Improved Self-Propelled Swarms Model with Enhanced Convergence Efficiency

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Abstract The paper deals with a biologically inspired model of self-propelled particles introduced by Vicsek. To solve the problem of low convergence efficiency in this model, an improved model based on distance weight is proposed in this paper. Particularly, distance weight function is designed in the form of polynomial function which is a monotone increasing function of distance. Moreover, a new index to evaluate the convergence efficiency called Vicsek algebraic connectivity is promoted. Finally, comprehensive comparative studies of the convergence properties among the improved model, original Vicsek model, and Degree model are investigated in the simulation part. The simulation results show that our modified model is better than other two models in convergence probability and consensus time. Our results may enlighten other researchers in revealing the mechanism of collective motion.

Keywords: Vicsek model, Convergency efficiency, Topological structure, Connectivity, Self-propelled particle

1 Introduction

In recent years, based on inspiration from the behaviors of biological clusters, researchers have proposed many swarm models. It is believed that schools of fish, flocks of birds, and group of bees are based on simple behavioral interactions between group members to develop coherent, intelligent behaviors at the collective level [1]. Besides the realm of biology, related achievements have been gradually applied to the field of computer algorithms [2] and robotic self-assembly [3]. Research of swarm system may also be used to explain the generation of swarm intelligence which has critical value in engineering [4, 5].

In 1986, by observing the assembly behavior of the natural flock of fish, Reynolds promoted Boid model based on three rules: separation, alignment and cohesion [6]. Vicsek et al. [7], put forward another related but simplified model called Vicsek model (OVM) which was based on alignment rules of Boid model. Barbies et al. [8], provided a minimal cognitive flocking model which lacked velocity-velocity alignment. Shirazi [9] introduced passive sensing and active sensing to improve collective behaviors of the group. Yang et al. [10] proposed a model with multiple constraint factors to improve the collaborative and continuous of the swarm in the moving and steering process.

Among all these models, OVM is preferred, because it is the simplest and the most efficient one. Due to the nonlinear updating equation and random noise, rigorous theoretical analysis towards OVM is challenging. One of the most crucial theoretical progress was given by Jadbabaie, Lin and Morse in [11], where they linearized the heading updating equation and omitted the noise disturbance. In [12], Savkin analyzed the model with discrete direction angle and showed that consensus can be obtained when the limit of the neighbor graphs is connected. In [13], it was pointed out that with the heterogeneity of the individual influence area, the heterogeneous network is more beneficial to the system to reach consensus.

As a typical theoretical problem, many attempts have been made to solve the problem of inefficient convergence efficiency in OVM [14–16]. In [14], considering the problem of panoramic vision of real individuals, Bao found that there was an optimum view angle which could accelerate the convergence of the OVM. Gao [15] proposed a new weight index model of degree (DM) which could improve the convergence speed. George et al. [16] proposed two improved versions of the model by introducing additional terms in the heading renewal equations.

In this paper, a modified version of the OVM called DWM, which exhibits significant improvement in enhancing the efficiency of consensus, is proposed. We present a weight function based on the distance between agents with the form of polynomial function which involves two additional terms in the heading update equation correlating with sense radius. It is shown that in various sense radius, there exist optimal values of the additional parameters which reduce convergence time effectively. The DWM has significantly greater reductions in synchronization time than DM proposed in [15]. Furthermore, due to the lack of evaluation parameters in convergence analysis, a new index called Vicsek algebraic connectivity (VAC) is proposed to evaluate the convergence efficiency of different rules.

The rest of the paper is organized as follows: Problem preliminaries and conception are given in Section 2. The improved model and the definition of VAC is proposed in Section 3. In Section 4, numerical simulation and analysis is presented, and conclusions are drawn in Section 5.

2 Problem Description

2.1 Orginal Vicsek Model

In OVM, n self-propelled agents move in a square-shaped region of length L with different motion direction but the same speed v_0. The velocity heading of agents are calculated by averaging headings of agents within perception radius R per unit of time step. The position $x_i(t)$ and velocity heading $\theta_i(t)(\in(-\pi,\pi])$ of agents update with the form of:

$$x_i(t+1) = x_i(t) + v_0 e^{i\theta_i(t)}\,, \tag{1}$$

$$\theta_i(t+1) = \arctan\left(\frac{\sum_{j\in N_i(t)} \sin\theta_j(t)}{\sum_{j\in N_i(t)} \cos\theta_j(t)}\right) + \delta_i(t)\,, \tag{2}$$

where $\delta_i(t)$ denotes the random noise chosen with a uniform probability from the interval $[-\delta_0, \delta_0]$ and $N_i(t) = \{j|\, \|x_i(t) - x_j(t)\| \leq R\}$.

To reduce the difficulty of theoretical analysis, Jadbabaie, Lin, and Morse in [11], simplified the heading updating equation (2) as follows:

$$\theta_i(t+1) = \frac{1}{|N_i(t)|} \sum_{j\in N_i(t)} \theta_j(t)\,, \tag{3}$$

Self-propelled particles continually update its states until the order parameter $\Psi(t)$ of the system:

$$\Psi(t) = \frac{1}{n} \left| \sum_{i=1}^{n} e^{i\theta(t)} \right|\,, \tag{4}$$

exceeds a given fixed value ψ_{sync} at time τ which is defined as convergence time.

2.2 Gaos Improved Vicsek Model [15]

In order to improve convergence in the Vicsek model, Gao developed a weighted rule based on the degree of agents. The direction updating rule is as follows:

$$\theta_i(t+1) = \arctan\left(\frac{\sum_{j\in N_i(t)} |N_j(t)| \sin\theta_j(t)}{\sum_{j\in N_i(t)} |N_j(t)| \cos\theta_j(t)}\right) + \delta_i(t)\,, \tag{5}$$

DM strengthens the heading connection with these agents which possess more neighbors and effectively improves the convergence time. However, this model requires additional communication in the updating process and leads to a relatively high failure probability in reaching consensus. The improved model promoted in this paper overcomes these disadvantages. This model will compare with our model in the simulation part.

3 Improved Model and Vicsek Algebraic Connectivity

3.1 Improved Model (DWM)

Velocity alignment is the key to unite all agents eventually moving with the same heading in OVM. Ideally, agents acquire mean directions of the whole swarm in which way agents reach consensus immediately. Under normal circumstances, due to the limited sense radius, reasonable utilization of neighbors information confers a large effect on the consensus process. In the OVM, the updating agents treat all its neighbors heading equally. However, closer agents headings are more likely to be adopted again in the next update. Thus, neighbors which are closer to the agent will exert greater influence in direction composition than distant neighbors for long time scales. In this way, it is prone to generate local consensus rather than global consensus. Instinctively, the strategy that increases the weight of distant neighbors is promoted to strengthen the global consensus.

Establishing stronger directional correlation with distant neighbors can effectively avoid the loss of neighbors. As is shown in Fig.1, for agent 4, adopting the original rule may lose neighbor agent 3 and agent 5 in the next step. If agent 4 pay more attention to the direction of the distant neighbors and follow them, these two neighbors may not leave neighborhood immediately, and the nearest agent 6 and agent 7 will remain its neighbors for a long time and be constantly influenced by agent 4. Furthermore, this preference retains closer direction links with agents beyond sense radius. In Fig.1 the direction correlation between agent 4 and agent 1 is strengthened through 3, which can more acutely perceive the direction change of distant individuals.

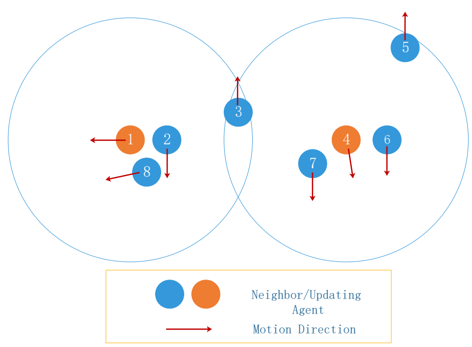


Figure 1. Agent distribution diagram

Based on the above analysis and proof, we propose DWM based on distance proportional weight as following form:

$$\theta_i(t+1) = \arctan(\frac{\sum_{j\in N_i(t)} f(x_{ij}) \sin\theta_j(t)}{\sum_{j\in N_i(t)} f(x_{ij}) \cos\theta_j(t)}) + \delta_i(t)\,, \tag{6}$$

where x_{ij} is the distance between i and j, $f(x)$ is the strictly monotonically increasing function.

The selection of function form will produce different effects. In this paper, we select the following mathematical expression:

$$f(x) = (1 + k_2 \frac{x}{R})^{k_1}\,, \tag{7}$$

where k_1 and k_2 is the weight coefficient. The optimal value of k_1 is influenced by the sense radius. For each value of k_1, the faster convergence performance can be obtained by adjusting k_2 flexibly.

3.2 Vicsek Algebraic Connectivity

Jadbabaie interpreted the OVM with graph theory as [11]:

$$\theta(t+1) = F_{\sigma(t)}\theta(t)\,, \tag{8}$$

$$F_{\sigma(t)} = (I + D_{\sigma(t)})^{-1}(A_{\sigma(t)} + I)\,, \tag{9}$$

where θ is the heading vector $\theta = [\theta_1\theta_2 \ldots \theta_n]$, $\sigma(t)$ is the index of the graph representing the agents neighbor relationships at time t , $A_{\sigma(t)}$ is the adjacency matrix of graph $G_{\sigma(t)}$ induced by the information flow, I is the identity matrix, and $D_{\sigma(t)}$ the degree matrix of whose i-th diagonal element d_i is the degree of vertex i and other elements are zero.

When the rate of change of θ is zero, agents obtain the convergence. We consider the first derivative of $\theta(t)$:

$$\dot{\theta}(t) = \theta(t+1) - \theta(t)\,, \tag{10}$$

Using Eqs. (8) and (9), we have:

$$\dot{\theta}(t) = -P_{\sigma(t)}\theta(t)\,, \tag{11}$$

$$P_{\sigma(t)} = I - (I + D_{\sigma(t)})^{-1}(A_{\sigma(t)} + I)\,. \tag{12}$$

According to inequality of arithmetic and geometric means theory, it could prove that the matrix $P_{\sigma(t)}$ is a positive semidefinite matrix. The eigenvalue of matrix $P_{\sigma(t)}$ are nonnegative denoted by $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$, always enumerated in increasing order and repeated according to their multiplicity. Referring to the definition of algebraic connectivity in graph Laplacian [17], we define the second smallest nonzero eigenvalue of $P_{\sigma(t)}$ as the OVMs VAC. Similarly, we know that a higher value of VAC means the faster synchronization convergence performance. The VAC for DWM and DM are defined similarly which will not repeat here. Due to the limited space, the detailed theoretical proof is not given in this paper.

The VAC could be used for evaluating convergence performance at fixed time point. Its application will be reflected in the simulation part.

4 Simulation

In order to verify that DWM has an improved convergence performance, this article simulates the model under different conditions with OVM and DM. In this paper, simulation parameters are selected as $n = 100, L = 10, R = 3.0, v_0 = 0.03, \delta_0 = 0.1, \psi_{sync} = 0.99, k_1 = 5, k_2 = 11$ unless otherwise stated. Agents move in the open box conditions rather than periodic box. Generally, agents can reach the steady state within 120 steps, so 200 steps are the max simulation time. Under this condition, the convergence steps are set as 200 when agents divided into several groups. Furthermore, each data in figures and table is the average of exceeding 100 replications.

4.1 Comparison of Convergence Properties

We investigate convergence steps of OVM, DM and DWM. As illustrated in Figs. 2(a), convergence steps decrease with the increasing of sense radius. Particularly, when sense radius is close to L, agents can achieve consensus in few steps. And comparing these three models, DWM has obvious advantages in convergence speed. In Figs. 2(b), as the increasing of velocity, the convergence superiority over DM and OVM is growing.

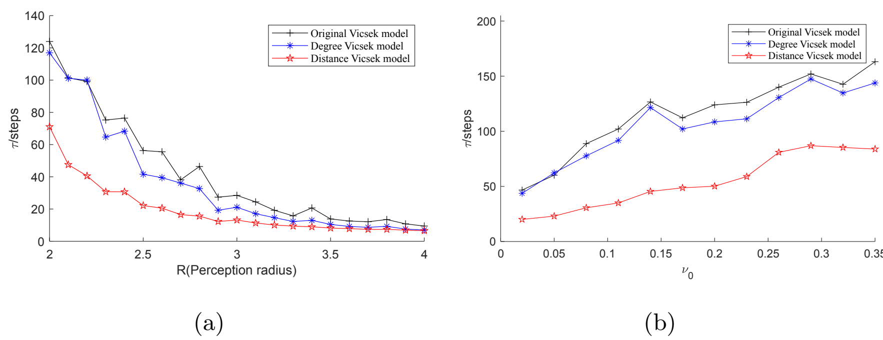


Figure 2. (a)The transient time step τ as a function of the neighborhood radius R. (b)The transient time step τ as a function of the velocity v_0. Here $R = 2.6$

In the course of heading renewal, different rules not only affect the convergence time, but also decide whether the whole group can reach consensus at the same initial conditions. According to different sense radius, 400 initial distributions are selected respectively to investigate convergence performance. As is presented in Table.1, this paper tallies the count of failures of group reaching agreement. It is shown that the DM offer a relatively higher probability of dividing into several groups although it shortens the convergence time, and our model realizes optimum performance of both convergence time and probability.

Table 1. Consensus simulation

Vicsek Model	**Perception radius**			
	$R = 2$	$R = 2.4$	$R = 2.8$	$R = 3.2$
OM	170	47	12	6
DM	182	59	13	3
DWM	56	11	1	0

The next we consider this three models VAC. According to different sense radius, 1000 initial distributions are selected respectively to observe the value of VAC. As is shown in Figs. 3, the VAC increases as the sensor radius increases. It is obvious that DWM is an efficient model in enhancing the VAC of self-propelled particles which results in faster convergence speed.

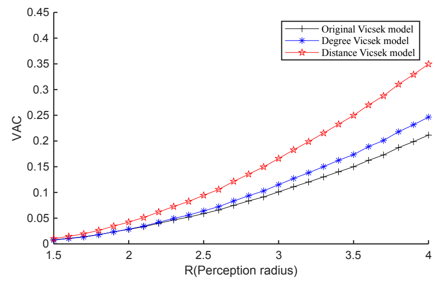


Figure 3. The VAC as a function of neighborhood radius$R$.

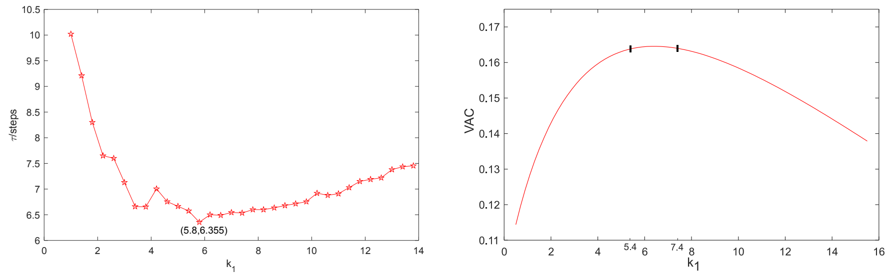
4.2 Influence of Weight Coffeicient

We then investigate the influence of the weight coefficient on the consensus process. The average convergence steps for the DWM significantly decreases with increasing values of k_1 till $k_1 = 5.8$ As can be observed in Figs. 4(a), $k_1 = 5.8$ is the optimal weight coefficient where the fastest consensus speed is obtained. The VAC as a function of k_1 is shown in Fig. 4(b). It is shown that there is an optimal value of k_1 where results in the maximum VAC. Moreover, there is an optimal value range which is $[5.4, 7.4]$ in figure where VACs difference is within 0.001. These results in Figs. 4(b) correspond to the results in Figs. 4(a) which indicates VACs effectiveness in convergence analysis.

Figure 5(a) shows the VAC as the function of k_2, which shows that increases the value of k_2 can enhance the convergence efficiency. It also shows that there is a threshold of k_2 over which the value of VAC will converge to a max value which is determined by the value of k_1. The variation of the value of k_1, where maximum VAC is obtained, is plotted against varying perception radius in Figs. 5(b). This optimal value of k_1 is denoted by k_{1opt}. It is obvious that the optimal weight coefficient increases as the perception radius increases till $R = 3.7$. The DWM will gradually degenerate into an OVM with continuously increasing sensing radius. In the simulation process, we noticed that there is not directly relevance between optimal weight coefficient and velocity.

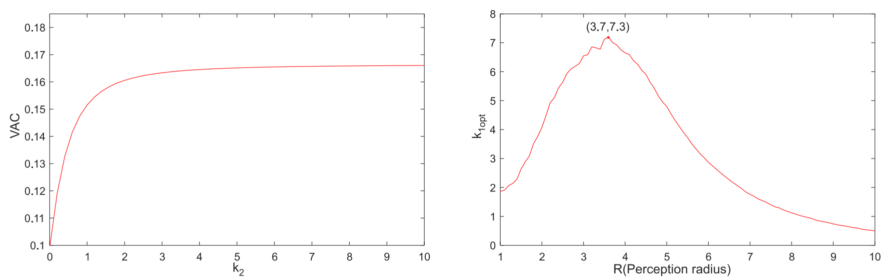
5 Conclusion and Future Work

Increasing the proportion of agents headings which are far away from the updating particle can accelerate consensus. Comparing with DM, it does not require



(a) (b)

Figure 4. (a)The transient time step τ as a function of weight coefficient k_1. (b)The VAC as a function of weight coefficient k_1.



(a) (b)

Figure 5. (a) The VAC as a function of k_2. Here $k_1 = 6$ (b)The optimal value of k_1 as a function of R.

additional communication between the agents. The new updating rule introduced in this paper ensures higher convergence probability and lower average convergence steps for a system of self-propelled swarm with open boundaries. Moreover, proposed parameter VAC effectively evaluates the convergency performance and matches the real results. These results can enlighten other researchers to design the manmade swarms. In this work, we have just considered this method in Vicsek model. Extending these results to other models are currently in progress.

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