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Traffic Signal Control Based on Reinforcement Learning and Fuzzy Neural Network

Hongxia Zhao, Songhang Chen, Fenghua Zhu and Haina Tang

Abstract—**For traffic signal control of intersections in cities, a new controller based on reinforcement learning and fuzzy neural network is proposed in this paper. The fuzzy neural network has the advantages of both fuzzy control and neural network, and overcome the former's lack of self-learning and generalization ability, and the latter's lack of understandability. Meanwhile, the reinforcement learning can make the controller improve itself on line continually by the simple feedback of environment. The result of computational experiments shows that the proposed traffic signal control algorithm can achieve a more effective optimization control.**

I. Introduction

Urban traffic flow has many complicated characteristics such as uncertainty, time-varying, hysteresis, *et al*., which brings great challenges for traffic signal control. Fuzzy control is suitable for the control of nonlinear and uncertain systems, because it does not need to establish an accurate mathematical model of the controlled object. Furthermore, it is good at expressing human experience and knowledge, and can simulate the logical reasoning and decision-making process of human brain. As early as 1976, *Pappis* and *Mamdani* used fuzzy control for the traffic signal control of the single intersection. Compared with the traditional fixed-time control method, the average vehicle delay is reduced by about 7% [1]. On their basis, *Chen et al*. conducted further research, considered the influence of noncritical traffic flow, and improved the algorithm, the effect of which is 12.5% higher than Pappis's algorithm [2].

However, the setting of fuzzy sets and the determination of fuzzy rules are very dependent on the experience of experts, which has great subjectivity and randomness. This affects the performance and popularization of fuzzy control to a certain extent. Generally speaking, fuzzy control lacks the ability of self-learning and generalization. Compared with fuzzy control, fuzzy neural network control has strong learning and generalization ability, and enhances the adaptability to the environment. So far, fuzzy neural network has made great progress in research, and is widely used in many fields, such as industrial control, information processing, fault detection, pattern recognition, automated vehicle control, spacecraft control and so on [3]-[5]. In the field of traffic signal control based on fuzzy neural network, *Li et al.,* used high-order generalized neural networks and fuzzy rules to coordinate the traffic signal for two adjacent intersections [6]. *Srinivasan et al.,* developed the traffic signal control multiagent system with fuzzy neural network for a large traffic network [7]. *Nae* and *Dumitrache* proposed a neuro-fuzzy traffic signal controller which used neural network to generate fuzzy logical rules in constant and changing traffic volumes conditions [8].

However, the selection and setting of membership function in its fuzzy layer still depends on expert experience, so there is still room and possibility for further optimization. Nowadays various methods of artificial intelligence (AI) were applied to the intelligent transportation field, such as deep learning [9]-[11], reinforcement learning [12]-[13], adaptive dynamic programming [15], and human machine hybrid augmented intelligence [16][17], and showed good application effect. Meanwhile, the new research theory and framework of transportation systems, namely parallel transportation systems [18][19], was proposed and has attracted more and more attentions for its integration of AI into transportation operations [20]-[22].Therefore, a traffic signal controller based on reinforcement learning (RL) and fuzzy neural network (FNN) is designed in this paper. By detecting the queue length of vehicles in the current and next release phases, the controller uses FNN to find the best green light time on the basis of timing control, so as to improve the traffic efficiency of the intersection as much as possible. At the same time, the online learning of the controller is realized by reinforcement learning method to further improve the adaptive and self-learning ability. Finally, the targeted computational experiments are carried out on a simulation platform based on the theory of parallel transportation systems.

II. Reinforcement Learning

Reinforcement learning is an unsupervised machine learning method that enables agents to learn to choose reasonable actions under different environmental states through online, quantified feedback signals [23]. The learning process can be expressed as: an agent performs an action that acts on the environment; the state of the environment will change, and immediately or delay feedback a reinforcement signal of reward or punishment; the agent will choose the next

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action based on the reinforcement signal, and the principle of selection is to maximize the probability of obtaining the reward signal.

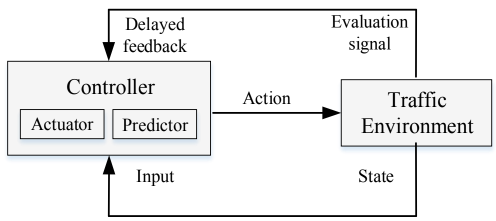

Figure 1. Reinforcement learning mechanism of the traffic signal controller.

In the paper, a traffic signal controller is an agent, and Figure 1 illustrates the reinforcement learning mechanism of the traffic signal controller. In the controller, the actuator is responsible for outputting actions to the intersection. The change of traffic flow has certain hysteresis, so the reinforcement signal cannot be given in time. In this case, it is suitable to use evaluation and prediction rules to realize the reinforcement learning of traffic signal control. The characteristic of evaluation and prediction learning rules is that it needs to predict the external strengthened signal, and the predicted value will be used as the basis of decision making. Therefore, an evaluation predictor needs to be designed into the traffic signal controller.

III. Design

As shown in Figure 2, the traffic signal controller is composed of FNN-based actuator, evaluation & prediction network (EPN), and random exploration. At time t, the controller obtains the current state (QC, QN) of the intersection through interaction with the environment, where $QC$and QN are the vehicle queue length corresponding to the current release phase and the next release phase respectively. Then, (QC, QN) is sent to the FNN-based actuator and the EPN at the same time. The former calculates the green light time $y(t)$ according to the state, while the latter predicts the value of the external reinforcement signal $p(t+1)$ at the next time. Finally, the controller uses the random exploration method to generate the actual green light time $\hat{y}(t)$.

When the green light time is executed, the controller first obtains the evaluation of control effect $r(t+1)$ from the intersection. When executing the yellow light and red light time of the current phase, the EPN and FNN-based actuator will start online training based on the $r(t+1)$ asynchronously.

A. FNN-based Actuator

The actuator adopts the standard fuzzy neural network structure of two inputs and one output, as shown in Figure 3.

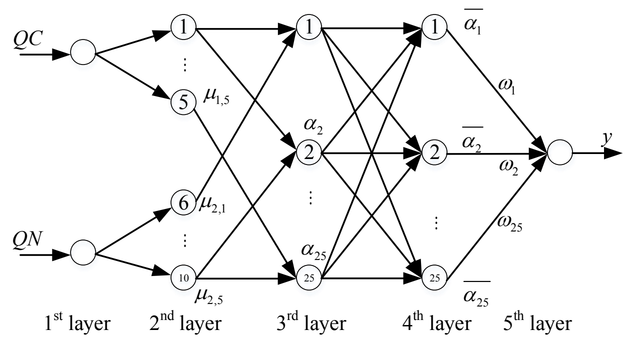

Figure 3. Structure diagram of fuzzy neural network

The first layer is the input layer. The second layer is the fuzzy language layer, which is divided into two groups with five nodes in each group, representing the values of five language variables: VF (very few), F (few), Z (moderate), M (much), VM (very much). Gaussian function is used to define their membership function noted as $\mu_{i,j}$.

$$\mu_{i,j} = exp\left(-\frac{\left(x_i - c_{ij}\right)^2}{\sigma_{ij}^2}\right) \quad i = 1,2; j = 1,2, \dots ,5 \qquad (1)$$

where c_{ij} and σ_{ij} are called the center and width of Gaussian membership function respectively. In reverse learning, the function is differentiable.

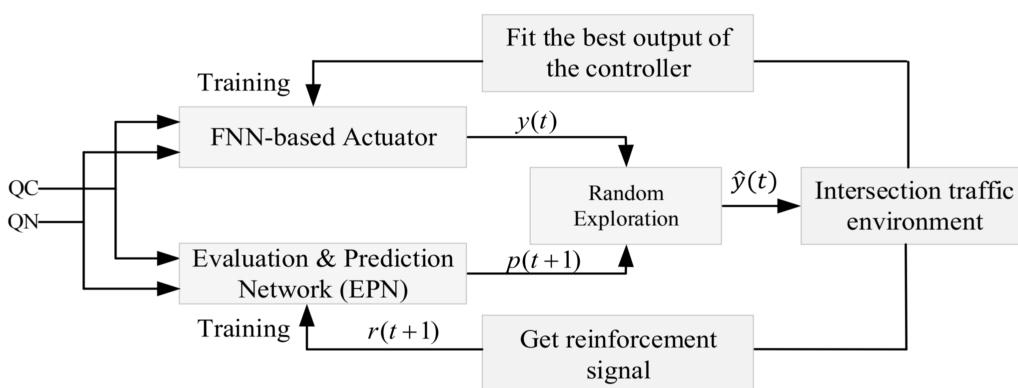

Figure 2. Structure diagram of traffic signal controller based on FNN and reinforcement learning

The third layer is the fuzzy rule layer, which has 25 nodes, representing 25 rules respectively. Its function is to match the precursors of fuzzy rules and calculate the applicability of each rule; The form of the rule is: IF $x_1 = \mu_{1,j}$ and $x_2 = \mu_{2,k}$, THEN $z_l = c_l$, where $j, k = 1,2,\ldots,5$, $l = 1,2,\ldots,25$. The fourth layer realizes normalization operation and the number of nodes in it is the same as that in the third layer. The fifth layer is the output layer to realize clear calculation.

The EPN can learn according to the actual reinforcement signal of environment, while the FNN-based actuator cannot obtain similar "guidance". If the desired control output can be fitted with a simple evaluation signal, the idea and method of learning based on training data can be used to solve the problem. Therefore, this paper fits the expected output of the actuator as:

$$y_d(t) \approx y(t) + \gamma \frac{\partial r}{\partial y} \tag{2}$$

$$\frac{\partial r}{\partial y} \approx sign(r(t+1) - p(t+1)) \frac{\hat{y}(t) - y(t)}{\sigma(t)} \tag{3}$$

where $\gamma \in [0,1]$ is the scale factor. If $r(t+1) > p(t+1)$, then $\hat{y}(t)$ is better than $y(t)$, and the desired control output $y_d(t)$ obtained according to equation (8) will be close to $\hat{y}(t)$, on the contrary, it will be far away from $\hat{y}(t)$.

After fitting the expected output of the actuator, the error back propagation algorithm can be used to adjust the center and width of the Gaussian membership function of each neuron in the fuzzy language layer and the connection weight of the output layer, so as to realize the online learning of the FNN-based actuator.

B. Evaluation & Prediction Network

The traditional back propagation network is used as the EPN, and the first and second layers of the FNN-based actuator are reused. The structure is shown in Figure 4.

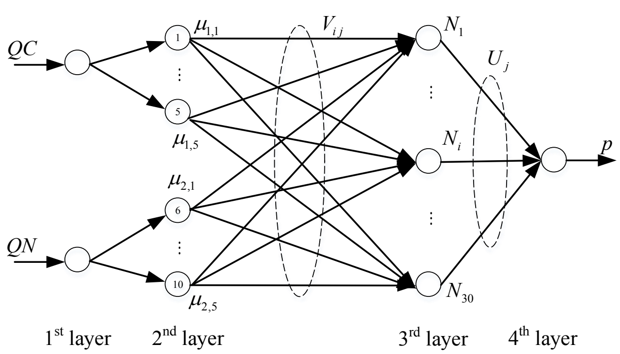

Figure 4. Structure diagram of evaluation & prediction network.

The third layer of the network is the hidden layer, and the number of nodes is taken as 30. The fourth layer is the output layer. The formal description of these two layers is as follows:

$$s_i^{(3)} = \sum_{j=1}^{10} x_j^{(2)} V_{ij} \quad i \in [1,30] \tag{4}$$

$$x_i^{(3)} = f_i^{(3)}\left(s_i^{(3)}\right) = \frac{1-e^{-s_i^{(3)}}}{1+e^{-s_i^{(3)}}} \quad i \in [1,30] \tag{5}$$

$$s_i^{(4)} = \sum_{j=1}^{30} x_j^{(3)} U_j \quad i = 1 \tag{6}$$

$$p = x_i^{(4)} = f_i^{(4)}\left(s_i^{(4)}\right) = \frac{1-e^{-s_i^{(4)}}}{1+e^{-s_i^{(4)}}} \quad i = 1 \tag{7}$$

where s and x are the input and output of neurons respectively, and the superscript of the symbol indicates the number of layers where the neuron node is located, and the subscript of the symbol indicates the index of the neuron node in the layer. The $f(*)$ are the output transformation functions.

For online learning, take $E = (r-p)^2/2$ as the performance function, where r is the actual evaluation signal, and then use the chain derivation rule and error back propagation algorithm to derive the correction amount of network weight as follows:

$$\frac{\partial E}{\partial U_i} = -(r-p) \frac{2e^{-s_i^{(4)}}}{\left(1+e^{-s_i^{(4)}}\right)^2} x_i^{(3)} \tag{8}$$

$$\frac{\partial E}{\partial V_{ij}} = -(r-p) \frac{2U_i e^{-s_i^{(4)}} \cdot 2x_j^{(2)} e^{-s_i^{(3)}}}{\left(1+e^{-s_i^{(4)}}\right)^2 \left(1+e^{-s_i^{(3)}}\right)^2} \tag{9}$$

where U_j and V_{ij} are initialed as random numbers between -0.1 and 0.1. The vehicle passing rate at the intersection within the extended green light time is used as the evaluation signal r fed back from the environment. On this basis, we can also add momentum factor [24] or variable step size [25] to accelerate the convergence of neural network.

C. Random Exploration

The information given by the reinforcement signal is often very little, only the "evaluation" signal. For a certain action in a certain state, a good evaluation does not mean that there is no better choice than the action. Therefore, the controller must have a strategic trade-off process, that is, choose to explore unknown states and actions, or choose to use the states and actions that it has learned and can produce high returns. In order to effectively solve the problem of utilization and exploration, a random exploration method [26][27] is used to generate the actual control quantity $\hat{y}(t)$ according to the output $y(t)$ of the actuator and the predicted evaluation signal $p(t+1)$. The specific steps are as follows:

Step 1: Determine the scope of random exploration $\sigma(t)$:

$$\sigma(t) = \frac{1}{1+e^{Kp(t+1)}} \tag{10}$$

where K and A are normal quantities.

Step 2: Taking $y(t)$ as the mean and $\sigma(t)$ as the variance, a normally distributed random value is generated as $\hat{y}(t)$, i.e.,

$$\hat{y}(t) \sim N(y(t), \sigma(t)) \tag{11}$$

It shows that when the external evaluation prediction signal $p(t+1)$ is small and the variance $\sigma(t)$ is large, the variation range of $\hat{y}(t)$ around the mean value $y(t)$ will increase, which can deviate from $y(t)$ with greater probability. On the contrary, it will approach $y(t)$ with greater probability.

IV. Implementation

The implementation of the designed controller is mainly divided into two modules: control decision-making and online learning. These two parts are described in detail below.

A. Control Decision-making Module

When the green light time is executed to two-thirds, the designed controller starts to decide the final green light time. The flow chart of decision-making is shown in Figure 5. The controller first obtains QC and QN through vehicle detectors, and then outputs the optimized green light time after a series of operations. At present, the widely used traffic video detector can obtain the queue length. Meanwhile, in order to avoid too long waiting time in other directions, the final green light time cannot exceed the maximum green light time g_{max}.

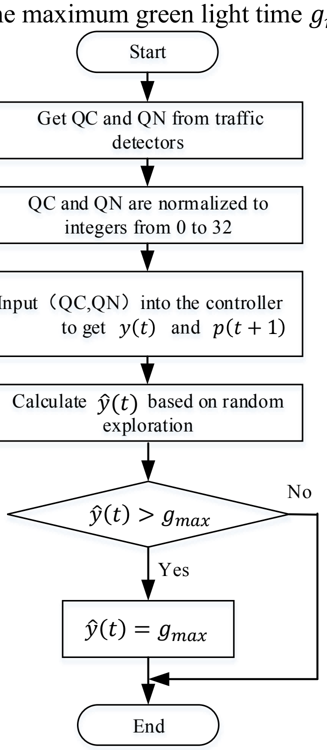

Figure 5. Flow chart of control decision-making.

B. Online Learning Module

The online learning module includes two parts: the learning of FNN-based actuator EPN and the learning of EPN. The FNN-based actuator uses equation (2) (take $\gamma = 1.0$) to fit the expected output of the controller, and the difference between the expected output and the actual control output is taken as the learning error. The EPN uses the difference between the predicted reinforcement signal and the actual one as the learning error. The error back propagation algorithm is used to correct the network weight and the center and width of Gaussian membership function. For each learning, the iterations of the EPN and FNN-based actuator are set to 20 times. The flow chart of online learning is shown in Figure 6.

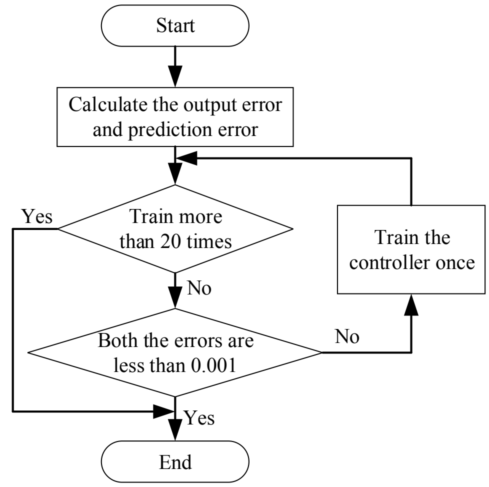

Figure 6. Flow chart of online learning.

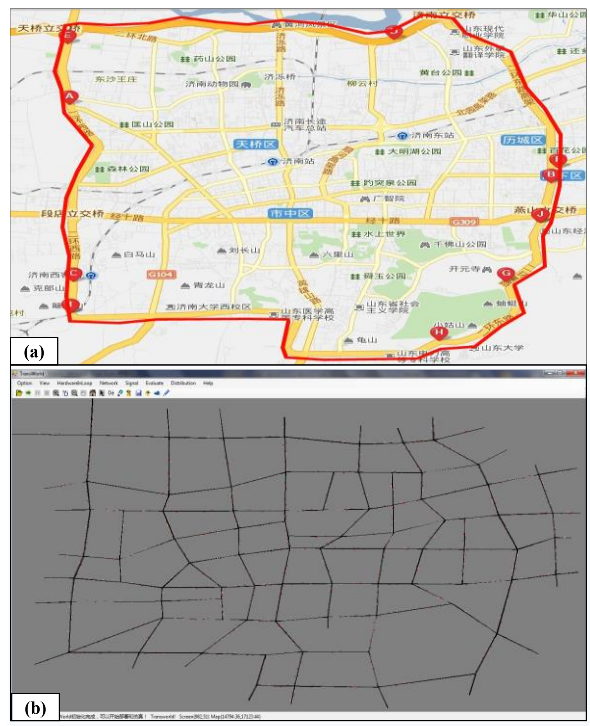

Figure 7. (a) Actual environment and (b) corresponding artificial environment

V. Computational Experiments

The paper uses the self-developed transportation simulation platform called *TransWorld* [18][19] based on the theory of parallel transportation systems to test the effectiveness of traffic signal control algorithm. Taking the main roads in the second ring of Jinan City, Shandong Province, China as the real physical model, the simulation environment is established. The main road network is shown in Figure 7(a), and Figure 7(b) shows the artificial traffic environment established in *Transworld*. The road network includes 419 intersections and 330 road sections with a total length of 570.81 kilometers. At present, the morning and evening peak in this area is very congested. The corresponding situation can be simulated by collecting the actual traffic timing scheme into the simulation environment. The traffic environment of this area is relatively complicated, so it is persuasive to test the performance of the proposed algorithm.

The direct result of road network congestion is that a large number of vehicles are stranded in the road network, and people often need to stop while driving. Therefore, the number of vehicles and average number of stops in the road network are two evaluation indicators that can directly reflect the traffic conditions of the road network [28].This paper selects these two typical indicators to evaluate the effect of traffic signal control algorithm. In *TransWorld*, the simulation period is set from 0:00 to 23:59, the simulation step is 1 second, and the size of the artificial population [29]-[31] is set to 50,000. In order to eliminate random errors, we run each experiment 10 times, and then take the mean value of relevant indicators. The results compared with the traditional fixed-time control algorithm are as follows.

A. Changes in the number of vehicles in the road network

For convenience, the proposed control method is abbreviated as the FN-RL control, and fixed-time control is abbreviated as the FT control in the following. As shown in Figure 8, when the FN-RL traffic signal control algorithm is adopted, the number of vehicles in the road network has changed greatly during the morning and evening peak hours.

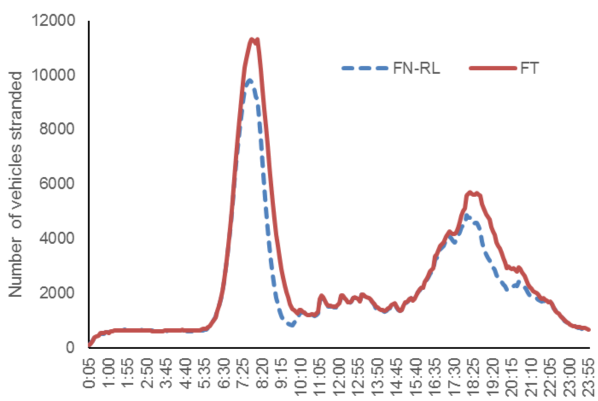

Figure 8. Comparison of the number of vehicles in the road network under the FN-RL and FT control

According to statistics, the average number of vehicles in the road network has decreased from 2504.8 under FT control to 2154.2 under FN-RL control, with a decrease ratio of 13.9%. It can also be found from the graph that the maximum number of vehicles in the road network under FN-RL control is significantly less than that under FT control during rush hours, which also shows that FN-RL control can effectively alleviate traffic congestion.

B. Average number of stops for different types of travel behaviors

Transworld divides people's travel behaviors in a day into nine categories: home, work, school, shopping, entertainment, sport, dining out, medical treatment and business. During the experiment, *TransWorld* can automatically classify and count the average number of stops times of various travel behaviors. As shown in Figure 9, for various travel activities, FN-RL control algorithm can effectively reduce the average number of stops.

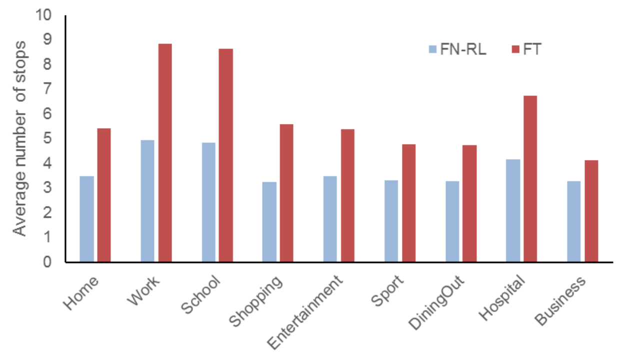

Figure 9. Comparison of average number of stops for different types of travel behaviors

According to the statistics, the average number of stops for all travel activities are reduced by 35.71%, where the average number of stops for home, shopping, and entertainment activities are reduced by more than 35%, and the average number of stops for work and school activities are reduced by about 44%.

VI. Conclusion

For the traffic environment of intersections in cities, the paper designs and implements a traffic signal control algorithm based on reinforcement learning and fuzzy neural network. The algorithm not only gives full play to many advantages of FNN control, but also realizes the online learning of traffic signal control algorithm by using reinforcement learning, so as to further improve the self-learning and adaptive ability of traffic signal control algorithm. Finally, the effectiveness of the algorithm is verified by the transportation simulation platform.

At present, we only compare the designed algorithm with fixed-time control, and we plan to compare it with other algorithms in the next step, including classical induction control, green band control and so on. Since all traffic signal control algorithms involve parameter settings, especially when comparing within an area, there are many parameters involved. In this case, how to make a fair enough comparison will be a challenge. On the other hand, we will study the incorporation of coordination mechanisms into the algorithm in order to achieve coordinated regional traffic signal control.

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