{i,t}\bar{\lambda}_{j,t})(E_i\otimes E_j)\Big]^T\Big\}\triangleq M_t
\end{aligned}\tag{52}$$

$$\mathbb{E}\{(\Lambda_t-\mathbb{E}\{\Lambda_t\})H_t x_t (y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}=-2\rho_t M_t \tag{53}$$

and

$$\mathbb{E}\{D_t v_t (y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}=-2\rho_t D_t \mathrm{sti}(\phi_{v_t}^{(3)})(D_t^{[2]})^T \tag{54}$$

where ρ_t equals to 0 when t is an triggering time instant, and 1 otherwise. Then, substituting (52)−(54) into (50) yields the expression of $Q_{\mathcal{V}_{12,t}}$ in (15).

Next, we are going to discuss the term $Q_{\mathcal{V}_{22,t}}$. It is straightforward to verify that

$$\begin{aligned}
Q_{\mathcal{V}_{22,t}}=&\,\mathbb{E}\{(\Lambda_t^{[2]}-\mathbb{E}\{\Lambda_t^{[2]}\})H_t^{[2]}x_t^{[2]}(x_t^{[2]})^T\\
&\times(H_t^{[2]})^T(\Lambda_t^{[2]}-\mathbb{E}\{\Lambda_t^{[2]}\})^T\}\\
&+\tilde{\Gamma}_{m,m}\mathbb{E}\{(\Lambda_t H_t x_t x_t^T H_t^T\Lambda_t^T)\otimes(D_t v_t v_t^T D_t^T)\}\tilde{\Gamma}_{m,m}^T\\
&+\mathbb{E}\{D_t^{[2]}(v_t^{[2]}-\phi_{v_t}^{(2)})(v_t^{[2]}-\phi_{v_t}^{(2)})^T(D_t^{[2]})^T\}\\
&+\tilde{\Gamma}_{m,m}\mathbb{E}\{(y_t\otimes u_t)(y_t\otimes u_t)^T\}\tilde{\Gamma}_{m,m}^T\\
&+\mathrm{Sym}\Big\{\mathbb{E}\{(\Lambda_t^{[2]}-\mathbb{E}\{\Lambda_t^{[2]}\})H_t^{[2]}x_t^{[2]}(y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}\\
&+\mathbb{E}\{\tilde{\Gamma}_{m,m}(\Lambda_t H_t x_t\otimes D_t v_t)(y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}\\
&+\mathbb{E}\{D_t^{[2]}(v_t^{[2]}-\phi_{v_t}^{(2)})(y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}\Big\}.
\end{aligned}\tag{55}$$

Based on Lemma 1, we can see that

$$\begin{aligned}
&\mathbb{E}\{(\Lambda_t^{[2]}-\mathbb{E}\{\Lambda_t^{[2]}\})H_t^{[2]}x_t^{[2]}(x_t^{[2]})^T\\
&\qquad\times(H_t^{[2]})^T(\Lambda_t^{[2]}-\mathbb{E}\{\Lambda_t^{[2]}\})^T\}\\
&\quad=N_t\circ(H_t^{[2]}\mathrm{sti}(\phi_{x_t}^{(4)})(H_t^{[2]})^T)\\
&\qquad-\mathbb{E}\{\Lambda_t^{[2]}\}H_t^{[2]}\mathrm{sti}(\phi_{x_t}^{(4)})(H_t^{[2]})^T(\mathbb{E}\{\Lambda_t^{[2]}\})^T.
\end{aligned}\tag{56}$$

Meanwhile, we have

$$\begin{aligned}
&\tilde{\Gamma}_{m,m}\mathbb{E}\{(\Lambda_t H_t x_t x_t^T H_t^T\Lambda_t^T)\otimes(D_t v_t v_t^T D_t^T)\}\tilde{\Gamma}_{m,m}^T\\
&\quad=\tilde{\Gamma}_{m,m}(T_t\circ(H_t\mathrm{sti}(\phi_{x_t}^{(2)})H_t^T))\otimes(D_t\mathrm{sti}(\phi_{v_t}^{(2)})D_t^T)\tilde{\Gamma}_{m,m}^T
\end{aligned}\tag{57}$$

and

$$\begin{aligned}
&\mathbb{E}\{D_t^{[2]}(v_t^{[2]}-\phi_{v_t}^{(2)})(v_t^{[2]}-\phi_{v_t}^{(2)})^T(D_t^{[2]})^T\}\\
&\quad=D_t^{[2]}(\mathrm{sti}(\phi_{v_t}^{(4)})-\phi_{v_t}^{(2)}(\phi_{v_t}^{(2)})^T)(D_t^{[2]})^T.
\end{aligned}\tag{58}$$

On the other hand,

$$\begin{aligned}
\mathbb{E}\{y_t y_t^T\}&=\mathbb{E}\{\Lambda_t H_t x_t x_t^T H_t^T\Lambda_t^T\}+\mathbb{E}\{D_t v_t v_t^T D_t^T\}\\
&=T_t\circ(H_t\mathrm{sti}(\phi_{x_t}^{(2)})H_t^T)+D_t\mathrm{sti}(\phi_{v_t}^{(2)})D_t^T.
\end{aligned}\tag{59}$$

Therefore, the following result holds:

$$\begin{aligned}
&\tilde{\Gamma}_{m,m}\mathbb{E}\{(y_t\otimes u_t)(y_t\otimes u_t)^T\}\tilde{\Gamma}_{m,m}^T\\
&\quad=\tilde{\Gamma}_{m,m}\mathbb{E}\{(y_t y_t^T)\otimes(u_t u_t^T)\}\tilde{\Gamma}_{m,m}^T\\
&\quad\leq\tilde{\Gamma}_{m,m}\Big\{(T_t\circ(H_t\mathrm{sti}(\phi_{x_t}^{(2)})H_t^T)\\
&\qquad+D_t\mathrm{sti}(\phi_{v_t}^{(2)})D_t^T)\otimes\bar{\Omega}_t^{(u,1)}I\Big\}\tilde{\Gamma}_{m,m}^T.
\end{aligned}\tag{60}$$

Following the similar line of the derivation of (53) and (54), one has:

$$\begin{aligned}
&\mathbb{E}\{(\Lambda_t^{[2]}-\mathbb{E}\{\Lambda_t^{[2]}\})H_t^{[2]}x_t^{[2]}(y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}\\
&\quad=-2\rho_t[N_t\circ(H_t^{[2]}\mathrm{sti}(\phi_{x_t}^{(4)})(H_t^{[2]})^T)\\
&\qquad-\mathbb{E}\{\Lambda_t^{[2]}\}H_t^{[2]}\mathrm{sti}(\phi_{x_t}^{(4)})(H_t^{[2]})^T(\mathbb{E}\{\Lambda_t^{[2]}\})^T]
\end{aligned}\tag{61}$$

$$\begin{aligned}
&\mathbb{E}\{\tilde{\Gamma}_{m,m}(\Lambda_t H_t x_t\otimes D_t v_t)(y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}\\
&\quad=-2\rho_t\tilde{\Gamma}_{m,m}[(T_t\circ(H_t\mathrm{sti}(\phi_{x_t}^{(2)})H_t^T))\\
&\qquad\otimes(D_t\mathrm{sti}(\phi_{v_t}^{(2)})D_t^T)]\tilde{\Gamma}_{m,m}^T
\end{aligned}\tag{62}$$

and

$$\begin{aligned}
&\mathbb{E}\{D_t^{[2]}(v_t^{[2]}-\phi_{v_t}^{(2)})(y_t\otimes u_t)^T\tilde{\Gamma}_{m,m}^T\}\\
&\quad=-2\rho_t D_t^{[2]}(\mathrm{sti}(\phi_{v_t}^{(4)})-\phi_{v_t}^{(2)}(\phi_{v_t}^{(2)})^T)(D_t^{[2]})^T.
\end{aligned}\tag{63}$$

Substituting (56)−(63) into (55), it is straightforward to see that the term $\bar{Q}_{\mathcal{V}_{22,t}}$ can be expressed as in (15). ■

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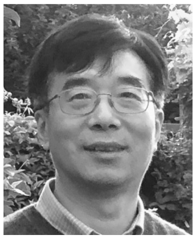

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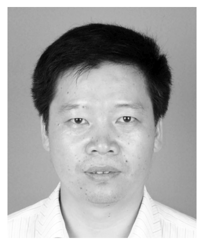

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