Letter

Long Duration Coverage Control of Multiple Robotic Surface Vehicles Under Battery Energy Constraints

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Dear Editor,

This letter addresses long duration coverage problem of multiple robotic surface vehicles (RSVs) subject to battery energy constraints, in addition to uncertainties and disturbances. An anti-disturbance energy-aware control method is proposed for performing coverage task of RSVs. Firstly, a centroidal Voronoi tessellation (CVT) is used to optimize the partition of the given coverage area. The optimal position for each vehicle corresponds to the centroid of the Voronoi cell. Secondly, by consisting two battery energy control barrier functions, an energy-aware kinematic guidance law is designed to drive each RSV to the optimal position. Finally, an anti-disturbance fixed-time kinetic control law is designed for each RSV to track the desired speed based on the fixed-time extended state observer and nonlinear tracking differentiator. By the control method, RSVs are capable of achieving the long duration coverage within the given coverage area. Simulation results verify the effectiveness of the proposed anti-disturbance energy-aware control method for multi-RSV.

There is a surge of interest in motion control of RSVs because they are vital tools in exploring the oceans [1]–[3]. Cooperative control enables RSVs to perform more challenging task over a single RSV in terms of enhanced efficiency and effectiveness [4], [5]. In particular, coverage control is a typical cooperative control of RSVs. Such motion control scenario can find numerous missions in practice such as maritime patrol and environment monitoring [1], [6]–[9]. Coverage control is proposed for unmanned ground vehicles [6], unmanned aerial vehicles [7], and robotic surface vehicles [8]. However, the aforementioned works [6]–[9] do not explicitly guarantee the persistent coverage performance. In practice, the vehicles may not execute task persistently because of the battery energy constraints.

Motivated by the above observations, this letter aims to design an anti-disturbance energy-aware control method for RSVs to execute long duration coverage task. The main contributions of this letter can be summarized as: 1) In contrast to the works in [4], [5] where the objectives are to maintain formation, the proposed control method for multi-RSV to achieve coverage task within a given coverage area. 2) In contrast to the existing works [6]–[9] where the persistent coverage performance may not be guaranteed definitely, an energy-aware control method based on two battery energy control barrier functions (BE-CBFs) are designed to achieve long duration coverage mission. 3) In contrast to the existing coverage control methods in [8], [9] where the convergence time of estimation errors can not be calculated in a prescribed time, a fixed time extended state observer (FTESO) is proposed such that the estimation errors are convergent to zero within a fixed time.

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Problem formulation: Consider the dynamics of RSVs as

$$\begin{cases}\dot{x}_i = u_i\cos(\psi_{bi}) - \upsilon_i\sin(\psi_{bi})\\ \dot{y}_i = u_i\sin(\psi_{bi}) + \upsilon_i\cos(\psi_{bi})\\ \dot{\psi}_{bi} = r_i\\ m_{ui}\dot{u}_i = f_{ui}(u_i,\upsilon_i,r_i) + \tau_{ui} + \tau_{wui}\\ m_{\upsilon i}\dot{\upsilon}_i = f_{\upsilon i}(u_i,\upsilon_i,r_i) + \tau_{w\upsilon i}\\ m_{ri}\dot{r}_i = f_{ri}(u_i,\upsilon_i,r_i) + \tau_{ri} + \tau_{wri}\end{cases} \tag{1}$$

where x_i, y_i, and ψ_{bi} are the position and orientation in an earth-fixed inertial frame; u_i, v_i, and r_i are the velocities in the surge, sway, and yaw directions respectively in a body-fixed frame; m_{ui}, $m_{\upsilon i}$, and m_{ri} are the mass of RSVs; $f_{ui}(\cdot)$, $f_{\upsilon i}(\cdot)$, and $f_{ri}(\cdot)$ represent unknown nonlinear functions; τ_{ui} and τ_{ri} denote the surge force and yaw moment; τ_{wui}, $\tau_{w\upsilon i}$, and τ_{wri} denote the time-varying environmental disturbances. Denote $U_i = \sqrt{u_i^2+\upsilon_i^2}$ as the total speed of the ith RSV, and the dynamics (1) of the RSV can be rewritten as

$$\begin{cases}\dot{x}_i = U_i\cos(\psi_i), \quad \dot{y}_i = U_i\sin(\psi_i), \quad \dot{\psi}_i = r_i + \dot{\beta}_i\\ m_{ui}\dot{U}_i = \cos(\beta_i)(f_{ui}+\tau_{wui}) - 2\sin^2(\dfrac{\beta_i}{2})\tau_{ui}\\ \qquad + \sin(\beta_i)(f_{\upsilon i}+\tau_{w\upsilon i})m_{ui}/m_{\upsilon i} + \tau_{ui}\\ m_{ri}\dot{r}_i = f_{ri} + \tau_{ri} + \tau_{wri}\end{cases} \tag{2}$$

where $\psi_i = \psi_{bi} + \beta_i$ is the course angle with $\beta_i = \mathrm{atan2}(\upsilon_i, u_i)$.

The charging and discharging model of the battery is stated as $\dot{E}_i = -\delta K_{ch} + (1-\delta)K_{en}$, where E_i is the battery energy level of the ith RSV; $K_{ch} > 0/K_{en} > 0$ represents the case of the worst/fastest battery discharging/charging; $\delta \in [0,1]$ is the switching coefficient of battery charging and discharging.

Design and analysis: Step 1: The first step is optimizing the partition of the given coverage area. CVT is a powerful tool for the area partition. The centroid of each Voronoi cell can be defined as the optimal position for each RSV to track. Consider that N RSVs are randomly distributed in a closed and connected area $G \in \mathbb{R}^2$ that has to be covered. Associate a density function $\phi(g): g \in \mathbb{R}^+$ with each point $g \in G$. Define locational cost function of the multi-RSV as $H(P) = \sum_{i=1}^N \int_{V_i} \|p_i - g\|^2 \phi(g)dg$, where $p_i = [x_i, y_i]^T$ is the position of ith RSV; $P = [p_1, \cdots, p_N]$ is position set of RSVs; $\|p_i - g\|$ denotes the Euclidean distance metric. Next, the Voronoi cell of the ith RSV can be defined as $V_i = \{g \in G \,|\, \|g - p_i\| \le \|g - p_j\|, \forall i \ne j\}$. Then, it follows from [8] that the optimal position p_{ci} corresponds to the centroid of the Voronoi cell in a CVT is:

$$p_{ci} = \int_{V_i} g\phi(g)dg / M_{V_i} \tag{3}$$

where M_{V_i} is the mass of the V_i with $M_{V_i} = \int_{V_i}\phi(g)dg$.

Step 2: Define the target tracking error as $e_{pi} = p_i - p_{ci}$. Take the derivative of e_{pi} along (2), and one has $\dot{e}_{pi} = q_i - \dot{p}_{ci}$, where

$q_i = [q_{xi}, q_{yi}]^T = U_i[\cos(\psi_{bi}), \sin(\psi_{bi})]^T$. To stabilize e_{pi}, an optimal total speed guidance law is designed as $q_{ci} = -K_{qi}e_{pi}/\sqrt{\|e_{pi}\|^2+\Delta_{qi}^2}+\dot{p}_{ci}$, where $K_{qi} \in \mathbb{R}^2$ is the guidance law parameter vector and Δ_{qi} is a positive constant. Define the velocity tracking error as $e_{qi} = q_i - q_{ci}$ and the time derivative of the e_{pi} can be rewritten

$$\dot{e}_{pi} = -K_{qi}e_{pi}/\sqrt{\|e_{pi}\|^2+\Delta_{qi}^2}+e_{qi}. \quad (4)$$

To implement the long duration coverage control task, the position of charging station p_{bi} should satisfy $E_{\text{charge}} = \{\varepsilon_i(p_i) \in \mathbb{R}^2 | \|p_i - p_{bi}\| \le d_{ch}\}$, where $\varepsilon_i(p_i)$ is the minimum energy required for the ith RSV to return to the charging station and d_{ch} is the radius of the charging station. In order to maintain a desired energy reserve, the following inequalities $E_i - E_{\min} > \varepsilon_i(p_i)$ and $E_{\max} - E_i > 0$ hold, where $E_{\min}$ denotes the minimum capacity of battery storage and $E_{\max}$ is the maximum capacity of battery storage. It means that each RSV can return to the charging station before limited battery energy depletion and leave the charging station before overcharging. According to the definition of the CBF, the functions h_{en} and h_{cn} are defined as $h_{en}(z_i) = E_i - E_{\min} - \varepsilon_i(p_i)$ and $h_{ch}(z_i) = E_{\max}\delta(s_{i0}) - E_i$, where $z_i = [p_i^T, E_i]^T$, $s_{i0} = \|p_i - p_{i0}\|$, and $\delta(s_{i0})$ is a step function. If $s_{i0} > d_{ch}$, $\delta(s_{i0}) = 1$; otherwise, $\delta(s_{i0}) = 0$. The safe set C_{zi} in this letter is defined as $C_{zi} = \{z_i | h_{en}(z_i) \ge 0, h_{ch}(z_i) \ge 0\}$.

It follows from the safe set that the functions h_{en} and h_{cn} can be described by an equivalent $h(z_i)$ as $h(z_i) = \min\{h_{en}(z_i), h_{ch}(z_i)\}$. Thus, the speed guidance law $q_{ci}^* = [q_{xi}^*, q_{yi}^*]^T$ can be obtained by the following quadratic programming problem:

$$\begin{aligned} \arg \min_{q_{ci}^* \in \mathbb{R}^2} & J_i(q_{ci}^*) = \|q_i - q_{ci}^*\|^2 \\ \text{s.t. } & \dot{h}(z_i) \ge -\alpha h(z_i) \end{aligned} \quad (5)$$

where $\dot{h}(z_i) = (\frac{\partial h}{\partial p_i})^T q_{ci}^* - \delta K_{ch} + (1-\delta)K_{en}$. The optimal total speed of the ith RSV is derived as $U_{ci} = q_{xi}^* \cos(\psi_i) + q_{yi}^* \sin(\psi_i)$.

Step 3: Define $\psi_{ci} = \text{atan2}(q_{yi}^*, q_{xi}^*)$ and $e_{\psi i} = \psi_i - \psi_{ci}$, and the dynamics of $e_{\psi i}$ is $\dot{e}_{\psi i} = r_i + \dot{\beta}_i - \dot{\psi}_{ci}$. To stabilize $e_{\psi i}$, a yaw guidance law is designed as

$$r_{ci} = -k_{ri}e_{\psi i}/\sqrt{|e_{\psi i}|^2+\Delta_{ri}^2} - \dot{\beta}_i + \dot{\psi}_{ci} \quad (6)$$

where $k_{ri} \in \mathbb{R}$ is the guidance law parameter and Δ_{ri} is a positive constant. Define $e_{ri} = r_i - r_{ci}$, and the dynamics of the yaw target tracking error is rewritten as

$$\dot{e}_{\psi i} = -k_{ri}e_{\psi i}/\sqrt{|e_{\psi i}|^2+\Delta_{ri}^2} + e_{ri}. \quad (7)$$

Step 4: Due to the fact that the uncertainties and disturbances can affect the performance of the control system seriously [10]–[12], it is necessary to recover the uncertainties and disturbances. First, to obtain a smoother motion profile for RSV, let U_{ci} and r_{ci} pass through two nonlinear tracking differentiators as

$$\begin{cases} \dot{\hat{U}}_{ci} = U_{di} - k_{i1}^U \text{sig}^{1-1/k_{i3}^U}(\hat{U}_{ci} - U_{ci}) \\ \dot{U}_{di} = -k_{i2}^U \text{sig}^{1-2/k_{i3}^U}(\hat{U}_{ci} - U_{ci}) \\ \dot{\hat{r}}_i = r_{di} - k_{i1}^r \text{sig}^{1-1/k_{i3}^r}(\hat{r}_i - r_{ci}) \\ \dot{r}_{di} = -k_{i2}^r \text{sig}^{1-2/k_{i3}^r}(\hat{r}_i - r_{ci}) \end{cases} \quad (8)$$

where $\hat{U}_{ci}$ is the estimate of the U_{ci}; $\hat{r}_i$ is the estimate of the r_{ci}; U_{di} is the estimate of the $\dot{U}_{ci}$, and r_{di} is the estimate of the $\dot{r}_{ci}$. $k_{i1}^U \in \mathbb{R}^+$, $k_{i2}^U \in \mathbb{R}^+$, $k_{i3}^U > 2$, $k_{i1}^r \in \mathbb{R}^+$, $k_{i2}^r \in \mathbb{R}^+$, and $k_{i3}^r > 2$ are the design parameters. Define $e_{Ui}^c = \hat{U}_i - U_{ci}$, $e_{ri}^c = \hat{r}_i - r_{ci}$, $e_{Ui}^d = U_{di} - \dot{U}_{ci}$, and $e_{ri}^d = r_{di} - \dot{r}_{ci}$. It follows from (8) that the dynamics of the e_{Ui}^c, e_{ri}^c, e_{Ui}^d, and e_{ri}^d are obtained as:

$$\begin{cases} \dot{e}_{Ui}^c = e_{Ui}^d - k_{i1}^U \text{sig}^{1-1/k_{i3}^U} e_{Ui}^c \\ \dot{e}_{Ui}^d = -k_{i2}^U \text{sig}^{1-2/k_{i3}^U} e_{Ui}^c U_{di} - \ddot{U}_{ci} \\ \dot{e}_{ri}^c = e_{ri}^d - k_{i1}^r \text{sig}^{1-1/k_{i3}^r} e_{ri}^c \\ \dot{e}_{ri}^d = -k_{i2}^r \text{sig}^{1-2/k_{i3}^r} e_{ri}^c r_{di} - \ddot{r}_{ci}. \end{cases} \quad (9)$$

From (2), the kinetics of the ith RSV can be rewritten as $\dot{U}_i = g_{ui} + m_{ui}^{-1}\tau_{ui}$ and $\dot{r}_i = g_{ri} + m_{ri}^{-1}\tau_{ri}$, where $g_{ui} = m_{ui}^{-1}(\cos(\beta_i)(f_{ui} + \tau_{wui}) - 2\sin^2(\frac{\beta_i}{2})\tau_{ui} + \sin(\beta_i)(f_{vi} + \tau_{wvi})m_{ui}/m_{vi})$ and $g_{ri} = m_{ri}^{-1}(f_{ri} + \tau_{wri})$. Two fixed-time ESOs are designed to estimate the unknown functions g_{ui} and g_{ri}

$$\begin{aligned} \dot{\hat{U}}_i &= -k_{i1}^u \lceil \tilde{U}_i \rfloor^{\alpha_{ui}} - k_{i2}^u \lceil \tilde{U}_i \rfloor^{\beta_{ui}} + \hat{g}_{ui} + m_{ui}^{-1}\tau_{ui} \\ \dot{\hat{g}}_{ui} &= -k_{i3}^u \lceil \tilde{U}_i \rfloor^{2\alpha_{ui}-1} - k_{i4}^u \lceil \tilde{U}_i \rfloor^{2\beta_{ui}-1} + k_{i0}^u \text{sign}(\tilde{U}_i) \\ \dot{\hat{r}}_i &= -k_{i1}^r \lceil \tilde{r}_i \rfloor^{\alpha_{ri}} - k_{i2}^r \lceil \tilde{r}_i \rfloor^{\beta_{ri}} + \hat{g}_{ri} + m_{ri}^{-1}\tau_{ri} \\ \dot{\hat{g}}_{ri} &= -k_{i3}^r \lceil \tilde{r}_i \rfloor^{2\alpha_{ri}-1} - k_{i4}^r \lceil \tilde{r}_i \rfloor^{2\beta_{ri}-1} + k_{i0}^r \text{sign}(\tilde{r}_i) \end{aligned} \quad (10)$$

where $k_{ia}^u \in \mathbb{R}^+$ and $k_{ia}^r \in \mathbb{R}^+ (a = 1,2,3,4)$ are the observer gains; $\hat{U}_i$, $\hat{r}_i$, $\hat{g}_{ui}$, and $\hat{g}_{ri}$ are the estimates of U_i, r_i, g_{ui}, and g_{ri} respectively; $\tilde{U}_i = \hat{U}_i - U_i$, $\tilde{r}_i = \hat{r}_i - r_i$, $\alpha_{ui} \in (1-\varepsilon_{ui}, 1)$, $\beta_{ui} = 1/\alpha_{ui}$, $\alpha_{ri} \in (1-\varepsilon_{ri}, 1)$, and $\beta_{ri} = 1/\alpha_{ri}$ with ε_{ui} and ε_{ri} being the small constants; $k_{i0}^u > 0$ and $k_{i0}^r > 0$.

Assumption 1: For unknown functions g_{ui} and g_{ri}, there are $g_{ui}^* \in \mathbb{R}^+$ and $g_{ri}^* \in \mathbb{R}^+$ such that $|\dot{g}_{ui}| \le g_{ui}^*$ and $|\dot{g}_{ri}| \le g_{ri}^*$.

Step 5: Let $\tilde{g}_{ui} = \hat{g}_{ui} - g_{ui}$ and $\tilde{g}_{ri} = \hat{g}_{ri} - g_{ri}$. It follows from equation (10) that the error dynamics can be expressed as:

$$\begin{aligned} \dot{\tilde{U}}_i &= -k_{i1}^u \lceil \tilde{U}_i \rfloor^{\alpha_{ui}} - k_{i2}^u \lceil \tilde{U}_i \rfloor^{\beta_{ui}} + \tilde{g}_{ui} \\ \dot{\tilde{g}}_{ui} &= -k_{i3}^u \lceil \tilde{U}_i \rfloor^{2\alpha_{ui}-1} - k_{i4}^u \lceil \tilde{U}_i \rfloor^{2\beta_{ui}-1} - \dot{g}_{ui} + k_{i0}^u \text{sign}(\tilde{U}_i) \\ \dot{\tilde{r}}_i &= -k_{i1}^r \lceil \tilde{r}_i \rfloor^{\alpha_{ri}} - k_{i2}^r \lceil \tilde{r}_i \rfloor^{\beta_{ri}} + \tilde{g}_{ri} \\ \dot{\tilde{g}}_{ri} &= -k_{i3}^r \lceil \tilde{r}_i \rfloor^{2\alpha_{ri}-1} - k_{i4}^r \lceil \tilde{r}_i \rfloor^{2\beta_{ri}-1} - \dot{g}_{ri} + k_{i0}^r \text{sign}(\tilde{r}_i). \end{aligned} \quad (11)$$

Define the tracking errors as $e_{Ui} = U_i - \hat{U}_{ci}$ and $e_{ri} = r_i - \hat{r}_{ci}$. Taking the derivatives of e_{Ui} and e_{ri} yields

$$\begin{cases} \dot{e}_{Ui} = g_{ui} + m_{ui}^{-1}\tau_{ui} - \dot{\hat{U}}_{ci} \\ \dot{e}_{ri} = g_{ri} + m_{ri}^{-1}\tau_{ri} - \dot{\hat{r}}_{ci}. \end{cases} \quad (12)$$

Two anti-disturbance fixed-time kinetic control laws are designed

$$\begin{cases} \tau_{ui} = m_{ui}(-k_{i1}^\tau \lceil e_{Ui} \rfloor^{\alpha_{ui}^\tau} - k_{i2}^\tau \lceil e_{Ui} \rfloor^{\beta_{ui}^\tau} - \hat{g}_{ui} + \dot{\hat{U}}_{ci}) \\ \tau_{ri} = m_{ri}(-k_{i3}^\tau \lceil e_{ri} \rfloor^{\alpha_{ri}^\tau} - k_{i4}^\tau \lceil e_{ri} \rfloor^{\beta_{ri}^\tau} - \hat{g}_{ri} + \dot{\hat{r}}_{ci}) \end{cases} \quad (13)$$

where $k_{ia}^\tau \in \mathbb{R}^+ (a = 1,2,3,4)$ are the control gains. Then, the tracking error systems (12) can be put into $\dot{e}_{Ui} = -k_{i1}^\tau \lceil e_{Ui} \rfloor^{\alpha_{ui}^\tau} - k_{i2}^\tau \lceil e_{Ui} \rfloor^{\beta_{ui}^\tau} - \tilde{g}_{ui}$ and $\dot{e}_{ri} = -k_{i3}^\tau \lceil e_{ri} \rfloor^{\alpha_{ri}^\tau} - k_{i4}^\tau \lceil e_{ri} \rfloor^{\beta_{ri}^\tau} - \tilde{g}_{ri}$.

Lemma 1: The tracking error subsystem consisting of (4) and (7): $[e_{qi}, e_{ri}] \mapsto [e_{pi}, e_{\psi i}]$ is input-to-state stable (ISS).

Proof: Choose the Lyapunov function as $V_{i1}^e = \sum_{i=1}^N (1/2)(e_{pi}^T e_{pi} + e_{\psi i}^2)$. The time derivative of V_{i1}^e is $\dot{V}_{i1}^e \le \sum_{i=1}^N (-\lambda_{\max}(K_{i1})\|H_{i1}\|^2/\sqrt{\|H_{i1}\|^2+\delta_{i\max}} + \|h_{i1}\|\|H_{i1}\|)$, where $H_{i1} = [e_{pi}^T, e_{\psi i}]$, $K_{i1} = \text{diag}\{K_{qi}, K_{ri}\}$, $\delta_{i\max} = \max\{\delta_{iq}, \delta_{ir}\}$, $h_{i1} = [e_{qi}, e_{ri}]$, and $\lambda_{\min}(K_{i1})$ is the minimum eigenvalue of a square matrix K_{i1}. Note that as $\|H_{i1}\|^2/\sqrt{\|H_{i1}\|^2+\delta_{i\max}} \ge \|h_{i1}\|/(a_{i1}\lambda_{\min}(K_{i1}))$, $\dot{V}_{i1}^e \le -\sum_{i=1}^N \{\lambda_{\min}(K_{i1})(1-a_{i1})\}$. ■

Lemma 2: Under Assumption 1, with the FTESO (10), the states $\hat{U}_i$, $\hat{g}_{ui}$, $\hat{r}_i$, and $\hat{g}_{ri}$ can achieve the estimations in a fixed time.

Proof: Take the following part from error dynamics (11):

$$\begin{cases} \dot{\tilde{U}}_i = -k_{i1}^u \lceil \tilde{U}_i \rfloor^{\alpha_{ui}} + \tilde{g}_{ui} \\ \dot{\tilde{g}}_{ui} = -k_{i3}^u \lceil \tilde{U}_i \rfloor^{2\alpha_{ui}-1}. \end{cases} \quad (14)$$

Define $F_{i1} = [\tilde{U}_i, \tilde{g}_{ui}]^T$ and $\zeta_{i1} = [\tilde{U}_i, ((\tilde{g}_{ui})^{1/\alpha_{ui}})]^T$. Consider a Lyapunov function $V^u_{i1}(\zeta_{i1}) = \zeta^T_{i1}P_{i1}\zeta_{i1}$, where P_{i1} is a symmetric positive definite matrix satisfying $P_{i1}A_{i1} + A^T_{i1}P_{i1} = -I$ and A_{i1} is a Hurwitz matrix with $A_{i1} = [-k^u_{i1}, 1; -k^u_{i3}, 0]$. Setting $\alpha_{ui} = 1$, the equation (14) becomes $\dot{F}_{i1} = A_{i1}F_{i1}$. Taking the time derivatives of $V^u_{i1}(F_{i1})$ gives $\dot{V}^u_{i1}(F_{i1}) = F^T_{i1}(P_{i1}A_{i1} + A^T_{i1}P_{i1})F_{i1} = -F^T_{i1}F_{i1} \le 0$. Define σ_{ui} as a small constant, and (14) is locally asymptotically stable by choosing α_{ui} in the interval $(1-\sigma_{ui}, 1)$. Then, from Lemma 2 in [5], $V^u_{i1}(\zeta_{i1})$ is homogeneous of degree ϱ_{i1} with $(0, \alpha_{ui})$ and its derivative $\dot{V}^u_{i1}(\zeta_{i1})$ is homogeneous of degree $\varrho_{i1} + \alpha_{ui} - 1$. One has $V^u_{i1}(F_{i1}) \le \lambda_{\max}(P_i)\|F_{i1}\|^2$ and $\dot{V}^u_{i1}(F_{i1}) \le -\|F_{i1}\|^2$. It renders that $\dot{V}^u_{i1}(F_{i1}) \le -V^u_{i1}(F_{i1})/\lambda_{\max}(P_{i1})$. The second part of (11) is expressed as $\dot{\tilde{U}}_i = -k^u_{i2}\lceil\tilde{U}_i\rfloor^{\beta_{ui}} + \tilde{g}_{ui}$ and $\dot{\tilde{g}}_{ui} = -k^u_{i4}\lceil\tilde{U}_i\rfloor^{2\beta_{ui}-1}$. Define $\zeta_{i2} = [\tilde{U}_i, ((\tilde{g}_{ui})^{1/\beta_{ui}})]^T$. Consider a Lyapunov function $V_{i2}(\zeta_{i2}) = \zeta^T_{i2}P_{i2}\zeta_{i2}$. Similar to the analysis in (14), one has $\dot{V}_{i2}(\zeta_{i2}) \le -V_{i2}(\zeta_{i2})^{(\varrho_{i2}+\beta_{ui}-1)/\varrho_{i2}}/\lambda_{\max}(P_{i2})$. For

$$\begin{cases}\dot{\tilde{U}}_i = -k^u_{i1}\lceil\tilde{U}_i\rfloor^{\alpha_{ui}} - k^u_{i2}\lceil\tilde{U}_i\rfloor^{\beta_{ui}} + \tilde{g}_{ui}\\ \dot{\tilde{g}}_{ui} = -k^u_{i3}\lceil\tilde{U}_i\rfloor^{2\alpha_{ui}-1} - k^u_{i4}\lceil\tilde{U}_i\rfloor^{2\beta_{ui}-1}\end{cases} \tag{15}$$

there exists ι_{ui} such that $V^u_{i2}(\zeta_{i2}(t_0)) > \iota_{ui}$ with ι_{ui} being $0 < \iota_{ui} < \lambda_{\min}(P_{i2})$. It renders that V^u_{i2} reaches ι_{ui} within a fixed-time $T^u_{i1} = \varrho_{i2}\lambda_{\max}(P_{i2})/(\beta_{ui}-1)\iota_{ui}^{(\beta_{ui}-1)/\varrho_{i2}}$. Besides, the inequalities $\|\zeta_{i2}\|^2 \le V^u_{i2}(\zeta_{i2})/\lambda_{\min}(P_{i2}) \le \iota_{ui}/\lambda_{\min}(P_{i2}) \le 1$ and $V^u_{i1}(\zeta_{i1}) \le \lambda_{\max}(P_{i1})\|\zeta_{i1}\|^2$ hold. If $V^u_{i1}(\zeta_{i1}) < \iota_{ui}$ and $\|\zeta_{i1}\| = 1$, $\|\zeta_{i1}\|^2 \le \|\zeta_{i2}\|^2 \le 1$ and $V_{i1}(\zeta_{i1}) \le \lambda_{\max}(P_{i1})$. The settling time of the system (14) satisfies $T^u_{i2} \le \varrho_{i1}\lambda_{\max}^{(\varrho_{i1}+\alpha_{ui}-1)/\varrho_{i1}}(P_{i1})/(1-\alpha_{ui})$. As a result, the states of the system (15) are convergent to zero within a fixed time $T^u_{i1} + T^u_{i2}$. If $t \ge T^u_{i1} + T^u_{i2}$, the system (11) becomes $\dot{\tilde{g}}_{ui} = k^u_{i0}\mathrm{sgn}(\tilde{U}_i) - \dot{g}_{ui}$. Consider a Lyapunov function $V^u_{i3} = (1/2)(\tilde{g}_{ui})^2$. The derivative of V^u_{i3} can be obtained as $\dot{V}^u_{i3} \le -(k^u_{i0} - g^*_{ui})\sqrt{(2V^u_{i3})}$. In summary, $\tilde{g}_{ui}$ is convergent to zero within the time $T^u_{i3} \le \sqrt{2V^u_{i3}(T^u_{i1}+T^u_{i2})}/(k^u_{i0} - g^*_{ui})$. Thus, the states $\tilde{U}_i$ and $\tilde{g}_{ui}$ are convergent to zero within a fixed time $T_{ui} \le T^u_{i1} + T^u_{i2} + T^u_{i3}$ regardless of the initial conditions. Similarly, the states $\tilde{r}_i$ and $\tilde{g}_{ri}$ achieve the estimations within the fixed time $T_{ri} \le T^r_{i1} + T^r_{i2} + T^r_{i3}$ with $T^r_{i1} = \varrho_{i4}\lambda_{\max}(P_{i4})/(\beta_{ri}-1)\iota_{ri}^{(\beta_{ri}-1)/\varrho_{i4}}$, $T^r_{i2} \le \varrho_{i3}\lambda_{\max}^{(\varrho_{i3}+\alpha_{ri}-1)/\varrho_{i3}}(P_{i3})/(1-\alpha_{ri})$, and $T^r_{i3} \le \sqrt{2V^r_{i3}(T^r_{i1}+T^r_{i2})}/(k^r_{i0} - g^*_{ri})$ regardless of the initial conditions. The definitions of P_{i2}, P_{i3}, P_{i4}, A_{i2}, A_{i3}, A_{i4}, ϱ_{i2}, ϱ_{i3}, ϱ_{i4}, and ι_{ri} analogize the definitions of P_{i1}, A_{i1}, ϱ_{i1}, and ι_{ui}. ■

Theorem 1: Under Assumption 1, by using the optimal total speed guidance law (5), the yaw guidance law (6), the nonlinear tracking differentiator (8), the fixed-time ESOs (10), and the anti-disturbance kinetic fixed-time control laws (12), the RSVs described by (2) are able to cover the centroid of the Voronoi cell (3) at any initial sates with the safe set C_{zi}. Besides, all the errors in the closed-loop system are uniformly ultimately bounded.

Proof: According to Lemma 1, the tracking errors e_{qi}, e_{pi}, and $e_{\psi i}$ are uniformly ultimately bounded (UUB). By [5], it can obtain that the errors e^c_{Ui}, e^c_{ri}, e^d_{Ui}, and e^d_{ri} can converge to a small neighborhood of the origin. Construct the Lyapunov function as $V_{i4} = (1/2)(e^2_{Ui} + e^2_{ri})$. Taking the time derivative of the V_{i4} along (12), it can be obtained that $\dot{V}_{i4} \le -2^{(1+\alpha^\tau_{ui})/2}k^\tau_{i1}(1/2|e_{Ui}|^2)^{1+\alpha^\tau_{ui}} - 2^{(1+\beta^\tau_{ui})/2}k^\tau_{i2} \times (1/2|e_{Ui}|^2)^{1+\beta^\tau_{ui}} - 2^{(1+\alpha^\tau_{ri})/2}k^\tau_{i3}((1/2)|e_{ri}|^2)^{1+\alpha^\tau_{ri}} - 2^{(1+\beta^\tau_{ri})/2}k^\tau_{i4}((1/2)|e_{ri}|^2)^{1+\beta^\tau_{ri}} + g^*_{ui}|e_{Ui}| + g^*_{ri}|e_{ri}|$. Then, the following inequality holds $\dot{V}_{i4} \le -k_{i1}(V_{i4})^{\alpha_i} - 2^{1-\beta_i}k_{i2}(V_{i4})^{\beta_i} + g^*_{ui}|e_{Ui}| + g^*_{ri}|e_{ri}|$, where $k_{i1} > 0$, $k_{i2} > 0$, $1 > \alpha_i > 0$, and $\beta_i > 1$. It follows from Lemma 2 that the errors $|\tilde{g}_{ui}|$ and $|\tilde{g}_{ri}|$ converge to zero within a fixed-time and $\dot{V}_{i4} \le -k_{i1}(V_{i4})^{\alpha_i} - 2^{1-\beta_i}k_{i3}(V_{i4})^{\beta_i}$, where $k_{i3} = 2^{1-\beta_i}k_{i2}$. Thus, all the tracking errors in the entire closed-loop system are UUB. ■

Simulation: Consider a multi-RSV system including six RSVs. The environment has the xy dimensions of $100\,\mathrm{m} \times 100\,\mathrm{m}$ and $\phi(q) = 1$. The initial states of six charging station and six RSVs are set to $p_{bi} = [2, 85 - 15(i-1)]^T$ $(i = 1, \dots, 6)$, $p_1 = [28, 15]^T$, $p_2 = [65, 43]^T$, $p_3 = [37, 20]^T$, $p_4 = [12, 15]^T$, $p_5 = [55, 50]^T$, $p_6 = [51, 12]^T$, $\psi_{b1} = \psi_{b6} = 0$, $\psi_{b2} = \pi/2$, $\psi_{b3} = -4\pi/5$, $\psi_{b4} = \pi/4$, $\psi_{b5} = \pi$, $q_i = [0, 0]^T$, $r_i = 0$, $E_1 = 3600$ mV, $E_2 = 3900$ mV, $E_3 = 4100$ mV, $E_4 = 3800$ mV, $E_5 = 4000$ mV, and $E_6 = 3700$ mV. The parameters in this case are set to $K_{ch} = 10$, K_{en}, $k = 0.02$, d_{ch}, $c = 0.5$, $E_{\max} = 4200$ mV, and $E_{\min} = 3000$ mV, $k^U_{i1} = k^U_{i2} = k^r_{i1} = k^U_{i2} = 10$, $k^U_{i3} = k^r_{i3} = 2.5$, $k^u_{i1} = k^u_{i2} = k^r_{i1} = k^r_{i2} = 20$, $k^u_{i3} = k^u_{i4} = k^r_{i3} = k^r_{i4} = 100$, $k^u_{i0} = k^r_{i0} = 0.05$, $\alpha_{ui} = \alpha_{ri} = 0.9$, and $\beta_{ui} = \beta_{ri} = 1.1$, $K_{qi} = \mathrm{diag}\{1, 1\}$, $k_{ri} = 0.5$, $k^\tau_{i1} = k^\tau_{i2} = k^\tau_{i3} = k^\tau_{i4} = 1$, $\alpha^\tau_{ui} = \alpha^\tau_{ri} = 0.9$, $\beta^\tau_{ui} = \beta^\tau_{ri} = 1.1$, and $\Delta_{qi} = \Delta_{ri} = 1$. The long duration coverage behavior and the energy change curve of the RSV battery are depicted in Fig. 1. It shows that the optimal coverage of the given coverage area is achieved for the first time at 125 s. At 220 s, the red RSV returns to the charging station for charging, and other RSVs continue the long duration coverage task. Next, the red RSV leaves the charging station to perform the coverage task. Then, at 630 s, the RSVs complete the optimal coverage for the second time.

Conclusion: In this letter, an anti-disturbance energy-aware control method for RSVs is proposed based on a CVT, two BE-CBFs,

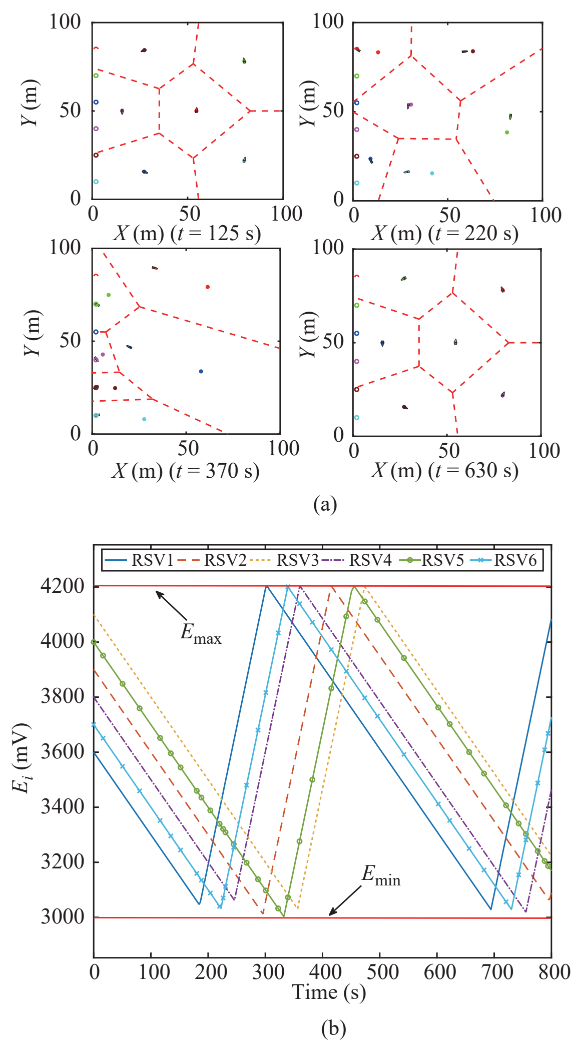

Fig. 1. Simulation results. (a) The coverage behavior; (b) The energy change curve.

and FTESO to achieve long duration coverage task. By the control method, RSVs are capable of achieving the long duration coverage within the given coverage area.

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