Privacy-Preserving Consensus-Based Distributed Economic Dispatch of Smart Grids via State Decomposition

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***Abstract*—This paper studies the privacy-preserving distributed economic dispatch (DED) problem of smart grids. An autonomous consensus-based algorithm is developed via local data exchange with neighboring nodes, which covers both the islanded mode and the grid-connected mode of smart grids. To prevent power-sensitive information from being disclosed, a privacy-preserving mechanism is integrated into the proposed DED algorithm by randomly decomposing the state into two parts, where only partial data is transmitted. Our objective is to develop a privacy-preserving DED algorithm to achieve optimal power dispatch with the lowest generation cost under physical constraints while preventing sensitive information from being eavesdropped. To this end, a comprehensive analysis framework is established to ensure that the proposed algorithm can converge to the optimal solution of the concerned optimization problem by means of the consensus theory and the eigenvalue perturbation approach. In particular, the proposed autonomous algorithm can achieve a smooth transition between the islanded mode and the grid-connected mode. Furthermore, rigorous analysis is given to show privacy-preserving performance against internal and external eavesdroppers. Finally, case studies illustrate the feasibility and validity of the developed algorithm.**

***Index Terms*—Consensus-based DED algorithm, privacy preservation, smart grids, state decomposition.**

I. Introduction

DRIVEN by the emerging global energy crisis and growing environmental issues, the past years have seen ever-increasing research enthusiasm devoted to the control and optimization of smart grids (SGs), which mainly stems from the outstanding merits of SGs in terms of flexibility, efficiency, scalability, and sustainability [1]–[4]. As a typical cyber-physical system, the SGs highly integrate physical electric systems and cyber communication. With the help of the cyber layer of SGs, a hierarchical control structure that contains the primary, secondary and tertiary control levels is established to achieve reliable operations and energy managements of SGs [5], [6]. It is worth noting that primary and secondary control is aimed at addressing the stability and synchronization issues, while tertiary control mainly deals with various optimization issues to implement the optimal operation of SGs [3], [7].

As one of the most important optimization issues in SGs, the economic dispatch (ED) problem has received wide research attention recently [8]–[12]. The main objective of the ED problem is to regulate the power outputs of all distributed generators (DGs) to achieve supply-demand balance with minimum generation cost under actual physical constraints. With the massive penetration of renewable energy resources (e.g., solar, wind and water) in modern SGs, the conventional centralized ED algorithm cannot meet the requirements of real-time computation and communication. Besides, the intermittent and random characteristics of renewable energy may give rise to serious challenges for the control and optimization of SGs. To this end, considerable efforts have been devoted to the distributed implementation scheme due to its prominent superiorities in terms of robustness, decentralization, autonomy, and scalability [12]–[14].

Recently, the consensus-based algorithms have been successful utilized to solve the DED problem of SGs since the pioneering works in [10], [12]. Compared with the existing distributed algorithms (e.g., alternating direction method of multipliers (ADMM) [15], [16]), consensus-based methods have distinguishing merits in simple coordination rules and easy-to-implement architecture. To date, a large amount of literature has been available on the consensus-based DED algorithm under various scenarios, such as lossy communication, event-triggered communication, and cyber attacks [17]–[21]. Furthermore, some consensus-based algorithms have been extended to solve the ED problem with non-convex constraints. For example, to maximize social welfare on both the generation and demand sides, a consensus-based algorithm combined with a relaxation technique has been developed in [22] to achieve the optimal energy management of SGs under transmission losses.

Note that the aforementioned results are presented under the assumption that the SG operates in the islanded mode. Unfortunately, such an assumption may be restrictive for the operation mode of SGs. Actually, the SG has two operation modes

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(i.e., the islanded mode and the grid-connected mode), and the mode is usually determined by energy routers (ERs) [5], [6]. To deal with some possible emergencies (e.g., cyber attacks in the main grid), the SG needs to smoothly switch between the two modes to avoid large-scale power system blackouts. In this regard, a feasible solution to the ED issue is to design an autonomous distributed algorithm, which is able to achieve smooth transitions between the islanded and the grid-connected mode. However, the corresponding results for the autonomous DED algorithm are very scattered, which spurs our present investigation.

In the context of the energy internet, data privacy preservation of SGs has become a research hotspot, which provides a prerequisite for the reliable and safe operation of SGs and the normative order of the power market [23], [24]. So far, some representative results on different types of privacy-preserving DED algorithms have been reported in [25]–[30]. For example, a differentially private scheme has been proposed in [25] by injecting independent noise sequences to mask sensitive information. However, such a scheme has a recognized weakness in terms of inaccuracy, and there exists a trade-off between the privacy level and the convergence accuracy. To overcome such a trade-off, a privacy-preserving protocol has been proposed in [27], where a well-designed correlated noise sequence has been utilized to obfuscate the privacy value. Nevertheless, privacy performance, as pointed out in [31], is usually compromised against external eavesdroppers. In [28], [29], a bounded and decaying noise sequence has been introduced to mask power-sensitive information, whereas the developed DED algorithms cannot achieve linear convergence due to the cost of privacy preservation. In addition, a homomorphic encryption-based protocol has been developed in [30] to achieve confidential communication between neighboring nodes by taking full advantage of the Pailler cryptosystem and random weight mechanism. However, the cryptology-based approach uses large amounts of computational and communication resources due to the complexity of encryption algorithms [31].

To overcome the aforementioned shortcomings of existing privacy-preserving algorithms, a output mask approach has been adopted in [32] by inserting dynamic vanishing affine information into exchanged data. Subsequently, [33] has proposed a perturbation-based mechanism by inserting an additive signal in the initial time period. It is worth noting that the aforementioned two privacy-preserving algorithms can exactly converge to the reference value. Furthermore, a state-decomposition scheme has been first proposed in [34] with guaranteed convergence accuracy and privacy properties. The main idea is that each node decomposes its initial sensitive information into two substates. External substate can communicate with its neighboring nodes, while internal substate only exchange information with the external substate and hence is thoroughly unknown to neighboring nodes. Furthermore, the initial values of these two substates are randomly generated but their sum is twice the initial value of the original state for the purpose of exact convergence and uncompromising privacy performance. To data, the state-decomposition approach has been extended into the dynamic average consensus of multi-robot formation systems [35], robust consensus of microgrid control [36], and distributed secondary control of AC microgrids [37]. Nevertheless, the corresponding privacy-preserving DED problem via such an approach has not yet been adequately investigated possibly due to the structural complexity of consensus-based DED algorithms. This motivates us to bridge such a gap.

Summarizing the above discussions, in this paper, we endeavor to address the privacy-preserving DED problem of SGs. Three substantial difficulties that we going to face are identified as follows: 1) How to design an effective DED scheme that can cover the islanded and grid-connected modes of SGs; 2) How to develop a privacy-preserving algorithm with exact convergence, well privacy, and low complexity; and 3) How to establish a unified analysis framework that takes the performance of convergence, optimality, and privacy preservation into account simultaneously. To overcome the aforementioned challenges, we develop an autonomous privacy-preserving DED algorithm via the state-decomposition approach to achieve the optimal ED without privacy disclosure. The main contributions of this paper are listed as follows.

1) Based on the leaderless and leader-following consensus algorithms developed for multi-agent systems, an autonomous consensus-based DED algorithm with a constant step size is developed to achieve the supply-demand balance with the minimum generation cost, which can realize smooth transitions between the grid-connected and islanded modes of SGs;

2) With the help of the matrix eigenvalue analysis technique, a comprehensive analysis is provided in terms of the convergence and the optimality of the proposed DED algorithm with and without state decomposition;

3) A state-decomposition-based scheme is, for the first time, incorporated into the framework of the DED algorithm, which is privacy-preserving against internal honest-but-curious agents and external eavesdroppers.

The remainder of this paper is organized as follows. Section II formulates the privacy-preserving ED problem of SGs and provides the primary objective of this paper. In Section III, a novel consensus-based DED algorithm is proposed and its convergence and optimality are analysed in the islanded and grid-connected modes, respectively. Section IV gives a privacy-preserving DED algorithm, and analyses its performance in view of convergence and privacy. Simulation studies are given to demonstrate the theoretical results in Section V. Finally, Section VI states conclusions.

Notations: $\mathrm{col}_N\{a_i\}$ is an N-dimensional column vector with a_i being the ith element, $\mathrm{diag}_N\{C_i\}$ refers to a block-diagonal matrix. $L = [l_{ij}]_N$ represents an N-dimensional matrix whose elements are expressed by l_{ij}. $\mathbf{1}_N$ is N-dimensional column vector of ones. $\rho(Q)$ refers to the spectral radius of the matrix Q. The symbol "\" represents the set subtraction.

II. Problem Formulation and Preliminaries

A. Structure of SGs

In the context of the energy internet, the SGs, composed of main grid, ERs, and interconnected microgrids, usually have two operation modes (i.e., the grid-connected mode and the islanded mode), where the operation mode is determined by the ERs. The microgrid mainly contains DGs, flexible loads, and local intelligent control units (ICUs). It should be pointed out that the ICU plays a key role in energy management and optimization control of SGs via local data exchange over sparse communication networks.

In this paper, the structure of SGs is modeled by a multi-agent system where each agent consists of a DG, a local ICU, and some loads. The communication topology of agents can be described by an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}=\{1,2,\ldots,N\}$ is the set of N agents, and $\mathcal{E}=\{(i,j)|i,j\in\mathcal{V}\}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges. $(i,j)\in\mathcal{E}$ means that agent i and agent j can communicate with each other. The neighborhood set of node i can be denoted by $\mathcal{N}_i=\{j|(i,j)\in\mathcal{E}, i\neq j\}$. In addition, the adjacency matrix of graph $\mathcal{G}$ is denoted by $\mathcal{A}=[a_{ij}]_N$, where $a_{ij}>0$ if $(i,j)\in\mathcal{E}$, and $a_{ij}=0$, otherwise. The Laplacian matrix is defined by $L=\mathcal{D}-\mathcal{A}$, where $\mathcal{D}\triangleq\text{diag}_N\{d_i\}$ is the degree matrix with $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$. Note that the matrix L is a symmetric matrix for the undirected graph $\mathcal{G}$. In this paper, the main grid is regarded as a leader labeled by 0, and the leader only exchanges data with its neighboring node i with the connected weight being defined as $a_{i0}=a_{0i}=1$, otherwise $a_{i0}=a_{0i}=0$.

Before proceeding, we present the following mild assumptions.

Assumption 1: The undirected graph $\mathcal{G}$ is connected.

Assumption 2: There exists at least a node $i\,(i\in\mathcal{V})$ connected to the leader 0.

B. Optimization Problem

In SGs, the objective of ED is to balance the supply and demand with the minimum generation cost under practical constraints. Note that the power transmission loss is not considered in this paper. Specifically, the ED of SGs can be characterized by the following optimization problem:

$$\begin{aligned}\underset{\{P_i,i=1,\ldots,N;P^{MG}\}}{\arg\min}\ &\sum_{i=1}^{N}F_i(P_i)+\tau\lambda_0P^{MG}\\ \text{s.t.}\ &\sum_{i=1}^{N}P_i+\tau P^{MG}=\sum_{i=1}^{N}P_i^D=P^D\\ &\underline{P}_i\leq P_i\leq\overline{P}_i\end{aligned}\tag{1}$$

where P_i^D is the local load, P_i is the local active power generated by DG i, P^D is the total load demand, P^{MG} is the output power of the main grid, λ_0 is the electricity price of the main grid, and $\tau\in\{0,1\}$ represents two operation modes of microgrids. If $\tau=1$, the microgrid operates in the grid-connected mode, and in the islanded mode, otherwise. In addition, the generation cost $F_i(P_i)$ of agent i can be described by the following quadratic function:

$$F_i(P_i)=\frac{(P_i-\alpha_i)^2}{2\beta_i}+\iota_i\tag{2}$$

where $\alpha_i<0$, $\beta_i>0$, and ι_i are appropriate cost parameters. The physical interpretation of (2) can see e.g., [38] for more details.

The optimal solution to the above optimization problem is obtained by the following lemma.

Lemma 1: Denote the incremental cost of agent i as

$$\lambda_i=\frac{dF_i(P_i)}{dP_i}=\frac{P_i-\alpha_i}{\beta_i}.\tag{3}$$

The optimal solution to the ED problem (1) is

$$\begin{aligned}\lambda^\star&=\begin{cases}\dfrac{\sum_{i=1}^{N}P_i^D-\sum_{i\in\mathcal{V}_1}P_i^\star-\sum_{i\in\mathcal{V}\setminus\mathcal{V}_1}\alpha_i}{\sum_{i\in\mathcal{V}\setminus\mathcal{V}_1}\beta_i}, & \tau=0\\ \lambda_0, & \tau=1\end{cases}\\ P_i^\star&=\begin{cases}\underline{P}_i, & \lambda^\star\leq\underline{\lambda}_i,\ i\in\mathcal{V}_1\\ \beta_i\lambda^\star+\alpha_i, & \underline{\lambda}_i<\lambda^\star<\overline{\lambda}_i,\ i\in\mathcal{V}\setminus\mathcal{V}_1\\ \overline{P}_i, & \lambda^\star\geq\overline{\lambda}_i,\ i\in\mathcal{V}_1\end{cases}\\ P^{MG\star}&=\begin{cases}0, & \tau=0\\ \sum_{i=1}^{N}P_i^D-\sum_{i=1}^{N}P_i^\star, & \tau=1.\end{cases}\end{aligned}\tag{4}$$

Here, $\mathcal{V}_1\subset\mathcal{V}$ is the subset of the nodes where the DG reaches its maximum capacity.

Proof: The optimal solution can be derived by the classic Lagrange multiplier method. Similar proofs can be found in [9]–[12], and thus are omitted here. ■

C. Objective

Before clarifying our primary objective, two types of adversaries are considered in this paper: 1) *An internal honest-but-curious agent* who can follow the DED algorithm but is curious to learn the sensitive information of the neighboring agents via the received data; 2) *An external eavesdropper* who can wiretap communication links of the whole network and tries to estimate the power-sensitive information of each agent by using the accessibly exchanged data among agents.

For these two types of adversaries, the privacy of the DED problem is defined as follows.

Definition 1 [34], [39]: For a SG of N agents, the privacy of agent i is said to be preserved if an external eavesdropper and an internal honest-but-curious agent cannot estimate/infer the power-sensitive information with any accuracy.

The objective of this paper is listed as follows.

1) Propose an autonomous consensus-based scheme to achieve the supply-demand balance with minimum generation cost for the ED problem (1), i.e., the developed DED algorithm converges to the optimal solution (4);

2) Develop a state-decomposition-based privacy-preserving algorithm to prevent sensitive information from being inferred or estimated by internal honest-but-curious agents and external eavesdroppers with any accuracy.

Remark 1: In the energy management system of SGs, the initial value of the DED algorithm plays an important role, which involves the privacy-sensitive information including the local demand P_i^D, generation power $P_{i,0}$, and generation cost coefficients $\{\alpha_i,\beta_i\}$ [23]. It should be pointed out that this information reflects essential market demand and consumer behavior, even the safe and reliable operation of power grids. For instance, if the generation information of some power providers is leaked to others, the competitors may manipulate the power supply to pursue more profit. Another example is that if household demand information is stolen, the thief might break into consumer's house, leading to property loss or damage. As such, it is preferable to explore a privacy-preserving approach to prevent the power sensitive-information from being leaked.

III. Distributed Economic Dispatch Algorithm

In this section, an autonomous distributed optimization algorithm is first developed for the ED problem (1) of SGs. Then, the convergence and optimality of the proposed DED algorithm are analysed with respect to the grid-connected and islanded modes, respectively. In addition, the smooth transition between the islanded mode and grid-connected mode is also achieved.

The autonomous DED algorithm is given as follows:

$$\begin{aligned}
&\lambda_{i,k+1} = \lambda_{i,k} + c_1\Big(\sum_{j\in\mathcal{N}_i} a_{ij,k}(\lambda_{j,k}-\lambda_{i,k}) \\
&\qquad + \tau a_{i0}(\lambda_0-\lambda_{i,k})\Big) + \epsilon\eta_{i,k} \\
&P_{i,k} = \begin{cases} \underline{P}_i, & \lambda_{i,k} \le \dfrac{\underline{P}_i-\alpha_i}{\beta_i} \\ \beta_i\lambda_{i,k}+\alpha_i, & \dfrac{\underline{P}_i-\alpha_i}{\beta_i} < \lambda_{i,k} < \dfrac{\overline{P}_i-\alpha_i}{\beta_i} \\ \overline{P}_i, & \lambda_{i,k} \ge \dfrac{\overline{P}_i-\alpha_i}{\beta_i} \end{cases} \\
&s_{i,k+1} = \eta_{i,k} + c_2\sum_{j\in\mathcal{N}_i} l_{ij,k}(\eta_{j,k}-\eta_{i,k}) - (P_{i,k+1}-P_{i,k}) \\
&\delta P^M_{i,k+1} = \tau a_{0i}s_{i,k+1} \\
&P^M_{i,k+1} = \tau(P^M_{i,k} + a_{i0}\delta P^M_{i,k+1}) \\
&\eta_{i,k+1} = s_{i,k+1} + a_{i0}(P^M_{i,k} - P^M_{i,k+1}) \\
&P^{MG}_k = \sum_{i=1}^{N} P^M_{i,k}
\end{aligned} \tag{5}$$

where $\lambda_{i,k}$ is the incremental cost; $P_{i,k}$ is the active power of DG i; $\delta P^M_{i,k}$, $P^M_{i,k}$, and P^{MG}_k are, respectively, the incremental power, the local power, and the total power that are exchanged with the main grid at the time instant k; $s_{i,k+1}$, $\eta_{i,k+1}$ are the local estimated mismatch between the supply and demand before and after directed power replenishment by the main grid. Note that if the agent i is not connected with the main grid, then $s_{i,k+1} = \eta_{i,k+1}$, and the mismatch $s_{i,k+1}$ is replenished by main grid and $\eta_{i,k+1}$ is set as 0, otherwise.

Here, $\epsilon > 0$ is a known small gain parameter, whose upper bound $\bar{\epsilon}$ has been discussed in *Proposition 2* of [11]. $c_1, c_2 \in (0, \frac{\overline{\varsigma}}{\max\{d_1,d_2,\ldots,d_N\}+1})$ are the coupling constants. The weights can be set as $a_{ij,k}, l_{ij,k} \in (\underline{\varsigma}, \overline{\varsigma}]$. Furthermore, the initial value of algorithm (5) are given as follows:

$$\begin{cases} \lambda_{i,0} = \dfrac{P_{i,0}-\alpha_i}{\beta_i}, \quad \underline{P}_i < P_{i,0} < \overline{P}_i \\ \eta_{i,0} = P^D_i - P_{i,0} \\ P^{MG}_0 = 0. \end{cases} \tag{6}$$

Remark 2: The proposed DED algorithm integrates two operation modes of SGs, which can achieve smooth transition between the islanded mode and the grid-connected mode. From the structural point of view, the developed algorithm can be divided into two main parts. In the first part, each agent calculates the incremental cost via local data exchange. Note that if $\tau = 1$, the SG operates in the grid-connected mode, and the corresponding DED algorithm is transformed into the leader-following consensus algorithm; if $\tau = 0$, the SG operates in the islanded mode, and the algorithm is degraded into the leaderless consensus algorithm. Then, each agent calculates the local active power in light of the established relationship between the incremental cost and the active power, and estimates the local mismatch via the consensus algorithm. In the second part, each agent, who is connected to the ER, transmits its estimated mismatch to the ER, and subsequently set its mismatch as 0. Next, the ER calculates local power and total power that are exchanged with the main grid.

For brevity, denote

$$\lambda_k = \mathrm{col}_N\{\lambda_{i,k}\},\ \eta_k = \mathrm{col}_N\{\eta_{i,k}\},\ P_k = \mathrm{col}_N\{P_{i,k}\}$$
$$s_k = \mathrm{col}_N\{s_{i,k}\},\ \delta P^M_k = \mathrm{col}_N\{\delta P^M_{i,k}\},\ P^M_k = \mathrm{col}_N\{P^M_{i,k}\}.$$

Then, the DED algorithm (5) can be rewritten as the following compact form:

$$\begin{cases} \lambda_{k+1} = (I_N - c_1(L_{1,k}+\tau A))\lambda_k + c_1\tau\lambda_0 A\mathbf{1}_N + \epsilon\eta_k \\ s_{k+1} = (I_N - c_2L_{2,k})\eta_k - (P_{k+1}-P_k) \\ \delta P^M_{k+1} = \tau A s_{k+1} \\ P^M_{k+1} = \tau(P^M_k + A\delta P^M_{k+1}) \\ \eta_{k+1} = s_{k+1} + A(P^M_k - P^M_{k+1}) \\ P^{MG}_k = \mathbf{1}^T_N P^M_k \end{cases} \tag{7}$$

where $A = \mathrm{diag}_N\{a_{i0}\} = \mathrm{diag}_N\{a_{0i}\}$, $L_{1,k} = [a_{ij,k}]_N$, and $L_{2,k} = [l_{ij,k}]_N$.

In what follows, we are going to show that the proposed autonomous distributed algorithm (5) can achieve the optimal ED with respect to the grid-connected and islanded modes, respectively. Furthermore, the proposed algorithm also achieves smooth transitions between these two modes.

A. Grid-Connected Operation Mode

Theorem 1: Under *Assumptions 1* and *2*, if $\tau = 1$, then the distributed algorithm (5) can converge to

$$\lim_{k\to\infty}\lambda_{i,k} = \lambda_0,\quad \lim_{k\to\infty}P_{i,k} = P^\star_i,\quad \lim_{k\to\infty}\eta_{i,k} = 0$$
$$\lim_{k\to\infty}P^{MG}_k = \sum_{i=1}^{N}P^D_i - \sum_{i=1}^{N}P^\star_i,\quad i\in\mathcal{V}. \tag{8}$$

Proof: The proof can be divided into two parts (i.e., the DED algorithm without and with practical constraints). Let us first consider the scenario without constraints. The augmented system (7) can be further expressed by

$$\begin{cases}\lambda_{k+1} = (I_N - c_1(L_{1,k}+A))\lambda_k + c_1\lambda_0 A\mathbf{1}_N + \epsilon\eta_k \\ \eta_{k+1} = \vec{A}((I_N - \epsilon\beta - c_2L_{2,k})\eta_k \\ \qquad + c_1\beta(L_{1,k}+A)\lambda_k - c_1\beta\lambda_0 A\mathbf{1}_N)\end{cases} \tag{9}$$

where $\beta = \mathrm{diag}_N\{\beta_i\}$, $\vec{A} = I_N - A$.

Define the consensus error as $e_k = \lambda_k - \lambda_0\mathbf{1}_N$, one has

$$\begin{cases}e_{k+1} = (I_N - c_1(L_{1,k}+A))e_k + \epsilon\eta_k \\ \eta_{k+1} = \vec{A}((I_N - \epsilon\beta - c_2L_{2,k})\eta_k \\ \qquad + c_1\beta(L_{1,k}+A)e_k).\end{cases} \tag{10}$$

Denote $\bar{x}_k = [\begin{array}{cc} e_k^T & \eta_k^T \end{array}]^T$, one has

$$\bar{x}_{k+1} = T\bar{x} \tag{11}$$

where

$$T \triangleq T_1 + \epsilon T_2$$

with

$$T_1 \triangleq \begin{bmatrix} I_N - c_1(L_{1,k}+A) & 0_{N\times N} \\ c_1\vec{A}\beta(L_{1,k}+A) & \vec{A}(I_N - c_2L_{2,k})\end{bmatrix}$$

$$T_2 \triangleq \begin{bmatrix} 0_{N\times N} & I_N \\ 0_{N\times N} & -\vec{A}\beta\end{bmatrix}.$$

It is observed from (11) that 1) As a well-known nonsingular M-matrix [40] the matrix $L_{1,k}+A$ is a positive definition matrix (i.e., $L_{1,k}+A > 0$ at any time instant k); 2) In light of the classic Gershgorin disc theorem, one has $\rho(c_1(L_{1,k}+A)) < 2$, and thus $\rho(I_N - c_1(L_{1,k}+A)) < 1$ for $\forall k > 0$; 3) It is obtained from *Theorem 3.2* of [41] (or *Lemma 1* of [9]) that $\rho(\vec{A}(I_N - c_2L_{2,k})) < 1$ for $\forall k > 0$; 4) The matrix T_1 is a lower block triangular matrix, and thus one has $\rho(T_1) = \max\{\rho(I_N - c_1(L_{1,k}+A)), \rho(\vec{A}(I_N - c_2L_{2,k}))\} < 1$; and 5) The matrix T can be regarded as T_1 perturbed by ϵT_2. According to the eigenvalue perturbation theory [42], [43], eigenvalues continuously depend on each entry of a matrix. In this paper, the eigenvalues of matrix $T = T_1 + \epsilon T_2$ are affected by parameter ϵ continuously, and hence there exists an upper bound $\bar{\epsilon}$ such that $\rho(T) < 1$ for $\forall\epsilon \in (0, \bar{\epsilon})$. Note that the value of $\bar{\epsilon}$ has been obtained in *Proposition 2* of [11].

Based on the above observations, we can conclude that $\lim_{k\to\infty}\bar{x}_k = 0$ due to the fact that the eigenvalues of matrix T are all inside the unit disk. Furthermore, it is obtained that

$$\lim_{k\to\infty}\lambda_{i,k} = \lambda_0, \lim_{k\to\infty}\eta_{i,k} = 0, \lim_{k\to\infty}P_{i,k} = P_i^\star,\ i \in \mathcal{V}. \tag{12}$$

Note that $\mathbf{1}_N^T L_{2,k} = 0$, it follows from (7) that:

$$\begin{aligned}&\mathbf{1}_N^T(\eta_{k+1} + P_{k+1}) + P_{k+1}^{MG} = \mathbf{1}_N^T(\eta_{k+1} + P_{k+1} + AP_{k+1}^M) \\ &= \mathbf{1}_N^T(s_{k+1} - As_{k+1} + P_{k+1} + AP_k^M + As_{k+1}) \\ &= \mathbf{1}_N^T(\eta_k + P_k + AP_k^M) = \cdots = \mathbf{1}_N^T(\eta_0 + P_0 + AP_0^M) \\ &= \mathbf{1}_N^T(\eta_0 + P_0) + P_0^{MG} \\ &= \sum_{i=1}^N P_i^D.\end{aligned} \tag{13}$$

When $k \to \infty$, one has

$$\lim_{k\to\infty} P_k^{MG} = \sum_{i=1}^N P_i^D - \sum_{i=1}^N P_i^\star. \tag{14}$$

In what follows, we turn to investigate the general case that the physical constraints is considered. During algorithm evolution, some DGs may arrive at their maximum output power. After a sufficient period of time T, if $P_{i,k}$ is saturated, then $P_{i,k}$ is always saturated for $k \ge T+1$. To this end, the algorithm (7) can be transformed into the following composite system:

$$\begin{cases}\lambda_{k+1} = (I_N - c_1(L_{1,k}+A))\lambda_k + c_1\lambda_0 A\mathbf{1}_N + \epsilon\eta_k \\ \eta_{k+1} = \vec{A}((I_N - \epsilon\tilde{\beta} - c_2L_{2,k})\eta_k \\ \qquad + c_1\beta(L_{1,k}+A)\lambda_k - c_1\tilde{\beta}\lambda_0 A\mathbf{1}_N)\end{cases} \tag{15}$$

where

$$\tilde{\beta} \triangleq \mathrm{diag}_N\{\tilde{\beta}_i\}, \quad \tilde{\beta}_i \triangleq \begin{cases}0, & \text{if } P_{i,k} \text{ is saturated} \\ \beta_i, & \text{otherwise.}\end{cases}$$

Following the above analysis approach, we can derive that system (15) can converge to the optimal solution (8) with the help of the eigenvalue perturbation approach. Overall, the distributed algorithm (5) can converge to the global optimal solution (8) in the grid-connected mode. ■

B. Islanded Operation Mode

Theorem 2: Under *Assumptions 1* and *2*, if $\tau = 0$, then the distributed algorithm (5) can converge to

$$\lim_{k\to\infty}\lambda_{i,k} = \lambda^\star,\ \lim_{k\to\infty}\eta_{i,k} = 0,\ \lim_{k\to\infty}P_{i,k} = P_i^\star,\ i \in \mathcal{V}. \tag{16}$$

where $\lambda^\star$ and $P_i^\star$ are given in *Lemma 1*.

Proof: Note that when $\tau = 0$, the proposed algorithm (5) is degenerated into that of [12]. For corresponding convergence and optimality analysis, one can refer to *Theorems 2* and *3* of [12], and hence is skipped here. ■

C. Smooth Transition Between Two Modes

Theorem 3: Under *Assumptions 1* and *2*, the distributed algorithm (5) can achieve smooth transitions between the islanded mode and the grid-connected mode. In other words, the total estimated mismatch is always equal to the real power mismatch, i.e., $\sum_{i=1}^N \eta_{i,k} = \sum_{i=1}^N P_i^D - \sum_{i=1}^N P_{i,k} - \tau P_k^{MG}$ always holds for $\forall\tau \in \{0,1\}, \forall k \ge 0$.

Proof: Without loss of generality, we assume that the SG switches its operation from the grid-connected mode to the islanded mode at the time instant $k = T+1$.

When $0 \le k \le T$, the distributed algorithm (5) is carried out in the grid-connected mode, i.e., $\tau = 1$. It follows from (13) that:

$$\mathbf{1}_N^T(\eta_k + P_k) + P_k^{MG} = \sum_{i=1}^N P_i^D \tag{17}$$

which means that

$$\sum_{i=1}^N \eta_{i,k} = \sum_{i=1}^N P_i^D - \sum_{i=1}^N P_{i,k} - P_k^{MG}. \tag{18}$$

When $k > T$, the distributed algorithm (5) is carried out in the islanded mode, i.e., $\tau = 0$. It is observed from (5) that $\eta_{i,T+1}$ depends on $P^M_{i,T}$ where node i is connected to the main grid. To this end, we have the following discussions.

When $k = T+1$, it follows from (7) that:

$$\eta_{T+1} = (I_N - c_2 L_{2,k})\eta_T - (P_{T+1} - P_T) + AP^M_T. \tag{19}$$

Note that $P^{MG}_T = \mathbf{1}^T_N AP^M_T$ and $\mathbf{1}^T_N L_{2,k} = \mathbf{0}^T_N$, one has

$$\begin{aligned}\mathbf{1}^T_N \eta_{T+1} &= \mathbf{1}^T_N \eta_T - \mathbf{1}^T_N (P_{T+1} - P_T) + P^{MG}_T \\ &= \mathbf{1}^T_N \eta_T + \mathbf{1}^T_N P_T + P^{MG}_T - \mathbf{1}^T_N P_{T+1} \\ &= \sum_{i=1}^N P^D_i - \mathbf{1}^T_N P_{T+1}\end{aligned} \tag{20}$$

which means that

$$\sum_{i=1}^N \eta_{i,T+1} = \sum_{i=1}^N P^D_i - \sum_{i=1}^N P_{i,T+1}. \tag{21}$$

When $k > T+1$, the distributed algorithm is operated in the islanded mode, i.e., $\tau = 1$. Recall (7), one has

$$\begin{aligned}\mathbf{1}^T_N \eta_k + \mathbf{1}^T_N P_k &= \mathbf{1}^T_N \eta_{k-1} + \mathbf{1}^T_N P_{k-1} \\ &= \cdots \\ &= \mathbf{1}^T_N \eta_{T+1} + \mathbf{1}^T_N P_{T+1} \\ &= \sum_{i=1}^N P^D_i, k = T+2, T+3, \ldots\end{aligned} \tag{22}$$

By combining (18), (21), and (22), we can conclude that the distributed algorithm can achieve a seamless transition from the grid-connected mode to the islanded mode. Following a similar process, such a conclusion also holds when τ changes from 0 to 1 at any time instant k. ■

Remark 3: It should be pointed out that the developed distributed algorithm is scalable, which not only achieves smooth switching between islanded and grid-connected modes, but also has the plug-and-play adaptability. Compared with the existing result [9] with a diminishing step size, the proposed scheme with a constant step size can achieve fast convergence. To be more specific, our work can achieve linear convergence, while [9] asymptotically converges to the optimal solution. The second essential difference is the convergence analysis approach. The employed algorithm achieves the optimal ED by means of the matrix eigenvalue analysis, while [9] utilizes the inductive approach to obtain the main results.

IV. Privacy-Preserving Scheme

In this section, the privacy-preserving DED scheme is first proposed via a state decomposition approach, and then the privacy performance is analysed to verify the security against two types of of adversaries.

A. Privacy-Preserving DED Algorithm via State Decomposition

As shown in Fig. 1, the main idea of the proposed approach is to randomly decompose state variables $\lambda_{i,k}, P_{i,k}, \eta_{i,k}, P^D_i$ into two parts $\lambda^\mu_{i,k}, P^\mu_{i,k}, \eta^\mu_{i,k}, P^{D\mu}_i$ and $\lambda^\nu_{i,k}, P^\nu_{i,k}, \eta^\nu_{i,k}, P^{D\nu}_i$ with the initial supply and demand satisfying

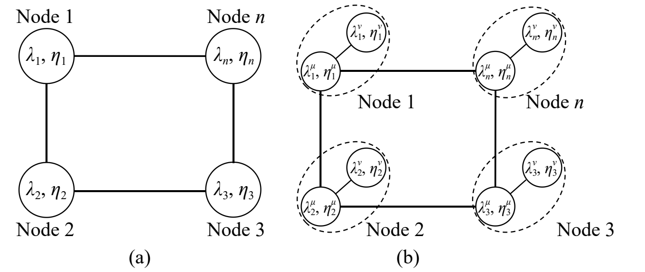

Fig. 1. Explanation of state decomposition: (a) Before decomposition; (b) After decomposition.

$$P^\mu_{i,0} + P^\nu_{i,0} = 2P_{i,0}, P^{D\mu}_i + P^{D\nu}_i = 2P^D_i, \ (i \in \mathcal{V}). \tag{23}$$

The substates $\lambda^\mu_{i,k}, \eta^\mu_{i,k}$ can exchange data with their neighboring nodes $\lambda^\mu_{j,k}, \eta^\mu_{j,k}$ ($j \in \mathcal{N}_i$), while the substates $\lambda^\nu_{i,k}, \eta^\nu_{i,k}$ only interact with $\lambda^\mu_{i,k}, \eta^\mu_{i,k}$. Note that the substates $\lambda^\mu_{i,k}, \eta^\mu_{i,k}$ can be observed by neighboring nodes, while the substates $\lambda^\nu_{i,k}, \eta^\nu_{i,k}$ are invisible to neighboring nodes. In addition, the construction of coupling weights between the substate $\lambda^\mu_{i,k}(\eta^\mu_{i,k})$ and the substate $\lambda^\nu_{i,k}(\eta^\nu_{i,k})$ is similar to that of weights between nodes.

Based on the above descriptions, the state-decomposition-based privacy-preserving algorithm can be designed as follows:

$$\begin{aligned}
\lambda^\mu_{i,k+1} &= \lambda^\mu_{i,k} + c_1\Big(\sum_{i=1}^N a_{ij,k}(\lambda^\mu_{j,k} - \lambda^\mu_{i,k}) \\
&\quad + a^i_{\mu\nu,k}(\lambda^\nu_{i,k} - \lambda^\mu_{i,k}) + \tau a_{i0}(\lambda_0 - \lambda^\mu_{i,k})\Big) + \epsilon \eta^\mu_{i,k} \\
\lambda^\nu_{i,k+1} &= \lambda^\nu_{i,k} + c_1 a^i_{\nu\mu,k}(\lambda^\mu_{i,k} - \lambda^\nu_{i,k}) + \epsilon \eta^\nu_{i,k} \\
P^\mu_{i,k} &= \begin{cases} \underline{P}_i, & \beta_i \lambda^\mu_{i,k} + \alpha_i \le \underline{P}_i \\ \beta_i \lambda^\mu_{i,k} + \alpha_i, & \underline{P}_i < \beta_i \lambda^\mu_{i,k} + \alpha_i < \overline{P}_i \\ \overline{P}_i, & \beta_i \lambda^\mu_{i,k} + \alpha_i \ge \overline{P}_i \end{cases} \\
P^\nu_{i,k} &= \begin{cases} \underline{P}_i, & \beta_i \lambda^\nu_{i,k} + \alpha_i \le \underline{P}_i \\ \beta_i \lambda^\nu_{i,k} + \alpha_i, & \underline{P}_i < \beta_i \lambda^\nu_{i,k} + \alpha_i < \overline{P}_i \\ \overline{P}_i, & \beta_i \lambda^\nu_{i,k} + \alpha_i \ge \overline{P}_i \end{cases} \\
s^\mu_{i,k+1} &= \eta^\mu_{i,k} + c_2\Big(\sum_{i=1}^N l_{ij,k}(\eta^\mu_{j,k} - \eta^\mu_{i,k}) \\
&\quad + l^i_{\mu\nu,k}(\eta^\nu_{i,k} - \eta^\mu_{i,k})\Big) - (P^\mu_{i,k+1} - P^\mu_{i,k}) \\
\eta^\nu_{i,k+1} &= \eta^\nu_{i,k} + c_2 l^i_{\nu\mu,k}(\eta^\mu_{i,k} - \eta^\nu_{i,k}) - (P^\nu_{i,k+1} - P^\nu_{i,k}) \\
\delta P^M_{i,k+1} &= a_{0i}\tau s^\mu_{i,k+1} \\
P^M_{i,k+1} &= \tau(P^M_{i,k} + a_{i0}\delta P^M_{i,k+1}) \\
\eta^\mu_{i,k+1} &= s^\mu_{i,k+1} + a_{i0}(P^M_{i,k} - P^M_{i,k+1}) \\
P^{MG}_k &= \frac{1}{2}\sum_{i=1}^N P^M_{i,k}
\end{aligned} \tag{24}$$

where the coupling weights $a^i_{\mu\nu,k}, l^i_{\mu\nu,k} \in (\underline{\varsigma}, \overline{\varsigma}]$.

The privacy-preserving DED algortihm via state decomposition is outlined in Algorithm 1.

Algorithm 1 Privacy-Preserving DED Algorithm

▸ **Initialization**:

1) Set the initial value $P_{i,0} \in [\underline{P}_i, \overline{P}_i]$, P_i^D, $P_0^{MG} = 0$, scalars c_1, c_2, ϵ; randomly generate $P_{i,0}^{\mu} \in [\underline{P}_i, \overline{P}_i]$, $P_{i,0}^{\nu} \in [\underline{P}_i, \overline{P}_i]$, $P_i^{D\mu} > 0$, $P_i^{D\nu} > 0$ such that (23) is satisfied;

2) Calculate $\lambda_{i,0}^{\mu} = \frac{P_{i,0}^{\mu} - \alpha_i}{\beta_i}$, $\lambda_{i,0}^{\nu} = \frac{P_{i,0}^{\nu} - \alpha_i}{\beta_i}$, $\eta_{i,0}^{\mu} = P_i^{D\mu} - P_{i,0}^{\mu}$, and $\eta_{i,0}^{\nu} = P_i^{D\nu} - P_{i,0}^{\nu}$.

▸ **Loop**:

1) Each agent i transmits $\lambda_{i,k}^{\mu}$, $\eta_{i,k}^{\mu}$ to its neighboring nodes and receives $\lambda_{j,k}^{\mu}$, $\eta_{j,k}^{\mu}$ $(j \in \mathcal{N}_i)$ from its neighboring nodes;

2) Each agent i updates $\lambda_{i,k+1}^{\mu}$, $\lambda_{i,k+1}^{\nu}$, $\eta_{i,k+1}^{\mu}$, $\eta_{i,k+1}^{\nu}$, $P_{i,k+1}^{\mu}$, and $P_{i,k+1}^{\nu}$ via (24);

3) If $|\lambda_{i,k+1}^{\mu} - \lambda_{i,k}^{\mu}| < \chi_1$, $|\lambda_{i,k+1}^{\nu} - \lambda_{i,k}^{\nu}| < \chi_1$, $|\eta_{i,k+1}^{\mu}| < \chi_2$, and $|\eta_{i,k+1}^{\nu}| < \chi_2$, $\forall i \in \mathcal{V}$ where χ_1, χ_2 are the error tolerance, break.

▸ **Output**:

1) $P_{i,k} = P_{i,k}^{\mu} = P_{i,k}^{\nu}$.

Theorem 4: Under *Assumptions 1* and *2*, the state-decomposition-based DED algorithm (24) can converge to

$$\lim_{k\to\infty} \lambda_{i,k}^{\mu} = \lim_{k\to\infty} \lambda_{i,k}^{\nu} = \lambda^{\star},\ \lim_{k\to\infty} P_{i,k}^{\mu} = \lim_{k\to\infty} P_{i,k}^{\nu} = P_i^{\star}$$
$$\lim_{k\to\infty} \eta_{i,k}^{\mu} = \lim_{k\to\infty} \eta_{i,k}^{\nu} = 0,\ \lim_{k\to\infty} P_k^{MG} = P^{MG\star},\ i \in \mathcal{V}. \tag{25}$$

Proof: It should be pointed out the number of nodes (variables) is doubled whereas the connected condition of communication network remains unchanged via the state-decomposition approach (i.e., Assumption 1 is satisfied). Furthermore, the variables $\lambda_{i,k}, \lambda_{i,k}^{\mu}, \lambda_{i,k}^{\nu}$ and $P_{i,k}, P_{i,k}^{\mu}, P_{i,k}^{\nu}$ are homogeneous under the same constraints. In light of the constructed initial conditions (23) and the structure of optimal solution (4), the distributed algorithm (24) can also converge to the optimal solution (4) of the ED problem (1). The proof is similar to that of *Theorems 1−3*, and thus is omitted here. ■

B. Analysis of Privacy

In this subsection, we are in position to show that the proposed algorithm (24) can prevent sensitive information leakage from internal honest-but-curious nodes and external eavesdroppers.

1) Privacy Preservation Against Honest-But-Curious Agents

The information set accessible to the honest-but-curious node $i \in \mathcal{V}$ at time instant k is defined as

$$\Upsilon_{i,k} = \{a_{ij,k}, l_{ij,k}, a_{\mu\nu,k}^{i}, l_{\mu\nu,k}^{i}, \lambda_{i,k}, \lambda_{i,k}^{\mu}, \lambda_{i,k}^{\nu}, \eta_{i,k}, \eta_{i,k}^{\mu}, \eta_{i,k}^{\nu}, \lambda_{j,k}^{\mu}, \eta_{j,k}^{\mu} | j \in \mathcal{N}_i\}.$$

Furthermore, the cumulated information set is defined as $\Upsilon_i = \cup_{k=0}^{\infty} \Upsilon_{i,k}$.

Theorem 5: Consider the state-decomposition-based DED algorithm (24) under *Assumptions 1* and *2*, an honest-but-curious node i $(i \in \mathcal{V})$ cannot evaluate its neighboring power supply $P_{j,0}$ and demand P_j^D $(j \in \mathcal{N}_i)$ by using the available information set Υ_i if node j has at least a legitimate neighboring node m.

Proof: To prove that the sensitive information P_j^D and $P_{j,0}$ cannot be estimated by the honest-but-curious node i with any accuracy, we are going to show that the information set Υ_i remains unchanged under different initial values (i.e., $\bar{P}_j^D \neq P_j^D$ and $\bar{P}_{j,0} \neq P_{j,0}$). It should be stressed that Υ_i is the only information set available for the honest-but-curious node i to infer sensitive information, and if $\bar{\Upsilon}_i$ is the same as Υ_i under conditions $\bar{P}_j^D \neq P_j^D$ and $\bar{P}_{j,0} \neq P_{j,0}$, then the node i cannot infer initial values P_j^D and $P_{j,0}$ via Υ_i. As a result, it suffices to prove that $\bar{\Upsilon}_i = \Upsilon_i$ holds for $\bar{P}_j^D \neq P_j^D$ and $\bar{P}_{j,0} \neq P_{j,0}$, where $\bar{\Upsilon}_i$ is the available information set under initial values $\bar{P}_j^D$ and $\bar{P}_{j,0}$.

To show the privacy-preserving performance against internal honest-but-curious nodes, combined with (23), the possible initial conditions can be constructed as follows:

$$\begin{aligned}
&\bar{P}_{j,0} \neq P_{j,0},\ \bar{P}_{m,0} = P_{j,0} + P_{m,0} - \bar{P}_{j,0},\ \bar{P}_{j,0}^{\mu} = P_{j,0}^{\mu}\\
&\bar{P}_{j,0}^{\nu} = 2\bar{P}_{j,0} - \bar{P}_{j,0}^{\mu},\ \bar{P}_m^{\mu} = P_m^{\mu},\ \bar{P}_m^{\nu} = 2\bar{P}_m - \bar{P}_m^{\mu}\\
&\bar{P}_j^D \neq P_j^D,\ \bar{P}_m^D = P_j^D + P_m^D - \bar{P}_j^D,\ \bar{P}_j^{D^{\mu}} = P_j^{D^{\mu}}\\
&\bar{P}_j^{D^{\nu}} = 2\bar{P}_j^D - \bar{P}_j^{D^{\mu}},\ \bar{P}_m^{D^{\mu}} = P_m^{D^{\mu}},\ \bar{P}_m^{D^{\nu}} = 2\bar{P}_m^D - \bar{P}_m^{D^{\mu}}\\
&\bar{P}_{l,0} = P_{l,0},\ \bar{P}_l^D = P_l^D,\ \bar{P}_{l,0}^{\mu} = P_{l,0}^{\mu},\ \bar{P}_{l,0}^{\nu} = P_{l,0}^{\nu}\\
&\bar{P}_l^{D^{\mu}} = P_l^{D^{\mu}},\ \bar{P}_l^{D^{\nu}} = P_l^{D^{\nu}},\quad \forall l \in \mathcal{V}\backslash\{j,m\}.
\end{aligned} \tag{26}$$

Furthermore, it follows from (6) that:

$$\begin{aligned}
&\bar{\lambda}_{m,0} = (\beta_j\lambda_{j,0} + \beta_m\lambda_{m,0} - \beta_j\bar{\lambda}_{j,0})/\beta_m,\ \bar{\lambda}_{j,0}^{\mu} = \lambda_{j,0}^{\mu}\\
&\bar{\lambda}_{m,0}^{\mu} = \lambda_{m,0}^{\mu},\ \bar{\lambda}_{j,0}^{\nu} = 2\bar{\lambda}_{j,0} - \bar{\lambda}_{j,0}^{\mu},\ \bar{\lambda}_{m,0}^{\nu} = 2\bar{\lambda}_{m,0} - \bar{\lambda}_{m,0}^{\mu}\\
&\bar{\eta}_{m,0} = \eta_{j,0} + \eta_{m,0} - \bar{\eta}_{j,0},\ \bar{\eta}_{j,0}^{\mu} = \eta_{j,0}^{\mu},\ \bar{\eta}_{m,0}^{\mu} = \eta_{m,0}^{\mu}\\
&\bar{\eta}_{j,0}^{\nu} = 2\bar{\eta}_{j,0} - \bar{\eta}_{j,0}^{\mu},\ \bar{\eta}_{m,0}^{\nu} = 2\bar{\eta}_{m,0} - \bar{\eta}_{m,0}^{\mu}\\
&\bar{\lambda}_{l,0} = \lambda_{l,0},\ \bar{\eta}_{l,0} = \eta_{l,0},\ \bar{\lambda}_{l,0}^{\mu} = \lambda_{l,0}^{\mu},\ \bar{\lambda}_{l,0}^{\nu} = \lambda_{l,0}^{\nu}\\
&\bar{\eta}_{l,0}^{\mu} = \eta_{l,0}^{\mu},\ \bar{\eta}_{l,0}^{\nu} = \eta_{l,0}^{\nu},\ \ l \in \mathcal{V}\backslash\{j,m\}.
\end{aligned} \tag{27}$$

To ensure the information set Υ_i is unchanged, we let

$$\begin{aligned}
&\bar{\lambda}_{j,1}^{\mu} = \lambda_{j,k}^{\mu},\ \bar{\lambda}_{j,1}^{\nu} = \lambda_{j,1}^{\nu},\ \bar{\lambda}_{m,1}^{\mu} = \lambda_{m,k}^{\mu}\\
&\bar{\lambda}_{m,1}^{\nu} = \lambda_{m,1}^{\nu},\ \bar{a}_{ij,0} = a_{ij,0},\ \bar{a}_{im,0} = a_{im,0}.
\end{aligned} \tag{28}$$

For the algorithm (24) under the conditions (27) and (28), the coupling weights can be calculated as

$$\bar{a}_{jm,0} = \frac{c_1 a_{jm,0}(\lambda_{m,0}^{\mu} - \lambda_{j,0}^{\mu}) + \lambda_{j,0}^{\nu} - \bar{\lambda}_{j,0}^{\nu} + \epsilon(\eta_{j,0}^{\nu} - \bar{\eta}_{j,0}^{\nu})}{c_1(\bar{\lambda}_{m,0}^{\mu} - \bar{\lambda}_{j,0}^{\mu})}$$

$$\bar{a}_{mj,0} = \frac{c_1 a_{mj,0}(\lambda_{j,0}^{\mu} - \lambda_{m,0}^{\mu}) + \lambda_{m,0}^{\nu} - \bar{\lambda}_{m,0}^{\nu} + \epsilon(\eta_{m,0}^{\nu} - \bar{\eta}_{m,0}^{\nu})}{c_1(\bar{\lambda}_{j,0}^{\mu} - \bar{\lambda}_{m,0}^{\mu})}$$

$$\bar{l}_{jm,0} = \frac{c_2 l_{jm,0}(\eta_{m,0}^{\mu} - \eta_{j,0}^{\mu}) + \eta_{j,0}^{\nu} - \bar{\eta}_{j,0}^{\nu} + P_{j,0}^{\nu} - \bar{P}_{j,0}^{\nu}}{c_2(\bar{\eta}_{m,0}^{\mu} - \bar{\eta}_{j,0}^{\mu})}$$

$$\bar{l}_{mj,0} = \frac{c_2 l_{mj,0}(\eta_{j,0}^{\mu} - \eta_{m,0}^{\mu}) + \eta_{m,0}^{\nu} - \bar{\eta}_{m,0}^{\nu} + P_{m,0}^{\nu} - \bar{P}_{m,0}^{\nu}}{c_2(\bar{\eta}_{j,0}^{\mu} - \bar{\eta}_{m,0}^{\mu})}$$

$$\bar{a}^s_{\mu\nu,0}=\frac{\lambda^\nu_{s,0}-\bar{\lambda}^\nu_{s,0}+c_1a^s_{\mu\nu,0}(\lambda^\mu_{s,0}-\lambda^\nu_{s,0})+\epsilon(\eta^\nu_{s,0}-\bar{\eta}^\nu_{s,0})}{c_1(\bar{\lambda}^\mu_{s,0}-\bar{\lambda}^\nu_{s,0})}$$
$$\bar{l}^s_{\mu\nu,0}=\frac{\eta^\nu_{s,0}-\bar{\eta}^\nu_{s,0}+c_2l^s_{\mu\nu,0}(\eta^\mu_{s,0}-\eta^\nu_{s,0})+P^\nu_{s,0}-\bar{P}^\nu_{s,0}}{c_2(\bar{\eta}^\mu_{s,0}-\bar{\eta}^\nu_{s,0})} \tag{29}$$

where $s\in\{j,m\}$, and other coupling weights stay unchanged for any time instant k.

Note that for $k\geq 1$, the privacy-preserving DED algorithm carries out the same state update due to the same condition at time instant $k=1$. To this end, the distributed algorithm (24) under conditions (26)–(29) can converge to the optimal solution (4), and $\bar{\Upsilon}_i=\Upsilon_i$ holds for $\bar{P}^D_i\neq P^D_i$ and $\bar{P}_{i,0}\neq P_{i,0}$, which means that the proposed algorithm can prevent privacy disclosure from honest-but-curious nodes if node j has at least a legitimate neighboring node. ■

Remark 4: In this paper, the legitimate neighbor is defined as a neighboring node who follows the DED algorithm faithfully without attempting to infer other nodes' states. In this case, all agents can be divided into legitimate nodes and honest-but-curious nodes. Actually, this is a safe connection (i.e., node j has at least a legitimate neighboring node), which can increase the difficulty in estimating the power-sensitive information for honest-but-curious nodes. More specifically, compared with the case that the agent j has no legitimate neighboring node, it follows from (24) that the node j has two additional terms $a_{jm,k}(\lambda^\mu_m-\lambda^\mu_j)$ and $l_{jm,k}(\eta^\mu_m-\eta^\mu_j)$ unknown to honest-but-curious nodes, which effectively reduces the risk of privacy leakage. So far, such a connection configuration has been widely adopted in [31], [34], [39].

2) Privacy Preservation Against External Eavesdroppers

The information set accessible to an external eavesdropper is defined as

$$\Theta=\{\mathcal{G},\lambda^\mu_{i,k},\eta^\mu_{i,k}|i\in\mathcal{V},k\geq 0\}.$$

Theorem 6: Consider the state-decomposition-based DED algorithm (24) under *Assumptions 1* and *2*, an external eavesdropper cannot evaluate local power supply $P_{i,0}$ and demand P^D_i for $\forall i\in\mathcal{V}$ by using the available information set Θ.

Proof: Following the similar line in *Theorem 5*, we only to show that $\bar{\Theta}=\Theta$ holds for $\bar{P}^D_i\neq P^D_i$ and $\bar{P}_{i,0}\neq P_{i,0}$, $\forall i\in\mathcal{V}$.

Specifically, the initial conditions are set as

$$\bar{\lambda}^\mu_{i,0}=\lambda^\mu_{i,0},\ \bar{\eta}^\mu_{i,0}=\eta^\mu_{i,0}$$
$$\bar{\lambda}^\nu_{i,0}=\lambda^\nu_{i,0}-\frac{\kappa_1c_1}{1-\epsilon\beta_i}\sum_{j\in N_i}a_{ij,0}(\lambda^\mu_{j,0}-\lambda^\mu_{i,0})+\frac{\kappa_2c_2\epsilon}{1-\epsilon\beta_i}\sum_{j\in N_i}l_{ij,0}(\eta^\mu_{j,0}-\eta^\mu_{i,0})$$
$$\bar{\eta}^\nu_{i,0}=\eta^\nu_{i,0}+\frac{\kappa_1c_1\beta_i}{1-\epsilon\beta_i}\sum_{j\in N_i}a_{ij,0}(\lambda^\mu_{j,0}-\lambda^\mu_{i,0})-\frac{\kappa_2c_2}{1-\epsilon\beta_i}\sum_{j\in N_i}l_{ij,0}(\eta^\mu_{j,0}-\eta^\mu_{i,0})$$
$$\bar{P}^\mu_{i,0}=P^\mu_{i,0},\ \bar{P}^\nu_{i,0}=\beta_i\bar{\lambda}^\nu_{i,0}+\alpha_i,\ \bar{P}_{i,0}=\frac{1}{2}(\bar{P}^\mu_{i,0}+\bar{P}^\nu_{i,0})$$
$$\bar{P}^{D^\mu}_i=P^{D^\mu}_i,\bar{P}^{D^\nu}_i=\bar{\eta}^\nu_{i,0}+\bar{P}^\nu_{i,0},\bar{P}^D_i=\frac{1}{2}(\bar{P}^{D^\mu}_i+\bar{P}^{D^\nu}_i)$$
$$\bar{a}_{ij,0}=(1+\kappa_1)a_{ij,0},\ \bar{l}_{ij,0}=(1+\kappa_2)l_{ij,0}$$
$$\bar{a}^i_{\mu\nu,0}=\frac{a^i_{\mu\nu,0}(\lambda^\nu_{i,0}-\lambda^\mu_{i,0})-\kappa_1\sum_{i=1}^N a_{ij,0}(\lambda^\mu_{j,0}-\lambda^\mu_{i,0})}{(\bar{\lambda}^\nu_{i,0}-\bar{\lambda}^\mu_{i,0})}$$
$$\bar{l}^i_{\mu\nu,0}=\frac{l^i_{\mu\nu,0}(\eta^\nu_{i,0}-\eta^\mu_{i,0})-\kappa_2\sum_{i=1}^N l_{ij,0}(\eta^\mu_{j,0}-\eta^\mu_{i,0})}{(\bar{\eta}^\nu_{i,0}-\bar{\eta}^\mu_{i,0})}$$
$$\bar{a}_{ij,k}=a_{ij,k},\ \bar{l}_{ij,k}=l_{ij,k}$$
$$\bar{a}^i_{\mu\nu,k}=a^i_{\mu\nu,k},\ \bar{l}^i_{\mu\nu,k}=l^i_{\mu\nu,k},\ \forall i,j\in\mathcal{V},\ k=1,2,\ldots \tag{30}$$

where $\kappa_1,\ \kappa_2\in\mathbb{R}\backslash\{0\}$.

It is obvious that the distributed algorithm (24) under conditions (30) can converge to optimal solution (4), and $\bar{\Theta}=\Theta$ holds for $\bar{P}^D_i\neq P^D_i$ and $\bar{P}_{i,0}\neq P_{i,0}$, $\forall i\in\mathcal{V}$, which implies that the proposed algorithm can prevent privacy disclosure from external eavesdroppers. ■

Remark 5: Actually, the main idea of privacy analysis is the same against two kinds of adversaries. However, in light of the definition of two kinds of adversaries, there exists an essential difference in the available information set. In this case, we need to search different-but-feasible weights and substates such that two kinds of adversaries are indistinguishable for different initial values in privacy analysis. In smart grids, the honest-but-curious agent and the external eavesdropper can be regarded as a competitor and an attacker, respectively. The competitor may unfairly strike rivals and disrupt market order for more interests by stealing the opponent's power-sensitive information. The attacker wants to know the power grid operation law, and further inject a stealthy attack signal to destroy the stability and reliability of the power grid, resulting in serious economic losses and security incidents.

Remark 6: In this paper, we have developed a privacy-preserving DED algorithm via state decomposition. In comparison with existing privacy-preserving schemes, our algorithm presents the following three advantages. Compared with the differentially private approach [25], our approach can achieve accurate convergence owing to special algorithm construction. In contrast to [28], our algorithm can achieve a higher privacy level where sensitive information cannot be eavesdropped by adversaries with any accuracy. Different from the homomorphically encrypted privacy-preserving scheme [30], our scheme has lower computation complexity in the sense that the proposed algorithm only operates simple multiplication and addition operations.

Remark 7: Up to now, a state-decomposition-based consensus algorithm has been developed to achieve the privacy-preserving optimal DED of SGs with respect to the islanded and grid-connected modes. Moreover, smooth mode transitions can be guaranteed. Compared with the existing literature, the distinctive merits of this work are listed as follows: 1) The proposed DED algorithm is novel in covering the islanded and grid-connected modes of SGs by virtue of the leaderless and leader-following consensus algorithms of multi-agent systems; 2) The established analysis framework is comprehensive, which includes the convergence and optimality analysis of the proposed DED algorithm with and without state decomposition as well as the privacy analysis against internal and external eavesdroppers; and 3) The proposed state-decomposition-based DED algorithm exhibits better performance in terms of accurate convergence, uncompromised privacy-preserving performance, and low computational complexity.

V. Simulation Study

In this section, we provide a simulation example to verify the theoretical results of this work. The considered SGs consist of five DGs, five loads and one ER, whose physical structure and communication topology are given in Fig. 2. The parameters of each DG are listed in Table I. The coupling scalars are selected as $c_1 = c_2 = 0.2$, the gain parameter is set as $\epsilon = 0.03$, the electricity price of the main grid is set as $\lambda_0 = 6$ \$/KWh. The local load P_i^D $(i \in \mathcal{V})$ is chosen as follows: $P_1^D = 10$ KW, $P_2^D = 30$ KW, $P_3^D = 25$ KW, $P_4^D = 35$ KW, $P_5^D = 20$ KW, and the total demand is $P^D = 120$ KW.

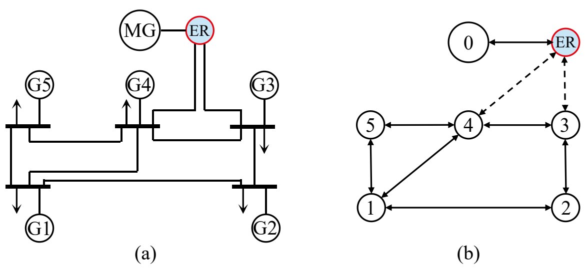

Fig. 2. (a) Test system of SGs; (b) Communication topology.

TABLE I
GENERATION PARAMETERS [11]

DGi	α_i	β_i	ι_i	$\underline{P}_i$	$\overline{P}_i$
1	−21.42	6.76	28.05	4.2	18
2	−24.51	6.10	−7.27	5.4	45
3	−12.05	4.76	62.76	3.8	40
4	−6.49	5.32	47.04	10	80
5	−21.86	6.41	−6.27	8	60

A. Case 1: Convergence and Optimality of the DED Algorithm

We first verify the convergence and optimality of the proposed DED algorithm. Set the initial active power $P_{i,0}$ $(i \in \mathcal{V})$ as $P_{1,0} = 7$ KW, $P_{2,0} = 13$ KW, $P_{3,0} = 12$ KW, $P_{4,0} = 20$ KW, and $P_{5,0} = 12$ KW. In light of the established results in *Lemma 1*, if the SGs operate in the islanded mode, then one has the optimal incremental cost as $\lambda^\star = 7.39$ \$/KWh, the optimal local active power as $P_1^\star = 18$ KW, $P_2^\star = 20.54$ KW, $P_3^\star = 23.14$ KW, $P_4^\star = 32.81$ KW, and $P_5^\star = 25.51$ KW. If the SGs operate in the grid-connected mode, then one has the optimal incremental cost as $\lambda^\star = 6$ \$/KWh, the optimal local active power as $P_1^\star = 18$ KW, $P_2^\star = 12.07$ KW, $P_3^\star = 16.52$ KW, $P_4^\star = 25.43$ KW, and $P_5^\star = 16.60$ KW, respectively. The output power of the main grid is $P^{MG\star} = 31.37$ KW.

Assume that the ER switches the mode τ from 0 to 1 (i.e., the operation mode is switched from the islanded mode to the grid-connected mode) at time instant $k = 200$, and switches the mode τ from 1 to 0 at time instant $k = 600$. The simulation results are presented in Fig. 3. It is observed from Fig. 3 that the incremental cost $\lambda_{i,k}$ $(i \in \mathcal{V})$ converges to the optimal value $\lambda^\star$ for both the islanded mode and the grid-connected mode, the local mismatches between the supply and demand approach 0, the local active power $P_{i,k}$ $(i \in \mathcal{V})$ converges to the optimal value $P_i^\star$, and the supply-demand balance of the whole network is achieved. Note that at the time instants $k = 200$ and $k = 600$, the operation mode of the SG is changed. It can be seen that all variables converge to the new optimal values, which shows that the proposed DED algorithm can achieve smooth transitions between the islanded mode and the grid-connected mode.

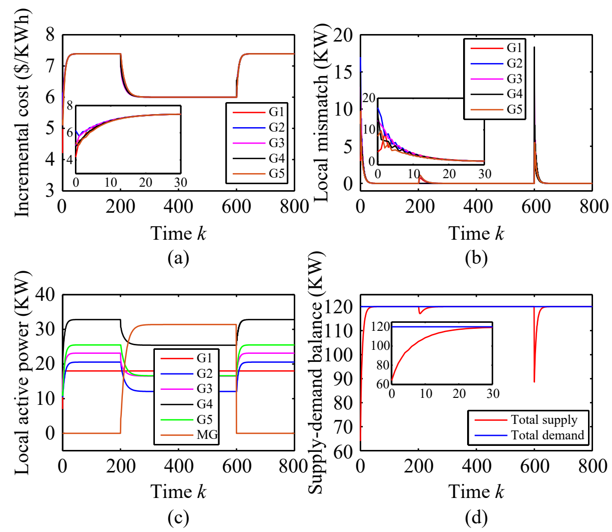

Fig. 3. Test results of the proposed DED algorithm without state decomposition.

B. Case 2: Convergence and Optimality of the Privacy-Preserving DED Algorithm via State Decomposition

In this case, we carry out the state-decomposition-based privacy-preserving DED algorithm (24). In light of the construction of initial conditions in Section IV-A, the local demand is set as $P_1^{D\mu} = 12$ KW, $P_1^{D\nu} = 8$ KW, $P_2^{D\mu} = 34$ KW, $P_2^{D\nu} = 26$ KW, $P_3^{D\mu} = 24$ KW, $P_3^{D\nu} = 26$ KW, $P_4^{D\mu} = 37$ KW, $P_4^{D\nu} = 33$ KW, $P_5^{D\mu} = 15$ KW, and $P_5^{D\nu} = 25$ KW, respectively. The initial active power is selected as $P_{1,0}^\mu = 6$ KW, $P_{1,0}^\nu = 8$ KW, $P_{2,0}^\mu = 10$ KW, $P_{2,0}^\nu = 16$ KW, $P_{3,0}^\mu = 15$ KW, $P_{3,0}^\nu = 9$ KW, $P_{4,0}^\mu = 17$ KW, $P_{4,0}^\nu = 23$ KW, $P_{5,0}^\mu = 14$ KW, and $P_{5,0}^\nu = 10$ KW, respectively. Similar to above case, the SG switches the operation mode from the islanded mode to the grid-connected mode at the time instant $k = 200$, and from the grid-connected mode to the islanded mode at the time instant $k = 600$. As shown in Fig. 4, all state variables converge to corresponding optimal values, where the solid lines refer to the trajectories of the substates $\lambda_{i,k}^\mu$, $\eta_{i,k}^\mu$, and $P_{i,k}^\mu$, respectively, while the dotted lines are the trajectories of the substates $\lambda_{i,k}^\nu$, $\eta_{i,k}^\nu$, and $P_{i,k}^\nu$, respectively.

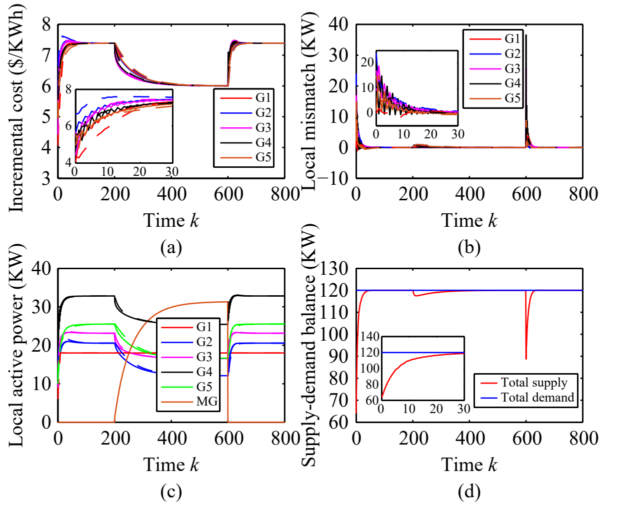

Fig. 4. Test results of the proposed DED algorithm with state decomposition.

C. Case 3: Privacy Preservation against Internal Honest-But-Curious Agents

In what follows, we turn to validate the privacy-preserving performance of the developed DED algorithm. Assume that node 5 is the honest-but-curious agent who can try to infer the sensitive information of node 4, and the node 1 is a legitimate neighbor of node 4. In light of the established results in *Theorem 5*, we set

$$\bar{P}_{4,0} = P_{4,0} - 4.2\text{ KW} = 15.8\text{ KW},\ \bar{P}_{1,0} = 11.2\text{ KW}$$
$$\bar{P}^{\mu}_{4,0} = P^{\mu}_{4,0} = 17\text{ KW},\ \bar{P}^{\nu}_{4,0} = 2\bar{P}_{4,0} - \bar{P}^{\mu}_{4,0} = 14.6\text{ KW}$$
$$\bar{P}^{\mu}_{1,0} = P^{\mu}_{1,0} = 6\text{ KW},\ \bar{P}^{\nu}_{1,0} = 2\bar{P}_{1,0} - \bar{P}^{\mu}_{1,0} = 16.4\text{ KW}$$
$$\bar{P}^{D}_{4} = P^{D}_{4} - 2.3\text{ KW} = 32.7\text{ KW},\ \bar{P}^{D}_{1} = 12.3\text{ KW}$$
$$\bar{P}^{D^{\mu}}_{4} = P^{D^{\mu}}_{4} = 37\text{ KW},\ \bar{P}^{D^{\nu}}_{4} = 2\bar{P}^{D}_{4} - \bar{P}^{D^{\mu}}_{4} = 28.4\text{ KW}$$
$$\bar{P}^{D^{\mu}}_{1} = P^{D^{\mu}}_{1} = 12\text{ KW},\ \bar{P}^{D^{\nu}}_{1} = 2\bar{P}^{D}_{1} - \bar{P}^{D^{\mu}}_{1} = 12.6\text{ KW}$$
$$\bar{P}_{l,0} = P_{l,0},\ \bar{P}^{D}_{l} = P^{D}_{l},\ \bar{P}^{\mu}_{l,0} = P^{\mu}_{l,0},\ \bar{P}^{\nu}_{l,0} = P^{\nu}_{l,0}$$
$$\bar{P}^{D^{\mu}}_{l} = P^{D^{\mu}}_{l},\ \bar{P}^{D^{\nu}}_{l} = P^{D^{\nu}}_{l},\quad \forall l \in \{2,3,5\}.$$

In light of the construction of initial values, we can obtain $\bar{\lambda}_{l,0}$, $\bar{\lambda}^{\mu}_{l,0}$, $\bar{\lambda}^{\nu}_{l,0}$, $\bar{\eta}_{l,0}$, $\bar{\eta}^{\mu}_{l,0}$ and $\bar{\eta}^{\nu}_{l,0}$ $(l \in \mathcal{V})$.

In addition, the initial coupling weights are selected as $a_{ij,0} = l_{ij,0} = a^{i}_{\mu\nu,0} = l^{i}_{\mu\nu,0} = 1$ $(i \in \mathcal{V},\ j \in \mathcal{N}_i)$, then we have

$$\bar{a}^{4}_{\mu\nu,0} = 11.0306,\ \bar{a}^{1}_{\mu\nu,0} = 3.2491,\ \bar{l}^{4}_{\mu\nu,0} = 4.7043$$
$$\bar{l}^{1}_{\mu\nu,0} = -1.3435,\ \bar{a}_{41,0} = -16.0531,\ \bar{a}_{14,0} = -12.1425$$
$$\bar{l}_{41,0} = \bar{l}_{14,0} = -0.3690$$

and other coupling weights stay unchanged for any time instant k. The simulation results are presented in Fig. 5. Fig. 5 sketches the evolution of incremental cost and local power mismatch of the proposed privacy-preserving DED algorithm under different initial conditions. It is observed that the available information set of node 5 is identical (i.e., $\bar{\Upsilon}_{5,k} = \Upsilon_{5,k}$) holds under conditions $\bar{P}_{4,0} \neq P_{4,0}$, $\bar{P}^{D}_{4,0} \neq P^{D}_{4,0}$. Hence, we can conclude that the developed algorithm is privacy-preserving against honest-but-curious agents.

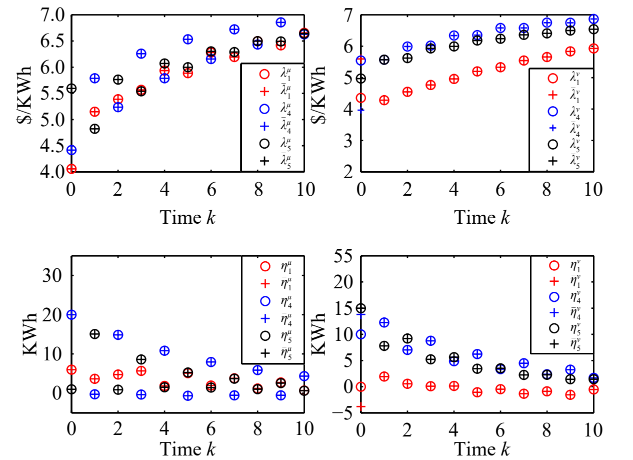

Fig. 5. Test results of the privacy-preserving DED algorithm under different initial conditions.

D. Case 4: Privacy Preservation against External Eavesdroppers

In this case, we are in position to show the privacy-preserving performance against external eavesdroppers. Without loss of generality, we set $\kappa_1 = 0.08$, $\kappa_2 = 0.12$.

For brevity, denote

$$\lambda^{z}_{0} = \mathrm{col}_N\{\lambda^{z}_{i,0}\},\ \bar{\lambda}^{z}_{0} = \mathrm{col}_N\{\bar{\lambda}^{z}_{i,0}\},\ \eta^{z}_{0} = \mathrm{col}_N\{\eta^{z}_{i,0}\}$$
$$\bar{\eta}^{z}_{0} = \mathrm{col}_N\{\bar{\eta}^{z}_{i,0}\},\ P^{z}_{0} = \mathrm{col}_N\{P^{z}_{i,0}\},\ \bar{P}^{z}_{0} = \mathrm{col}_N\{\bar{P}^{z}_{i,0}\}$$
$$P^{D^{z}} = \mathrm{col}_N\{P^{D^{z}}_{i}\},\ \bar{P}^{D^{z}} = \mathrm{col}_N\{\bar{P}^{D^{z}}_{i}\},\ z \in \{\mu, \nu\}.$$

According to the construction in *Theorem 6*, we have

$$\bar{P}^{\mu}_{0} = P^{\mu}_{0} = \begin{bmatrix} 6 & 10 & 15 & 17 & 14 \end{bmatrix}^{T}$$
$$\bar{P}^{D^{\mu}}_{0} = P^{D^{\mu}} = \begin{bmatrix} 12 & 34 & 24 & 37 & 15 \end{bmatrix}^{T}$$
$$P^{\nu}_{0} = \begin{bmatrix} 8 & 16 & 9 & 23 & 10 \end{bmatrix}^{T}$$
$$\bar{P}^{\nu}_{0} = \begin{bmatrix} 7.475 & 16.381 & 9.180 & 22.856 & 10.420 \end{bmatrix}^{T}$$
$$P^{D^{\nu}} = \begin{bmatrix} 8 & 26 & 26 & 33 & 25 \end{bmatrix}^{T}$$
$$\bar{P}^{D^{\nu}} = \begin{bmatrix} 7.069 & 27.318 & 25.170 & 34.617 & 24.151 \end{bmatrix}^{T}$$
$$P_{0} = \begin{bmatrix} 7 & 13 & 12 & 20 & 12 \end{bmatrix}^{T}$$
$$\bar{P}_{0} = \begin{bmatrix} 6.738 & 13.191 & 12.090 & 19.928 & 12.210 \end{bmatrix}^{T}$$
$$P^{D} = \begin{bmatrix} 10 & 30 & 25 & 35 & 20 \end{bmatrix}^{T}$$
$$\bar{P}^{D} = \begin{bmatrix} 9.534 & 30.659 & 24.585 & 35.809 & 19.576 \end{bmatrix}^{T}$$
$$\bar{a}_{ij,0} = (1+\kappa_1)a_{ij,0} = 1.08,\ \bar{l}_{ij,0} = (1+\kappa_2)l_{ij,0} = 1.12$$
$$\bar{a}^{1}_{\mu\nu,0} = 0.068, \bar{a}^{2}_{\mu\nu,0} = 1.126, \bar{a}^{3}_{\mu\nu,0} = 0.960, \bar{a}^{4}_{\mu\nu,0} = 0.929$$
$$\bar{a}^{5}_{\mu\nu,0} = 0.762, \bar{l}^{1}_{\mu\nu,0} = 1.442,\ \bar{l}^{2}_{\mu\nu,0} = 0.769,\ \bar{l}^{3}_{\mu\nu,0} = 0.698$$
$$\bar{l}^{4}_{\mu\nu,0} = 0.573,\ \bar{l}^{5}_{\mu\nu,0} = 0.874,\ \bar{a}_{ij,k} = a_{ij,k},\ \bar{l}_{ij,k} = l_{ij,k}$$
$$\bar{a}^{i}_{\mu\nu,k} = a^{i}_{\mu\nu,k},\ \bar{l}^{i}_{\mu\nu,k} = l^{i}_{\mu\nu,k},\ \forall i \in \mathcal{V},\ j \in \mathcal{N}_i,\ k = 1, 2, \ldots$$

and further obtain λ_0^z, $\bar{\lambda}_0^z$, η_0^z, $\bar{\eta}_0$, and $\bar{\eta}_0^z$ $(z \in \{\mu, \nu\})$.

The simulation results are shown in Fig. 6. Fig. 6 sketches the dynamic trajectories of the incremental cost and the local power mismatch for the proposed privacy-preserving DED algorithm under different initial conditions. It is obvious that the available information sets are identical (i.e., $\bar{\Theta} = \Theta$) under different initial conditions $\bar{P}_{4,0} \neq P_{4,0}$, $\bar{P}_{4,0}^D \neq P_{4,0}^D$, which reveals that the developed algorithm is privacy-preserving against external eavesdroppers.

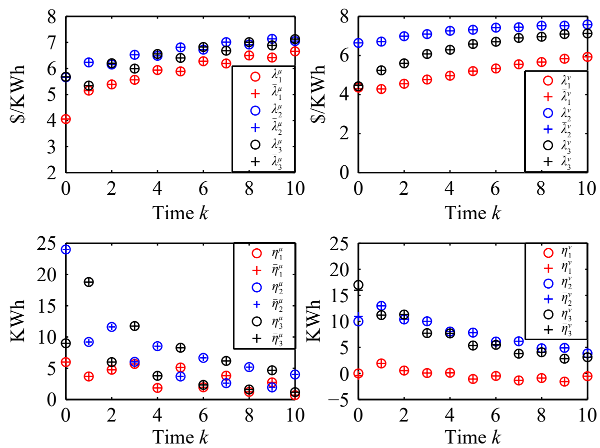

Fig. 6. Test results of the privacy-preserving DED algorithm under different initial conditions.

VI. Conclusion

In this paper, we have coped with the privacy-preserving ED problem of SGs. A consensus-based optimization algorithm has been developed to achieve the optimal power dispatch with the lowest generation cost under practical constraints. With the help of the consensus theory and the eigenvalue perturbation approach, we have shown that the developed distributed algorithm can converge to the optimal solution of the ED problem with respect to the islanded and grid-connected modes. Furthermore, the smooth transitions between these two modes can be achieved in the meantime. To protect initial supply and demand information, a privacy-preserving strategy has been successfully integrated into the DED algorithm via state decomposition, which has shown the security of the proposed algorithm against internal honest-but-curious and external eavesdroppers. Finally, a simulated example has been provided to demonstrate the feasibility and validity of the developed algorithm. Future directions would be the extensions of our results to more general privacy-preserving DED algorithms with intermediate state preservation implemented via an experimental platform [36], [44]–[46].

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