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A Simple and Analytical Procedure for Calibrating Extrinsic Camera Parameters

Fei-Yue Wang

***Abstract*—This paper presents a simple and analytical procedure for calibrating extrinsic camera parameters. First, a calibration equation that separates rotational and translational parameters is given. The calibrating equation involves only rotational parameters and requires no absolute position information. A four-point calibration procedure is proposed that involves three points on a line and one point out of the line, and leads to four possible calibration solutions, obtained analytically. Additional steps required to eliminate false solutions are also discussed. Once the true rotational parameters are identified, the translational parameters are obtained analytically and uniquely with additional absolute position information. The required absolute position information appears in a simple and explicit form. For extrinsic calibration among multiple cameras, such as stereo cameras, it is easy to show that the absolution position information is not needed.**

***Index Terms*—Camera calibration, extrinsic, multiple camera systems, parameter calibration.**

I. Introduction

Camera calibration is an important task in computer vision, robotics and automation, and computer-integrated manufacturing systems. Over the past few decades, considerable effort has been made on development of effective and accurate procedures and algorithms to identify the internal camera geometric and optical characteristics (intrinsic parameters) and/or the three-dimensional (3-D) position and orientation of the camera frame relative to a certain world coordinate system (extrinsic parameters) for various applications [1], [3]–[5], [8], [12]. Most work in this area is focused on the complete camera calibration that involves calibrating both intrinsic and extrinsic camera parameters simultaneously.

However, for many applications, intrinsic camera calibration is required only once, but extrinsic camera parameters have to be calibrated many times, and even constantly over extended operating periods. For those applications, we can assume that the intrinsic camera parameters have been calibrated, and the remaining task is to develop an effective and accurate procedure for calibrating extrinsic parameters frequently. Such applications can be found in vehicle guidance [2], [7], [10], robotic vision, visual security systems, 3-D traffic monitoring, automated 3-D surveys of road accident sites, field monitoring systems in mining operations, and motion estimation using airborne camera systems [9]. In those cases, position and orientation of cameras are subject to constant, random, and significant disturbances, due to motion, thermal effects, environmental variation, or other unpredictable factors, and thus, the relative position and orientation of cameras must be calibrated repeatedly for extracting any meaningful 3-D information from computer images. In many situations, such as vehicle guidance

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TABLE I
INTRINSIC CAMERA PARAMETERS

Symbol	meanings
f	focal length
x_p, y_p	image coordinate of the principal point
k_1, k_2, k_3	radial distortion
p_1, p_2	lens decentering distortion
a_1, a_2	affinity distortion

and robotic vision, real-time information is critical for successful applications. Therefore, an efficient extrinsic parameter calibration procedure that requires less preparation in setup and less computation in processing would be extremely useful.

Based on our previous work [11], the focus here is to specify a simple, analytical, and accurate procedure for calibrating extrinsic camera parameters that can meet the time and accuracy constraints imposed by various real-time camera applications. Section II presents the calibrating equation that involves rotational parameters only, Section III introduces a simple calibrating procedure and its four possible analytical solutions, and finally, Section IV gives the procedure for calibrating extrinsic parameters between cameras without using the absolute position information.

II. EXTRINSIC CAMERA CALIBRATION EQUATIONS

Let $W = (X, Y, Z)^T$ be the 3-D world coordinate system and $c = (x, y)^T$ be the corresponding 2-D image coordinate system of a camera. For the sake of simplicity, it has been assumed that the camera world and image coordinate axes are identical, and Z and z are the optical axis of the camera. Then image coordinate c and world coordinate W are related by [6]

$$d(c, \zeta) = \begin{pmatrix} d_x \\ d_y \end{pmatrix} = -\frac{f}{Z}\begin{pmatrix} X \\ Y \end{pmatrix} \tag{1}$$

where $\zeta = (k_1, k_2, k_3, p_1, p_2, a_1, a_2, x_p, y_p, f)$ is the vector of intrinsic camera parameters, and

$$\begin{aligned} d_x &= \Delta x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) \\ &\quad + p_1(r^2 + 2\Delta x^2) + 2p_2\Delta x\Delta y \\ d_y &= \Delta y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) \\ &\quad + p_2(r^2 + 2\Delta y^2) + a_1\Delta x + a_2\Delta y \\ \Delta x &= x - x_p \\ \Delta y &= y - y_p \\ r^2 &= \Delta x^2 + \Delta y^2. \end{aligned}$$

In (1), both the effects of symmetrical and asymmetrical lens distortions are considered. Table I gives the meaning of each intrinsic parameter. As we have assumed in the beginning, intrinsic camera parameter vector ζ is known here. Note that Tsai [8] had pointed out that, for many applications, not all intrinsic parameters are needed.

Let G be a reference world coordinate system, R be the rotation matrix, and p the translation vector from W to G, then

$$W = R \cdot P + p \tag{2}$$

where P is a point in the reference coordinate system G. In terms of G, (1) can be written as

$$fp(1) + d_x p(3) + [fR(1) + d_x R(3)]P = 0 \tag{3a}$$

$$fp(2) + d_x p(3) + [fR(2) + d_y R(3)]P = 0 \tag{3b}$$

where $p(j)$ and $R(j), j = 1, 2, 3$ are the jth element of translation vector p and the jth row of rotation matrix R, respectively. (p, R) represent the extrinsic parameters of the camera, and the objective here is to find a procedure to determine their value.

To this end, let $G_i, i = 0, 1, 2, \ldots, n$ be $(n+1)$ points in G, called calibration points. G_0 and relative position information

$$\Delta G_i = G_i - G_0, \quad i = 1, 2, \ldots, n$$

are assumed to be known. The corresponding image coordinates c_i of G_i, and thus $d(c_i, \zeta) = (d_{xi}, d_{yi})^T$, are also given for $i = 0, 1, \ldots, n$. Therefore, for any two calibration points G_j and G_k, we have

$$\begin{bmatrix} f & 0 & d_{xj} \\ 0 & f & d_{yj} \\ f & 0 & d_{x0} \end{bmatrix} p + \begin{bmatrix} [fR(1) + d_{xj}R(3)]G_j \\ [fR(2) + d_{yj}R(3)]G_j \\ [fR(1) + d_{x0}R(3)]G_0 \end{bmatrix} = 0$$

$$\begin{bmatrix} f & 0 & d_{xk} \\ 0 & f & d_{yk} \\ 0 & f & d_{y0} \end{bmatrix} p + \begin{bmatrix} [fR(1) + d_{xk}R(3)]G_k \\ [fR(2) + d_{yk}R(3)]G_k \\ [fR(2) + d_{y0}R(3)]G_0 \end{bmatrix} = 0$$

which leads to

$$\begin{aligned} &\Delta u_j(p + RG_0) \\ &\quad = \begin{bmatrix} u_0 R(1)\Delta G_j + u_o u_j R(3)\Delta G_j \\ v_j R(1)\Delta G_j - \Delta u_j R(2)\Delta G_j + u_o v_j R(3)\Delta G_j \\ -R(1)\Delta G_j - u_j R(3)\Delta G_j \end{bmatrix} \end{aligned} \tag{4a}$$

$$\begin{aligned} &\Delta v_k(p + RG_0) \\ &\quad = \begin{bmatrix} u_k R(2)\Delta G_k - \Delta v_k R(1)\Delta G + v_o u_k R(3)\Delta G_k \\ v_0 R(2)\Delta G_k + v_0 v_k R(3)\Delta G_k \\ -R(2)\Delta G_k - v_k R(3)\Delta G_k \end{bmatrix} \end{aligned} \tag{4b}$$

where

$$u_j = d_{xj}/f, \quad v_j = d_{yj}/f, \quad \Delta u_j = u_j - u_0, \quad \Delta v_j = v_j - v_0.$$

Since (4a) and (4b) must lead to the same solution for p, we conclude that

$$\begin{aligned} &\Delta v_k R(1)(u_0\Delta G_j + \Delta u_j \Delta G_k) - u_k \Delta u_j R(2)\Delta G_k \\ &\quad + R(3)(u_0 u_j \Delta v_k \Delta G_j - v_0 u_k \Delta u_j \Delta G_k) = 0 \end{aligned} \tag{5a}$$

$$\begin{aligned} &v_j \Delta v_i R(1)\Delta G_j - \Delta u_j R(2)(\Delta v_k \Delta G_j + v_0 \Delta G_k) \\ &\quad + R(3)(u_0 v_j \Delta v_k \Delta G_j - v_0 v_k \Delta u_j \Delta G_k) = 0 \end{aligned} \tag{5b}$$

$$\begin{aligned} &\Delta v_k R(1)\Delta G_j - \Delta u_j R(2)\Delta G_k \\ &\quad + R(3)(u_j \Delta v_k \Delta G_j - v_k \Delta u_j \Delta G_k) = 0. \end{aligned} \tag{5c}$$

When $j = k$, all three equations in (5) lead to

$$\Delta v_j R(1)\Delta G_j - \Delta u_j R(2)\Delta G_j + (u_j \Delta v_j - v_j u_j)R(3)\Delta G_j = 0.$$

Using the above equation to eliminate $R(2)$ in (5a) and $R(1)$ in (5b), we find

$$\begin{aligned} &R(1)(\Delta u_k \Delta G_j - \Delta u_j \Delta G_k) \\ &\quad + R(3)(u_j \Delta u_k \Delta G_j - u_k \Delta u_j \Delta G_k) = 0 \end{aligned} \tag{6a}$$

$$\begin{aligned} &R(2)(\Delta v_k \Delta G_j - \Delta v_j \Delta G_k) \\ &\quad + R(3)(v_j \Delta v_k \Delta G_j - v_k \Delta v_j \Delta G_k) = 0 \end{aligned} \tag{6b}$$

$$\begin{aligned} &R(1)\Delta v_k \Delta G_j - R(2)\Delta u_j \Delta G_k \\ &\quad + R(3)(u_j \Delta v_k \Delta G_j - v_k \Delta u_j \Delta G_k) = 0 \end{aligned} \tag{6c}$$

where $j = 1, 2, \ldots, n$, which is identical to the result given in [11] for a different camera model.

Equations in (6) involve a rotation matrix only and relative calibration information ΔG only. Therefore, they can be used to find the three rotational parameters based on relatively positional information. Once they are found, (4) can be used to obtain the translation parameter p. Note that the absolute position information G_0 appears independently and explicitly in (4). As we can see later in Section IV, G_0 is not needed if the calibration is performed only between several cameras.

III. A Simple Calibration Procedure and Its Analytical Solution

Let the rotation about the X, Y, and Z axes be denoted by ω, ϕ, and φ, then

$$\begin{aligned} R &= R_Z(\varphi)R_Y(\phi)R_X(\omega) \\ &= \begin{bmatrix} c\phi c\varphi & s\omega s\phi c\varphi - c\omega s\varphi & c\omega s\phi c\varphi + s\omega s\varphi \\ c\phi s\varphi & s\omega s\phi s\varphi + c\omega c\varphi & c\omega s\phi s\varphi - s\omega c\varphi \\ -s\phi & s\omega c\phi & c\omega c\phi \end{bmatrix} \end{aligned}$$

where ω, ϕ, and φ are called yaw, pitch, and roll, respectively, and $c\phi = \cos\phi, s\phi = \sin\phi$, etc.

Now let us assume that G_0, G_1, and G_2 all lie on the X axis of G. Then

$$\Delta G_1 = (L_1, 0, 0)^T, \quad \Delta G_2 = (L_2, 0, 0)^T$$

and (6) leads to

$$a_1 \cos\phi \cos\varphi - e_1 \sin\phi = 0 \tag{7a}$$
$$b_1 \cos\phi \sin\varphi - e_2 \sin\phi = 0 \tag{7b}$$
$$a_2 \cos\phi \cos\varphi - b_2 \cos\phi \sin\varphi - e_3 \sin\phi = 0 \tag{7c}$$

where

$$\begin{aligned} a_1 &= \Delta u_2 L_1 - \Delta u_1 L_2 \\ a_2 &= \Delta v_2 L_1 \\ b_1 &= \Delta v_2 L_1 - \Delta v_1 L_2 \\ b_2 &= \Delta u_1 L_2 \\ e_1 &= u_1 \Delta u_2 L_1 - u_2 \Delta u_1 L_2 \\ e_2 &= v_1 \Delta v_2 L_1 - v_2 \Delta v_1 L_2 \\ e_3 &= u_1 \Delta v_2 L_1 - v_2 \Delta u_1 L_2. \end{aligned}$$

From (7a) and (7b), we have

$$(a_1 e_2 \cos\varphi - b_1 e_1 \sin\varphi)\cos\phi = 0$$

which leads to:

1) $\phi = \pm\pi/2$;
2) $\varphi + \alpha = \pm\pi/2$, or $\varphi = \pm\pi/2 - \alpha$, where

$$\begin{aligned} \cos\alpha &= a_1 e_2/\Delta_1, \quad \sin\alpha = b_1 e_1/\Delta_1 \\ \Delta_1 &= \sqrt{(a_1 e_2)^2 + (b_1 e_1)^2}. \end{aligned} \tag{8}$$

The first case could be a false solution in most cases, and in the second case, we have for ϕ that

$$a_1 b_1 \cos\phi \mp \Delta_1 \sin\phi = 0$$

which leads to

3) $\phi \pm \beta = \pm\pi/2$ or $\phi = \pm\pi/2 \mp \beta$ where

$$\begin{aligned} \cos\beta &= a_1 b_1/\Delta_2, \quad \sin\beta = \Delta_1/\Delta_2 \\ \Delta_2 &= \sqrt{\Delta_1^2 + (a_1 b_1)^2}. \end{aligned} \tag{9}$$

In summary, there are two possible analytical solutions for roll φ, and for each φ, there are two possible analytical solutions for pitch ϕ. Therefore, there are four possible analytical solutions for pitch ϕ and roll φ from relative information ΔG_1 and ΔG_2.

Note that the three equations in (7) are not independent, and it is easy to show that

$$a_2 b_1 e_1 - b_2 a_1 e_2 - a_1 b_1 e_3 \equiv 0.$$

Thus, only two of them in (7) should be used to solve ϕ, φ.

Once ϕ and φ are found, yaw ω can be obtained using an additional calibration point G_3 which must not lie on the X axis of G.

An additional calibration G_3 will lead to six more calibration equations according to (6), using combination $(\Delta G_1, \Delta G_3)$ and $(\Delta G_2, \Delta G_3)$. Using (6a) and (6b), yaw ω can be found uniquely using either $(\Delta G_1, \Delta G_3)$ or $(\Delta G_2, \Delta G_3)$

$$\begin{aligned} &\begin{bmatrix} a_{13}s_2c_3 - a_{12}s_3 + e_{13}c_2 & a_{12}s_2c_3 + a_{13}s_3 + e_{12}c_2 \\ b_{12}c_3 + b_{13}s_2s_3 + e_{23}c_2 & b_{12}s_2s_3 - b_{13}c_3 + e_{22}c_2 \end{bmatrix} \\ &\qquad \times \cdot \begin{bmatrix} \cos\omega \\ \sin\omega \end{bmatrix} = \begin{bmatrix} e_{11}s_2 - a_{11}c_2c_3 \\ e_{21}s_2 - b_{11}c_2s_3 \end{bmatrix} \end{aligned} \tag{10}$$

where

$$\begin{aligned} c_2 &= \cos\phi = \pm\sin\beta, \quad s_2 = \sin\phi = \pm\cos\beta \\ c_3 &= \cos\varphi = \pm\sin\alpha, \quad s_3 = \sin\varphi = \pm\cos\alpha \end{aligned}$$

and a_{ij}, b_{ij}, and e_{ij} are constant from (6a) and (6b).

Note that the combination of (ϕ, φ) must satisfy the constraint

$$\cos^2\omega + \sin^2\omega = 1 \tag{11}$$

where $\cos\omega$ and $\sin\omega$ are found directly from (10), as well as the other four unused equations in (6). Therefore, we have five constraints to eliminate the false calibration solution among the four possible analytical solutions for (ϕ, φ). Note that constraint (11) should be used as the most important one in selecting the correct calibration solution.

A simple and straightforward method to determine the correct calibration solution is to use (3), and define the correct solution as the one leads to the minimum total residual in (3).

IV. Extrinsic Calibration of Multiple Cameras

To extract 3-D information, one needs multiple cameras or images at different poses. In this case, we need to find the relative orientation and position between two cameras or poses, the so-called extrinsic parameter calibration problem for multiple camera systems. Let R_{12} and p_{12} be the rotation matrix and translation vector between two cameras, i.e.,

$$W_1 = R_{12}W_2 + p_{12}. \tag{12}$$

(R_1, p_1) and (R_2, p_2) are the rotation and translation of the two cameras obtained by the procedure described in Sections II and III. Then

$$R_{12} = R_1 R_2^T \tag{13}$$
$$\begin{aligned} p_{12} &= p_1 - R_{12}p_2 \\ &= (\underline{p_1 + R_1 G_0}) - R_{12}(\underline{p_2 + R_2 G_0}). \end{aligned} \tag{14}$$

Clearly, from (4) and (6), (13) and (14) are independent of the selection of the particular reference world coordinate system G and calibration point G_0. Therefore, no absolution position information is needed for the extrinsic calibration of multiple cameras.

As a numerical example with synthetic data, consider two cameras and four calibration points, as shown in Fig. 1. Table II presents the corresponding image coordinates of the four calibration points for the two cameras.

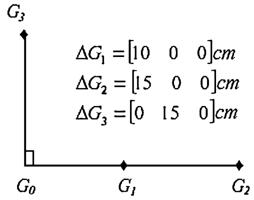

Fig. 1. Four calibration points.

TABLE II
IMAGE COORDINATES FROM TWO CAMERAS

Camera		G_0	G_1	G_2	G_3
Camera one	x_1	-0.2361	-0.2921	-0.3209	-0.1542
	y_1	-0.1180	-0.2101	-0.2574	-0.2278
Camera two	x_2	-0.2016	-0.2170	-0.2249	-0.1336
	y_2	-0.1210	-0.1892	-0.2246	-0.1827

Using the procedure described above, we find the following.

CAMERA 1

$$\Theta_1 = [\omega \phi \varphi] = [45^\circ \; 30^\circ \; 60^\circ] \text{ or } [-135^\circ \; 150^\circ \; -120^\circ]$$
$$p_1 + R_1 G_0 = [20\ 10\ 300]^{\mathrm{T}} \text{ cm.}$$

CAMERA 2

$$\Theta_2 = [\omega \phi \varphi] = [53^\circ \; 37^\circ \; 81^\circ] \text{ or } [-127^\circ \; 143^\circ \; -99^\circ]$$
$$p_2 + R_2 G_0 = [25\ 15\ 250]^{\mathrm{T}} \text{ cm.}$$

Note that two correct solutions have been found for both cameras, and both lead to the same rotation matrix, i.e.,

$$R_1 = \begin{bmatrix} 0.4330 & -0.4356 & 0.7891 \\ 0.7500 & 0.6597 & -0.0474 \\ -0.5000 & 0.6124 & 0.6124 \end{bmatrix} \quad \text{for camera 1}$$
$$R_2 = \begin{bmatrix} 0.1249 & -0.5192 & 0.8455 \\ 0.7888 & 0.5686 & 0.2328 \\ -0.6018 & 0.6378 & 0.4806 \end{bmatrix} \quad \text{for camera 2.}$$

Therefore

$$R_{12} = \begin{bmatrix} -0.5334 & 0.0073 & 0.8456 \\ 0.8432 & 0.0796 & 0.5316 \\ -0.0634 & 0.9968 & -0.0486 \end{bmatrix}$$
$$p_{12} = [-178.1640 \quad -145.1759 \quad 298.7785]^{\mathrm{T}} \text{ cm.}$$

V. CONCLUDING REMARKS

Finally, it is necessary to point out the major difference in the motivation between the current research and many other camera calibration works. The focus here is on **real-time, repetitive extrinsic parameter calibration** tasks (for example, caused by constant vehicle motion during the driving process), not **offline, one-time full parameter calibration** (extrinsic and intrinsic) problems, as in most other studies. Therefore, a simple and analytical calibration procedure is extremely desired in those situations.

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Profile Sensing With an Actuated Whisker

Gregory R. Scholz and Christopher D. Rahn

***Abstract*—Obstacle avoidance and object identification are important tasks for robots in unstructured environments. This paper develops an actuated whisker that determines contacted object profiles using a hub load cell. The shape calculation algorithm numerically integrates the elastica equations from the measured hub angle, displacement, forces, and torque until the bending moment vanishes, indicating the contact point. Sweeping the whisker across the object generates a locus of contact points that can be used for object identification. Experimental results demonstrate the ability to identify and differentiate square and curved objects at various orientations.**

***Index Terms*—Contact sensing, flexible beam, shape sensing.**

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