# Prescribed Performance Tracking Control of Time-Delay Nonlinear Systems With Output Constraints

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***Abstract*—The problem of prescribed performance tracking control for unknown time-delay nonlinear systems subject to output constraints is dealt with in this paper. In contrast with related works, only the most fundamental requirements, i.e., boundedness and the local Lipschitz condition, are assumed for the allowable time delays. Moreover, we focus on the case where the reference is unknown beforehand, which renders the standard prescribed performance control designs under output constraints infeasible. To conquer these challenges, a novel robust prescribed performance control approach is put forward in this paper. Herein, a reverse tuning function is skillfully constructed and automatically generates a performance envelop for the tracking error. In addition, a unified performance analysis framework based on proof by contradiction and the barrier function is established to reveal the inherent robustness of the control system against the time delays. It turns out that the system output tracks the reference with a preassigned settling time and good accuracy, without constraint violations. A comparative simulation on a two-stage chemical reactor is carried out to illustrate the above theoretical findings.**

***Index Terms*—Nonlinear systems, output constraints, prescribed performance, reference tracking, time delays.**

## I. Introduction

IT seems to be insufficient to describe the evolutions of some engineering systems, if the future state of the system is determined only by the present state and is independent of the past [1]. Instead, some information on the past state should be included in the mathematical model of the system. Such systems are generally called delay systems [1]. Common delay systems in engineering incorporate but are not limited to continuous stirred tank reactors, heat exchangers, and cold rolling mills [1]–[3], as a result of transport delay. For example, in a continuous stirred tank reactor, the conversion from input to output is never complete, so a recycle stream is necessary to recycle the unreacted reagents. In order to increase overall conversion and reduce reaction cost, the unreacted reagents are returned into the reactor again by travelling through a pipe. This progress needs a finite amount of time and thus introduces a delay into the system dynamics [1], [4]. Moreover, the time-delay phenomenon can be seen in economic systems, mechanics, population dynamics, etc., [1]. Therefore, in a real world, time delay is sometimes inevitable, and for a control system, it significantly challenges the control development. If it is not addressed well, the control system will suffer from performance degradation and even instability.

At present, there are two main methods for control designs of time-delay systems. They are based on the Lyapunov-Krasovskii functionals [5]–[14] and the Razumikhin functions [15]–[17], respectively. Both of them work well, if the time delay meets the following assumption or condition. First, the time delay or its bound is known [7], [8], [12], [13], [15]. Second, the bounding functions of the time-delay nonlinearities are in the parametric form [5], [6], [16], [17] with unknown parameters but known functions. Third, the derivative of the time delay should be less than 1 [7]–[11], [13], [14]. As pointed out by [18], the above assumptions are restrictive from both theoretical and practical perspectives. Therefore, the control development for time-delay systems without the requirements mentioned above is considerably significant and appears to be open.

On the other hand, the outputs of many practical systems are subject to constraints from operational specifications and/or safety considerations [19]. For instance, the reagents of a continuous stirred tank reactor have to be maintained within certain ranges to avoid environmental pollution and security incidents. Barrier Lyapunov functions (BLFs) are a main tool to address output constraints [19]–[24] which guarantees that the system output evolves strictly within the given constraint boundaries throughout. Nevertheless, in terms of output tracking, only the boundedness of the tracking error is obtained and the specific tracking behavior (e.g., the settling time and the accuracy) can not be predetermined, in the case of unmatched disturbances and/or unknown nonlinearities [20]–[23]. The prescribed performance control (PPC) method [25]–[27] pro-

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vides a way to quantitatively preassign the transient and steady-state response of reference tracking. Intuitively, the combination of PPC and BLFs yields a solution to the above problem. As substantiated by [28], however, this makes it necessary for the reference to be known in advance for the preselection of performance functions. Nevertheless, this may not be met in some applications. A typical example is leader-following of mobile vehicles, where the position of the leader is sent to the follower in real time, which is unavailable before data reception [29], [30]. Other examples encompass, but are not exculsive to robotic interception [31], [32] and adaptive extremum seeking [33], [34]. Recently, a practical tracking control approach for Euler-Lagrange systems with output constraints without prior knowledge of the specific reference was proposed [28]. However, transient performance was lost. Moreover, it remains unknown whether or not this approach can be applied to time-delay systems.

Inspired by the above discussion, a novel PPC approach for the time-delay output-constrained nonlinear systems is developed in this paper. To overcome the challenge caused by absent prior information of the reference, a reverse tuning function is skillfully constructed. On this basis, a unified performance analysis framework based on proof by contradiction and barrier functions is established to reveal the inherent robustness of the resulting control system. The contribution of this paper and the superiority of the proposed approach are enumerated as follows.

1) The admissible time delays by our approach need only to be bounded and locally Lipschitz continuous, without the aforesaid assumptions [5]–[17].

2) It achieves PPC under output constraints without the need for prior knowledge of the reference, unlike [25]–[28].

3) It preserves the robustness and simplicity of the PPC method. For unknown time-delay nonlinear systems, the controller design does not invoke approximation [6]–[9], [11], [35], [36] identification [5], [10], [15], estimation [37], [38], or filtering [39]–[41].

The rest of this paper is organized as follows. The problem under consideration is formulated in Section II. A solution is given in Section III. Its feasibility is substantiated in Section IV and illustrated by the simulation results in Section V. Section VI concludes this paper.

*Notations:* The notations used in this paper are standard that are summarized as follows. $\mathbb{R}^i$ denotes the $i$-dimensional Euclidean space, where $\mathbb{R}^1 = \mathbb{R}$; $\sup(\cdot)$ is the supremum of the function; $L^\infty$ denotes the Banach space of all Lebesgue measurable, i.e., $L^\infty$ is a space consisting of all bounded series.

## II. Problem Formulation

### A. System Description

$$\begin{cases} \dot{x}_i(t) = f_i(\bar{x}_i(t),\bar{x}_i(t-\tau_i),d(t)) + g_i(\bar{x}_i(t),\bar{x}_i(t-\tau_i),d(t))x_{i+1}(t) \\ \qquad i = 1,\dots,n-1 \\ \dot{x}_n(t) = f_n(\bar{x}_n(t),\bar{x}_n(t-\tau_n),d(t)) + g_n(\bar{x}_n(t),\bar{x}_n(t-\tau_n),d(t))u(t) \\ y(t) = x_1(t) \\ x_i(t-\tau_j) = \varphi_{ij}(t),\ \ t\in[0,\tau_j],\ \ i=1,\dots,n,\ \ j=1,\dots,n. \end{cases} \tag{1}$$

The time-delay nonlinear systems under consideration are described by (1) at the bottom of this page, where $\bar{x}_i(t) = [x_1(t),\dots,x_i(t)]^T \in \mathbb{R}^i$, $i = 1,\dots,n$; $\bar{x}_n(t)$ is the system state; $u(t)\in\mathbb{R}$ and $y(t)\in\mathbb{R}$ represent the control input and the system output, respectively; $d(t)\in\mathbb{R}^m$ consists of the bounded external disturbances; $\varphi_{ij}(t)\in\mathbb{R}$ is the initial value of the state variable, $i=1,\dots,n$, $j=1,\dots,n$; $\tau_i\in\mathbb{R}$ with $\tau_i \ge 0$ stands for the time delay of the state, $i=1,\dots,n$; $f_i(\cdot)\in\mathbb{R}$ and $g_i(\cdot)\in\mathbb{R}$, $i=1,\dots,n$, denote the system nonlinearities, both of which are continuous in their arguments.

Three common assumptions for the system in (1) are made as follows.

*Assumption 1:* Let $|g_i(\cdot)| \ge \underline{g} > 0$, $i=1,\dots,n$, where $\underline{g}$ is a constant. Without loss of generality, it is assumed that $g_i(\cdot)\ge \underline{g}$, $i=1,\dots,n$ [19], [26].

*Assumption 2:* The time delays are bounded [10], [18].

*Assumption 3:* The function $\varphi_{ij}(t)$ is continuous in $[0,\tau_j]$, $i=1,\dots,n$, $j=1,\dots,n$ [5], [15].

*Remark 1:* In related works, the time delay was required to be fixed [5], [6], [12], [35], bounded by a known constant [7], [8], [13], [15], [36], or differentiable with the derivative less than 1 [7]–[11], [13], [14]. In this study, both time-varying and heterogeneous time delays are taken into account, which need only to meet Assumption 2, where the upper bound is not necessarily known. Moreover, (1) describes a general family of time-delay nonlinear systems, in the sense that the state variables, the time delays and the disturbances are lumped by the unknown nonlinear mappings together in a non-affine form. It not only covers time-delay strict-feedback systems [6], [35], [36], but also include for example,

$$\dot{x} = x^2 x^3(t-\tau)d + u \tag{2}$$

where $\tau$ and $d$ as well as their bounds are totally unknown. Commonly used tools, e.g., adaptive identification [5], [10], [15] and neural/fuzzy approximation [6]–[9], [11], [35], [36], cannot be applied to dealing with the unknown nonlinear function in (2) straightforward. Nevertheless, it should be noted that Assumption 1 requires the control direction of each subsystem to be known a priori.

### B. Output Constraints

On the purpose of safe operation, the output constraint on (1) is taken into consideration, i.e.,

$$\underline{y}(t) < y(t) < \bar{y}(t),\ \ t\ge 0 \tag{3}$$

where $\bar{y}(t)$ and $\underline{y}(t)$ stand for the upper and lower boundaries, respectively, which are permitted to be both time-varying and asymmetric. The initial output is reasonably required to satisfy

$$\underline{y}(0) < y(0) < \bar{y}(0). \tag{4}$$

A relevant assumption [28] is introduced.

*Assumption 4:* There hold

1) $\bar{y}(t), \underline{y}(t), \dot{\bar{y}}(t), \dot{\underline{y}}(t) \in L^\infty$;

2) $0 < \underline{a} < \bar{y}(t) - \underline{y}(t) < \bar{a}$, $t\ge 0$;

with $\bar{a}$ being a known constant.

### C. Control Objective

The control goal for (1) is twofold. First, keep the output constraint intact the whole time. On this basis, steer the system output to track a reference, $r(t)$, which meets the following assumption [28].

*Assumption 5:* There hold $\underline{y}(t) < r(t) < \bar{y}(t)$, $t \geq 0$, and $\dot{r}(t) \in L^{\infty}$.

Specifically, the requisite transient and steady-state tracking response is formulated by

$$|y(t) - r(t)| < \varepsilon, \quad t > T \tag{5}$$

where $\varepsilon$ and $T$ are nonzero constants which denote the allowable bounds on the accuracy and settling time, respectively.

Now, we summarize the problem treated in this paper as follows.

*Problem 1:* Develop a control method for the system in (1) to guarantee the output constraint in (3), the performance requirement in (5), and the boundedness of all the signals involved in the closed loop.

*Problem 2:* The classical PPC methods [25], [26], [42] guarantee the tracking error within a performance tube as follows:

$$\underline{p}(t) < y(t) - r(t) < \bar{p}(t)$$

where $\underline{p}(t)$ and $\bar{p}(t)$ should be predetermined by the designer. Under output constraints, $\underline{p}(t)$ and $\bar{p}(t)$ should satisfy

$$\underline{y}(t) - r(t) < \underline{p}(t) < y(t) - r(t) < \bar{p}(t) < \bar{y}(t) - r(t).$$

Obviously, the selections of $\underline{p}(t)$ and $\bar{p}(t)$ depend on $r(t)$, which makes it necessary that $r(t)$ is known a priori. However, in some practical applications, $r(t)$ is only available in real-time, such as a leader-following formation [29], [30], robotic interception of moving objects [31], [32], and adaptive extremum seeking control [33], [34]. Although a solution to this problem was given [28], it guarantees merely the prescribed tracking accuracy, and thus fails to achieve fast tracking. Moreover, it works for delay-free systems and thus may fail in the presence of time delays.

## III. Control Design

To solve Problem 1, a reverse tuning function-based PPC approach is developed in this section.

### A. Constraint Transformation

In order to guarantee prescribed performance, a reverse tuning function, $\varphi(t)$, is devised and adopted to online generate a pair of boundaries for the tracking error. It fulfills

1) $\varphi(0) = 1$;
2) $\varphi(t) = \varphi_{\infty}$, $t \geq T$;
3) $-\infty < \dot{\varphi}(t) \leq 0$, $0 \leq t \leq T$;

where $T$ is predefined in (5) and $0 < \varphi_{\infty} < 1$ is constant and determined by the designer. Through this paper, we select

$$\varphi(t) = \begin{cases} 1 + (\varphi_{\infty} - 1)\sin(\frac{\pi}{2} \times \frac{t}{T}), & 0 \leq t < T \\ \varphi_{\infty}, & t \geq T. \end{cases} \tag{6}$$

Define

$$e_1(t) = x_1(t) - r(t), \quad t \geq 0. \tag{7}$$

Based on (7), (3) is equivalent to

$$\underline{y}(t) - r(t) < e_1(t) < \bar{y}(t) - r(t), \quad t \geq 0. \tag{8}$$

Adopt the reverse tuning function to produce

$$\begin{aligned} \bar{k}(t) &= (\bar{y}(t) - r(t))\varphi(t) \\ \underline{k}(t) &= (\underline{y}(t) - r(t))\varphi(t). \end{aligned} \tag{9}$$

*Remark 3:* In the related constraint-handling studies [19]–[21], $\underline{y}(t) - r(t)$ and $\bar{y}(t) - r(t)$ are imposed on $e_1(t)$ directly. In this paper, a reverse tuning function is newly constructed to adjust the transformed constraint boundaries on the tracking error. It is stressed that the original tuning function [42], [43] is used to tackle the initialization problem of PPC designs. This online automatically generates a tube with a contractive trend. As long as the tracking error evolves inside this tube, both the output constraint and the performance requirement are fulfilled. Therefore, a unified framework to address the above problem is established, as substantiated by Lemma 1 below.

*Lemma 1:* If there exists

$$\underline{k}(t) < e_1(t) < \bar{k}(t), \quad t \geq 0 \tag{10}$$

then (3) and (5) hold.

*Proof:* Due to $0 < \varphi(t) \leq 1$, (9) is scaled by

$$\underline{y}(t) - r(t) \leq \underline{k}(t) < \bar{k}(t) \leq \bar{y}(t) - r(t), \quad t \geq 0.$$

Therefore, (8) certainly holds under (10). Since (8) and (3) are equivalent, (3) is established under the same condition. Assumptions 4 and 5 show

$$0 < \bar{y}(t) - r(t) < \bar{a}, \quad -\bar{a} < \underline{y}(t) - r(t) < 0, \quad t \geq 0.$$

By (6) and (9), one has

$$\bar{k}(t) < \bar{a}\varphi_{\infty}, \quad \underline{k}(t) > -\bar{a}\varphi_{\infty}, \quad t \geq T.$$

Obviously, (10) yields

$$|e_1(t)| < \bar{a}\varphi_{\infty}, \quad t \geq T. \tag{11}$$

Since $\varphi_{\infty}$ is freely chosen and $\bar{a}$ is known, let $\varepsilon = \bar{a}\varphi_{\infty}$. Then (11) becomes (5). ■

### B. Controller Design

According to Lemma 1, employ a barrier function to confine $e_1(t)$

$$\eta_1(t) = \ln\left(\frac{\bar{k}(t) - e_1(t)}{e_1(t) - \underline{k}(t)}\right). \tag{12}$$

Following the recursive design philosophy, (12) yields the first intermediate control law:

$$\alpha_1(t) = c_1\eta_1(t) \tag{13}$$

where $c_1$ denotes the positive constant control gain. Proceed with

$$e_i(t) = x_i(t) - \alpha_{i-1}(t) \tag{14}$$

$$\eta_i(t) = \tan\left(\frac{\pi}{2}\frac{e_i(t)}{k_i(t)}\right) \tag{15}$$

$$\alpha_i(t) = -c_i\eta_i(t) \tag{16}$$

where $c_i$ is the positive constant control gain and $k_i(t)$ denotes the prescribed boundary on $e_i(t)$ which should satisfy

$$0 < \underline{k} < k_i(t) < \infty \tag{17}$$

$$\left|\dot{k}_i(t)\right| < \infty \tag{18}$$

$$|e_i(0)| < k_i(0) \tag{19}$$

where $\underline{k}$ is constant. A typical example of $k_i(t)$ is the exponentially decaying time function [25]. Starting from $i = 2$, execute (14)–(16), with $i = i+1$ recursively, till $i = n$. This yields the final control

$$u(t) = \alpha_n(t). \tag{20}$$

*Remark 4:* Careful inspection of (6), (9), (12)–(16) and (20) reveals that the controller design is independent of the explicit information about the system nonlinearities, or their bounding functions, or the bounds of the time delay, or the disturbance bounds, or the prior knowledge of the reference. Nonetheless, no adaptive algorithms [5], [10], [15], disturbance observers [37], [38], approximation structures [6]–[9], [11], [35], [36], etc., are employed to acquire such knowledge. Besides, the control law does not involve the derivatives of the reference or the intermediate control signals. However, this is accomplished without the aid of dynamic surface control [39] or auxiliary filters [40], [41]. Therefore, the developed controller is less demanding and strikingly simple.

## IV. Theoretical Analysis

For ease of theoretical analysis, a pair of lemmas are given as follows.

*Lemma 2:* For any $M > 0$, there holds

$$\left|x_i\left(t-\tau_j\right)\right| < \infty,\ \ 0 \le t < M \tag{21}$$

if

$$|x_i(t)| < \infty,\ \ 0 \le t < M \tag{22}$$

with $i, j = 1,\dots,n$.

*Proof:* Note from (1) and Assumption 3 that

$$|x_i(t)| < \infty,\ \ -\tau_j \le t \le 0 \tag{23}$$

for $i, j = 1,\dots,n$. By variable substitution, (21) is equivalent to

$$|x_i(t)| < \infty,\ \ -\tau_j \le t < M-\tau_j \tag{24}$$

with $i, j = 1,\dots,n$. For each $\tau_j$, $j = 1,\dots,n$, two cases are discussed by comparing it with $M$.

1) If $\tau_j \ge M$, then $-\tau_j \le t-\tau_j < M-\tau_j \le 0$ as $0 \le t < M$. By (23), (24) holds.

2) If $\tau_j < M$, the interval $[-\tau_j, M-\tau_j)$ covers $[-\tau_j, 0)$ and $[0, M-\tau_j)$. According to (23) and (22), respectively, (24) holds.

Therefore, (21) as an equivalent of (24) holds for any $M > 0$, under (22). ■

*Lemma 3:* For any $q > 0$, $\dot{\alpha}_i(t)$ is bounded on $[0, q)$, if

1) $e_i(t)$ evolves inside $\left(\underline{k}_i(t), \bar{k}_i(t)\right)$ but keeps away from the boundaries on $[0, q)$;

2) $\dot{e}_i(t)$ is bounded on $[0, q)$;

with $i = 1,\dots,n$, where $\underline{k}_1(t) = \underline{k}(t)$, $\bar{k}_1(t) = \bar{k}(t)$, $\underline{k}_i(t) = -k_i(t)$ and $\bar{k}_i(t) = k_i(t)$, $i = 2,\dots,n$.

*Proof:* Differentiating (13) and (16) by (12) and (15), respectively, yields

$$\dot{\alpha}_1(t) = c_1\dot{\eta}_1(t) \tag{25}$$

$$\dot{\alpha}_i(t) = -c_i\dot{\eta}_i(t),\ \ i = 2,\dots,n \tag{26}$$

where

$$\dot{\eta}_1(t) = \beta_{11}(t) \times (\lambda_1(t) + \dot{e}_1(t)\beta_{12}(t)) \tag{27}$$

$$\dot{\eta}_i(t) = \frac{\pi}{2} \times \frac{1}{k_i(t)} \times \frac{1}{\beta_i(t)} \times \rho_i(t) \tag{28}$$

with

$$\lambda_1(t) = \dot{\bar{k}}(t)e_1(t) - \dot{\bar{k}}(t)\underline{k}(t) + \bar{k}(t)\dot{\underline{k}}(t) - e_1(t)\dot{\underline{k}}(t) \tag{29}$$

$$\beta_{11}(t) = \frac{1}{\bar{k}(t)-e_1(t)} \times \frac{1}{e_1(t)-\underline{k}(t)} \tag{30}$$

$$\beta_{12}(t) = \underline{k}(t) - \bar{k}(t) \tag{31}$$

$$\beta_i(t) = \cos^2\left(\frac{\pi}{2}\frac{e_i(t)}{k_i(t)}\right) \tag{32}$$

$$\rho_i(t) = \dot{e}_i(t) - \frac{e_i(t)\dot{k}_i(t)}{k_i(t)}. \tag{33}$$

Note from Assumptions 4 and 5 and the definition of $\varphi(t)$ that

$$\underline{y}(t), \bar{y}(t), \dot{\underline{y}}(t), \dot{\bar{y}}(t), r(t), \dot{r}(t), \varphi(t), \dot{\varphi}(t) \in L^\infty.$$

Therefore, it follows from (9) and (31) that:

$$\bar{k}(t), \underline{k}(t), \dot{\bar{k}}(t), \dot{\underline{k}}(t), \beta_{12}(t) \in L^\infty.$$

Under the first assumed condition for $i = 1$, $\lambda_1(t)$ in (29) and $\beta_{11}(t)$ in (30) are bounded on $[0, q)$. As a result, $\dot{\eta}_1(t)$ in (27) and $\dot{\alpha}_1(t)$ in (25) are bounded on $[0, q)$ in the assumed cases for $i = 1$.

Similarly, the first assumed condition for $i = 2,\dots,n$ implies the reciprocal of (32) is bounded on $[0, q)$. Recalling (17) and (18), we have $|\rho_i(t)| < \infty$, $t < q$, under the assumed conditions for $i = 2,\dots,n$. Therefore, $\dot{\eta}_i(t)$ in (28) and $\dot{\alpha}_i(t)$ in (26) are bounded over $[0, q)$ under the assumed conditions of Lemma 3, $i = 2,\dots,n$. ■

The theoretical result of this paper is presented as follows.

*Theorem 1:* Under Assumptions 1–5 and the initial conditions in (4) and (19), the control scheme composed of (6), (9), (12)–(16) and (20) solves Problem 1.

*Proof:* It starts from claiming that

$$\begin{cases} \underline{k}(t) < e_1(t) < \bar{k}(t), & t \ge 0 \\ |e_i(t)| < k_i(t),\ \ i = 2,\dots,n, & t \ge 0 \end{cases} \tag{34}$$

which is shown by contradiction. It follows from (4) and (6)–(10) that:

$$\underline{k}(0) < e_1(0) < \bar{k}(0),\ \ t \ge 0.$$

Combining it with (19), (34) is met at $t = 0$. Note that $\bar{x}_n(t)$ is continuous. The same holds for $r(t)$ by Assumption 5. Thus, $e_1(t)$ in (7) does so. This together with the continuity of $\underline{k}(t)$ and $\bar{k}(t)$ established above ensures that $\eta_1(t)$ in (12) and $\alpha_1(t)$ in (13) are continuous, if $\underline{k}(t) < e_1(t) < \bar{k}(t)$. Further, $e_2(t)$ in (14) is continuous under the same condition. Follow the same line to examine $e_3(t),\dots,e_n(t)$ one by one. Then, it can be recursively confirmed that each $e_i(t)$, $i \in \{3,\dots,n\}$, is continuous if $\underline{k}(t) < e_1(t) < \bar{k}(t)$ and $\left|e_j(t)\right| < k_j(t)$, $j = 2,\dots,i-1$.

These facts imply that if (34) is violated, then there exists $t^* > 0$ so that

$$\begin{cases} \underline{k}(t) < e_1(t) < \bar{k}(t), & t < t^* \\ |e_i(t)| < k_i(t), \ \ i = 2, \ldots, n, & t < t^* \end{cases} \tag{35}$$

and at least one of the following equations holds

$$\begin{cases} \lim_{t \to t^*} e_1(t) = \bar{k}(t^*) \\ \lim_{t \to t^*} e_1(t) = \underline{k}(t^*) \\ \lim_{t \to t^*} \left| e_j(t) \right| = k_j(t^*), \ \ j \in \{2, \ldots, n\}. \end{cases} \tag{36}$$

Next, we suppose (36) and enumerate each case therein. This is prefaced with a deep analysis of $\eta_i(t)$, $i = 1, \ldots, n$. For brevity, the dependence of some functions on time, state or disturbances may be omitted in the sequel. Rewrite (14) as follows:

$$x_{i+1} = e_{i+1} + \alpha_i, \ \ i = 1, \ldots, n. \tag{37}$$

Differentiating (7) and (14), by (1) and (37), leads to

$$\dot{e}_1 = f_1 + g_1 e_2 + g_1 \alpha_1 - \dot{r} \tag{38}$$

$$\dot{e}_i = f_i + g_i e_{i+1} + g_i \alpha_i - \dot{\alpha}_{i-1}, \ \ i = 2, \ldots, n \tag{39}$$

where $e_{n+1} = 0$. Substituting (38) with (13) into (27) gives

$$\dot{\eta}_1 = \beta_{11} \times (\omega_1 + c_1 \beta_{12} g_1 \eta_1) \tag{40}$$

where

$$\omega_1 = \beta_{12} \times (f_1 + g_1 e_2 - \dot{r}) + \lambda_1. \tag{41}$$

Similarly, inserting (33) with (39) and (16) into (28) yields

$$\dot{\eta}_i = \frac{\pi}{2} \times \frac{1}{k_i} \times \frac{1}{\beta_i} \times (\omega_i - c_i g_i \eta_i), \ \ i = 2, \ldots, n \tag{42}$$

where

$$\omega_i = f_i + g_i e_{i+1} - \dot{\alpha}_{i-1} - \frac{e_i \dot{k}_i}{k_i}, \ \ i = 2, \ldots, n. \tag{43}$$

*Case 1:* At the outset, we analyze the evolution of $e_1$ by constructing

$$V_1 = \frac{1}{2} \eta_1^2. \tag{44}$$

It is straightforward to see from (40) that

$$\dot{V}_1 = \eta_1 \times \beta_{11} \times (\omega_1 + c_1 \beta_{12} g_1 \eta_1). \tag{45}$$

Due to $\bar{k}, \underline{k}, \dot{\bar{k}}, \dot{\underline{k}} \in L^\infty$, $\lambda_1$ in (29) is bounded for $t < t^*$ under (35). Continue to analyze $f_1$ and $g_1$, where

$$f_1 = f_1(x_1, x_1(t - \tau_1), d) \tag{46}$$

$$g_1 = g_1(x_1, x_1(t - \tau_1), d). \tag{47}$$

The boundedness of $e_1$ and $r$ on $[0, t^*)$ implies by (7) that $|x_1| < \infty$, $t < t^*$. According to Lemma 2 for $i = 1$ and $j = 1$, $x_1(t - \tau_1)$ is bounded over $[0, t^*)$. Note that $d \in L^\infty$ and (46) and (47) are continuous with respect to their arguments. There thus hold $|f_1| < \infty$ and $|g_1| < \infty$ on $[0, t^*)$. Recall $\beta_{12} \in L^\infty$ and $\dot{r} \in L^\infty$. The above facts together with (35) for $i = 2$ imply the boundedness of $\omega_1$ in (41) for $t < t^*$. Then, let

$$\sup_{t \in [0, t^*)} |\omega_1(t)| = \overline{\omega}_1. \tag{48}$$

Note from (30), (31) and (35) that $\beta_{11} > 0$ and $\beta_{12} < 0$ on $[0, t^*)$. This together with (48) and Assumption 1 enables us to scale (45) as follows:

$$\dot{V}_1 \leq \beta_{11} \times |\eta_1| \times \left( \overline{\omega}_1 + c_1 \beta_{12} \underline{g} |\eta_1| \right), \ \ t < t^*$$

which shows $\dot{V}_1 < 0$ as $|\eta_1| > -\overline{\omega}_1 / (c_1 \beta_{12} \underline{g})$ for $t < t^*$. This means by (44) that $|\eta_1|$ decreases once $|\eta_1| > -\overline{\omega}_1 / (c_1 \beta_{12} \underline{g})$ for $t < t^*$. It thus holds that

$$|\eta_1(t)| \leq \max \left\{ |\eta_1(0)|, \frac{-\overline{\omega}_1}{c_1 \beta_{12} \underline{g}} \right\}, \ \ t < t^*.$$

The boundedness of $\eta_1$ in (12) implies the existence of a pair of constants $\bar{\delta}_1 > 0$ and $\underline{\delta}_1 > 0$ so that

$$\underline{k}(t) < \underline{k}(t) + \underline{\delta}_1 \leq e_1(t) \leq \bar{k}(t) - \bar{\delta}_1 < \bar{k}(t), \ \ t < t^*. \tag{49}$$

As a result, $\alpha_1$ in (13) is bounded for $t < t^*$. Further, owing to the boundedness of $f_1$, $g_1$, $e_2$ and $\dot{r}$ on $[0, t^*)$, the same holds for $\dot{e}_1$ in (38). This together with (49) yields by Lemma 3 the boundedness of $\dot{\alpha}_1$ on $[0, t^*)$, which enables us to analyze the behavior of $e_2$ next.

*Case 2:* Construct

$$V_2 = \frac{1}{2} \eta_2^2.$$

The derivative of $V_2$ along (42) and (43) for $i = 2$ is given by

$$\dot{V}_2 = \frac{\pi}{2} \times \frac{1}{k_2} \times \frac{1}{\beta_2} \times \eta_2 \times (w_2 - c_2 g_2 \eta_2) \tag{50}$$

where

$$w_2 = f_2 + g_2 e_3 - \dot{\alpha}_1 - \frac{e_2 \dot{k}_2}{k_2}. \tag{51}$$

Unfold $f_2$ and $g_2$ as follows:

$$f_2 = f_2(x_1, x_1(t - \tau_2), x_2, x_2(t - \tau_2), d) \tag{52}$$

$$g_2 = g_2(x_1, x_1(t - \tau_2), x_2, x_2(t - \tau_2), d). \tag{53}$$

The boundedness of $\alpha_1$ on $[0, t^*)$ in conjunction with (35) for $i = 2$ implies by (14) for $i = 2$ that $|x_2| < \infty$, $t < t^*$. Invoking Lemma 2, there thus holds $|x_2(t - \tau_2)| < \infty$, $t < t^*$. Similarly, the boundedness of $x_1$ on $[0, t^*)$, established above, implies that $|x_1(t - \tau_2)| < \infty$, $t < t^*$. These facts together with $d \in L^\infty$ and the continuity of (52) and (53) in their arguments guarantee $|f_2| < \infty$ and $|g_2| < \infty$, $t < t^*$. Recall (17), (18), (35) and the boundedness of $\dot{\alpha}_1$ on $[0, t^*)$. Therefore, $w_2$ in (51) is bounded over $[0, t^*)$. Denote

$$\sup_{t \in [0, t^*)} |\omega_2(t)| = \overline{\omega}_2. \tag{54}$$

Note from (17), (32) and (35) that $k_2 > 0$ and $\beta_2 > 0$ as $t < t^*$. With (54) and Assumption 1, (50) is scaled by

$$\dot{V}_2 \leq \frac{\pi}{2} \times \frac{1}{k_2} \times \frac{1}{\beta_2} \times |\eta_2| \times \left( \overline{\omega}_2 - c_2 \underline{g} |\eta_2| \right), \ \ t < t^*.$$

Obviously, $\dot{V}_2 < 0$ if $|\eta_2| > \overline{\omega}_2 / (c_2 \underline{g})$ as $t < t^*$. Consequently,

$$|\eta_2(t)| \le \max\left\{|\eta_2(0)|, \frac{\overline{\omega}_2}{c_2\underline{g}}\right\},\ \ t < t^*.$$

This means by (15) and (35) for $i=2$ that $|e_2(t)|$ keeps away from $k_2(t)$ on $[0,t^*)$, i.e.,

$$|e_2(t)| \le k_2(t)-\delta_2 < k_2(t),\ \ t < t^* \tag{55}$$

where $\delta_2 > 0$ is a constant. Moreover, $\alpha_2$ in (16) is bounded over $[0,t^*)$. Recall the boundedness of $f_2$, $g_2$, $e_3$ and $\dot{\alpha}_1$ on $[0,t^*)$ established above. Thus, $\dot{e}_2$ in (39) does so in the same interval. This in company with (55) ensures by Lemma 3 that $\dot{\alpha}_2$ in (26) is bounded on $[0,t^*)$.

*Case $j$ ($j=3,\ldots,n$):* Follow the same line as in Case 2 to analyze $\eta_j$, $j=3,\ldots,n$, one by one. Then, we can conclude that there exists a set of constant $\delta_j>0$, $j=3,\ldots,n$, so that

$$\left|e_j(t)\right| \le k_j(t)-\delta_j < k_j(t),\ \ j=3,\ldots,n,\ \ t<t^*. \tag{56}$$

As seen, (49), (55) and (56) contradict (36). Therefore, (36) is false, and instead

$$\underline{k}(t) < \underline{k}(t)+\underline{\delta}_1 \le e_1(t) \le \bar{k}(t)-\overline{\delta}_1 < \bar{k}(t),\ \ t\ge 0 \tag{57}$$

and

$$|e_i(t)| \le k_i(t)-\delta_i < k_i(t),\ \ i=2,\ldots,n,\ \ t\ge 0. \tag{58}$$

Apparently, (34) is true. This means that the controller not only ensures error constraints but also precludes boundary contact. Lemma 1 further demonstrates that the output constraint in (3) and the prescribed tracking performance requirement in (5) are guaranteed.

It remains for us to verify that the rest of the signals in the control system are bounded. They include the virtual control signals, $\alpha_1,\ldots,\alpha_{n-1}$, the actual control input, $u$, and the state variables, $x_2,\ldots,x_n$. Under (57) and (58), one sees from (12) and (15) that $\eta_i \in L^\infty$, $i=1,\ldots,n$. This guarantees the boundedness of $\alpha_1$ in (13), $\alpha_i$ in (16) and $u$ in (20), $i=2,\ldots,n-1$. Further, under (34), $x_i$ in (37) is bounded, $i=2,\ldots,n$. ■

*Remark 5:* Different from the classical Lyapunov stability theory, a unified performance analysis based on proof by contradiction and BLFs is carried out in this paper. It reveals the robustness of the control system in the face of time delays, model uncertainties, disturbances, etc. As shown in (48), the integration of these unknown terms is bounded under the error constraints. This means that a finite (virtual) control input, as a linear function of the barrier function as shown in (16), is sufficient to counteract the effects of the unknown terms in the dynamics of the closed-loop system. Further, the boundedness of the barrier function in turn implies that the error keeps away from the prescribed boundary, such that it is confined in the predefined interval. The above analysis explains why the additional requirements for the time delays alluded to in Remark 1 and the dependence on commonly used tools for nonlinear control mentioned in Remark 4 are not included in this paper.

*Remark 6:* On the basis of the proposed approach, the regulation of the overshoot of the tracking error is achievable in a simple skillful way. Modify the first intermediate control law and introduce a switching rule as follows:

$$\alpha_1(t)=\begin{cases} c_1\ln\left(\dfrac{\bar{k}(t)-e_1(t)}{e_1(t)+\bar{k}(t)}\right), & \text{if } e_1(0)>0 \text{ and } t<T^* \\ c_1\ln\left(\dfrac{-\underline{k}(t)-e_1(t)}{e_1(t)-\underline{k}(t)}\right), & \text{if } e_1(0)<0 \text{ and } t<T^* \\ c_1\ln\left(\dfrac{\varepsilon-e_1(t)}{e_1(t)+\varepsilon}\right), & \text{otherwise} \end{cases} \tag{59}$$

where $T^*$ is the first time instant at which $e_1(t)=0$. Then, no overshoot of $e_1(t)$ occurs before $t=T^*$. One sees from (59) that after $t=T^*$, $\varepsilon$ serves as the bound of $|e_1|$, and thus $|e_1|<\varepsilon$ holds for $t>T^*$ which can be warranted as substantiated in the above proof. Therefore, the overshoot of $e_1$ is always less than $\varepsilon$. It is also noted from (59) that

$$\lim_{t\to T^{*-}}\alpha_1(t)=\lim_{t\to T^{*+}}\alpha_1(t)=0.$$

This means that $\alpha_1(t)$ is continuous at $T^*$, which has no impact on the recursive PPC design. In this way, a complete performance specification for reference tracking is realized, like [25], [26].

## V. Simulation Study

$$\begin{cases} \dot{x}_1(t) = -\dfrac{1}{\Theta_1}x_1(t)-K_1x_1(t)+\dfrac{1-R_2}{V_1}x_2(t) \\ \qquad +\delta_1(x_1(t-\tau_1))+d_1(t) \\ \dot{x}_2(t) = -\dfrac{1}{\Theta_2}x_2(t)-K_2x_2(t)+\dfrac{F}{V_2}u(t)+\dfrac{R_1}{V_2}x_2(t-\tau_2) \\ \qquad +\delta_2(x_1(t-\tau_2))+d_2(t). \end{cases} \tag{60}$$

In order to illustrate the above theoretical findings, a simulation study on a two-stage chemical reactor with delayed recycle streams is conducted.

The plant is modeled [36], [44] by (60) below, where $x_1$ and $x_2$ denote the reaction compositions; $K_1$ and $K_2$ are the reaction constants; $R_1$ and $R_2$ represent the recycle flow rates; $\Theta_1$ and $\Theta_2$ stand for the reactor residence times; $F$ is the feed rate; $V_1$ and $V_2$ are the reactor volumes; $\tau_1$ and $\tau_2$ are the time delays; $\delta_1$ and $\delta_2$ represent the system nonlinearities; $d_1$ and $d_2$ denote the disturbances. In the simulation, let $x_1(0)=0.5$, $x_2(0)=0.5$, $\Theta_1=\Theta_2=2$, $R_1=R_2=0.5$, $V_1=V_2=0.5$, $F=0.5$, $K_1=K_2=0.3$, $d_1=0.2\cos(0.65\pi t)$, $d_2=0.2\sin(0.65\pi t)$, $\delta_1=0.5\sin(t)x_1^2(t-\tau_1)$, $\tau_1=1+\sin(t)$, $\delta_2=0.5\sin(t)x_1^3(t-\tau_2)$, and $\tau_2=1.2+1.2\cos(t)$.

One of the control goals for (60) is to ensure the following constraint:

$$\underline{y}=-1.5<y(t)=x_1(t)<\bar{y}=1.4,\ \ \forall t\ge 0.$$

Meanwhile, steer $y(t)$ to track $r(t)=\sin(t)$ with the following transient and steady-state performance guarantees:

$$|y(t)-r(t)|<0.05,\ \ \forall t>2.5. \tag{61}$$

According to Theorem 1, a model-free controller is obtained with $c_1=13$, $c_2=5$ and

$$\varphi(t)=\begin{cases} 1-0.9833\sin(\frac{\pi t}{5}), & \text{if } t<2.5 \\ 0.0167, & \text{otherwise} \end{cases}$$

$$k_2(t)=(18-0.15)e^{-t}+0.15.$$

Applying the above controller to the time-delay nonlinear system in (60), the simulation results are displayed in Figs. 1–5. It is seen from Fig. 1 that the system output evolves within the constraint band and almost tracks the reference after 2.5 s. Certainly, it is substantiated by Fig. 2 where one can see that the tracking performance fulfills the predefined specification in (61), despite the a priori unknown reference to the controller. Similarly, the evolution of the intermediate error is also inside the prescribed performance funnel, as shown in Fig. 3. Finally, Figs. 4 and 5 show that the state variable, the intermediate control law and the control input are all bounded. Accordingly, the simulation results illustrate the effectiveness of our approach.

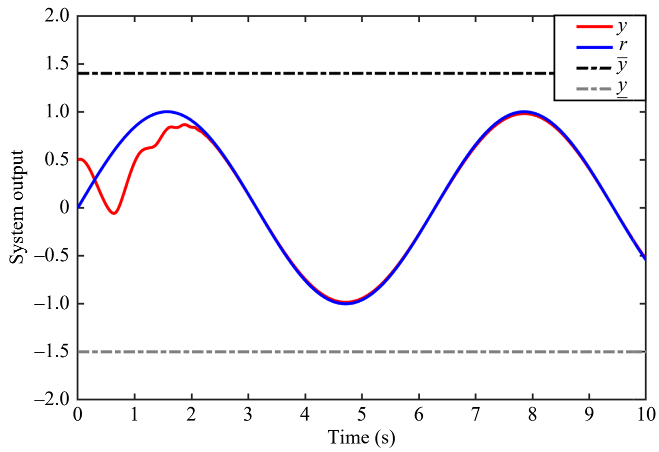


Fig. 1. Output tracking and output constraint.

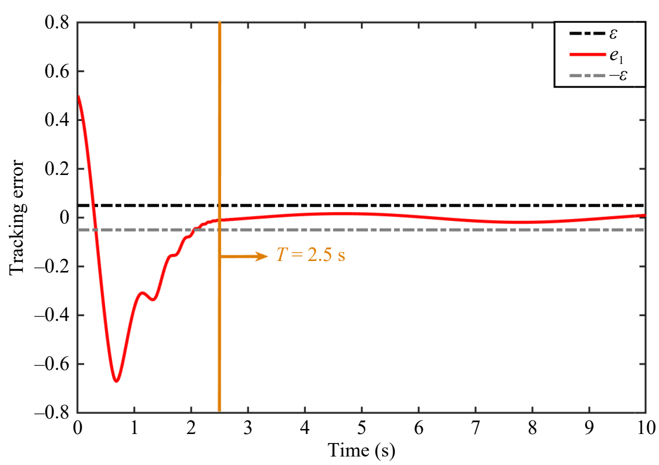


Fig. 2. Error evolution and performance requirement.

To perform a comparative study, an adaptive fuzzy controller [36] is applied to (60) with the same control objective and under the same simulation condition. The simulation results are depicted in Figs. 6 and 7. Fig. 6 shows that fast accurate reference tracking is basically realized, however, a minor performance violation occurs on [7.5 s, 8 s]. Furthermore, Fig. 7 shows three control peaks near $t = 0$ s with $u > 13\,000$, $t = 1.8$ s with $u > 4000$ and $t = 2.3$ s with $u > 1000$, respectively. However, Fig. 5 shows that the supremum of the control input by our approach is no more than 25

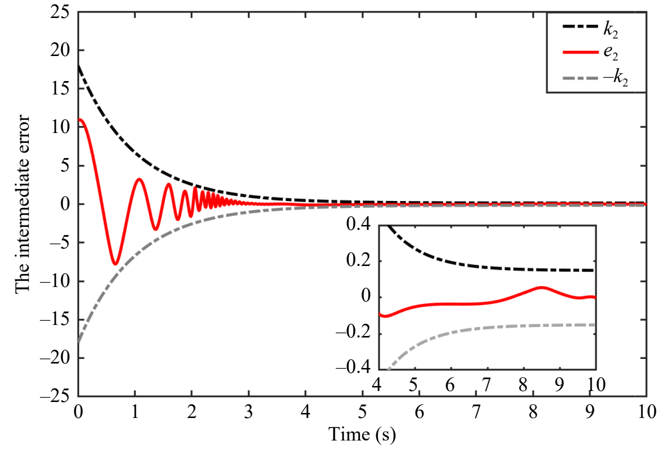


Fig. 3. The intermediate error and the prescribed boundaries.

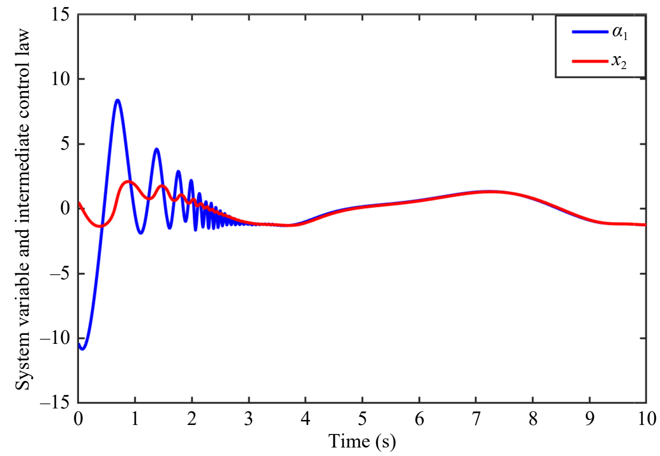


Fig. 4. The state variable and the intermediate control law.

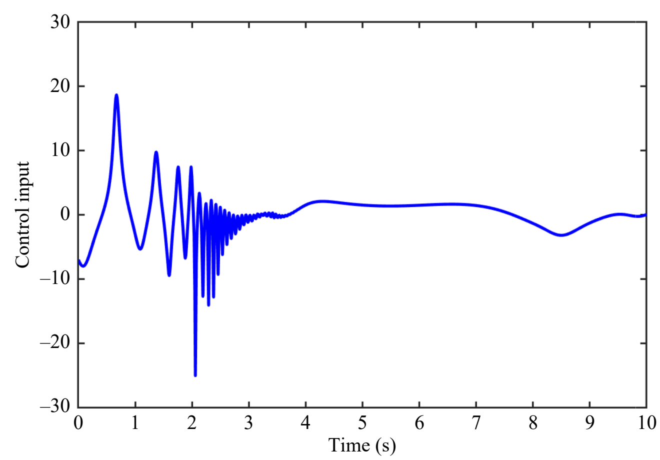


Fig. 5. The control input.

throughout. Therefore, comparative results show the superiority of our approach with its higher tracking performance and lower control cost.

## VI. Conclusion

A novel robust prescribed performance control approach for the output-constrained time-delay lower-triangular nonlinear

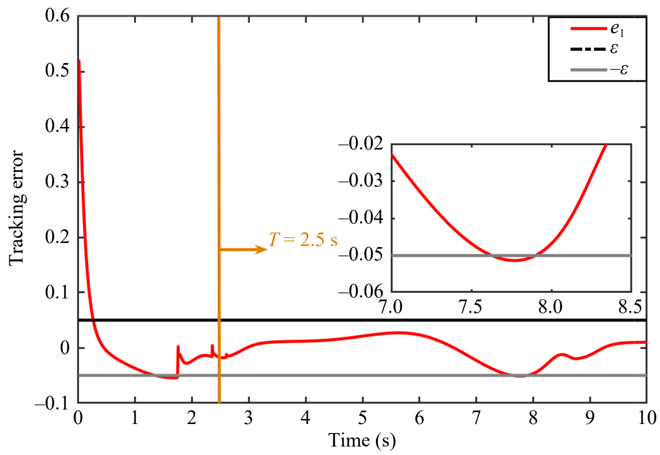


Fig. 6. The tracking error by the comparative controller [36].

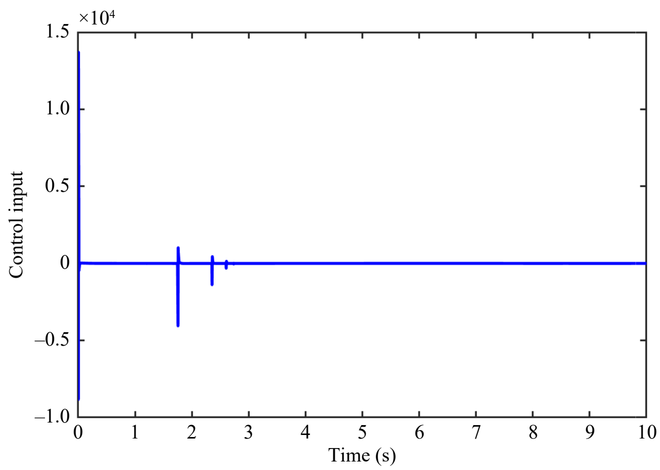


Fig. 7. The control input by the comparative controller [36].

systems with mismatched disturbances is put forward in this paper. It is suitable for cases where the reference is not known a priori; the time delays satisfy only the most fundamental requirements; the output constraints are time-varying and asymmetric; the nonlinear functions are unknown. It achieves reference tracking with the arbitrarily predefined setting time and accuracy. On the other hand, the proposed control exhibits a significant simplicity in the sense that it does not invoke techniques for estimation, adaption, identification, approximation, filtering, etc. This is attributed to the inherent robustness of our approach, which is revealed by the unified performance analysis framework based on proof by contradiction and barrier functions. For future work, it is of interest to extend such a control strategy to more complex systems, e.g., multi-agent systems.

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