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Immersion and Invariance Based Composite Adaptive Control for Nonlinear Systems with Both Parametric and Non-Parametric Uncertainties

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Abstract: The design of an immersion and invariance (I&I) based composite adaptive control for a class of uncertain nonlinear systems is presented in this paper. The key feature of this control scheme lies in the construction of the novel adaptive laws, aiming to address both parametric and non-parametric uncertainties simultaneously. Composite adaptive laws, which are driven by both the information of tracking error and prediction error, are first proposed using I&I theory for the estimations of parametric uncertainties. Then the technique of $\boldsymbol{\sigma}$-modification is used to guarantee the stability in the presence of non-parametric uncertainties. Stability analysis is presented using the Lyapunov theory. Improved performance of the proposed control scheme is observed via numerical simulations.

Keywords: Immersion and invariance, Composite adaptive, Adaptive control, Parametric and non-parametric uncertainties

1. INTRODUCTION

The problem of control design for uncertain nonlinear systems has been widely discussed in the literature (Astrom & Wittenmark, 2008; Hassan, 2001; Ioannou & Sun, 1996; Krstic, Kanellakopoulos, & Kokotovic, 1995). Various control system architectures have been proposed to address the issue of uncertainty. Adaptive control is one of the pioneer works, which is originally developed to achieve the control of high-performance aircraft over a large envelope. Many efforts have been made for the development of this technology during the past decades, and many applications have been reported as well(Hu, Yao, Chen, & Wang, 2011; Li, Lu, Liu, & Li, 2018; Ma, Lum, & Ge, 2007; Roy & Baldi, 2019). Despite years of research, it is extensively recognized that more effective and physical adaptive methods are required for its further application to engineering practice.

The basic objective of adaptive control is to maintain consistent performance of a system in the presence of parameter uncertainty(Slotine & Li, 1991), which shows great potential in many practical applications. New knowledge to advance adaptive control has appeared constantly. One remarkable contribution is made by Astolfi (Astolfi, Karagiannis, & Ortega, 2008; Astolfi & Ortega, 2003), who proposed a new framework of adaptive control called immersion and invariance (I&I). With I&I, the knowledge of a control Lyapunov function is no longer necessary at the control/adaptive law design level. Moreover, an additional nonlinear term, together with the parameter adaptive law, is introduced for the unknown parameter estimate, which allows a flexible construction of the estimate error dynamics. It is of increasing interest in the research of I&I theory since it was first proposed, and many results have been reported in various fields(Chen & Astolfi, 2018; X. B. Liu, Ortega, Su, & Chu, 2010; Lou & Zhao, 2018; Monaco & Normand-Cyrot, 2015; Tagne, Tali, & Charara, 2016). However, only parameter information from the state tracking errors is used in the above results to update the adaptive laws, while no information of the parameter estimation errors, namely the prediction errors, is considered. Those adaptive laws who are driven by both tracking errors and prediction errors are called *composite adaptive* ones, which usually yield better parameter estimations (Slotine & Li, 1991). Therefore, it is important for control engineers to discuss how to use the information of these two parameter sources for the design of I&I-based adaptive laws.

Compared with constant improvements of fundamental research, the engineering applications of adaptive control haven't developed too much. One significant case is that the most commonly used control method in actual aircraft control so far is gain-scheduling rather than adaptive control. The reason behind this is that the standard adaptive control is mainly used to deal with parametric uncertainty, while it cannot guarantee the stability in the presence of non-parametric uncertainties(Ioannou & Sun, 1996; Slotine & Li, 1991). However, non-parametric uncertainties, like model uncertainties and external disturbances, exist widely in practice. This motivates many researchers to address this issue, which builds up a new work known as *robust adaptive control*. It appears that no attempt has been made to develop a robust adaptive scheme based on the composite I&I adaptive theory.

Building upon our recent work(Han, Liu, & Yi, 2018; Z. Liu, Han, Yuan, Fan, & Yi, 2017), a composite robust adaptive control based on I&I is proposed here to address this problem. Two sources of parameter information, including tracking error and prediction error, are used for the design of adaptive laws. Moreover, the technique of σ-modification is introduced to further modify the adaptive laws so that the

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control scheme can guarantee the stability in the presence of non-parametric uncertainties. The rest of the paper is organized as follows. First, the main theoretical results are presented in Section II, where the composite robust adaptive control for uncertain nonlinear systems is derived using I&I. Then simulation results are shown in Section III, and finally conclusions are offered in Section IV.

2. MAIN RESULTS

2.1 Problem Definition

Consider the following nth-order nonlinear system

$$\begin{cases} \dot{x}_1 = f_1(\boldsymbol{x}) + g_1(\boldsymbol{x})x_2 + \Delta_1 \\ \quad\vdots \\ \dot{x}_i = f_i(\boldsymbol{x}) + g_i(\boldsymbol{x})x_{i+1} + \Delta_i \\ \quad\vdots \\ \dot{x}_n = f_n(\boldsymbol{x}) + g_n(\boldsymbol{x})u + \Delta_n \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T$ is the state vector, u is the scalar control input, Δ_i is the lumped disturbances, and $f_i(\boldsymbol{x})$, $g_i(\boldsymbol{x})$ are nonlinear functions of the states. Two assumptions are expressed as follows.

Assumption 1. The functions $f_i(\boldsymbol{x})$ and $g_i(\boldsymbol{x})$ can be described by a regressive form

$$\begin{cases} f_i(\boldsymbol{x}) = \boldsymbol{\theta}_{fi}^T \boldsymbol{\phi}_{fi}(\boldsymbol{x}) \\ g_i(\boldsymbol{x}) = \boldsymbol{\theta}_{gi}^T \boldsymbol{\phi}_{gi}(\boldsymbol{x}) \end{cases} \tag{2}$$

where $\boldsymbol{\theta}_{fi}$ and $\boldsymbol{\theta}_{gi}$ are the vectors of unknown constant parameters, and $\boldsymbol{\phi}_{fi}(\boldsymbol{x})$ and $\boldsymbol{\phi}_{gi}(\boldsymbol{x})$ are the known "regressors". This is the so-called linear parametrization condition which is common in adaptive control to denote parametric uncertainties. Since not all system uncertainties can be parameterized, Δ_i is introduced for non-parametric uncertainties.

Assumption 2. The non-parametric uncertainty Δ_i in the system (1) is bounded,

$$|\Delta_i| \le \eta_i, \quad i = 1, 2, \cdots, n \tag{3}$$

where η_i is unknown positive constant.

The control problem is to find a continuous adaptive control law of the form

$$\begin{cases} \dot{\boldsymbol{\vartheta}}_{fi} = \dot{\hat{\boldsymbol{\theta}}}_{fi\_T}(\boldsymbol{x}, \boldsymbol{\vartheta}_{fi}) + \dot{\hat{\boldsymbol{\theta}}}_{fi\_P}(\boldsymbol{x}, \boldsymbol{\vartheta}_{fi}), \ i = 1, 2, \cdots n \\ \dot{\boldsymbol{\vartheta}}_{gi} = \dot{\hat{\boldsymbol{\theta}}}_{gi\_T}(\boldsymbol{x}, \boldsymbol{\vartheta}_{gi}) + \dot{\hat{\boldsymbol{\theta}}}_{gi\_P}(\boldsymbol{x}, \boldsymbol{\vartheta}_{gi}), \ i = 1, 2, \cdots n \\ u = u(\boldsymbol{x}, \boldsymbol{\vartheta}) \end{cases} \tag{4}$$

such that all trajectories of the closed-loop system (1) with (4) are bounded and $\lim_{t\to\infty}\left[x_1(t) - x_{1c}\right] = 0$, where $\boldsymbol{\vartheta}_{fi}$ is the estimate of $\boldsymbol{\theta}_{fi}$, with $\hat{\boldsymbol{\theta}}_{fi\_T}$, $\hat{\boldsymbol{\theta}}_{fi\_P}$ denoting tracking-error based estimation and prediction-error based estimation respectively, and $\boldsymbol{\vartheta}_{gi}, \hat{\boldsymbol{\theta}}_{gi\_T}, \hat{\boldsymbol{\theta}}_{gi\_P}$ have the same denotation as the above.

2.2 Composite Adaptive Control Based on Immersion and Invariance

To begin with, the following expressions are presented as

$$\boldsymbol{\theta}_i = \begin{bmatrix} \boldsymbol{\theta}_{fi} \\ \boldsymbol{\theta}_{gi} \end{bmatrix}, \ \boldsymbol{\vartheta}_i = \begin{bmatrix} \boldsymbol{\vartheta}_{fi} \\ \boldsymbol{\vartheta}_{gi} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{fi\_T} + \hat{\boldsymbol{\theta}}_{fi\_P} \\ \hat{\boldsymbol{\theta}}_{gi\_T} + \hat{\boldsymbol{\theta}}_{gi\_P} \end{bmatrix}$$

where $\boldsymbol{\theta}_i$ consists of unknown parameters and $\boldsymbol{\vartheta}_i$ consists of their estimates. Then the estimate error can be written as

$$\boldsymbol{z}_i = \begin{bmatrix} \boldsymbol{z}_{fi} \\ \boldsymbol{z}_{gi} \end{bmatrix} = \boldsymbol{\vartheta}_i - \boldsymbol{\theta}_i = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{fi\_T} + \hat{\boldsymbol{\theta}}_{fi\_P} - \boldsymbol{\theta}_{fi} \\ \hat{\boldsymbol{\theta}}_{gi\_T} + \hat{\boldsymbol{\theta}}_{gi\_P} - \boldsymbol{\theta}_{gi} \end{bmatrix} \tag{5}$$

Before further discussion, the compensated tracking errors, which are used in the command filtered backstepping approach to calculate the derivatives of intermediate virtual controls(Farrell, Polycarpou, & Sharma, 2004), are defined as

$$\bar{x}_i = \tilde{x}_i - \zeta_i, \ i = 1, 2, \cdots n \tag{6}$$

where $\tilde{x}_i = x_i - x_{ic}$ is the actual tracking error, and ζ_i is defined as

$$\begin{aligned} \dot{\zeta}_i &= -k_i\zeta_i + \boldsymbol{\vartheta}_{gi}{}^T \cdot \boldsymbol{\phi}_{gi}(\boldsymbol{x}) \cdot (x_{i+1,c} - x_{i+1,cd} + \zeta_{i+1}), \ i = 1, 2, \cdots n-1 \\ \dot{\zeta}_n &= -k_n\zeta_n + \boldsymbol{\vartheta}_{gn}{}^T \cdot \boldsymbol{\phi}_{gn}(\boldsymbol{x}) \cdot (u_c - u_{cd}) \end{aligned} \tag{7}$$

with $k_i > 0$. $x_{i+1,cd}$ shall be designed by the dynamics of $\bar{x}_i$, and $x_{i+1,c}$ is generated by passing $x_{i+1,cd}$ through a command filter.

Step 1: The dynamics of the actual tracking error $\tilde{x}_1$ can be written as

$$\dot{\tilde{x}}_1 = \boldsymbol{\theta}_{f1}^T \cdot \boldsymbol{\phi}_{f1}(\boldsymbol{x}) + \boldsymbol{\theta}_{g1}^T \cdot \boldsymbol{\phi}_{g1}(\boldsymbol{x}) \cdot x_2 + \Delta_1 - \dot{x}_{1c} \tag{8}$$

Select the nominal virtual control signal x_{2cd} as

$$\boldsymbol{\vartheta}_{g1}{}^T \boldsymbol{\phi}_{g1}(\boldsymbol{x}) x_{2cd} = -k_1\tilde{x}_1 - \boldsymbol{\vartheta}_{fi}{}^T \boldsymbol{\phi}_{f1}(\boldsymbol{x}) + \dot{x}_{1c} \tag{9}$$

Passing x_{2cd} through a command filter can generate the signal of x_{2c} and $\dot{x}_{2c}$, where x_{2c} is the new variable yet to be tracked in the next step. Using (6), (8), and (9) gives the dynamics of the compensated tracking error

$$\dot{\bar{x}}_1 = -k_1\bar{x}_1 - \boldsymbol{z}_1{}^T \boldsymbol{\Phi}_1 + \boldsymbol{\vartheta}_{g1}{}^T \boldsymbol{\phi}_{g1}(\boldsymbol{x})\bar{x}_2 + \Delta_1 \tag{10}$$

where $\boldsymbol{\Phi}_1 = \left[\boldsymbol{\phi}_{f1}^T(\boldsymbol{x}), \boldsymbol{\phi}_{g1}^T(\boldsymbol{x})x_2\right]^T$.

Now consider the design of the parameter adaptive laws. Differentiating the estimate error $\boldsymbol{z}_1$ yields

$$\dot{\boldsymbol{z}}_1 = \dot{\boldsymbol{\vartheta}}_1 - \dot{\boldsymbol{\theta}}_1 = \dot{\hat{\boldsymbol{\theta}}}_{1\_T} + \dot{\hat{\boldsymbol{\theta}}}_{1\_P} \tag{11}$$

where

$$\hat{\boldsymbol{\theta}}_{1\_T} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{f1\_T} \\ \hat{\boldsymbol{\theta}}_{g1\_T} \end{bmatrix}, \quad \hat{\boldsymbol{\theta}}_{1\_P} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{f1\_P} \\ \hat{\boldsymbol{\theta}}_{g1\_P} \end{bmatrix}$$

represent tracking-error based estimation and prediction-error based estimation respectively.

First is the design of the tracking-error based estimation law. According to the I&I method, the tracking-error based estimation consists of two parts

$$\hat{\boldsymbol{\theta}}_{1\_T} = \hat{\boldsymbol{\theta}}_1 + \boldsymbol{\beta}_1 \tag{12}$$

where $\boldsymbol{\beta}_1(\cdot)$ is a nonlinear vector function to be determined. Substitute (12) into (11) using (10), which yields

$$\dot{\boldsymbol{z}}_1 = \dot{\hat{\boldsymbol{\theta}}}_1 + \frac{\partial \boldsymbol{\beta}_1}{\partial \bar{x}_1}\left[-k_1\bar{x}_1 - \boldsymbol{z}_1{}^T\boldsymbol{\Phi}_1 + \boldsymbol{\vartheta}_{g1}{}^T\boldsymbol{\phi}_{g1}(\boldsymbol{x})\bar{x}_2 + \Delta_1\right] + \dot{\hat{\boldsymbol{\theta}}}_{1\_P} \tag{13}$$

Now the I&I based adaptive law using the information of tracking-error can be designed as

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_1 = \dfrac{\partial \boldsymbol{\beta}_1}{\partial \bar{x}_1}\left[k_1\bar{x}_1 - \boldsymbol{\vartheta}_{g1}{}^T\boldsymbol{\phi}_{g1}(\boldsymbol{x})\bar{x}_2\right] - r_1\sigma_{1\_T}\boldsymbol{\vartheta}_1 \\ \dfrac{\partial \boldsymbol{\beta}_1}{\partial \bar{x}_1} = r_1\boldsymbol{\Phi}_1 \end{cases} \tag{14}$$

where $r_1 > 0$, $\sigma_{1\_T} > 0$. $r_1\sigma_{1\_T}\boldsymbol{\vartheta}_1$ is the so-called σ-modification term which ensures the update law bounded in the presence of non-parametric uncertainties.

Next the prediction-error based estimation law $\dot{\hat{\boldsymbol{\theta}}}_{1\_P}$ is to be designed. Filtering both sides of (8), one has

$$\frac{\dot{\tilde{x}}_1}{s+\lambda_f} = \boldsymbol{\theta}_{f1}^T\frac{\boldsymbol{\phi}_{f1}(\boldsymbol{x})}{s+\lambda_f} + \boldsymbol{\theta}_{g1}^T\frac{\boldsymbol{\phi}_{g1}(\boldsymbol{x})x_2}{s+\lambda_f} - \frac{\dot{x}_{1c}}{s+\lambda_f} + \frac{\Delta_1}{s+\lambda_f} \tag{15}$$

where s is the Laplace operator and $\lambda_f > 0$. Rearrange (15) as follows

$$Y_1 = \boldsymbol{\theta}_1^T\boldsymbol{\Phi}_{f1} + \Delta_{f1} \tag{16}$$

where f denotes filtered signal, and

$$Y_1 = \tilde{x}_1 + \frac{\dot{x}_{1c} - \lambda_f\tilde{x}_1}{s+\lambda_f}, \Delta_{f1} = \frac{\Delta_1}{s+\lambda_f},$$

$$\boldsymbol{\Phi}_{f1} = \left[\frac{\boldsymbol{\phi}_{f1}{}^T(\boldsymbol{x})}{s+\lambda_f}, \frac{\boldsymbol{\phi}_{g1}{}^T(\boldsymbol{x})x_2}{s+\lambda_f}\right].$$

Now the prediction-error can be specified as

$$e_1 = \hat{Y}_1 - Y_1 = \boldsymbol{\vartheta}_1^T\boldsymbol{\Phi}_{f1} - \left(\tilde{x}_1 + \frac{\dot{x}_{1c} - \lambda_f\tilde{x}_1}{s+\lambda_f}\right) \tag{17}$$

where $\hat{Y}_1 = \boldsymbol{\vartheta}_1^T\boldsymbol{\Phi}_{f1}$. It should be noted that all the signals in the rightmost side of (17) are available. Therefore, the information of prediction error, e_1 , can be obtained from (17).

Thus, one can design the prediction-error based estimation law as

$$\dot{\hat{\boldsymbol{\theta}}}_{1\_P} = -r_1\boldsymbol{\Phi}_{f1}e_1 - r_1\sigma_{1\_P}\boldsymbol{\vartheta}_1 \tag{18}$$

where $\sigma_{1\_P} > 0$ is also the σ-modification parameter. Substituting (14) and (18) into (13) yields

$$\begin{aligned} \dot{\boldsymbol{z}}_1 = & -r_1\boldsymbol{\Phi}_1\boldsymbol{z}_1{}^T\boldsymbol{\Phi}_1 + r_1\boldsymbol{\Phi}_1\Delta_1 - r_1\sigma_{1\_T}\boldsymbol{\vartheta}_1 \\ & -r_1\boldsymbol{\Phi}_{f1}\boldsymbol{z}_1{}^T\boldsymbol{\Phi}_{f1} + r_1\boldsymbol{\Phi}_{f1}\Delta_{f1} - r_1\sigma_{1\_P}\boldsymbol{\vartheta}_1 \end{aligned} \tag{19}$$

Step i: Now x_{ic} is the new variable to be tracked. The dynamics of the actual tracking error $\tilde{x}_i$ can be written as

$$\dot{\tilde{x}}_i = \boldsymbol{\theta}_{fi}^T\cdot\boldsymbol{\phi}_{fi}(\boldsymbol{x}) + \boldsymbol{\theta}_{gi}^T\cdot\boldsymbol{\phi}_{gi}(\boldsymbol{x})\cdot x_{i+1} + \Delta_i - \dot{x}_{ic} \tag{20}$$

The nominal virtual control signal $x_{i+1,cd}$ can be chosen as

$$\boldsymbol{\vartheta}_{gi}{}^T\boldsymbol{\phi}_{gi}(\boldsymbol{x})x_{i+1,cd} = -k_i\tilde{x}_i - \boldsymbol{\vartheta}_{fi}{}^T\boldsymbol{\phi}_{fi}(\boldsymbol{x}) + \dot{x}_{ic} - \boldsymbol{\vartheta}_{g,i-1}^T\boldsymbol{\phi}_{g,i-1}(\boldsymbol{x})\bar{x}_{i-1} \tag{21}$$

Similar as *Step 1*, using (20) and (21), the dynamics of the compensated tracking error $\bar{x}_i$ can be written as

$$\dot{\bar{x}}_i = -k_i\bar{x}_i - \boldsymbol{z}_i{}^T\boldsymbol{\Phi}_i + \boldsymbol{\vartheta}_{gi}{}^T\boldsymbol{\phi}_{gi}(\boldsymbol{x})\bar{x}_{i+1} + \Delta_i - \boldsymbol{\vartheta}_{g,i-1}^T\boldsymbol{\phi}_{g,i-1}(\boldsymbol{x})\bar{x}_{i-1} \tag{22}$$

where $\boldsymbol{\Phi}_i = \left[\boldsymbol{\phi}_{fi}^T(\boldsymbol{x}), \boldsymbol{\phi}_{gi}^T(\boldsymbol{x})x_{i+1}\right]^T$.

For the design of the parameter adaptive laws, the estimate error $\boldsymbol{z}_i$ can be expressed as

$$\dot{\boldsymbol{z}}_i = \dot{\boldsymbol{\vartheta}}_i - \dot{\boldsymbol{\theta}}_i = \dot{\hat{\boldsymbol{\theta}}}_{i\_T} + \dot{\hat{\boldsymbol{\theta}}}_{i\_P} \tag{23}$$

where

$$\hat{\boldsymbol{\theta}}_{i\_T} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{fi\_T} \\ \hat{\boldsymbol{\theta}}_{gi\_T} \end{bmatrix}, \quad \hat{\boldsymbol{\theta}}_{i\_P} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{fi\_P} \\ \hat{\boldsymbol{\theta}}_{gi\_P} \end{bmatrix},$$

represent tracking-error based estimation and prediction-error based estimation respectively.

First is the design of the tracking-error based estimation law using I&I method. As (12), the estimation is of the form

$$\hat{\boldsymbol{\theta}}_{i\_T} = \hat{\boldsymbol{\theta}}_i + \boldsymbol{\beta}_i \tag{24}$$

Using (22), (23) and (24) yields

$$\begin{aligned} \dot{\boldsymbol{z}}_i = \dot{\hat{\boldsymbol{\theta}}}_i + \frac{\partial \boldsymbol{\beta}_i}{\partial \bar{x}_i}[& -k_i\bar{x}_i - \boldsymbol{z}_i{}^T\boldsymbol{\Phi}_i + \boldsymbol{\vartheta}_{gi}{}^T\boldsymbol{\phi}_{gi}(\boldsymbol{x})\bar{x}_{i+1} \\ & + \Delta_i - \boldsymbol{\vartheta}_{g,i-1}{}^T\boldsymbol{\phi}_{g,i-1}(\boldsymbol{x})\bar{x}_{i-1}] + \dot{\hat{\boldsymbol{\theta}}}_{i\_P} \end{aligned} \tag{25}$$

Similarly, the I&I based adaptive law can be designed as

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_i = \dfrac{\partial \boldsymbol{\beta}_i}{\partial \bar{x}_i}\left[k_i\bar{x}_i - \boldsymbol{\vartheta}_{gi}{}^T\boldsymbol{\phi}_{gi}(\boldsymbol{x})\bar{x}_{i+1} + \boldsymbol{\vartheta}_{g,i-1}^T\boldsymbol{\phi}_{g,i-1}(\boldsymbol{x})\bar{x}_{i-1}\right] - r_i\sigma_{i\_T}\boldsymbol{\vartheta}_i \\ \dfrac{\partial \boldsymbol{\beta}_i}{\partial \bar{x}_i} = r_i\boldsymbol{\Phi}_i \end{cases} \tag{26}$$

where $r_i > 0$, $\sigma_{i\_T} > 0$.

Next is to design the prediction-error based estimation law $\dot{\hat{\boldsymbol{\theta}}}_{i\_P}$. Multiplying both sides of (20) by $1/(s+\lambda_f)$, one has

$$Y_i = \boldsymbol{\theta}_i^T \boldsymbol{\Phi}_{fi} + \Delta_{fi} \tag{27}$$

where f denotes filtered signal, and

$$Y_i = \tilde{x}_i + \frac{\dot{x}_{ic} - \lambda_f \tilde{x}_i}{s+\lambda_f}, \quad \Delta_{fi} = \frac{\Delta_i}{s+\lambda_f},$$

$$\boldsymbol{\Phi}_{fi} = \left[\frac{\boldsymbol{\phi}_{fi}{}^T(\boldsymbol{x})}{s+\lambda_f}, \frac{\boldsymbol{\phi}_{gi}{}^T(\boldsymbol{x})x_{i+1}}{s+\lambda_f} \right]^T.$$

Now the prediction-error can be defined as

$$e_i = \hat{Y}_i - Y_i = \boldsymbol{\vartheta}_i^T \boldsymbol{\Phi}_{fi} - (\tilde{x}_i + \frac{\dot{x}_{ic} - \lambda_f \tilde{x}_i}{s+\lambda_f}) \tag{28}$$

where $\hat{Y}_i = \boldsymbol{\vartheta}_i^T \boldsymbol{\Phi}_{fi}$. Then the prediction-error based estimation law can be designed as

$$\dot{\hat{\boldsymbol{\theta}}}_{i\_P} = -r_i \boldsymbol{\Phi}_{fi} e_i - r_i \sigma_{i\_P} \boldsymbol{\vartheta}_i \tag{29}$$

where $\sigma_{i\_P} > 0$ is used to address the non-parametric uncertainties. With (25), (26) and (29), the dynamics of the estimate error $\boldsymbol{z}_i$ can be written as

$$\begin{aligned} \dot{\boldsymbol{z}}_i = & -r_i \boldsymbol{\Phi}_i \boldsymbol{z}_i{}^T \boldsymbol{\Phi}_i + r_i \boldsymbol{\Phi}_i \Delta_i - r_i \sigma_{i\_T} \boldsymbol{\vartheta}_i \\ & -r_i \boldsymbol{\Phi}_{fi} \boldsymbol{z}_i{}^T \boldsymbol{\Phi}_{fi} + r_i \boldsymbol{\Phi}_{fi} \Delta_{fi} - r_i \sigma_{i\_P} \boldsymbol{\vartheta}_i \end{aligned} \tag{30}$$

Step n: The last step is to design the control input u so that $\tilde{x}_n = x_n - x_{nc}$ can asymptotically converge to zero. The dynamics of $\tilde{x}_n$ can be written as

$$\dot{\tilde{x}}_n = \boldsymbol{\theta}_{fn}^T \cdot \boldsymbol{\phi}_{fn}(\boldsymbol{x}) + \boldsymbol{\theta}_{gn}^T \cdot \boldsymbol{\phi}_{gn}(\boldsymbol{x}) \cdot u + \Delta_n - \dot{x}_{nc} \tag{31}$$

Select u_{cd} as

$$\boldsymbol{\vartheta}_{gn}{}^T \boldsymbol{\phi}_{gn}(\boldsymbol{x}) u_{cd} = -k_n \tilde{x}_n - \boldsymbol{\vartheta}_{fn}{}^T \boldsymbol{\phi}_{fn}(\boldsymbol{x}) + \dot{x}_{nc} - \boldsymbol{\vartheta}_{g,n-1}^T \boldsymbol{\phi}_{g,n-1}(\boldsymbol{x}) \bar{x}_{n-1} \tag{32}$$

Pass u_{cd} through a command filter to generate the signal of u_c and $\dot{u}_c$. Since u_c is achievable by the actuator, we have $u_c = u$. Then using (31) and (32), the dynamics of the compensated tracking error $\bar{x}_n$ can be written as

$$\dot{\bar{x}}_n = -k_n \bar{x}_n - \boldsymbol{z}_n{}^T \boldsymbol{\Phi}_n + \Delta_n - \boldsymbol{\vartheta}_{g,n-1}^T \boldsymbol{\phi}_{g,n-1}(\boldsymbol{x}) \bar{x}_{n-1} \tag{33}$$

where $\boldsymbol{\Phi}_n = \left[\boldsymbol{\phi}_{fn}^T(\boldsymbol{x}), \boldsymbol{\phi}_{gn}^T(\boldsymbol{x}) u \right]^T$.

The estimate error $\boldsymbol{z}_n$ can be expressed as

$$\dot{\boldsymbol{z}}_n = \dot{\boldsymbol{\vartheta}}_n - \dot{\boldsymbol{\theta}}_n = \dot{\hat{\boldsymbol{\theta}}}_{n\_T} + \dot{\hat{\boldsymbol{\theta}}}_{n\_P} \tag{34}$$

where

$$\hat{\boldsymbol{\theta}}_{n\_T} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{fn\_T} \\ \hat{\boldsymbol{\theta}}_{gn\_T} \end{bmatrix}, \quad \hat{\boldsymbol{\theta}}_{n\_P} = \begin{bmatrix} \hat{\boldsymbol{\theta}}_{fn\_P} \\ \hat{\boldsymbol{\theta}}_{gn\_P} \end{bmatrix},$$

represent tracking-error based estimation and prediction-error based estimation respectively.

First is the design of the tracking-error based estimation law using I&I method. As (12), the estimation is of the form

$$\hat{\boldsymbol{\theta}}_{n\_T} = \hat{\boldsymbol{\theta}}_n + \boldsymbol{\beta}_n \tag{35}$$

Using (33), (34) and (35) yields

$$\dot{\boldsymbol{z}}_n = \dot{\hat{\boldsymbol{\theta}}}_n + \frac{\partial \boldsymbol{\beta}_n}{\partial \bar{x}_n} \left[-k_n \bar{x}_n - \boldsymbol{z}_n{}^T \boldsymbol{\Phi}_n + \Delta_i - \boldsymbol{\vartheta}_{g,n-1}{}^T \boldsymbol{\phi}_{g,n-1}(\boldsymbol{x}) \bar{x}_{n-1} \right] + \dot{\hat{\boldsymbol{\theta}}}_{n\_P} \tag{36}$$

Similarly, the I&I based adaptive law can be designed as

$$\begin{cases} \dot{\hat{\boldsymbol{\theta}}}_n = \dfrac{\partial \boldsymbol{\beta}_n}{\partial \bar{x}_n} \left[k_n \bar{x}_n + \boldsymbol{\vartheta}_{g,n-1}^T \boldsymbol{\phi}_{g,n-1}(\boldsymbol{x}) \bar{x}_{n-1} \right] - r_n \sigma_{n\_T} \boldsymbol{\vartheta}_n \\ \dfrac{\partial \boldsymbol{\beta}_n}{\partial \bar{x}_n} = r_n \boldsymbol{\Phi}_n \end{cases} \tag{37}$$

where $r_n > 0$, $\sigma_{n\_T} > 0$.

Next the design of the prediction-error based estimation law $\dot{\hat{\boldsymbol{\theta}}}_{n\_P}$ is presented. Multiplying both sides of (31) by $1/(s+\lambda_f)$, one has

$$Y_n = \boldsymbol{\theta}_n{}^T \boldsymbol{\Phi}_{fn} + \Delta_{fn} \tag{38}$$

where

$$Y_n = \tilde{x}_n + \frac{\dot{x}_{nc} - \lambda_f \tilde{x}_n}{s+\lambda_f}, \ \Delta_{fn} = \frac{\Delta_n}{s+\lambda_f},$$

$$\boldsymbol{\Phi}_{fn} = \left[\frac{\boldsymbol{\phi}_{fn}{}^T(\boldsymbol{x})}{s+\lambda_f}, \frac{\boldsymbol{\phi}_{gn}{}^T(\boldsymbol{x}) u}{s+\lambda_f} \right]^T.$$

Now the prediction-error can be defined as

$$e_n = \hat{Y}_n - Y_n = \boldsymbol{\vartheta}_n{}^T \boldsymbol{\Phi}_{fn} - (\tilde{x}_n + \frac{\dot{x}_{nc} - \lambda_f \tilde{x}_n}{s+\lambda_f}) \tag{39}$$

where $\hat{Y}_n = \boldsymbol{\vartheta}_n{}^T \boldsymbol{\Phi}_{fn}$. The prediction-error based estimation law is designed as

$$\dot{\hat{\boldsymbol{\theta}}}_{n\_P} = -r_n \boldsymbol{\Phi}_{fn} e_n - r_n \sigma_{n\_P} \boldsymbol{\vartheta}_n \tag{40}$$

where $\sigma_{n\_P} > 0$ is used to address the non-parametric uncertainties. With (36), (37), and (40), the dynamics of the estimate error $\boldsymbol{z}_n$ can be written as

$$\begin{aligned} \dot{\boldsymbol{z}}_n = & -r_n \boldsymbol{\Phi}_n \boldsymbol{z}_n{}^T \boldsymbol{\Phi}_n + r_n \boldsymbol{\Phi}_n \Delta_n - r_n \sigma_{n\_T} \boldsymbol{\vartheta}_n \\ & -r_n \boldsymbol{\Phi}_{fn} \boldsymbol{z}_n{}^T \boldsymbol{\Phi}_{fn} + r_n \boldsymbol{\Phi}_{fn} \Delta_{fn} - r_n \sigma_{n\_P} \boldsymbol{\vartheta}_n \end{aligned} \tag{41}$$

2.3 Stability Analysis

The stability of the above closed-loop system is analyzed here, which is summarized in the following statement.

Theorem 1: Consider the system (1) with Assumptions 1 and 2. Then there exist adaptive control laws described by (14), (18), (26), (29), (32), (37) and (40), which guarantee all signals of the closed-loop system are bounded and $\lim_{t\to\infty} \bar{x}_1(t) = 0$

Proof. Consider the following Lyapunov candidate function

$$V = \frac{1}{2}\sum_{j=1}^{n}\bar{x}_j^2 + \frac{1}{2}\sum_{j=1}^{n}k_j^{-1}\boldsymbol{z}_j^T r_j^{-1}\boldsymbol{z}_j \tag{42}$$

whose time-derivative along (10), (19), (22), (30), (33) and (41), is given by

$$\begin{aligned}\dot{V} = &-\sum_{i=1}^{n}k_i\bar{x}_i^2 - \sum_{i=1}^{n}\bar{x}_i\boldsymbol{z}_i^T\boldsymbol{\Phi}_i + \sum_{i=1}^{n}\bar{x}_i\Delta_i - \sum_{i=1}^{n}k_i^{-1}(\boldsymbol{z}_i^T\boldsymbol{\Phi}_i)^2 + \\ &\sum_{i=1}^{n}k_i^{-1}(\boldsymbol{z}_i^T\boldsymbol{\Phi}_i)\Delta_i - \sum_{i=1}^{n}k_i^{-1}(\boldsymbol{z}_i^T\boldsymbol{\vartheta}_i)\sigma_{i\_T} - \sum_{i=1}^{n}k_i^{-1}(\boldsymbol{z}_i^T\boldsymbol{\Phi}_{fi})^2 \\ &+\sum_{i=1}^{n}k_i^{-1}(\boldsymbol{z}_i^T\boldsymbol{\Phi}_{fi})\Delta_{fi} - \sum_{i=1}^{n}k_i^{-1}(\boldsymbol{z}_i^T\boldsymbol{\vartheta}_i)\sigma_{i\_P}\end{aligned} \tag{43}$$

By Young's inequality

$$\begin{aligned}-\bar{x}_i\boldsymbol{z}_i^T\boldsymbol{\Phi}_i &\le k_i\bar{x}_i^2/2 + k_i^{-1}\left|\boldsymbol{z}_i^T\boldsymbol{\Phi}_i\right|^2/2 \\ \bar{x}_i\Delta_i &\le \bar{x}_i^2/2 + \eta_i^2/2 \\ (\boldsymbol{z}_i^T\boldsymbol{\Phi}_i)\Delta_i &\le \left|\boldsymbol{z}_i^T\boldsymbol{\Phi}_i\right|^2/2 + \eta_i^2/2 \\ -\boldsymbol{z}_i^T\boldsymbol{\vartheta}_i &\le -\left\|\boldsymbol{z}_i\right\|^2/2 + \left\|\theta_i\right\|^2/2 \\ (\boldsymbol{z}_i^T\boldsymbol{\Phi}_{fi})\Delta_{fi} &\le \left|\boldsymbol{z}_i^T\boldsymbol{\Phi}_{fi}\right|^2/2 + \eta_{fi}^2/2\end{aligned} \tag{44}$$

it can be observed that

$$\dot{V} \le -\alpha V + \mathrm{X} \tag{45}$$

where $\alpha_i = \min\left[k_i - 1, r_i(\sigma_{i\_T} + \sigma_{i\_P})\right]$, $\alpha = \min \alpha_i$, $i = 1, 2, \cdots n$, and

$$\mathrm{X} = \frac{1}{2}\sum_{i=1}^{n}(k_i^{-1}(\sigma_{i\_T} + \sigma_{i\_P})\left\|\theta_i\right\|^2 + (k_i^{-1}+1)\eta_i^2 + k_i^{-1}\eta_{fi}^2 .$$

It implies that for $V \ge \mathrm{X}/\alpha$, $\dot{V} \le 0$. Thus, it can be concluded that all the signals are bounded and $\lim_{t\to\infty}\bar{x}_i(t) = 0$, $i = 1, 2, \cdots n$, thereby completing the proof.

3. NUMERICAL SIMULATION

To demonstrate the performance of the proposed control scheme, a buck converter system modelled in(Ding, Zheng, Sun, & Wang, 2018) is used for numerical simulation here. The model is described as

$$\begin{cases}\dot{x}_1 = f_1(\boldsymbol{x}) + g_1(\boldsymbol{x})x_2 + \Delta_1 \\ \dot{x}_2 = f_2(\boldsymbol{x}) + g_2(\boldsymbol{x})u + \Delta_2\end{cases} \tag{46}$$

where the states x_1 and x_2 represent the capacitor output voltage v_0 and the inductor current i_L, u is the control input, Δ_1 and Δ_2 denote the lumped disturbances, and

$$f_1(\boldsymbol{x}) = -x_1/(RC),\ g_1(\boldsymbol{x}) = 1/C,$$

$$f_2(\boldsymbol{x}) = -x_1/L,\ g_2(\boldsymbol{x}) = V_{in}/L.$$

The parameters R, C, L and V_{in} are the load resistor, the capacitor, the inductor and the voltage source respectively, and they are treated as unknown constant parameters. The control objective is to design u such that the capacitor output voltage x_1 can track a desired reference voltage.

Following the above procedure, a composite robust adaptive controller based on I&I is designed for the buck converter system. To illustrate the robustness of the proposed control system, two simulation cases with different model uncertainties are considered here. The first simulation is carried out on the model whose lumped disturbance Δ_1 and Δ_2 are both expressed by a zero-mean Gaussian white noise (GWN) signal with variance 10, while the second one is on the model whose lumped disturbance Δ_1 and Δ_2 are both expressed by a GWN signal with variance 100. Moreover, the results simulated on the standard I&I-based adaptive control of the buck converter system are illustrated simultaneously, which are used for comparison.

First, simulation is conducted on the model with lumped disturbance expressed by a zero-mean GWN signal with variance 10. The simulation results are shown in Figs. 1-3. It can be observed from Fig. 1 that the capacitor output voltage x_1 converges to the reference command fast for both standard and composite I&I-based adaptive control system. However, there are significant differences in terms of the transient response of the two control systems. The overshoot of the standard I&I-based adaptive control system is serious, which maybe unacceptable for the electrical closed-loop system. On the contrary, the dynamical behaviour of the composite I&I-based adaptive control system is damped. Similarly, more stable and damped behaviours of the estimates of the unknown parameters for the composite I&I-based adaptive control system than the ones of the standard I&I-based adaptive control system can be observed from Fig. 2 and Fig. 3, which further implies a better stability of the composite adaptive control system. Actually, a faster adaptation without getting the oscillatory behaviour is the key feature of composite adaptive control. Therefore, it clearly demonstrates the superiority of the proposed composite I&I adaptive control system.

Next, the model with lumped disturbance expressed by a zero-mean GWN signal with variance 100 is used for simulation. The simulation results are shown in Figs. 4-6. The tracking performances of both the standard and composite I&I-based adaptive control system are depicted in Fig. 4. A damped dynamical behaviour is still observed from the result of the composite I&I-based adaptive control system, while more serious oscillation can be found from the standard I&I-based adaptive control system. Moreover, from Fig.5 and Fig. 6, the estimates of the unknown parameters for the composite I&I-based adaptive control system are shown more stable and damped than the ones for the standard I&I-

based adaptive control system, which is corresponding to the first simulation case. With these results, it can be concluded that the proposed composite I&I adaptive control scheme is effective and robust.

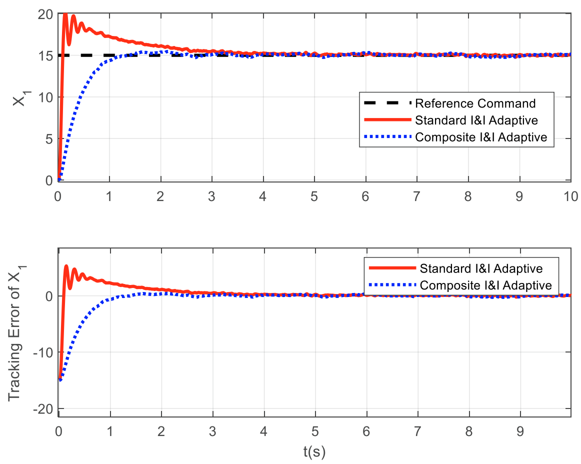

Fig. 1. Trajectories of x_1 and tracking error (GWN with variance 10)

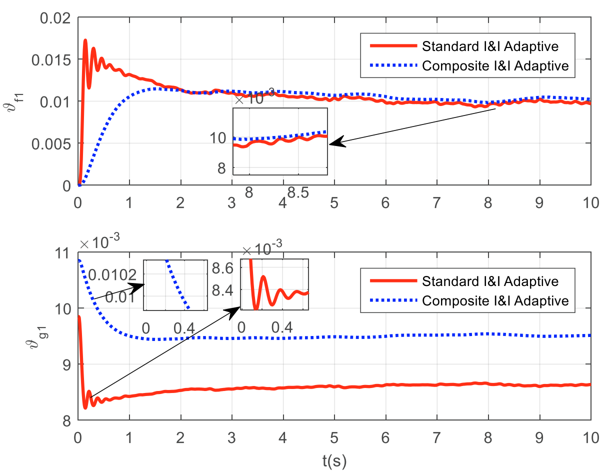

Fig. 2. Trajectories of ϑ_{f1} and ϑ_{g1} (GWN with variance 10)

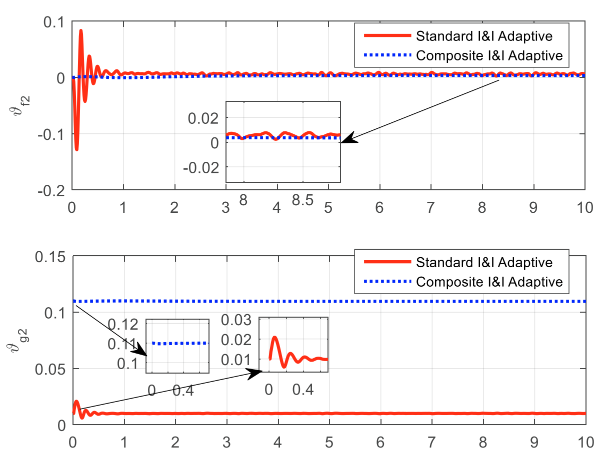

Fig. 3. Trajectories of ϑ_{f2} and ϑ_{g2} (GWN with variance 10)

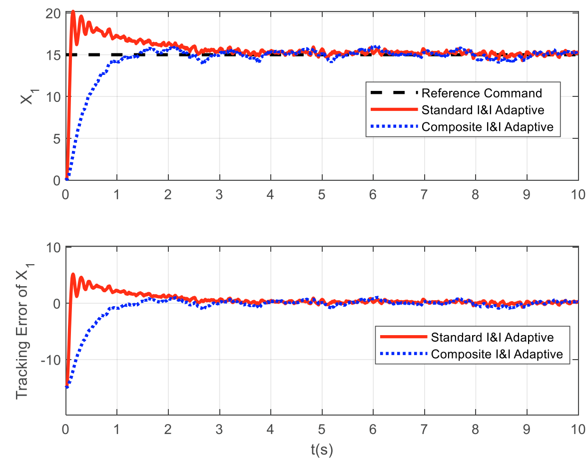

Fig. 4. Trajectories of x_1 and tracking error (GWN with variance 100)

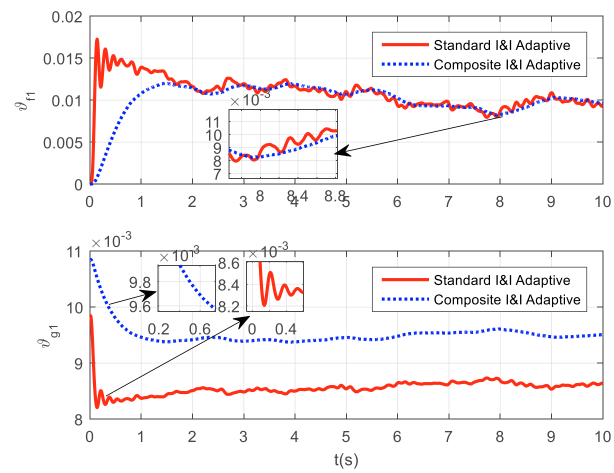

Fig. 5. Trajectories of ϑ_{f1} and ϑ_{g1} (GWN with variance 100)

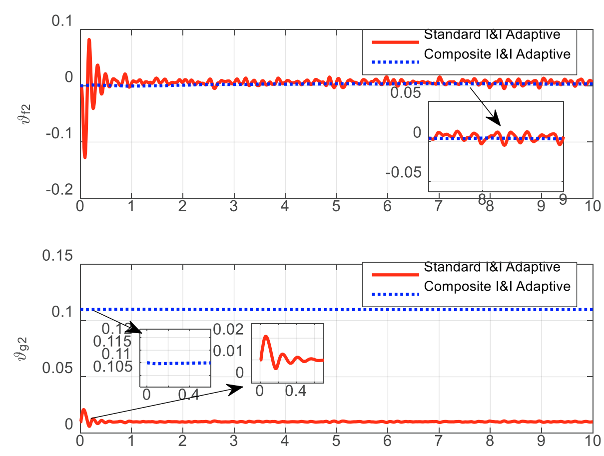

Fig. 6. Trajectories of ϑ_{f2} and ϑ_{g2} (GWN with variance 100)

4. CONCLUSIONS

A new composite robust adaptive control for uncertain nonlinear systems is proposed in this paper. The interest here is to achieve an immersion and invariance (I&I) based adaptive control in the presence of both parametric and non-parametric uncertainties. Both the information of the tracking error and prediction error are considered in the design of I&I adaptive laws, thus building up a composite adaptive control. The term of σ-modification is added to the composite adaptive laws, which makes this technique robust to the non-parametric uncertainties. Stability analysis of the whole closed-loop system is presented using Lyapunov theory. Numerical simulations are performed, which illustrate the superiority of the proposed control scheme.

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