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Form-closure caging grasps of polygons with a parallel-jaw gripper

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SUMMARY
Parallel-jaw gripper finds wide applications in various industrial sectors. In this paper, we mainly focus on the problem of form closure caging grasps of polygons with a parallel-jaw gripper equipped with four fingers. The form closure caging grasp is helpful for the fingers placements and contact region selections of a pneumatic gripper, as it is less sensitive to fingers misplacements.

We firstly prove that there is always a path from a cage to a form closure grasp of the object that never breaks the cage, as long as the attractive region constructed in the configuration space has a local minimum. If such a minimum cannot be found, we further adjust the fingers arrangements to produce the form closure grasp. Meanwhile, we also develop an algorithm to compute the initial cage of the form closure grasp. Simulations of the grasping process witness the effectiveness of the above analysis results.

KEYWORDS: Parallel-jaw gripper; Caging grasps; Attractive region.

1. Introduction

Grasp is very important in many manufacturing processes, such as handling, sorting and packing. Computation of grasps is usually based on sufficient conditions, such as the form or force closure configurations, to restrain all the motions of the objects. Nguyen[9] presented a computation algorithm to obtain a subset of form closure configurations consisting of independent edge regions. Given a polygonal part with n edges and a maximum diameter d, Brost and Goldberg[2] presented an algorithm for computing all of the form closure configurations. Moreover, there are many other efforts on form or force closure grasps investigations and computations with 2D and 3D objects. Comprehensive reviews on grasping were referred to[1] and[20].

Compared with the previous work motivated by high precision applications that require objects to be firmly grasped, caging provides a way to manipulate an object without immobilizing it. Kuperberg[7] originally posed a formal definition for the caging problem. They defined the caging set as the set of placements of fingers that prevents a polygon to move arbitrarily from a given position. Kriegman[8] regarded the cage as the set of system configurations which may not immobilize the object being manipulated but prevent it from escaping to infinity. Based on this concept, Sudsang *et al.* worked on how to grasp polygonal objects,[15] and on sufficient condition for capturing objects with disc-shaped mobile robots in the plane.[16] Rimon *et al.*[13] proposed caging grasps for a one-parameter two-finger gripper, where the caging set was defined as the hand configurations which maintain the object caged or confined between the fingers. Davidson *et al.*[3] analyzed the problem of caging 2D objects with a three-finger gripper, and they[4] also presented the error-tolerant visual 2D grasping strategy based on caging grasps. Erickson *et al.*[6] studied the problem of caging a convex object with three fingers, and proposed both exact and approximate algorithms to render the caging region, assuming that two fingers were fixed on the boundary of the convex object. Vahedi and Stappen[17,18] developed algorithms for computing all possible placements of two fingers and three fingers that cage a given closed polygon

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bounded by n edges. When caging a polygon object with three fingers, they also computed all placements of the third finger, in which the placements of two base fingers were given. Diankov *et al.*[5] presented the grasp planning of a multi-finger gripper by computing caging grasps specific to the objects. They regarded a cage as the condition where a robot hand constrains the configuration space of an object to a finite volume. Pipattanasomporn and Sudsang[10] worked on caging concave polygons with two fingers. They presented a combinatorial algorithm to report all caging sets for two point fingers and a given concavities of planar polygons, by identifying the associated representative finger placements and critical distances. Rodriguez[14] discussed the relationship between the grasps and cages of a rigid 2D object with two or three fingers. They illustrated that some cages especially suited as waypoints to grasp an object, that is, there is a set of configurations of the manipulator where the object will never escape the reach of the manipulator. Wan *et al.*[19] performed a grasp with three or four fingers to manipulate planar convex objects in a grasping-by-caging way. They demonstrated that the minimum caging is immobilization.

Pneumatic gripper finds wide applications in various industrial sectors. The most popular types of pneumatic grippers are the 2-finger, 3-finger, and 4-finger grippers. As discussed by Qian[12], the 4-finger gripper has many advantages. For example, the gripper is versatile to grasp different kinds of parts, and the state of the grasped object is usually unique and can be predicted.

This paper aims to discuss the form-closure caging grasps of polygons with a pneumatic four-finger gripper. The form closure caging grasp is helpful for the fingers placements and contact region selections of a pneumatic gripper, as it is less sensitive to fingers misplacements. We prove that there is always a path from a cage to a form-closure grasp of the object that never breaks the cage, as long as the attractive region has a V-bottom (i.e., there is only local minima of the attractive region).

Relevant to our work are the concepts of grasping-by-caging proposed in recent algorithmic approaches to the three-finger or four-finger caging problems.[14,19] They aim to find the initialization of caging of an immobilization grasp or a form closure grasp, where the placements of the fingers are flexible in the configuration space. The primary difference between their work and ours is that we focus on the strategy of the adjustment of the fingers placements to produce the form closure grasp. Also, we present the strategy to find the initial cage of the form closure grasp, where the pneumatic gripper geometry constraint is taken into account.

The rest of this paper is arranged as follows. In Section 2, concepts and definitions are introduced, and in Section 3 the sufficient condition of a form-closure grasp is discussed based on the attractive region. In Section 4, the adjustment of the fingers placements to produce a form closure caging grasp and the initialization of caging of the form closure grasp are explored. Finally, several simulations and experiments are given to illustrate the proposed method.

2. Terminology

In this section, we introduce some concepts and definitions related to attractive region, cage, form closure, and form closure caging grasps.

In our previous works[11,21] the concept of "attractive region" which concerns on the object's motion configuration space was presented. It was defined as a set of object configurations where the grasping system could be pulled to a stable state from any initial state under an active input.

Definition 1 (Attractive Region)[11,21] Assume that the state of a rigid object can be characterized as $\mathrm{d}X/\mathrm{d}t = f(X, u, t)$, X is the state of the object. For all X in the region Ω, if there exists a state-independent input u and a special function $g(X)$ satisfying

(a) $g(X) > 0$, $X \neq X_0$; $g(X_0) = 0$, $X \neq X_0$,
(b) $g(X)$ has continuous partial derivatives with respect to all components of X and
(c) $\mathrm{d}g(X)/\mathrm{d}t < 0$,

then the system is stable in the region Ω, which is called the attractive region in the environment.

Figure 1 explains the concept of "attractive region" by a dynamical system. Let us consider the example of the bean and the bowl from the view of a dynamical system. If the goal is to put the bean to the bottom of the bowl, then no matter where the initial position of the bean is, with the gravity, it would finally reach the bottom of the bowl. In this case, the position of the bean could be treated as

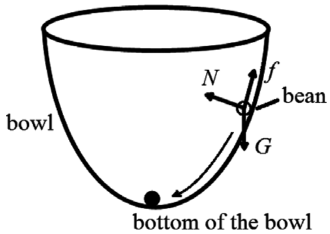


Fig. 1. Example of the attractive region in the motion region of a bean under the constraints of a bowl.

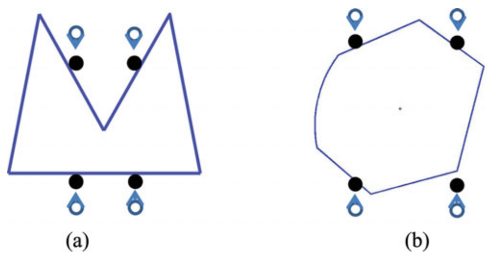


Fig. 2. (Colour online) Examples of (a) a form closure caging grasp, and (b) a caging grasp.

the state of a system, and the bowl as some constraints formed by environment. Under the effect of gravity, which is a state-independent input to the system, the state would finally converge to the goal region. In this process, it should be noted that the size of goal region is smaller than that of the initial region, that is to say, the uncertainty of the system is eliminated.

The grasping configuration space is determined by the fingers placements of the gripper and the geometric properties of the objects. Attractive region presents the state of the objects in the grasping configuration space. We will show how it can be applied to find a form-closure grasp in the next section.

Definition 2 (Cage).[18] A cage is a configuration of the gripper that bounds the mobility of the part when any motion of the object violates the rigidity of the part or the fingers.

Caging grasps guarantee that the object travels along with the fingers as these travels to their destination. The set of caging grasps is significantly larger than the set of form closure grasps. Moreover, caging grasps are considerably less sensitive to finger misplacements.

Definition 3 (Form closure).[1] A grasp is defined as form closure if and only if it is force closed with frictionless contacts.

Form closure is a condition of complete restraint in which the grasped body can resist any external disturbance wrench, irrespective of the magnitude of the contact forces.

Definition 4 (Form closure caging grasps). A form closure caging grasp is defined as a grasp where there is a path in the configuration space lead a caging configuration to a form closure configuration.

Figure 2(a) should be defined as a form closure caging grasp. The initial grasping configuration described by the four hollow circles is a cage, as they bound the mobility of the object. Meanwhile, the four solid circles constraint all motions of the object, so the finial grasping configuration is a form closure grasp. Figure 2(b) is a caging grasp but is not in form closure, because the four solid circles do not constraint all motions of the object. Moreover, in such a configuration space there does not exist a path that could produce a closure configuration (the simulation and experiment results will be discussed in Section 5.2).

It is noted that, first, a caging grasp is not sufficiently a fully immobilization of the object, and second, a form closure caging grasp is sufficiently a full immobilization of the object, and it is always a caging grasp.

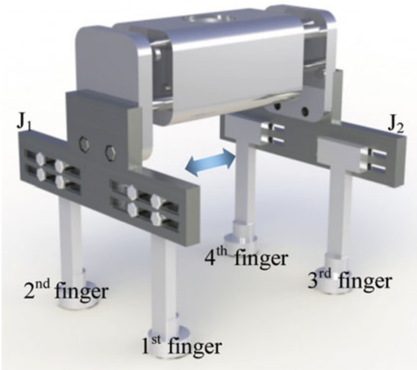


Fig. 3. (Colour online) The 4-finger gripper.

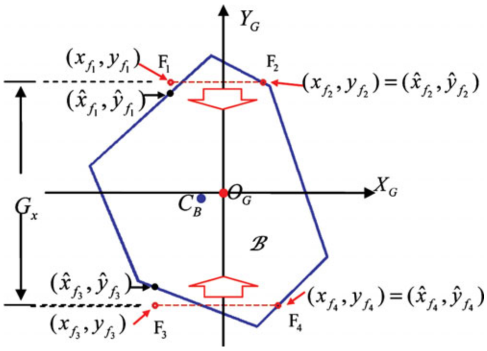


Fig. 4. (Colour online) The definition of the distance of the two base plates.

3. Construction of the Attractive Region

In this section, the attractive region formed in the grasping configuration space is analyzed, and then, based on the attractive region, the condition of form closure is also discussed.

3.1. Construction of the attractive function for a four-finger pneumatic gripper

The four-finger gripper used in this paper is shown in Fig. 3, where J_i (i = 1, 2) are the two base plates of the gripper, F_i (i = 1, 2, 3, 4) are the four fingers on the two base plates. The finger F_i is adjustable on the base plates. The placements of the four fingers implies that the arrangements of the 1st finger and the 2^{nd} finger on the base plateJ_1, and 3rd finger and 4th finger on the base plateJ_2. It should be noted that the adjustments of the four fingers on the two base plates are "off-line". The squeezing forces on the object are exerted by the two plates.

Denoting by B the object, and by C_B the center of B, the gripper's coordinate frame is built as shown in Fig. 3, where $O_G$is the center of the gripper. The position of finger F_i is denoted by (x_{f_i}, y_{f_i}). One can obtain

$$\begin{cases} y_{f_1} = y_{f_2} \\ y_{f_3} = y_{f_4} \\ |y_{f_1}| = |y_{f_3}| \end{cases}$$

Denoting by (x, y) and θ the position and orientation of the object in the gripper frame, respectively, we then use (F_1, F_2, F_3, F_4, X) to describe the grasping configuration, where $X = (x, y, \theta)$ is the pose of the object, and $F_i = (x_{f_i}, y_{f_i})$ (i = 1, 2, 3, 4) are the position of the four fingers.

We define the distance of the two base plates by G_x as follows,

$$G_x = y_{f_1} - y_{f_3} = 2|y_{f_1}| \tag{1}$$

where y_{f_1} and y_{f_3} are respectively the y-coordinate of F_1 and F_3 shown in Fig. 4.

Based on the discussion of the relationship between the pose of the object (i.e., X) and the distance of the two base plates of the gripper (i.e., G_x), below we are to establish the attractive function in the grasping configuration space.

Without loss of generality, we assume that initially not all fingers of the gripper contact with the object, for instance, only the 2nd and 4th finger touch the object at initial. Thus, the relationship between the pose of the object and the distance of the two base plates can be constructed as follows,

$$G_x = y_{f_1} - y_{f_3} = f\,(F, B; X) \tag{2}$$

where $F = (F_1, F_2, F_3, F_4)$ are the positions of the four fingers, B is the contour of the object, and $X = (x, y, \theta)$ is the pose of the object.

Denoting by $(\hat{x}_{f_i}, \hat{y}_{f_i})$ the projection of the i^{th} finger (x_{f_i}, y_{f_i}) on the edge of the object, as shown in Fig. 4, Eq. (1) can be then rewritten as

$$G_x = y_{f_1} - y_{f_3} = 2 \times \max_{i=1,\ldots,4} \left|\hat{y}_{f_i}\right| \tag{3}$$

where $(\hat{x}_{f_i}, \hat{y}_{f_i})$ satisfies the following equation

$$E_k\left(a_k, b_k, c_k; \hat{x}_{f_i}, \hat{y}_{f_i}\right) = 0, \quad k = 1, \ldots, N, \tag{4}$$

in which E_k denotes the line passing through $(\hat{x}_{f_i}, \hat{y}_{f_i})$, and N the number of the vertices of the object.

For a given position of the fingers, we can suppose

$$x_{f_i} = \hat{x}_{f_i} = c, \quad i = 1, \ldots, 4$$

where c is a constant, and meanwhile,

$|\hat{y}_{f_1}| = |y_{f_1}|$ or $|\hat{y}_{f_2}| = |y_{f_2}|$, if the plate J_1 touches the object.
$|\hat{y}_{f_3}| = |y_{f_3}|$ or $|\hat{y}_{f_4}| = |y_{f_4}|$, if the plate J_2 touches the object.

Thus, $B \cap F_i \neq \emptyset$.

The parameters of the straight line E_k, denoted by (a_k, b_k, c_k), is determined by the pose of the object. Thus, Eq. (2) can be written as

$$G_x = f\,(F, B; x, y, \theta) \tag{5}$$

Equation (5) describes the mapping between the distance G_x and the state (x, y, θ) of the object.

Because all the local minimum of space (x, y, θ, G_x) could be found in its subspace (x, θ, G_x) (see Appendix for details), we can use a three dimensional configuration space (x, θ, G_x) to describe the four dimensional grasping configuration space (x, y, θ, G_x).

In next section, based on the function (5) established in the configuration space, we will discuss: (a) whether there exists a form closure grasp related to the attractive region, and (b) how to detect the initial grasp in the attractive region that would lead to the form closure grasp.

3.2. *Find a form-closure grasp in the attractive region*

In the following, we prove that the local minimum of the attractive function indicates a form-closure grasp. That is, an object can be firmly grasped by the given configuration of the fingers, because the object can be "pushed" to the minimum of the attractive region by the fingers. We also use this condition to discuss whether there exists a form closure grasp for a given configuration of the fingers together with a given object.

Proposition 1: A V-bottom of the bowl-like attractive region indicates a form-closure grasp, where the V-bottom represents a local minimum of the attractive region.

Proof:

As shown in Fig. 4, denoting by $X = (x, y, \theta)$ the pose of the grasped object in the coordinate frame, the grasping system can be described as:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = f\,(X, F(t), t) \tag{6}$$

where $F(t)$ denotes the holding forces from the four fingers, and satisfies

$$\begin{cases} F_{\hat{y}_{f_i}}(t) > 0 \\ \mathrm{d}F_{\hat{y}_{f_i}}(t)/\mathrm{d}t > 0 \end{cases} \tag{7}$$

The contact points of the gripper and the object are denoted by $(\hat{x}_{f_i}, \hat{y}_{f_i})$, as shown in Fig. 4. We then define an energy function as below:

$$E_p\,(X, t) = \left[F_{\hat{y}_{f_i}}(t)\right]\left[\max_i |\hat{y}_{f_i}|\right]^T \tag{8}$$

where $F_{\hat{y}_{f_i}}(t)$ is the squeeze force applied by finger-i.

In the grasping process, $\max_i |\hat{y}_{f_i}|$satisfies:

$$\frac{\mathrm{d}(\,\max_i |\hat{y}_{f_i}|)}{\mathrm{d}t} \le 0 \tag{9}$$

From (6–9), we can obtain

$$\frac{\mathrm{d}E_p(X, t)}{\mathrm{d}t} \le 0$$

There exists a unique state X_0, which satisfies: for all X in the region $\|X - X_0\| < \varepsilon$ (ε is a positive number)

$$\begin{cases} E_p(X_s, t) > E_p(X_0, t), & X_s \ne X_0 \\ E_p(X_s, t) = E_p(X_0, t), & X_s{=}X_0 \end{cases}$$

Normally, an attractive region exists in the configuration space of X under the energy function $E_p(X, t)$. We can choose the energy function to design the attractive function, hence, we define

$$G_x = G_x(X) = 2 \times \max_{i=1,\ldots,4} \left|\hat{y}_{f_i}\right|$$

where G_x satisfies the following conditions:

(a) $G_x > G_0, X \ne X_0; G_x = G_0, X = X_0$;

(b) G_x has continuous partial derivatives with respect to all components of X and

(c) $\mathrm{d}G_x/\mathrm{d}X < 0$

Thus, $G_x$is an attractive function.

A point on the bottom of the attractive region corresponds to the object's state $X_s = X_0 = (x_0, \theta_0)$. If the object is within the neighborhood of X_0, it will transfer to X_0 under the squeeze forces.

Since the attractive region has a V-bottom, there is only one single state of the object X_0 corresponding to this V-bottom, which implies a full immobilization of the object. Thus, we achieve a form closure grasp.

An example of the attractive region of a polygonal object is given below, where the bowl-like shape represents an attractive region, and a point on the attractive region indicates a state of the object. It is obvious that the point on the bottom of the bowl (i.e., the minimum value of the attractive function) is a form-closure grasp.

The process of how to compute the attractive surface is described as follows.

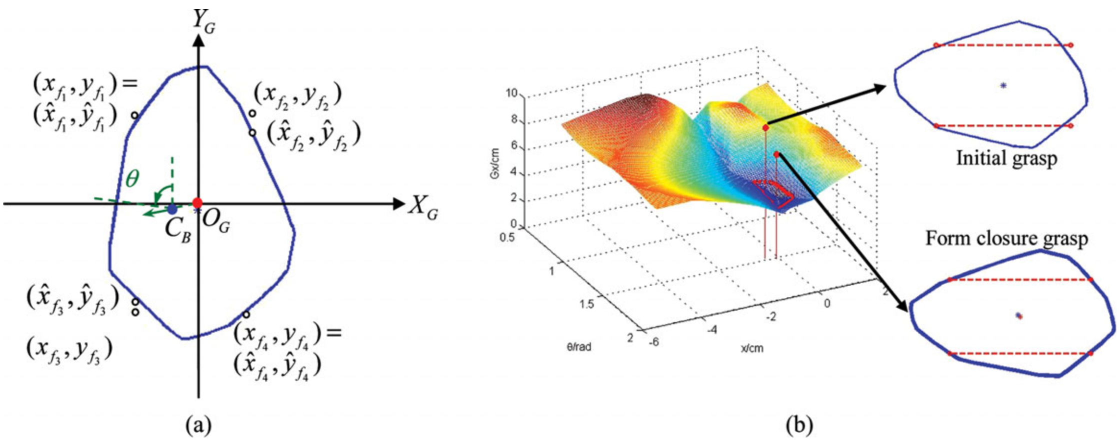


Fig. 5. (Colour online) (a) The grasped object in the gripper frame; (b) The attractive region for grasping of the polygon.

As shown in Fig. 5(a), the object is a convex polygon with 39 vertices, c_B the center of the polygon, O_G the center of the gripper, (x_{f_i}, y_{f_i}) the position of finger F_i. The pose of the object is described by (x, y, θ) in the gripper frame, where (x, y) is the position of the object, and θ is the orientation of the object. Consequently, the attractive shape can be generated based on Eq. (3) as follows,

$$G_x = y_{f_1} - y_{f_4} = 2 \times \max_{i=1,\ldots,4} \left|\hat{y}_{f_i}\right|,$$

where $\hat{y}_{f_i}$ is the projection of y_{f_i} on the edge of the quadrilateral, and satisfies

$$E_k\left(a_k, b_k, c_k; \hat{x}_{f_i}, \hat{y}_{f_i}\right) = 0, \quad k = 1, 2, 3, 4, \tag{10}$$

where E_k denotes the line passing through $(\hat{x}_{f_i}, \hat{y}_{f_i})$.

Meanwhile, the 39 edges E_k $(i = 1, \ldots, 39)$ can be formulated as follows:

$$\cos\theta_{i0} \cdot x + \sin\theta_{i0} \cdot y + c_i = 0 \tag{11}$$

where θ_{i0} denotes the angle between the i^{th} edge and X_G-axis, and c_i the perpendicular distance from the origin point to the i^{th} edge.

Therefore, when the pose of the object (i.e., x and θ coordinate) is changed, the shape of the attractive region can be obtained as shown in Fig. 5(b).

As shown in Fig. 5(b), the initial grasping configuration relates to a point inside the attractive region, and the finial grasping configuration corresponds to the minimum of the attractive region. If the attractive surface has a V-bottom, the finial grasping configuration then holds form closure. The initial state of the objects corresponds to a point on the attractive region, and the finial state of the objects relates to the bottom of the attractive region. Under the squeeze forces of the fingers, the object will definitely be pushed to the bottom of the attractive region.

However, in case the attractive region does not possess a V-bottom, the grasp cannot achieve the form-closure grasps for the given configuration of the fingers, as illustrated by Fig. 6.

As shown in Fig. 6, the bottom of the bowl-like shape is flat. The object is not constrained by the three contact points, since we can move it towards a new position, illustrated by the dashed line. That is, the state of the object is not determined when it arrive to the flat bottom, or in other word, a grasp is not always a form closure grasp.

4. Adjust the Placements of the Four Fingers for a form Closure Caging Grasp

In the previous section, we show that some grasps are not able to produce form-closure grasps for the special finger configuration of the gripper. In this section, a method to adjust the placements of the fingers to produce a form closure caging grasp is explored.

Table I. The algorithm to change the finger configuration.

Algorithm 1	
Step 0	Suppose the initial position of finger F_i is denoted by $\mathrm{f}_i = (x_{f_i}, y_{f_i})$, the adjustment distance of two base plates J_1 and J_2 is denoted by ΔL.
Step 1	Adjust the position of the i^{th} finger until it contacts with the object.
Step 2	Expand two base plates with distance ΔL, so, the distance of base plates changes from d_{min} to $(d_{min} + 2\Delta L)$.
Step 3	Suppose the state of the object keeps unchanged, compute the new position of the four fingers when they all touch the object.
Step 4	If the contact state at the new position forms a V-bottom, go to Step 5, otherwise go to Step 2.
Step 5	We can obtain the new placements of the four fingers, where a form closure caging grasp can be obtained.

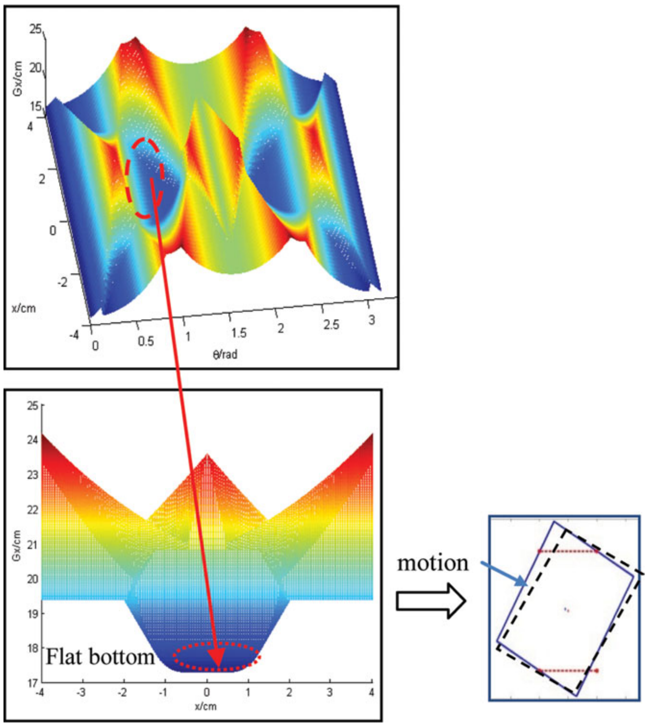


Fig. 6. (Colour online) The grasp cannot product a form-closure grasp.

4.1. Adjust the placements of the fingers to form a V-bottom

If the attractive region has a “flat bottom” or non- “V-bottom”, it is difficult to achieve a form closure grasp. In the following, a method to adjust the fingers placements to form the V-bottoms is discussed.

Suppose at the bottom of the attractive region, i^{th}-finger does not contact the object. The adjustment follows two steps: first to keep the existing contact state, and then to decrease the distance of two fingers on the same base plate. The algorithm to change the finger configuration is described as in Table I.

See Fig. 7 for an example, where the change in the configuration of the fingers will cause the observed change in the shape of the attractive region, and furthermore, the change from a flat bottom to a V-bottom will allow a form- closure grasp.

For the given objects, the adjustments of the fingers arrangements should ensure the V-bottoms exist in every attractive regions corresponding to the given objects.

4.2. Detect the initial cage to hold a form closure

As explained above, the attractive region formed by the contact constraints between the gripper and the object eliminates the state uncertainty of the object during the grasping. In this

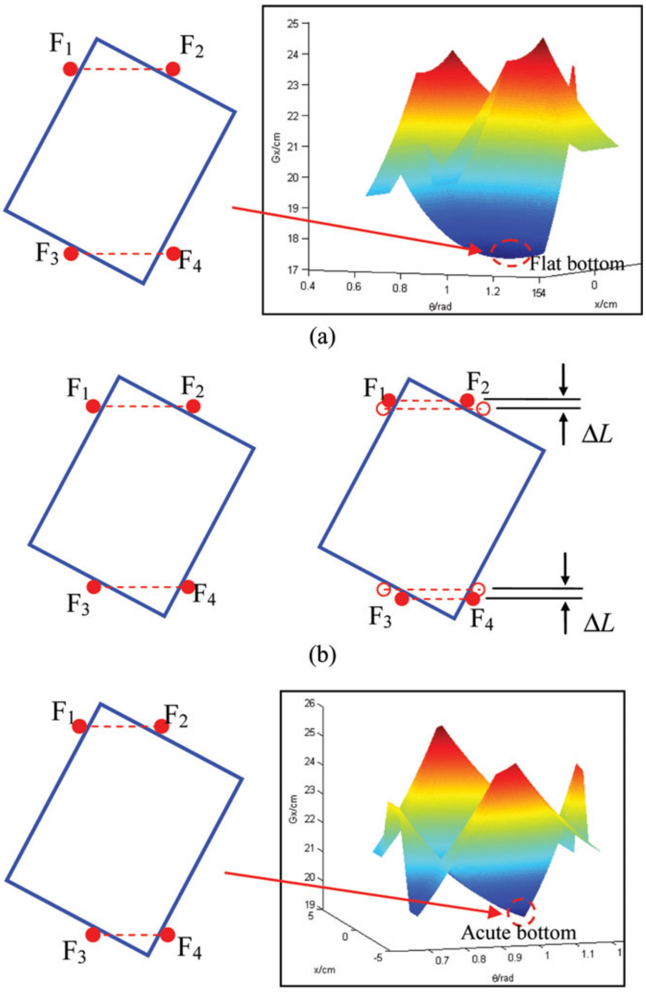


Fig. 7. (Colour online) Adjust the fingers configurations to form a "V-bottom". (a) The four contacts cannot prevent all motion of the grasped object, and the state of the object corresponds to a flat bottom on the attractive region. (b) According to Algorithm 1, we adjust F_4 to touch the object (Step 1), and then expand the two fingers with a small distance ΔL (Step 2). (c) The contact state at the new position forms a V-bottom, i.e., the four fingers formulate a form closure caging grasp (Step 3).

subsection, we explore how to determine the initial cage in order to find a form closure caging grasp.

The computation process of the initial cage is illustrated by Fig. 8, where the four edges of the five-edge polygon (shown in Fig. 8) which touches with the fingers are denoted by E_i (i = 1, 2, 3, 4) respectively, and vertexes on edge E_i are denoted by $P_{ir}(x_{ir}, y_{ir})$ (right) and $P_{il}(x_{il}, y_{il})$ (left).

The following two conditions are concluded to ensure that the object would be pushed into the attractive region

$$x_{il} \leq -\frac{r}{2} \quad \text{and} \quad x_{ir} \geq -\frac{r}{2}, \quad i = 1, 3 \tag{12a}$$

$$x_{jl} \leq \frac{r}{2} \quad \text{and} \quad x_{jr} \geq \frac{r}{2}, \quad j = 2, 4 \tag{12b}$$

The equations of the four edges E_i(i = 1, 2, 3, 4) have been defined by Eq. (11) previously.

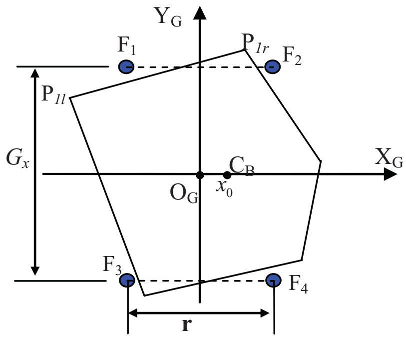


Fig. 8. (Colour online) The computation of the initial cage.

Then the initial cage can be presented as:

$$\mathbf{S}_\mathrm{I} = \left\{ (x_0, \theta) |\, x_{il} \leq -\frac{r}{2}, \quad x_{ir} \geq -\frac{r}{2}, \quad x_{jl} \leq \frac{r}{2}, \quad x_{jr} \geq \frac{r}{2}, \cos(\theta_{p_0} + \theta) \cdot x \right.$$
$$\left. + \sin(\theta_{p_0} + \theta) \cdot y + (c_p - x_0 \cdot \cos(\theta_{p_0} + \theta)) = 0 \right\} \tag{13}$$

where $i = 1, 3; j = 2, 4; p = 1, 2, 3, 4$.

It is noted that $x_{1r} \leq x_{2l}$ and $x_{3r} \leq x_{4l}$, thus we can obtain:

$$-\frac{r}{2} \leq x_{1r} \leq x_{2l} \leq \frac{r}{2} \tag{14a}$$

and

$$-\frac{r}{2} \leq x_{3r} \leq x_{4l} \leq \frac{r}{2} \tag{14b}$$

The above two inequalities implies that the range of the translation deviations of the object depends on the distance r of the two base plates. It is thus found that the "mouth" of the attractive region should be larger if the distance r increases.

In the following, an algorithm to figure out the initial cage on a given attractive region is described.

Suppose the initial cage is denoted by $C(X_0)$, where X_0 is the state of the object corresponds to the local minimum of the attractive region. Let v be a real number, S_v the set of object state where the distance d of two base plate J_1 and J_2 is less than v, and $C(X) = v$ a closed curved line. One can obtain

$$S_v = \{X\, |c(X) \leq v, c(X) = v \text{ is a closed curved line, } v \geq v_0, X \in U^{AR}(X_0)\}$$

where $v_0 = d(X_0)$, $U^{AR}(X_0)$ is the attractive region.

We project the curved surface S_v along -d direction and denote the projection area by $A(S_v)$. Then the maximum initial region is defined as:

$$C(X_0) = S_{v^*},\, v^* = \arg\max_v A(S_v)$$

Algorithm 2 describes the computation of the initial cage (Table II).

See Fig. 9 for an example, where the mouth of the attractive region denotes an initial cage.

As illustrated by Fig. 9, the computational algorithm of the initial cage is proposed as follows.

- Initially, we generate the attractive region shown in Fig. 9(a) based on Eq. (2), where x is the translation vector of the object, θ is the rotation of the object in the gripper frame, and d is the distance of the two base plates.

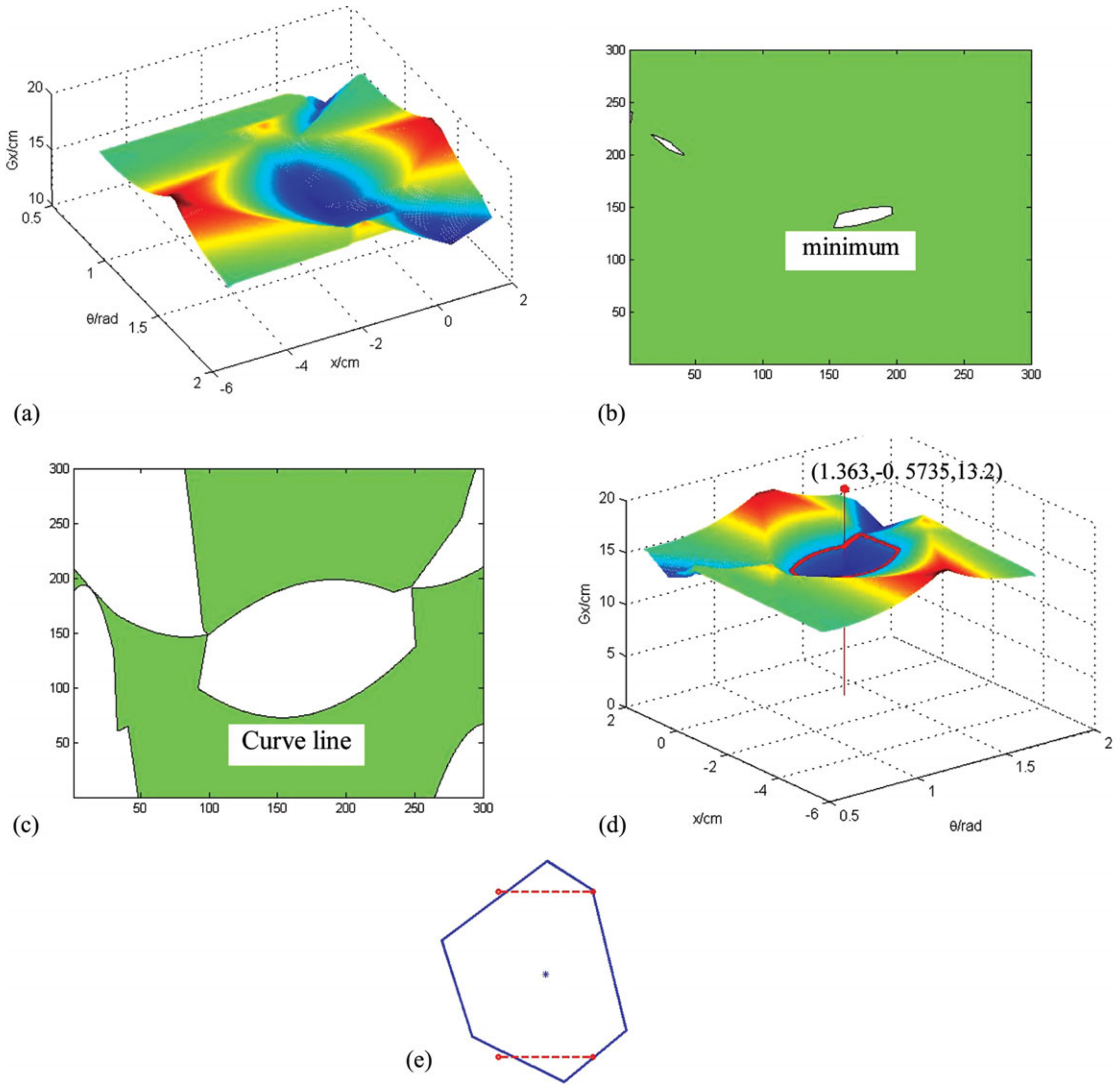


Fig. 9. (Colour online) Computation of the mouth of the attractive region. (a) The attractive region formed in the grasping; (b) The minimum of the attractive region; (c) Increasing the grasping distance with $v = v_0 + \Delta v$, then obtaining of the curve line $d(X) = v$. (d) The region marked by the red line on the attractive region is the initial cage, and a point below the curve represents a caging grasp. (e) The caging grasp corresponds to the point shown in (d).

- Then, we can obtain the minimum of the attractive region, as shown in Fig. 9(b). The minimum of the attractive region (i.e., a V-bottom) denotes a form closure grasp.
- We next increase the grasping distance, i.e., the distance of the two base plates by $v = v_0 + \Delta v$. Thus, we can obtain a closed curve line on the surface of the attractive region, as shown in Fig. 9(c).
- If the curve is not close, we stop the iteration, so, the region below the maximum closed curve, which is marked by the red line, is the initial caging region, as shown in Fig. 9(d).
- The four fingers in Fig. 9(e), which is marked by red circle, denote a caging configuration that corresponds to a point below the red curve.

5. Simulations and Experiments

In this part, several simulations and experiments are conducted to illustrate the proposed method, and then a vision-based grasp planning is proposed to grasp a given object.

The grasping problem is defined as below.

Table II. The algorithm of computation of the initial cage.

Algorithm 2	
Step 0	Given the attractive region $U^{AR}(X_0)$, the local minimum v_0, the adjustment step Δv. Initialize the grasping distance v as v_0.
Step 1	Update the grasping distance as $v = v_0 + \Delta v$.
Step 2	Slice the curved surface $U^{AR}(X_0)$ using the plane $d = v$ and obtain the curved line $d(X) = v$.
Step 3	If the curved line $d(X) = v$ is closed, then go to Step2; otherwise, go to step 4.
Step 4	The region enclosed by the curved line $d(X) = v - \Delta v$ is the desirable maximum initial state region.

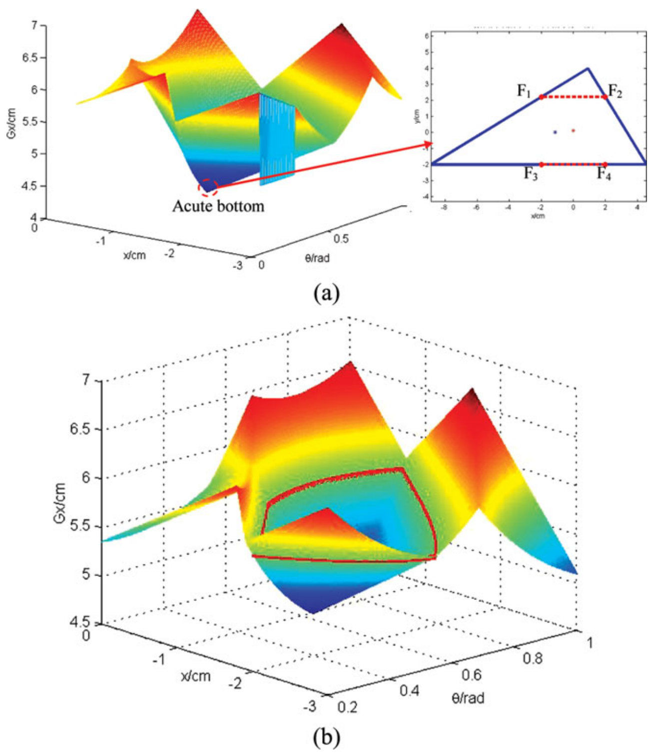


Fig. 10. (Colour online) Caging grasps of a triangle. (a) The attractive region of a triangle has a V-bottom. (b) The region marked by the red line on the attractive region is the initial cage.

- Construct the attractive regions of the given objects.
- Adjust the placements of fingers to find a form closure caging grasp.
- Obtain the initial cage based only on some image information, such as the edges and the position of the contour.

5.1. Caging grasps of convex polygons

We firstly discuss the attractive regions of two objects correspond to the given configuration of the fingers. The initial *x*-coordinate of 1st finger and 3rd finger are set by -*l*, the *x*-coordinate of 2^{nd} finger and 4th finger are set by *l*. In this case, we can obtain a form closure caging grasp of a triangle (shown in Fig. 10) and a convex polyhedron (shown in Fig. 11).

The grasping of the convex polyhedron is shown in Fig. 12. If we make the placements of the four fingers locate inside the mouth of the attractive region (as computed by Fig. 11(b)), the object can be pushed to the bottom of the attractive region.

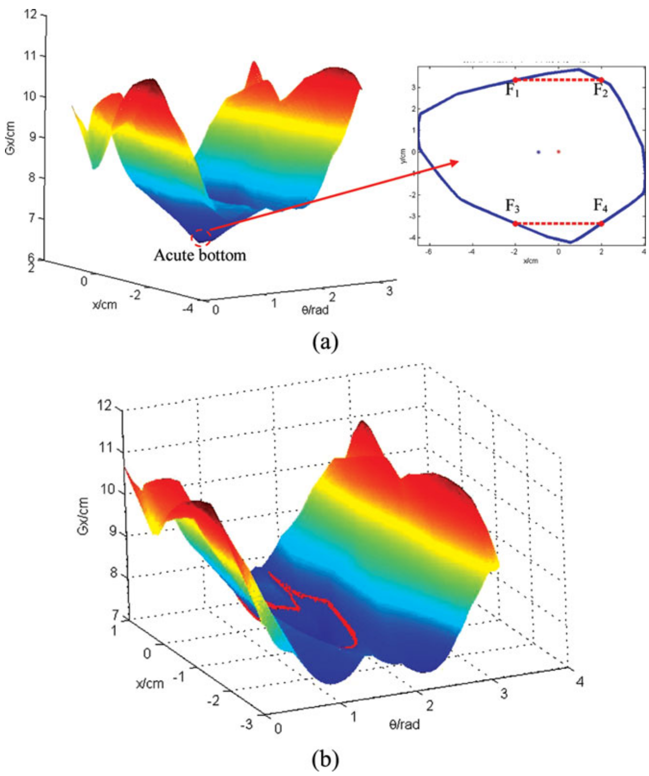
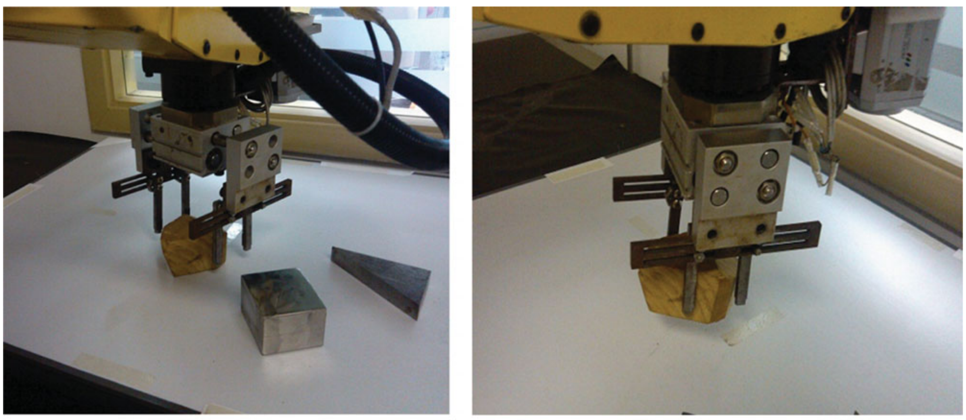


Fig. 11. (Colour online) Caging grasps of a convex polygon. (a) The attractive region of a convex polygon has a V-bottom. (b) The region marked by the red line on the attractive region is the initial cage.



(a) (b)

Fig. 12. (Colour online) Grasping of the polygon with the pneumatic gripper. (a)The gripper arrives to the initial grasping configuration. (b) Squeeze the object to a stable state.

5.2. Caging grasps of a convex polygon needing adjustments of fingers

Figures 13 and 14 illustrate the adjustment of the fingers placements to guarantee a form-closure grasp of a convex object.

The experimental grasping results of the polygonal object are shown in Fig. 14.

Figure 13 shows the computation of the attractive region and the adjustment of the fingers for a form closure caging grasp. Figure 14 shows the experimental grasping results of the polygon with or without adjustment of the fingers, in which the object is placed in allowed initial cage in the initial grasp configuration.

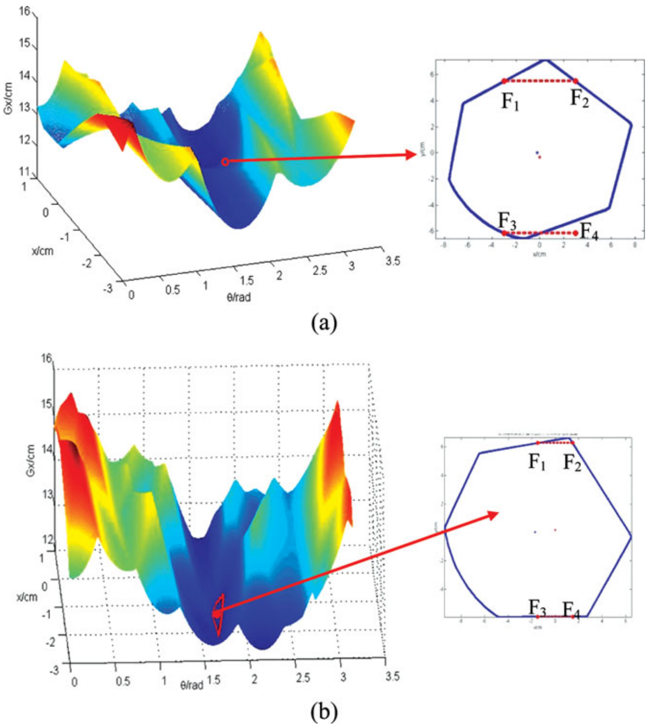
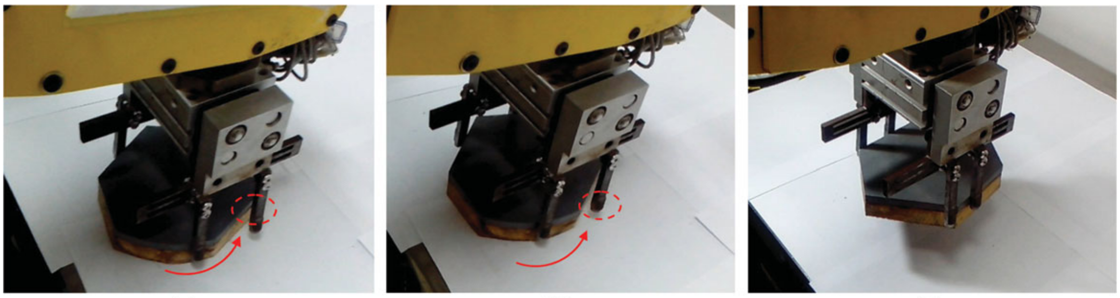


Fig. 13. (Colour online) Adjustments of the fingers configuration to produce a form closure grasp. (a) The four contacts do not prevent all motion of the grasped object, which corresponds to a flat bottom of the attractive region. (b) We adjust the distance of 1st finger and 2nd finger, the distance of 3rd finger and 4th finger to produce a V-bottom, so, the V-bottom of the attractive region notes a form-closure grasp. And, the region marked by the red line on the attractive region is the initial cage.



(a) (b) (c)

Fig. 14. (Colour online) Grasping of the polygon with the pneumatic gripper. (a) The experimental grasping result relates to Fig. 13(a), in which Finger-4 does not touch the object (marked by red circle) and we can rotate the object. (b) The experimental grasping result relates to Fig. 13(a), in which Finger-4 still does not touch the object(marked by red circle) after the object has been rotated. (c) Experimental grasping result relates to Fig. 13(b), in which the object is firmly grasped.

5.3. Caging grasps of concave polygons

In Fig. 15, we would discuss the grasps of a polygon with concave. Figure 15(a) and (b) show the simulation results. Figure 15(c) shows the experimental grasping results of the concave polygon, in which the object is placed in allowed initial cage in the initial grasp configuration.

The grasping of the polygon with concave is shown in Fig. 15. If we make the placements of the four fingers locate inside the mouth of the attractive region, the objects can be pushed to the bottom of the attractive region.

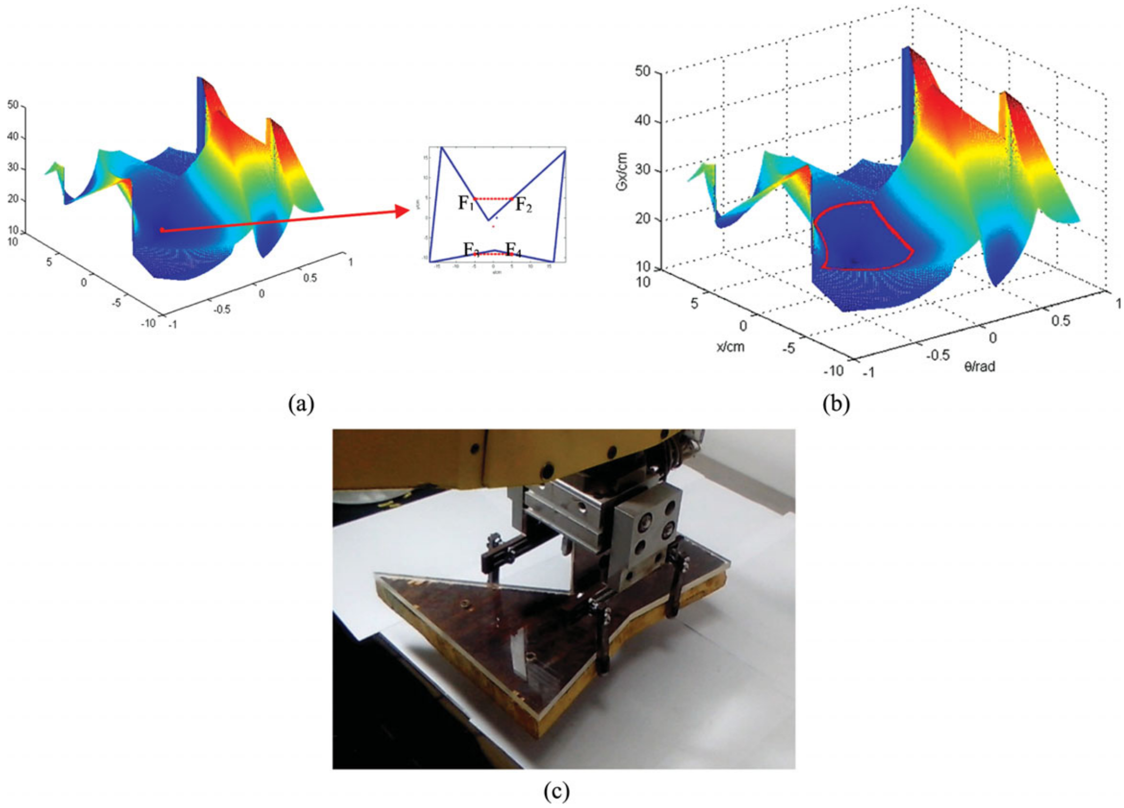


Fig. 15. (Colour online) Grasping of the concave polygon. (a) The attractive region of a convex polygon has a V-bottom. (b) The region marked by the red line on the attractive region is the initial cage. (c) The experimental grasping results of the concave polygon.

6. Conclusion

Gripper finger placements and contact region selections are particularly challenging when the natural resting pose of a part differs from the pose desired for grasp. Given a set of objects, how to design the finger configuration, rotate it into a desired orientation and then grasp it securely is very important for an industrial grasping system.

Based on our previous works on attractive region, we discuss the fingers arrangements of the pneumatic gripper. We analyze the decomposition of the four-dimensional grasping configuration space, such that the attractive region can be visibly constructed in the three dimensional subspace. Then, we discuss the relationship between the fingers configuration and the shape of the attractive region, and present a method to adjust the fingers configuration to produce a V-bottom on the attractive region, which corresponds to a form closure grasp. Last but not least, we explore how to determine the initial cage of the object to produce the form closure caging grasp.

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Appendix

In the Appendix, we will discuss the relationship between two configuration space S and $\tilde{S}$.

We suppose S is denoted by the set of the object's state when at least one finger touches the object.

$$S = \{X \mid B \cap F \neq \emptyset,\ x \in [x_{f,\min}, x_{f,\max}],\ y \in [y_{f,\min}, y_{f,\max}],\ \theta \in [-\pi, \pi)\}$$

where $x_{f,\min} = \min_i x_{f_i}$, $x_{f,\max} = \max_i x_{f_i}$, $y_{f,\min} = \min_i y_{f_i}$, $y_{f,\max} = \max_i y_{f_i}$.

Define S' by the set of the object's state when the two base plates of gripper touch the object (a base plate touches the object means any fingers on the base plate touch the object), one can obtain

$$S' = \{X \mid B \cap (F_1, F_2) \neq \emptyset,\ B \cap (F_3, F_4) \neq \emptyset,\quad x, y \in R,\ \theta \in [-\pi, \pi)\}$$

where $\{F_1, F_2\}$ are denoted by the 1st and the 2nd finger on the upper plate J_1, $\{F_3, F_4\}$ is denoted by the 3rd and the 4th finger on the lower plate J_2.

Denoted by $\tilde{S} = S \cap S'$, where S describes that at least one finger touches the object, S'denotes that at least one finger on upper plate J_1 and at least one finger on lower plate J_2 touch the object simultaneous.

The relationship between the two configurations spaces S and $\tilde{S}$ is described in proposition A.1.

Proposition A.1: $\forall X = (x, y, \theta) \in S$ satisfies $X \in \tilde{S}$, if and only if the following equation is satisfied:

$$\max\left\{\left|\hat{y}_{f_1}\right|, \left|\hat{y}_{f_2}\right|\right\} = \max\left\{\left|\hat{y}_{f_3}\right|, \left|\hat{y}_{f_4}\right|\right\}$$

Proof:

Without loss of generality, we assume that

$$\max\left\{\left|\hat{y}_{f_1}\right|, \left|\hat{y}_{f_2}\right|\right\} = \left|\hat{y}_{f_1}\right|, \max\left\{\left|\hat{y}_{f_3}\right|, \left|\hat{y}_{f_4}\right|\right\} = \left|\hat{y}_{f_3}\right|$$

so, we only require to prove the condition $|\hat{y}_{f_1}| = |\hat{y}_{f_3}|$.

If $|\hat{y}_{f_1}| \neq |\hat{y}_{f_3}|$, we suppose $|\hat{y}_{f_1}| > |\hat{y}_{f_3}|$, which notes finger F_1 touch the object B firstly, but finger F_3 do not touch the object. $\max\{|\hat{y}_{f_3}|, |\hat{y}_{f_4}|\} = |\hat{y}_{f_3}|$ notes that finger F_4 also do not touch B, i.e., $B \cap (\mathrm{F}_3, \mathrm{F}_4) = \emptyset$, which contradicts to the condition $X \in S$.

Similarly, when $|\hat{y}_{f_1}| < |\hat{y}_{f_3}|$, finger F_3 should touch B firstly. In this case, finger F_1 has no contact point with B, so, one can obtain$B \cap (F_1, F_2) = \emptyset$, which contradicts to $X \in S$.

Thus, we can obtain $|\hat{y}_{f_1}| = |\hat{y}_{f_3}|$, i.e., for any $X \in S$, $\max\{|\hat{y}_{f_1}|, |\hat{y}_{f_2}|\} = \max\{|\hat{y}_{f_3}|, |\hat{y}_{f_4}|\}$. $\max\{|\hat{y}_{f_1}|, |\hat{y}_{f_2}|\} = \max\{|\hat{y}_{f_3}|, |\hat{y}_{f_4}|\}$ implies that base plates J_1 and J_2 would touch object B simultaneous, i.e., $X \in S$. In the following, we prove that all the local minimum of space (x, y, θ, d) in its subspace (x, θ, d).

Proposition A.2:$\forall X = (x, y, \theta) \in S$, if

$$y^* = h(x, \theta) = \arg\min_y f(F, B; X)$$

$$X^* = (x, y^*, \theta) = (x, h(x, \theta), \theta),$$

thus$X^* \in S$.

Proof:

Suppose the state of the object is denoted by $X^* = (x, y^*, \theta)$, the desired position of ith finger, i.e., $\hat{y}^*_{f_i}(i = 1, \ldots, 4)$, can be presented by

$$\max\left\{\left|\hat{y}^*_{f_1}\right|, \left|\hat{y}^*_{f_2}\right|\right\} = \max\left\{\left|\hat{y}^*_{f_3}\right|, \left|\hat{y}^*_{f_4}\right|\right\}$$

We set

$$z_1 = \max\left\{\left|\hat{y}^*_{f_1}\right|, \left|\hat{y}^*_{f_2}\right|\right\}, z_2 = \max\left\{\left|\hat{y}^*_{f_3}\right|, \left|\hat{y}^*_{f_4}\right|\right\}$$

Without loss of generality, we suppose $z_1 \neq z_2$. When $z_1 > z_2$, the distance between two fingers can be expressed by $d = 2 \times z_1$. Thus, $B \cap (F_1, F_2) = \emptyset$,$B \cap (F_3, F_4) = \emptyset$.

Suppose another state of object by $X' = (x, y^* - \Delta/2, \theta)$, where $\Delta = |z_1 - z_2|$, then $B(X') \cap (F_1, F_2) = \emptyset$. In this case, the gripper would keep squeezing force on the object until $B(X') \cap (F_1, F_2) = \emptyset$.

Corresponding to the state $X' = (x, y^* - \Delta/2, \theta)$, one can obtain $d' = d - \Delta < d$, i.e.,

$$f\left(F, B; x, y^* - \Delta/2, \theta\right) < f\left(F, B; x, y^*, \theta\right)$$

But, it contradicts to

$$f(F, B; x, y^*, \theta) = \min_y f(F, B; x, y, \theta).$$

Similar, if $z_1 < z_2$, there exists a state $X'' = (x, y^* + \Delta/2, \theta)$

$$f\left(F, B; x, y^* + \Delta/2, \theta\right) < f\left(F, B; x, y^*, \theta\right)$$

It contradicts to

$$f(F, B; x, y^*, \theta) = \min_y f(F, B; x, y, \theta).$$

Therefore, $z_1 = z_2$, so, we can obtain $X^* \in S$. Illustrations of Proposition A.1 and Proposition A.2 are given as follows:

Proposition A.1 describes the relation of S and $\tilde{S}$, where $\tilde{S}$ consists of the components which belongs to S and obtains the local minimum of the distance on y direction. (x, θ, G_x) is a subspace of the space (x, y, θ, G_x), which keep the properties:

All the state describe the two fingers touch the object on space (x, y, θ, d) is also located in its subspace (x, θ, G_x);

All the local minimum of space (x, y, θ, G_x) can be find in its subspace (x, θ, G_x);

Proposition A.2 points that when $(x, y, \theta) \in S$, the translation of object along y-direction is a function of (x, θ), i.e., $y = h(x, \theta)$.